Characteristics of the dynamics of ferromagnetic liquid crystals

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A calculation is reported of the spectrum of spin waves in liquid and liquid-crystal ferromagnets. It is shown that at long wavelengths the damping of spin waves is due to fluctuation effects and in the exchange approximation is proportional to k^3 ($\mathbf{k}$ is the wave vector). This dependence of the spin wave damping on the wave vector gives rise, in turn, to logarithmic corrections to the viscosity of a liquid or a liquid crystal. Examples of ferromagnetic nematics and smectics are used to analyze the role of the relativistic effects which modify both the real and imaginary parts of the spin wave dispersion law. The relativistic effects in a ferromagnetic nematic result, even in the linear approximation, in the coupling of spin waves and oscillations of the director modifying radically the lf mode spectrum.

1. INTRODUCTION

Liquids found in nature do not have any magnetic order and, therefore, interact very weakly with a magnetic field. On the other hand, the ability to control the flow and other properties of liquids by a magnetic field is very attractive in a number of different technical applications, which has stimulated attempts to synthesize magnetic liquids. Although there are no fundamental reasons why such liquids should not exist, all the attempts so far have been unsuccessful.

However, for over 30 years it has been known that magnetic liquids can be formed from colloidal solutions of ferromagnetic particles in suitable carrier liquids; the properties of such ferromagnetic liquids are reviewed in Ref. 1. Similar systems composed of liquid crystals have recently become a subject of study.[2] The main shortcoming limiting possible applications of magnetic colloids is their instability, with a tendency for ferromagnetic particles to coagulate forming large nodules which settle rapidly under the influence of the force of gravity. Therefore, it is in practice possible to form a long-lived ferromagnetic colloid only if the particle concentration is low and, consequently, the magnetic susceptibility of such liquids is low. Special methods for the stabilization of colloids[1] have made it possible to reach susceptibilities of the order of 10^{-2}–10^{-1}, which are still much less than typical values of the susceptibilities of solid ferromagnets.

In view of this situation, we shall ignore colloidal systems. We concentrate our attention on the noncrystalline state with an intrinsic long-range magnetic order. The hope for experimental realization of such a state has come from the major progress made in the synthesis of paramagnetic[3] and metal-organic[4] liquid crystals. The present authors recently became aware of a preliminary report of the discovery of magnetic ordering in a nematic.[5] There is therefore hope that some liquid-crystal (and possibly liquid) magnetics will be synthesized in the nearest future.

This situation makes it a pressing task to formulate the problem of the properties of such systems. We shall consider ferromagnetic liquid crystals. Properties of liquid-crystal antiferromagnets are also very interesting. However, the latter substances have a number of special properties and should therefore be considered separately.

In our opinion, the present state of the experimental data makes premature any consideration of the various mechanisms of ferromagnetic ordering in liquid crystals or of related problems in microscopic theory. The various possibilities are in principle so numerous that their consideration without an analysis of specific experimental results is hardly useful. It is better to consider general phenomenological properties of liquid-crystal ferromagnets independently of any specific microscopic mechanisms and structure details, but governed by the very existence of magnetic and liquid-crystal ordering. It is the purpose of the present paper to consider such phenomenological properties.

The combination of both types of ordering is manifested most strikingly in the dynamic properties of liquid-crystal ferromagnets. It is known that in the ferromagnetic state a substance has a spontaneous magnetization characterized by a dynamics investigated many years ago by Landau and Lifshitz.[6] The existence of weakly damped spin waves in a ferromagnet is related to oscillations of the spontaneous magnetization direction. In particular, we shall study the characteristics of the spectrum of spin waves associated with the liquid-crystal nature of the state of these ferromagnets. It will be necessary therefore to formulate a system of dynamic equations for the magnetic and other soft degrees of freedom, typical of a liquid crystal. An investigation of this combined system of equations shows that the linearized equation for the magnetization describing spin waves in the exchange approximation becomes separated from the other equations of the system. Linear coupling of spin waves to other modes appears only if we allow for the relativistic terms, which may result in a drastic modification of the lf mode spectrum.

