Acoustic Cotton–Mouton effect in antiferromagnets

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A rotationally invariant theory of a magnetic acoustic birefringence is developed for tetragonal antiferromagnets with the rutile structure (fluorides) in a situation with $\mathbf{k}\|\mathbf{L}\|\mathbf{B}\|4\|\mathbf{z}$, where $\mathbf{k}$, $\mathbf{L}$, and $\mathbf{B}$ are, respectively, the wave vector of sound, and the antiferromagnetic and magnetic field vectors, and 4 is the tetragonal symmetry axis. The effect is proportional to the product L_zB_z and is governed by the combined effect of the piezomagnetism and magnetostriction. A quantitative estimate of the effect is obtained for CoF_2. Trigonal antiferromagnets in a state with $\mathbf{L}\perp 3$ are also considered, thus providing a quantitative explanation of the experimental results obtained by Gakel' [JETP Lett. **9**, 360 (1969)], who discovered oscillations of the amplitude of sound transmitted by an $MnCO_3$ plate as a function of $\mathbf{B}$.

1. INTRODUCTION

A symmetry analysis was used in Ref. 1 to show that antiferromagnetic linear birefringence (LB) should be exhibited by sound as well as by light.[2] In optics the magnetic LB effect is called, depending on the source, the Voigt or the Cotton–Mouton effect. It was observed experimentally[3] in acoustics over 20 years ago by Gakel' in $MnCO_3$. When the effect is sufficiently large, it may be of considerable interest in solid-state electronics, because it provides means for controlling sound (its polarization and intensity) by a magnetic field. We shall show that this may occur in some antiferromagnets.

Our aim is to develop a quantitative theory of the acoustic LB in tetragonal and trigonal antiferromagnets on the basis of the coupled equations of motion of the magnetic moments and elastic displacements. However, it should be noted that in the trigonal case (α-Fe_2O_3, $FeBO_3$, $MnCO_3$, etc.) the wave vectors of normal acoustic modes, governing the LB, can be calculated using the effective elastic moduli, known from the work of Ozhogin and Preobrazhenskiĭ[4] and allowing for the renormalization associated with the magnetoelastic interaction.

2. TETRAGONAL ANTIFERROMAGNETS

We first consider tetragonal antiferromagnets with the $I^+4_z^-2_x^+$ structure[1] in a state characterized by $\mathbf{l}\|\mathbf{B}\|\mathbf{z}$, $\mathbf{m}=0$ [$\mathbf{l}=(\mathbf{M}_1-\mathbf{M}_2)/2M_0$ and $\mathbf{m}=(\mathbf{M}_1+\mathbf{M}_2)/2M_0$ are the relative values of the antiferromagnetic and magnetization vectors]. Allowing for the rotational invariance,[5] we can describe the quadratic (in small oscillations of the dynamic variables) part of the free energy density corresponding to magnetoelastic waves propagating along the tetragonal axis ($\mathbf{k}\|4_z^-\|\mathbf{z}$) by the following expression

$$F_2(z)=F_2^{(x)}+F_2^{(y)}, \tag{1}$$

where

$$F_2^{(x)}={}^1/_2Em_x^2+{}^1/_2Kl_x^2+(h-D)m_xl_x+(2B_{44}e_{xz}-K\omega_{xz})l_x+(2\Pi_{44}e_{xz}+D\omega_{xz})m_x+2C_{44}e_{xz}^2+{}^1/_2K\omega_{xz}^2-2B_{44}e_{xz}\omega_{xz}, \tag{2}$$

whereas $F_2^{(y)}$ is obtained from the above expression by the substitutions $x\to y$, $D\to -D$, $\Pi_{44}\to -\Pi_{44}$. Here, E, K, and D are, respectively, the homogeneous exchange, anisotropy, and Dzyaloshinskii constants; C_{44} is an elastic modulus; B_{44} and Π_{44} are magnetostriction and piezomagnetism constants (representing the magnetoelastic interaction). Finally, the constant

$$h=L_zB_z\equiv 2M_0l_zB \tag{3}$$

determines the contribution of the external magnetic field $B_z\equiv B$ to the energy of the oscillations. The dynamic variables in Eq. (2) are, apart from the magnetic oscillations l_x and m_x, also dynamic strains $e_{xz}=(\partial u_x/\partial z+\partial u_z/\partial x)/2$ and local rotations $\omega_{xz}=(\partial u_x/\partial z-\partial u_z/\partial x)/2$.

