Kinetics of excited muonic hydrogen in mixtures of hydrogen and helium isotopes

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The de-excitation cascade of excited muonic hydrogen in a mixture of hydrogen and helium isotopes is considered. A method is proposed for determining the rates of transfer of a muon from excited muonic hydrogen to helium, and also the probability of direct atomic capture of a muon into isotopes of hydrogen.

The study of the processes which occur during the brief time of de-excitation of muonic atoms of hydrogen presents significant interest in the physics of muonic atoms. These processes are as follows: de-excitation of the muonic atom in Auger transitions during collisions with molecules of the target mixture and in radiative transitions,[1,2] elastic collisions, which are responsible for the thermalization of muonic atoms and for their acceleration in de-excitation processes,[3,4] and finally the transfer of muons from excited states of muonic hydrogen to other nuclei.[5–7] This large number of competing processes makes it complicated to identify each of them experimentally. To this it must be added that the rates of these processes are high ($\gtrsim 10^{11}$ sec^{-1} for the density of liquid hydrogen), which makes it difficult to study them experimentally. At the same time the development of the muon-atom cascade will depend on the initial conditions of formation of the muonic atom.[8] The processes of direct atomic capture of muons by nuclei with formation of a muonic atom and of charge exchange in these nuclei of muonic atoms of hydrogen isotopes in excited states are difficult to distinguish experimentally.

In the present work we discuss the possibility of experimental separation of these processes. As an illustration we discuss here a mixture of hydrogen isotopes and helium. Muon-atom processes in a hydrogen-helium mixture are attracting attention in connection with study of the problem of muon catalysis, which is accompanied by accumulation of helium as the result of fusion of nuclei of hydrogen isotopes and decay of tritium, in a deuterium-tritium mixture, which presents the greatest interest.[9] To assure the best conditions for muon catalysis for energy production purposes, i.e., to obtain the greatest number of catalysis cycles per muon, it is necessary to reduce to a minimum the possible admixture of helium accumulated in the $D_2 + T_2$ mixture. The evaluation of the conditions necessary to achieve purification from helium is determined by the information on the rates of transfer of muons from muonic hydrogen to helium and of direct atomic capture of muons by the helium nucleus. In regard to the atomic capture of muons by helium nuclei from unexcited muonic atoms of the hydrogen isotopes, there is exhaustive information, both theoretical[10] and experimental,[11–14] which permits estimation of the permissible admixture of helium which will not influence the number of muon catalysis cycles.

At the same time only partial experimental[15,16] and theoretical[17,18] information exists on the atomic capture of muons by helium nuclei from excited muonic atoms of the hydrogen isotopes. The question of the direct atomic capture of muons by helium nuclei remains equally ambiguous. It is very important to obtain independent experimental information on these processes.

Transfer of a muon from excited muonic hydrogen to helium and direct atomic capture of a muon by helium can be characterized by the quantity[11,16]

$$W = W_H W_0, \tag{1}$$

which determines the probability that a muon stopped in a mixture of hydrogen and helium will be captured by hydrogen and that the muonic hydrogen formed will reach its ground state without giving up the muon to a helium atom. This quantity depends on the concentration of the impurity and on the density of the target. The probability of atomic capture of a muon by a hydrogen isotope can be described by the expression

$$W_H = (1 + AC_{He})^{-1}, \tag{2}$$

where C_{He} is the relative concentration of helium and A is the ratio of the probabilities of atomic capture of the muon by helium and hydrogen. If we use the data on atomic capture of π^- mesons by hydrogen, then $A = A_\pi = 1.84$,[19] i.e., the probability of atomic capture in helium is about twice that for hydrogen.

The quantity $W_0 \equiv q_{1s}^{He}$ which characterizes the population of the ground state of muonic hydrogen with allowance for the atomic capture of the muon by helium from excited states of the muonic atom, is determined by the density of the target, the concentration of the impurity, and in general by the energy of the muonic atom in its excited state. Values of W_0 are found as the result of solution of a system of kinetic equations which determine the kinetics of excited muonic hydrogen with allowance for cascade transitions and atomic capture of the muon in a time of about 10^{-11} sec (for densities close to the density of liquid hydrogen). Generally speaking, determination of W_0 calls for solving a system of differential equations of large size (≈ 14), in which it is necessary to take into account the competing processes mentioned above. However, a simplification is made possible by Auger de-excitations for $n > 5$ (n is the principal quantum number of muonic hydrogen) in comparison with the rates of atomic capture of the muon by helium. Assuming, for example, that the population of the state with $n = 5$ of muonic hydrogen corresponds to the initial population, i.e., that $q_{1s}^{He} = 1$, the problem is simplified (Fig. 1) and reduces to solution of a system of substantially fewer equations ($n \approx 5$). However, in this case in order to find the atomic

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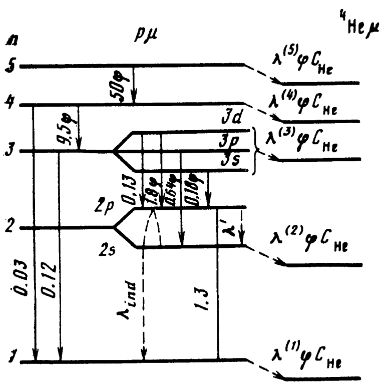

FIG. 1. Diagram of the cascade of the $p\mu$ atom in a H_2 + He mixture. Transitions rates are given in units of 10^{-11} sec^{-1}. The Stark de-excitation rates $\lambda_{\rm ind}$ and $\lambda' = \lambda(2p \to 2s)$ are discussed in the text; $\lambda_{\rm ex}^{(1)}$–$\lambda_{\rm ex}^{(5)}$ are the rates of transfer of muons from $p\mu$ atoms to He nuclei.