The law of conservation of spin, considered in the exchange approximation, means that the inherent damping of spin waves in a ferromagnet is anomalously weak. We shall demonstrate that in the case of a liquid-crystal (or liquid) ferromagnet there is a contribution from thermal fluctuations to the spin wave damping which exceeds the inherent damping. This thermal contribution is due to a nonlinear interaction of spin waves with viscous modes. In the case of a nematic it is necessary to allow also for the interaction of spin waves with orientational modes associated with relaxa-

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tion of the director. This fluctuation effect is absent in solid ferromagnets, because of the absence of soft diffusion modes mentioned above.

The present paper is organized as follows. The second and third sections deal with a liquid ferromagnet. The anomalous fluctuation contribution to the spin wave damping associated with viscous modes is calculated in Sec. 2. The logarithmic corrections to the viscosity associated with fluctuations of the magnetic degrees of freedom are calculated in Sec. 3.

The results obtained for a liquid ferromagnet are generalized in Sec. 4 to a liquid-crystal ferromagnet. In the exchange region an allowance for the anisotropy of a liquid crystal makes the final expression more cumbersome but does not alter the results qualitatively. The fluctuation contribution to the spin wave damping associated with orientational modes is calculated in this section.

Next, Secs. 5 and 6 deal with the role of the relativistic terms by considering the examples of ferromagnetic nematics and smectics. In a ferromagnetic nematic there is an interaction between spin waves and orientational modes even in the linear approximation, which leads to a drastic modification of the lf spectrum. This situation is even less trivial in the case of ferromagnetic smectics because linear coupling of modes or fluctuation effects may be important, depending on the direction of the wave vector.

The results are summarized in Sec. 7 and possible experimental consequences are discussed briefly.

2. SPIN WAVE SPECTRUM OF A LIQUID FERROMAGNET

In this and the following sections we consider a liquid ferromagnet. It represents a system which can be described by a simple model that can be used to study the main features of the dynamic effects of interest to us. Therefore, an analysis of a liquid ferromagnet is justified at least from the methodological point of view. However, we must stress that such ferromagnets are of interest for their own sake.

We describe the spontaneous magnetization direction by a unit vector $\mathbf{m}$. An inhomogeneity of the vector $\mathbf{m}$ is associated with a gradient energy which can be written in the form

$$E_{ex}=\frac{T}{2g}(\nabla\mathbf{m})^2. \tag{1}$$

Here, T is the absolute temperature and g is a certain constant representing the intensity of fluctuations of the vector $\mathbf{m}$.

The following nondissipative equation for the vector $\mathbf{m}$ can be derived from the Landau–Lifshitz equation[6]:

$$\frac{\partial\mathbf{m}}{\partial t}=-\mathbf{v}\nabla\mathbf{m}+\nabla[A\mathbf{m},\nabla\mathbf{m}]. \tag{2}$$

The right-hand side of Eq. (2) allows for what is known as the convective (or drift) term ($\mathbf{v}$ is the velocity). This term is shown to play an important role later. The coefficient A on the right-hand side of Eq. (2) can be expressed in terms of the parameters of a ferromagnet:

$$A=T/gS_0, \tag{3}$$

where S_0 is the modulus of the spin density related to the spontaneous magnetization by the gyromagnetic ratio.

Linearization of Eq. (2) with respect to deviations of the vector $\mathbf{m}$ from the equilibrium homogeneous value yields the dispersion law of spin waves:

$$\omega=\pm Ak^2. \tag{4}$$

Here, ω is the frequency and $\mathbf{k}$ is the wave vector. There is no damping in the dispersion law of Eq. (4) since the law was derived using the nondissipative equation. A dissipative term must be included on the right-hand side of Eq. (2) in order to calculate the spin wave damping.

An analysis allowing for the law of conservation of spin (valid in the case of a liquid because of its isotropy) shows that the regular damping of spin waves is proportional to k^4 (see, for example, Ref. 7). The same result follows also from a microscopic theory given in Ref. 8. This means that the spin wave damping in a ferromagnet is very weak. Therefore, the effects associated with fluctuations of the vector $\mathbf{m}$ are important in a crystalline ferromagnet. These effects are considered in Ref. 7 by one of the authors of the present paper.