It is worth noting that, firstly, in writing down Eqs. (1) and (2) the coordinate axes x and y are selected in the basal plane along the binary 2^+ symmetry axes and, secondly, that Eq. (2) ignores the inhomogeneous exchange energy. This means that only vibrations with sufficiently long wavelengths will be considered.

We assume that the frequency of these vibrations is much less than the gap (activation) of a spin-wave mode, so that we consider only zero-gap quasiacoustic waves. For these waves, l_x and m_x follow in a quasi-equilibrium manner the vibrations

$$e_{xz}=\omega_{xz}=\frac{1}{2}\frac{\partial u_x}{\partial z}\quad (\text{for } \mathbf{k}\|\mathbf{z}) \tag{4}$$

(similar expressions can be obtained for e_{yz} and ω_{yz}) and, consequently, the former can be expressed in terms of the latter by minimizing $F_2^{(x)}$ [or $F_2^{(y)}$.] The values of m_x and l_x (m_y and l_y) found in this way should be substituted into the initial expressions for $F_2^{(x,y)}$. Consequently, Eq. (1) subject to Eq. (4) becomes

$$F_2(z)=\frac{1}{2}C_{44}^{(x)}\left(\frac{\partial u_x}{\partial z}\right)^2+\frac{1}{2}C_{44}^{(y)}\left(\frac{\partial u_y}{\partial z}\right)^2, \tag{5}$$

where $C_{41}^{(x,y)}$ are the effective elastic moduli for the vibrations polarized along the $\mathbf{x}$ and $\mathbf{y}$ axes, respectively, renormalized allowing for the magnetoelastic interaction. This renormalization lifts the degeneracy of normal transverse waves with the polarizations given above and propagating along the $\mathbf{z}$ axis.

We shall not give the explicit form of the moduli $C_{44}^{(x,y)}$, but write down directly the difference between them because it governs the LB effect of interest to us:

$$\Delta C = C_{44}^{(x)} - C_{44}^{(y)}$$
$$= h\{D[(2B_{44}-K)(2B_{44}E+D^2-h^2)+K(2\Pi_{44}+D)^2]$$
$$+2\Pi_{44}(2B_{44}-K)(EK+D^2-h^2\}/$$
$$\{[((EK)^{1/2}+D)^2-h^2][((EK)^{1/2}-D)^2-h^2]\}. \quad (6)$$

We can use Eq. (6) to find the relative differences between the phase velocities $v^{(x,y)} = (C_{44}^{(x,y)}/\rho)^{1/2}$ and the wave vectors $k^{(x,y)} = \omega/v^{(x,y)}$ (at a given frequency ω) of normal waves which are polarized along the x and y axes. For $|\Delta C/C| \ll 1$ (which is not always true), then

$$\Delta v/v \approx -\Delta k/k \approx \Delta C/2C. \quad (7)$$

A linearly polarized transverse wave entering a plate along the normal $\mathbf{n}\|\mathbf{z}$ and characterized by a polarization vector $\mathbf{e} = \mathbf{u}/u$, tilted at an angle $\varphi_0 \neq 0$ and $\pi/2$ to the $\mathbf{x}$ axis, is generally elliptically polarized at the exit (see, for example, Ref. 6). This ellipticity is an oscillatory function either of the plate thickness $z = d$ for a given value of Δk or of the magnetic field B [because of the dependence $\Delta k(B)$] when the value of d is given. This means that, depending on B, we can expect oscillations of the amplitude A_ψ of vibrations measured along one of the directions of ψ at the exit from the plate. For example, if $\psi = \varphi_0 = \pi/4$ holds, we have

$$A_\psi = u_0|\cos(\Delta k d/2)|. \quad (8)$$

(Expressions for the more general situation are given in the Appendix.) The period ΔB of the amplitude oscillations is given by

$$\Delta k(B+\Delta B) - \Delta k(B) = 2\pi/d. \quad (9)$$

It is oscillations of this type that were observed in $MnCO_3$ (Ref. 3), but this point will be discussed later. At this stage let us obtain a quantitative estimate of the effect (i.e., of the quantity $\Delta k/k$) for CoF_2.