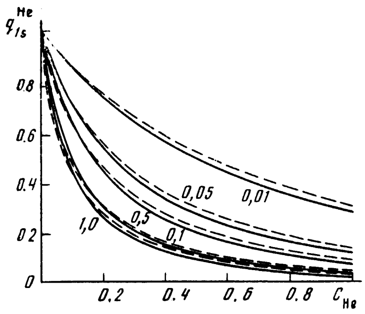

FIG. 3. Population of the ground state of muonic hydrogen $q_{1s}^{\rm He}$ as a function of the concentration of He for various densities of the H_2 + He mixture. The solid lines correspond to a collision energy 1 eV, and the dashed lines to an energy 0.04 eV. Values of the density of the H_2 + ^{4}He mixture in units of the density of liquid hydrogen $N_0 = 4.25 \cdot 10^{22}$ cm^{-3} are given on the corresponding curves.

capture rates it is necessary to solve a multichannel problem with channel coupling.

A further simplification was to calculate the muon atomic capture rates by quasiclassical methods.[6,20] Following the method proposed in Ref. 20 for a deuterium-tritium mixture, we applied it for a hydrogen-helium mixture as was done in Ref. 18. However, in contrast to Ref. 18, in the present work we take into account the energy dependence of the rates of direct charge exchange and also calculate the population of the ground state of muonic hydrogen.

In Fig. 2 we show energy dependences of the rates of atomic capture of a muon $\lambda_{\rm ex}^{(n)}$ from excited muonic hydrogen to helium for $n = 2$ and $n = 3$.

In solution of the system of kinetic equations (Fig. 1) we made the assumption that the rates of atomic capture of a muon from states with $n > 3$ are the same as for $n = 3$. For the ground state we used the molecular charge-exchange rates calculated in Ref. 10. The rate of Stark de-excitation of the 2s state is $\lambda_{\rm ind} = 0.04\varphi \cdot 10^{11}$ sec^{-1} for a muonic atom energy $\varepsilon = 0.04$ eV, where φ is the density of the mixture in units of the density of liquid hydrogen $N_0 = 4.25 \cdot 10^{22}$ cm^{-3}. In calculation for higher energies, as in Ref. 7, it was assumed that $\lambda(2p \to 2s) = (1/3)\lambda(2s \to 2p)$, with $\lambda(2p \to 2s) = 32\varphi \cdot 10^{11}$ sec^{-1} for $\varepsilon = 0.5$ eV and $\lambda(2s \to 2p) = 48\varphi \cdot 10^{11}$ sec^{-1} for $\varepsilon = 1$ eV.

Figure 3 gives calculated dependences of $q_{1s}^{\rm He}$ (obtained for the system $p\mu$ + He) on the relative concentration of helium $C_{\rm He}$ for various densities of the mixture in the range $0.01 \leqslant \varphi \leqslant 1$. The solid curves correspond to an energy of the excited muonic atom 1 eV, and the dashed curves correspond to 0.04 eV ($T \approx 300$ K). It can be seen that, in contrast to a deuterium-tritium mixture,[7] here there is no appreciable dependence of $q_{1s}^{\rm He}$ on ε. This is also illustrated in Fig. 4, where this same dependence is given for $C_{\rm He} = 0.5$ and $\varphi = 0.1$, 1.0, and 0.05. The very weak energy dependence of $q_{1s}^{\rm He}$ (in comparison with the similar dependence for a d–t mixture) is due to the relatively small rate of charge exchange of muonic hydrogen in the metastable $2s$ state in helium, about an order of magnitude smaller than in the case of charge exchange in tritium.

The point is that the de-excitation of the $2s$ state due to the $2s \to 2p$ Stark transition (with a subsequent $2p \to 2s$ transition) has a strong energy dependence characterized by an almost hundred–fold excess of the rate of de-excitation of a muonic atom for $\varepsilon > 0.2$ eV (the value of the $2p$–$2s$ Lamb shift) in comparison with the rate for $\varepsilon < 0.2$ eV. However, if the rate of transfer of a muon to helium from muonic hydrogen in the $2s$ state is low enough, this circumstance has only a weak effect on the value of $q_{1s}^{\rm He}$, which is illustrated in Figs. 3 and 4. Comparison of the results of calculation of $q_{1s}^{\rm He}$ (Figs. 3 and 4) with the phenomenological analysis data of the experiment of Ref. 16 (see Fig. 2 of Ref. 16) shows that some excess of the "experimental" values of W_0 is observed over

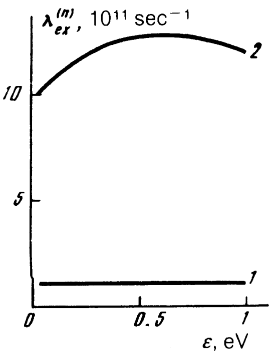

FIG. 2. Rates of charge exchange of an excited $p\mu$ atom in He as a function of the kinetic energy for principal quantum numbers $n = 2$ (curve 1) and $n = 3$ (curve 2).