It will be shown below that in the case of a liquid ferromagnet such fluctuation damping is much stronger than the inherent damping so that we can ignore the latter.

The fluctuation contribution to the spin wave damping can be calculated conveniently using a diagram technique proposed by Wyld.[9] We shall employ a generalized variant of the method proposed in Ref. 10 (see also our monograph[11]). A procedure for deriving the effective action I from macroscopic dynamic equations, which then leads to the required diagram technique in accordance with the standard rules used in field theory, is proposed in Refs. 10 and 11.

In this way Eq. (2) yields the following contribution to the effective action:

$$\int dt\,d^3r\left\{\mathbf{p}\frac{\partial\mathbf{m}}{\partial t}+\mathbf{p}\mathbf{v}\nabla\mathbf{m}+A\nabla\mathbf{p}[\mathbf{m},\nabla\mathbf{m}]\right\}. \tag{5}$$

Here $\mathbf{p}$ is an auxiliary Bose field which is subject to an additional condition

$$\mathbf{pm}=0. \tag{6}$$

The correlations of the physical quantities are found as functional integrals with respect to the fields of $\mathbf{m}$ and $\mathbf{p}$ with a weight $\exp(iI)$.

We shall assume that under equilibrium conditions the unit vector $\mathbf{m}$ is directed along the z axis. Its deviation from equilibrium is described by components m_μ of the vector $\mathbf{m}$ along the x and y axes. It follows from the condition (6) that the vector $\mathbf{p}$ has two independent components and it is convenient to select these components p_μ along the same axes x and y.

We are interested in the correlation function

$$D_{\mu\nu}(t,\mathbf{r})=\langle m_\mu(t,\mathbf{r})m_\nu(0,0)\rangle. \tag{7}$$

The inherent correlation function (7) is determined by the part of $I^{(2)}$ which is quadratic in m_μ and p_μ the effective action of Eq. (5), which must be supplemented by the dissipative terms omitted above if we are to obtain the correct analytic properties. Integration with respect to m_μ and p_μ with a weight $\exp(iI^{(2)})$ makes it possible to obtain, in the limit of infinitesimally weak damping, an expression for the correlation function (7) which can be written conveniently in the Fourier representation:

$$D_{\mu\nu}(\omega, \mathbf{k}) = 2\pi g A \delta_{\mu\nu}\delta(\omega^2 - A^2k^4). \quad (8)$$

We introduce the following notation for the velocity correlation function:

$$D_{ik}(t, \mathbf{r}) = \langle v_i(t, \mathbf{r}) v_k(0, 0)\rangle. \quad (9)$$

The expression for the above correlation function is well known (see, for example, Refs. 9 and 11). We are interested in the transverse part of the correlation function of Eq. (9), which in the Fourier approximation is

$$D_{ik}^{(t)}(\omega, \mathbf{k}) = \frac{2\eta T}{\rho^2\omega^2 + \eta^2 k^4}(k^2\delta_{ik} - k_i k_k), \quad (10)$$

where ρ is the mass density and η is the viscosity. The longitudinal part of the correlation described by Eq. (9) is associated with the acoustic degrees of freedom.

The fluctuation corrections to the dispersion law of spin waves can be studied conveniently as a manifestation of the fluctuation contribution to the part of the effective action which is a quadratic function of m_μ and p_μ. This contribution can be written in the form

$$i\int dt\, d^3r\,(p_\mu \Sigma_{\mu\nu} m_\nu + p_\mu \Pi_{\mu\nu} p_\nu). \quad (11)$$

Here, $\hat{\Sigma}$ and $\hat{\Pi}$ are operators which in what follows are called the self-energy and polarization operators. It follows from the fluctuation–dissipation theorem that these quantities are related by an expression which can be written conveniently in terms of the Fourier components:

$$\operatorname{Im}\Sigma_{\mu\nu}(\omega, \mathbf{k}) = -\frac{k^2}{g}\Pi_{\mu\nu}(\omega, \mathbf{k}). \quad (12)$$