3. ESTIMATE FOR CoF_2

Using Eq. (7) in fields such that

$$h \ll (EK)^{1/2} - |D| \quad (10)$$

(which in the case of CoF_2 corresponds to $B < 10$ T), we find approximately from Eq. (6) that

$$\delta \equiv \Delta k/k \approx 2B_{44}E\Lambda_1 B/(EK-D^2), \quad (11)$$

where

$$\Lambda_1 = -(2M_0/C_{44})(B_{44}D+\Pi_{44}K)/(EK-D^2) \quad (12)$$

is a coefficient deduced from the piezomagnetic effect. In fact, according to Borovik-Romanov,[7] we have

$$M_x = 2\Lambda_1 C_{44} e_{xz}, \quad (13)$$

where $\Lambda_1 \approx 2\cdot 10^{-5}$ T^{-1}. On the other hand, minimization of Eq. (2) with respect to m_x and l_x (when $\omega_{xz} = 0$) gives

$$M_x = -2M_0 \frac{2(B_{44}D+\Pi_{44}K)}{EK-D^2} e_{xz}, \quad (14)$$

which together with Eq. (13) yields Eq. (12). In Eqs. (11) and (12) the following constants (in J/m) are also known[8]: $E = 1.4\times 10^8$, $K = 3.8\times 10^6$, and $D = 1.7\times 10^7$.

It therefore follows that the knowledge of the magnetostriction constant B_{44} is insufficient to calculate δ from Eq. (11). This quantity can be estimated from Eq. (12) if we assume that B_{44} makes a contribution to Λ_1 which in any case is no less than the contribution of Π_{44}. Consequently, assuming also $2M_0 \approx 10^6$ A/m and $C_{44} \approx 10^{11}$ J/m^3, we find $B_{44} \approx 3\times 10^7$ J/m^3. Then, we finally obtain from Eq. (11)

$$\delta/B \approx 10^{-3}\ \mathrm{T}^{-1}. \quad (15)$$

When the condition (10) is satisfied, the period of the amplitude oscillations of Eq. (8) in terms of the field B is constant (independent of B), so that in Eqs. (6)–(11) we have

$$\Delta B = Q\lambda/d, \quad (16)$$

where $Q = (\delta/B)^{-1}$ and $\lambda = 2\pi/k$ is the wavelength of sound. When the frequency is $\omega = 10^9$ s^{-1}, the velocity is $v = 10^3$ m/s, and the plate thickness is $d = 1$ cm, we find from Eq. (16) subject to Eq. (15), in particular, $\Delta B \approx 0.6$ T.

Noting that independence of the oscillation period ΔB of B disappears in high fields (in the case of CoF_2 this happens in the range $B \gtrsim 10$ T) and also in the approach to its spin-flop point where $h = (EK)^{1/2} - |D|$ holds, the value of ΔB decreases rapidly because of an increase in ΔC given by Eq. (6).

Experimental investigations of the acoustic LB effect described above have not yet (to the best of our knowledge) been carried out, bearing in mind that it may be simply of purely physical interest as a new effect, but may also have practical applications.

The only new feature (singularity) of the effect is associated with the fact that we have $\Delta k \propto h = L_z B_z$, so that Δk changes its sign exactly when the sign of B_z or L_z is reversed. This clearly provides an opportunity for "acoustic visualization" of antiferromagnetic domains differing in the direction $\mathbf{L}$. The point is this: reversal of the sign of Δk alters the phase of the transmitted beam. [The latter is clear from Eq. (A2) in the Appendix.]