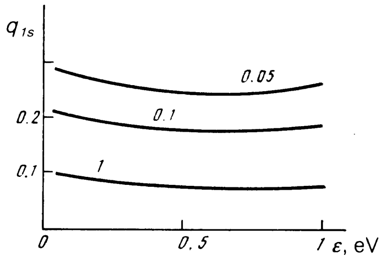

FIG. 4. Energy dependence of the population of the ground state of muonic hydrogen for a H_2 + ^{4}He mixture with a ^{4}He concentration $C_{\rm He} = 0.5$ and various densities of the mixture (the numbers on the curves).

the calculated values of q_{1s}^{He}. This may be due to the fact that $A_\mu < A_\pi$ (the values of W_H are greater than the corresponding values used in processing the data of Ref. 16), and consequently, according to Eq. (1), the values of W_0 found are somewhat smaller than the calculated values.

Comparison of the calculated values (Fig. 3) with the experimental values[16] leads to $A_\mu = 1.25$ (comparison of q_{1s}^{He} with the data of an experiment with $\varphi = 0.05$), i.e., the probabilities of atomic capture in hydrogen and helium are approximately the same. It can be seen that the value of A obtained from the experimental data is very sensitive to the value of q_{1s}^{He} (a decrease of q_{1s}^{He} by 10% leads to a reduction of A by 30%). This fact can be used to verify the mechanism of direct atomic capture, which is extremely important to take into account correctly in study of muon catalysis in a $D_2 + T_2$ mixture. The absence of a significant energy dependence of q_{1s}^{He} in comparison with q_{1s} in a $D_2 + T_2$ mixture does not permit study of the question of thermalization of excited muonic hydrogen in a hydrogen-helium mixture. However, this is a favorable circumstance for obtaining values of W_H from the experimental data, since the uncertainty due to the lack of information on the energy distribution of the muonic hydrogen atoms is excluded. For $C_{He} = 10^{-3}$–10^{-2} and $\varphi \approx 0.9$ it is possible to neglect the direct atomic capture of muons in helium (i.e., $W_H = 1$) and to determine the value of W_0.

Experimental data on the value of W_0 determined for several values of C_{He} in the range indicated (10^{-3}–10^{-2}) will permit in principle obtaining more accurate theoretical rates of muon atomic capture by helium from excited states of muonic hydrogen. Then, by increasing the helium concentration to the range $C_{He} \approx 0.1$–0.5 it will be possible to find the value of W_H, using the values of W_0 obtained above. By means of Eq. (2) the value of A can be calculated with accuracy 2–4%. It should be mentioned that carrying out experiments at high helium concentrations ($C_{He} = 0.3$–0.5) and low densities of the hydrogen-helium mixture ($\varphi \approx 0.05$–0.1) is undoubtedly desirable if one is to obtain more accurate information on the rates of atomic capture of muons from excited muonic hydrogen by helium.

According to our estimates, the transfer of a muon from excited states of muonic hydrogen to helium leads to an appreciable knockout of muons from the muon catalysis chain. Therefore to achieve efficient dt fusion it is necessary to take into account this process (in addition to transfer of the muon from the ground state) in evaluating the permissible impurity of helium in the $D_2 + T_2$ mixture. By carrying out this experiment for the mixtures $H_2 + He$, $D_2 + He$, and $T_2 + He$, it is possible to extract information on the value of A for each of the hydrogen isotopes and, consequently, to determine the isotopic dependence of the probability of direct atomic capture of a muon by hydrogen. It must be mentioned that experiments with use of deuterium and tritium are especially informative for two reasons: first, in this case it is not necessary (as in the case of a $H_2 + He$ mixture) to use a test gas (Xe, Ar) since the experimental determination of the characteristics of the muon-atom processes is based on analysis of the yields and time distributions of the products of dd and tt fusion reactions; second, in these mixtures the possibility arises of determining the rate of transfer of a muon from the ground state of muonic hydrogen to helium even with low concentrations of helium.

It is preferable that these experiments be carried out in a gaseous medium, since this permits one to avoid possible uncertainties due to inaccurate knowledge of the concentration of helium in liquid hydrogen.

Summing up, we can say that carrying out the proposed set of experiments will permit one to obtain extremely important information which is necessary for correct interpretation of data obtained in study of muon catalysis of nuclear fusion reactions in a $D_2 + T_2$ mixture.

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