The dispersion law of spin waves derived including the contribution made by Eq. (11) to the effective action is

$$\det(\omega\delta_{\mu\nu} - Ak^2\varepsilon_{\mu\nu} - \Sigma_{\mu\nu}(\omega, \mathbf{k})) = 0, \quad (13)$$

where $\varepsilon_{\mu\nu}$ is a two-dimensional antisymmetric tensor. It therefore follows that the self-energy function $\Sigma_{\mu\nu}(\omega,\mathbf{k})$ determines directly the fluctuation corrections to the dispersion law of spin waves. It follows from Eq. (12) that in determining the damping of these waves it is in fact sufficient to calculate the polarization operator $\hat{\Pi}$.

We are interested in the contribution made to $\hat{\Pi}$ by the diagram shown in Fig. 1. The continuous curve in this diagram is the correlation function of Eq. (7), the wavy line represents the correlation function of Eq. (10), and triple vertices (dots in the diagram) are governed by the second term in the action of Eq. (5), which is associated with convection. An analytic expression corresponding to this diagram is

$$\Pi_{\mu\nu}(\omega, \mathbf{k}) = \frac{1}{2}k_i k_k \int \frac{d\nu\, d^3q}{(2\pi)^4} D_{ik}^{(t)}(\nu, \mathbf{q}) D_{\mu\nu}(\omega+\nu, \mathbf{k}+\mathbf{q}). \quad (14)$$

The functions $D_{\mu\nu}$ and $D_{ik}^{(t)}$ should be described by Eqs. (8) and (10), whereas the longitudinal part D_{ik} makes only an unimportant contribution to $\hat{\Pi}$.

FIG. 1.

Below we show that the fluctuation corrections to the real part of the dispersion law of spin waves are small, so that in obtaining the integral of Eq. (14) we can assume directly, following Eq. (4), that $\omega = Ak^2$. We then obtain the law of proportionality $\Pi \propto k$, and it then follows from Eq. (12) that $\operatorname{Im}\Sigma \propto k^3$. This means that the fluctuation damping of spin waves is proportional to k^3, i.e., it is known to exceed the inherent damping (proportional to k^4) in the long-wavelength limit. A similar law of proportionality $\operatorname{Re}\Sigma \propto k^3$ applies also to the real part of Σ. This quantity describes the correction to the real dispersion law of spin waves. This correction is small in the long-wavelength limit ($\operatorname{Re}\Sigma/\omega \propto k$) and we shall ignore it.

An analytic calculation of the integral of Eq. (14) for the general case is difficult. We shall give its values in the limit $A \ll \eta/\rho$. In this case we have

$$\Pi_{\mu\nu} = gYk\delta_{\mu\nu}, \quad (15)$$

where

$$Y = T/16\eta. \quad (16)$$

Calculating now the dispersion law of spin waves in accordance with Eqs. (12) and (13) (and ignoring for the reasons given above $\operatorname{Re}\Sigma$) we obtain the final dispersion law of spin waves in a liquid ferromagnet:

$$\omega = \pm Ak^2 - iYk^3. \quad (17)$$

In the case $A \sim \eta/\rho$ the dispersion law of Eq. (17) is still valid, but the expression (16) for Y should then be regarded as an estimate.

3. CORRECTIONS TO THE VISCOSITY OF A LIQUID FERROMAGNET

The weak spin wave damping has the effect that fluctuations of $\mathbf{m}$ make a considerable contribution to the dynamic characteristics of the system. For example, it is shown in Ref. 7 that in solid isotropic ferromagnets the fluctuations of $\mathbf{m}$ are responsible for the long-wavelength divergence proportional to $k^{-1/4}$ of the viscosity (here, $\mathbf{k}$ is the characteristic wave vector). The actual law describing this divergence is associated with the fact that the spin wave damping is proportional to k^4. As demonstrated in the preceding section, in the case of a liquid ferromagnet such damping is proportional to k^3. Therefore, the fluctuation contribution to the viscosity of such a system is less divergent (as shown below, it diverges logarithmically). Let us calculate this logarithmic contribution.