It is interesting to note also that the magnitude of the LB effect (magnitude of $|\Delta k|$) depends on the sign of the constant D or, more exactly, on the relationship between the signs of D and Π_{44}: reversal of one of them changes the value of $|\Delta k|$. A quantitative theory of the effect should include the magnetoelastic energy contributions of both the terms with strains $e_{\alpha\beta}$ and with local rotation $\omega_{\alpha\beta}$.

Finally, a special feature of the situation in the case of the $I^+4_z^-2_x^+$ structure is that the magnetic LB effect (Cotton–Mouton effect) appears not in a transverse field $\mathbf{B}$ (or, more correctly, not only in a transverse field, as is usually assumed for paramagnets), but in a longitudinal field when $\mathbf{B}\|\mathbf{k}$.

4. TRIGONAL ANTIFERROMAGNETS

We now consider trigonal antiferromagnets with the $I^+3_z^+2_x^-$ structure (α-Fe_2O_3, $FeBO_3$, $MnCO_3$, etc.). In the $\mathbf{B}\|\mathbf{l}\|3_z\|\mathbf{z}$ state (which is known to occur, for example, in hematite α-Fe_2O_3 at temperatures $T < T_M = 250$ K) there is no LB and we shall consider only the easy-plane state with $\mathbf{l} \perp \mathbf{z}$. Let us assume that $\mathbf{B}$ also lies in the basal plane xy and is sufficiently strong to ensure that the condition $\mathbf{M} \| \mathbf{B} \perp \mathbf{l}$ is satisfied. This direction of $\mathbf{B}$, making an angle of φ_B with the $2_x^- \| \mathbf{x}$ axis, will be adopted as the new $\mathbf{X}$ axis. We then have $\mathbf{l} \| \mathbf{Y}$.

Let us consider magnetoelastic modes which are coupled to an lf spin-wave mode. In the case of the latter mode both $\mathbf{l}$ and $\mathbf{M}$ do not, in the first approximation, emerge on the XY plane and all the dynamic variables include now (in addition to the strains $e_{\alpha\beta}$) not only the angle $\delta\varphi$ by which the whole "construct" $\mathbf{l} \perp \mathbf{M}$ deviates from its equilibrium direction ($\mathbf{l} \| \mathbf{Y}$ and $\mathbf{M} \| \mathbf{X}$), but also the relative magnetization $m = M/2M_0$. Since $l_z = 0$, there are no terms with $\omega_{\alpha\beta}$ (those that need be allowed for). We shall again consider the lf range where we can ignore the inhomogeneous exchange and assume quasiequilibrium coupling between the elastic and magnetic variables.

Omitting calculations similar to those given in the preceding case, we write down immediately the result of Ref. 4 for the effective correction to the elastic energy associated with the magnetoelastic interaction

$$\Delta F_2 = -\frac{H_E[2B_{44}e_{XY} + 4B_{14}(e_{Xz}\cos 3\varphi_B - e_{Yz}\sin 3\varphi_B)]^2}{M_0[B(B+H_D) + 2H_E H_{me1}]}, \tag{17}$$

where H_D and H_E are the effective Dzyaloshinskiĭ and homogeneous exchange fields, whereas

$$H_{me1} = [C_{44}B_{66}^2 - 4C_{14}B_{14}B_{66} + 4C_{66}B_{14}^2]/2M_0(C_{44}C_{66} - C_{14}^2)$$

is the effective magnetoelastic field due to the spontaneous magnetostrictive strains ($2C_{66} = C_{11} - C_{12}$, $B_{66} = B_{11} - B_{12}$).

The effective magnetoelastic correction of Eq. (17) to the elastic energy not only renormalizes some of the elastic moduli, by differentiating the initially identical (for example, $C_{55}^{\text{eff}} \neq C_{44}^{\text{eff}}$) moduli, but it may also create additional nonzero moduli corresponding to the crystal symmetry disturbed by the antiferromagnetic ordering. Here we consider only the effect of the first type, which in the present case gives rise to an LB of purely antiferromagnetic origin because the magnetoelastic interaction lifts the degeneracy of the polarizations of two normal modes.

We again consider waves with the vector $\mathbf{k}$ directed along the principal symmetry axis ($\mathbf{k} \| 3_z \| \mathbf{z}$). The corresponding part of the elastic energy deduced allowing for Eq. (17) becomes

$$F_2^{\text{eff}} = 2C_{44}e_{\eta z}^2 + 2C_{55}^{\text{eff}}e_{\xi z}^2, \tag{18}$$

where

$$C_{55}^{\text{eff}} = C_{44} - 2H_E(2B_{14})^2/M_0[B(B+H_D) + 2H_E H_{me1}]. \tag{19}$$

It is convenient to introduce here new coordinate axes ξ and η in the basal plane instead of $\mathbf{X}$ and $\mathbf{Y}$ in such a way that

$$\xi = X\cos 3\varphi_B - Y\sin 3\varphi_B, \qquad \eta = Y\cos 3\varphi_B + X\sin 3\varphi_B. \tag{20}$$

This corresponds to an additional rotation of the coordinate axes $\mathbf{X}$ and $\mathbf{Y}$ about the $\mathbf{z}$ axis by an angle $-3\varphi_B$ and, consequently, the ξ axis makes an angle $-2\varphi_B$ with the 2_x^- symmetry axis.

The transformation (20) in fact separates variables reducing the energy F_2^{eff} to the diagonal form of Eq. (18), which allows us to determine directly the phase velocities and the polarization of the normal waves:

$$v \equiv v_\xi = (C_{55}^{\text{eff}}/\rho)^{1/2} \quad \text{for } \mathbf{e} \| \xi, \qquad v \equiv v_\eta = (C_{44}/\rho)^{1/2} \quad \text{for } \mathbf{e} \| \eta. \tag{21}$$

The magnetoelastic interaction does indeed lift the degeneracy of two transverse waves characterized by $\mathbf{k} \| \mathbf{z}$ and therefore gives rise to LB. We then have

$$\Delta k = \omega(1/v_\xi - 1/v_\eta) \approx k_\eta H_E H_{me2}/[B(B+H_D) + 2H_E H_{me1}], \tag{22}$$

where $k_\eta = \omega/v_\eta$ and

$$H_{me2} = (2B_{14})^2/M_0C_{44} \tag{23}$$

representing the second effective magnetoelastic field. The second (approximate) part of Eq. (22) corresponds to the case when $\Delta k/k \ll 1$.[1)]

The intensity of sound I transmitted by an $MnCO_3$ plate of thickness $d = 0.08$ cm and with the same polarization $\mathbf{e}$ as at the entry ($I \equiv I_e$) and also with the polarization in the transverse direction $\mathbf{t}$($I \equiv I_t$) was determined and reported in Ref. 3. The general expression (A1) in the Appendix applicable to such cases ($\psi = \varphi_0$ and $\psi = \varphi_0 + \pi/2$) yields the corresponding expressions

$$I_e/u_0^2 = \cos^2 2\varphi_0 + \sin^2 2\varphi_0 \cos^2(\Delta kd/2), \qquad I_t/u_0^2 = \sin^2 2\varphi_0 \sin^2(\Delta kd/2). \tag{24}$$

This indicates the appearance of ellipticity, which oscillates with B, because according to Eq. (22) Δk is a function of B. For $\varphi_0 = \pi/4$ the polarization ellipse has its semiaxes $a = u_0|\cos(\Delta kd/2)|$ and $b = u_0|\sin(\Delta kd/2)|$ along the directions $\mathbf{e}$ and $\mathbf{t}$, respectively. This means that if $\Delta kd = (2p+1)\pi$ and $\Delta kd = 2p\pi$ hold, the polarization is linear for only one or the other of these directions, whereas for $\Delta kd = \pi(2p+1)/2$ the polarization is circular (p is an integer).

We can estimate approximately the rate of oscillations of the ellipticity (and also I_e and I_t) as a function of B using Eqs. (9) and (22) provided we know their "period" ΔB. In fact, because of the nonlinearity of the function $\Delta k(B)$ this is true only if $\Delta B \ll B$. We then have

$$\Delta B \approx (\lambda_\eta/d)[B(B+H_D) + 2H_E H_{me1}]^2/(2B+H_D)H_E H_{me2}, \tag{25}$$

where $\lambda_\eta = 2\pi v_\eta/\omega$ is the wavelength of sound with the polarization η.

Before discussing the experimental results reported in Ref. 3 for $MnCO_3$, we first estimate ΔB for hematite (there are as yet no suitable experimental data for this compound). All the parameters occurring in Eq. (25) are however known for hematite[4] and, consequently, for $d = 1$ mm, then at a frequency $\omega/2\pi \approx 203$ MHz and in fields $B \approx 0.1$ T we find from Eq. (25) $\Delta B \approx 0.01$ T. Therefore, hematite is a unique material which can be used to analyze hypersound and to control it with a magnetic field. Transmission of

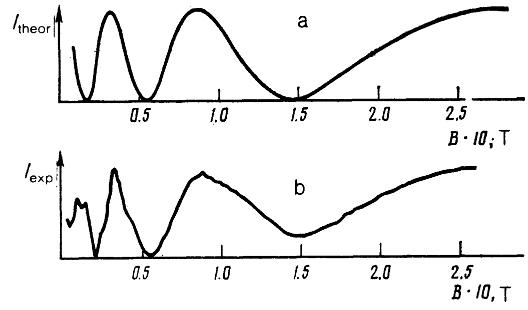

FIG. 1.

sound across a plate can alter the polarization and intensity of sound along a given polarization direction and can modulate it by small oscillations of $\mathbf{B}$.

5. MANGANESE CARBONATE

In the case of manganese carbonate $MnCO_3$ the situation is, on the one hand, less favorable because we do not know all the parameters necessary for a quantitative description of the LB. On the other hand, there is also an advantage to the experimental data reported in Ref. 3, because they allow us not only to check the theoretical predictions but even possibly estimate some unknown parameters.

Figure 1a shows the experimental (reported in Ref. 3) intensity $I \equiv I_{\rm exp}$ of sound of frequency $\omega/2\pi = 203$ MHz transmitted by a plate of thickness $d = 0.8$ mm in a geometry corresponding to Eq. (24), as a function of the field $\mathbf{B}$. (Unfortunately, the value of the angle φ_0 was not given.) Using Fig. 1 (very approximately!) to find the positions of two maxima ($B_1^{\max} \approx 0.033$ T and $B_2^{\max} \approx 0.089$ T) and two minima ($B_1^{\min} \approx 0.059$ T and $B_2^{\min} \approx 0.155$ T) and substituting the known values $H_D = 0.44$ T and $H_E = 32$ T (Ref. 7), we can use equations

$$\Delta k(B_1^{max}) - \Delta k(B_2^{max}) = \Delta k(B_1^{min}) - \Delta k(B_2^{min}) = 2\pi/d \quad (26)$$

in accordance with Eq. (9) and allowing for Eq. (22), which yields two quantities:

$$H_\Delta^2 = 2H_E H_{me1} \approx 0.03\ \mathrm{T}^2 \quad \text{и} \quad G = (d/\lambda_\eta) H_E H_{me2} \approx 0.12\ \mathrm{T}^2.$$

In addition to these data the agreement between the theoretical curve and the experimental results in the argument of the cosine in the second term of Eq. (24) (which is the only quantity that can be compared with the experimental results) requires introduction of an additional phase. It is somewhat surprising that this phase is not $\pi/2$, but this is clearly related to the method used to record the transmitted signal.

Our theoretical curve shown in Fig. 1b reproduces quite well the experimental results. The fitting parameters used were as follows: the first of them (H_Δ^2) governs the gap in the lf branch of spin waves and it can be compared with the corresponding H_Δ^2 (AFMR) value amounting to 0.016 T^2, obtained from an antiferromagnetic resonance (AFMR).[9] If we bear in mind the roughness of our estimate, then in spite of the difference between the value obtained and the AFMR data by a factor of 2, the agreement can be regarded as satisfactory. In both cases H_Δ^2 allows not only for the magnetoelastic contribution, but also for the hyperfine interaction (at $T = 4.2$ K, which was the temperature at which the experiments of Ref. 3 were carried out, the hyperfine contribution even predominated).

The second fitting parameter G subject to allowance for the velocity of sound $v_\eta = 3 \times 10^3$ m/s (Ref. 9) made it possible to find the second magnetoelastic field, which was $H_{me2} \approx 7 \times 10^{-5}$ T. Unfortunately, this value was deduced from Eq. (23), but could not be compared with the experimental value because it was not available.

6. RAY VELOCITY IN HEMATITE

We conclude by considering an additional effect typical of trigonal antiferromagnets. In the case of crystals with the symmetry axis 3 this axis is no longer "pure" acoustic. The ray or group velocity of transverse (or more exactly quasi-transverse) waves $\mathbf{V}$ is not equal to the phase velocity $\mathbf{v} \| \mathbf{k} \| 3 \| \mathbf{z}$ and it is described by the following expression[10]:

$$V_\alpha / v = C_{\alpha\beta z\delta} e_\beta e_\delta / \rho v^2, \quad (27)$$

where $\mathbf{e}$ is a unit polarization vector. In the absence of antiferromagnetism, two such waves are degenerate in the sense that their phase velocities are identical and the polarization in the XY plane can be arbitrary. Rotation of $\mathbf{e}$ in this plane makes the group velocity vector describe a cone of angle θ at its vertex given by the equation

$$\mathrm{tg}\,\theta = |C_{14}/C_{44}|.$$

In the presence of antiferromagnetism this degeneracy is lifted by the magnetoelastic interaction. Each of the normal quasitransverse waves with $\mathbf{k} \| 3_z$ can have not only its own definite polarization (along the ξ or η axes), as well as its own phase and group (V_ξ or V_η) velocities, but also its own group velocity direction, which is the energy transport velocity. These components can be found from Eq. (27) if in the elastic moduli $C_{\alpha\beta\gamma\delta}$ we include the effective magnetoelastic corrections found from Eq. (17), and adopt the axes ξ and η. It should be noted that v_ξ and v_η are the projections of the vectors $\mathbf{V}_\xi$ and $\mathbf{V}_\eta$ along the $\mathbf{z}$ axis. In terms of the polar (θ_ξ and θ_η) and azimuthal (φ_ξ and φ_η) angles of the vectors $\mathbf{V}_\xi$ and $\mathbf{V}_\eta$, the result is as follows:

$$\mathrm{tg}\,\theta_\xi \cos\varphi_\xi = -cb_\xi \sin 6\varphi_B, \quad \mathrm{tg}\,\theta_\xi \sin\varphi_\xi = cb_\xi \cos 6\varphi_B,$$
$$\mathrm{tg}\,\theta_\eta \cos\varphi_\eta = c \sin 6\varphi_B, \quad \mathrm{tg}\,\theta_\eta \sin\varphi_\eta = -c \cos 6\varphi_B, \quad (28)$$

where $c = C_{14}/C_{44}$ and

$$b_\xi = [B(B+H_D) + 2H_E(H_{me1} - H_{me2})] / [B(B+H_D) + 2H_E(H_{me1} - H_{me3})]. \quad (29)$$

In addition to H_{me1} and H_{me2} we can also deduce directly the third effective magnetoelastic field:

$$H_{me3} = 2B_{14}B_{66}/M_0 C_{14}.$$

In the case of hematite, we have $H_{me1} = 7 \times 10^{-5}$, $H_{me2} = 10 \times 10^{-5}$, and $H_{me3} = -19 \times 10^{-5}$, when $H_E = 920$ and $H_D = 2.2$ (all in teslas). This means that b_ξ of Eq. (29) shows sign reversal at $B \equiv B_0 \approx 0.023$ T. In fields $B > B_0$, which are the only ones that can reasonably be considered (assuming the samples to be of the single-domain type), $\varphi_\eta = \varphi_\xi + \pi$ follows from Eq. (28). The angle between the vectors $\mathbf{V}_\xi$ and $\mathbf{V}_\eta$ is then equal to $\theta = \theta_\xi + \theta_\eta$, whereas the angles φ_ξ, θ_ξ, and θ_η are described by the equations

$$\mathrm{tg}\, \varphi_\xi = -\mathrm{ctg}\, 6\varphi_B, \tag{30}$$

$$|\mathrm{tg}\, \theta_\xi| = |c| b_\xi, \quad |\mathrm{tg}\, \theta_\eta| = |c|. \tag{31}$$

These relationships show that an acoustic ray with a definite transverse polarization (which does not coincide with the ξ and η axes) directed along the symmetry axis 3_z splits in an antiferromagnet into two rays with the polarizations along the axis ξ and η and group velocities $\mathbf{V}_\xi$ and $\mathbf{V}_\eta$ differing in the magnitude ($V_\xi = v_\xi / \cos\theta_\xi$ and $V_\eta = v_\eta / \cos\theta_\eta$) and direction. A magnetic field $\mathbf{B} \perp 3_z$ can, in accordance with Eq. (30), be used to concentrate the polarizations and also the phase and group velocities of one of the rays (that with $\mathbf{e} \| \xi$) with the angle θ between $\mathbf{V}_\xi$ and $\mathbf{V}_\eta$.

We now give some numerical values of the angle $\theta = \theta_\xi + \theta_\eta$ calculated in degrees and deduced from Eqs. (29) and (31) ($c = -0.153$) for different values of B (in teslas):

$B \cdot 10$	0.5	1.0	2.0	5.0	10.0	20.0
θ	9.6	10.9	12.6	14.9	16.2	16.9

We should note finally that the cardinal difference between the antiferromagnetic LB effect in tetragonal and trigonal antiferromagnets, which follows from the present work, is not only associated with the different crystallographic symmetries, but also with the different parity of the principal symmetry axes (4^- and 3^+) and in different magnetic states ($\mathbf{l} \| \mathbf{z}$ and $\mathbf{l} \perp \mathbf{z}$).

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APPENDIX

Let us assume that the polarization vector $\mathbf{e}$ of sound of amplitude u_0, which enters a medium along the $\mathbf{z}$ direction, makes an angle φ_0 with the direction of polarization of the normal mode ξ. At the exit of a plate of thickness $z = d$ the projections of the displacement $\mathbf{u}$ along the ξ axis and η are, respectively,

$$u_\xi(d) = u_0 \cos\varphi_0 \cos(k_\xi d - \omega t),$$
$$u_\eta(d) = u_0 \sin\varphi_0 \cos(k_\eta d - \omega t).$$

Hence, we can find the total displacement in any one direction making an angle of ψ with the ξ axis:

$$u_\psi = A_\psi \cos[(k_\xi + k_\eta) d/2 - \omega t + \Phi_\psi],$$

where the amplitude A_ψ and phase Φ_ψ are described by the expressions

$$A_\psi^2 = u_0^2 [\cos^2(\varphi_0 - \psi)\cos^2(\Delta k d/2) + \cos^2(\varphi_0 + \psi)\sin^2(\Delta k d/2)]. \tag{A1}$$

$$\mathrm{tg}\, \Phi_\psi = \frac{\cos(\varphi_0 + \psi)}{\cos(\varphi_0 - \psi)} \mathrm{tg}\left(\frac{\Delta k}{2} d\right) \quad (\Delta k = k_\xi - k_\eta). \tag{A2}$$

Both the amplitude and phase oscillate as a function of B for $\Delta k = \Delta k(B)$.

[1)] This may not be satisfied in low fields B: for example, in the case of hematite we find $\Delta k / k \approx 25\%$ even for $B = 0.1$ T.

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