We can do this if we have an explicit expression for the contribution to the stress tensor associated with an inhomogeneity of the vector $\mathbf{m}$. The main contribution is to the nondissipative stress tensor, the expression for which is best deduced by the Poisson bracket method[12] (see also the monograph of Ref. 11). In addition to the well-known brackets for the mass density ρ, the specific entropy σ, and the momentum density $\mathbf{j} = \rho\mathbf{v}$, we also have to use the bracket

$$\{j_i(\mathbf{r}_1), \mathbf{m}(\mathbf{r}_2)\} = -\nabla_i \mathbf{m}\delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (18)$$

which has a standard structure.

Calculations are carried out using the Liouville equation

$$\partial j_i/\partial t + \nabla_k T_{ik}^{(r)} = 0, \qquad (19)$$

where the nondissipative stress tensor is described by

$$T_{ik}^{(r)} = \rho v_i v_k + P\delta_{ik} + \frac{\partial E}{\partial \nabla_k m} \nabla_i m. \qquad (20)$$

Here, E is the energy density and P is the applied pressure:

$$P = \rho\left(\frac{\partial E}{\partial \rho}\right)_\sigma - E. \qquad (21)$$

Using now the expression for the energy density (1), we find with the aid of Eq. (20) the magnetic contribution to the stress tensor:

$$T_{ik}^{(m)} = \frac{T}{g}\left[\nabla_i \mathbf{m} \nabla_k \mathbf{m} + \theta(\nabla \mathbf{m})^2 \delta_{ik}\right]. \qquad (22)$$

Here,

$$\theta = \frac{1}{2}\left[\left(\frac{\partial \ln T}{\partial \ln \rho}\right)_\sigma - \left(\frac{\partial \ln g}{\partial \ln \rho}\right)_\sigma - 1\right] \qquad (23)$$

is a constant of the order of unity.

The contribution to the effective action associated with the momentuum density is

$$\int dt\, d^3r \left(p_i \frac{\partial j_i}{\partial t} - \nabla_k p_i T_{ik}^{(r)} + i p_i \Sigma_{ij} v_j + i p_i \Pi_{ij} p_j\right). \qquad (24)$$

It includes the self-energy Σ_{ij} and polarization Π_{ij} operators which are coupled because of the fluctuation–dissipation theorem. This coupling can be written conveniently in the Fourier representation:

$$\Pi_{ij}(\omega, \mathbf{k}) = T \,\mathrm{Im}\, \Sigma_{ij}(\omega, \mathbf{k}). \qquad (25)$$

The inherent value of the self-energy operator Σ_{ij} is governed by the first η_0 and second ζ_0 viscosities:

$$\Sigma_{ij}^{(0)}(\omega, \mathbf{k}) = \eta_0(k^2\delta_{ij} + {}^1/_3 k_i k_j) + \zeta_0 k_i k_j. \qquad (26)$$

We are interested in the fluctuation contribution to Π_{ij} which is determined by the diagram shown in Fig. 2. In this diagram the continuous curves represent the operator (7) and the triple vertices (points in the diagram) are governed by the contribution (22) to the stress tensor, in agreement with Eq. (24). The explicit expression for this contribution is

$$\Pi_{ik}(\omega, \mathbf{k}) = \frac{T^2}{g^2} k_m k_n \int \frac{d\nu\, d^3q}{(2\pi)^4} (q_i q_m + \theta q^2 \delta_{im})(q_k q_n + \theta^2 q^2 \delta_{kn}) \times D_{\mu\nu}(\nu, \mathbf{q}) D_{\mu\nu}(\nu, \mathbf{q}). \qquad (27)$$

In view of the logarithmic nature of the integration process we have dropped from Eq. (27) the dependence on ω and $\mathbf{k}$ in the integrand.

In evaluating the integral in Eq. (27) we can no longer use the expression (8), which was obtained ignoring the spin wave damping. Allowing for the fluctuation contribution to the action (11), we find that the explicit expressions for $\hat{\Pi}$ and Σ are given by Eqs. (12) and (15), so that

$$D_{\mu\nu}(\omega, \mathbf{k}) = \frac{4A^2 g Y k^5 \delta_{\mu\nu}}{(\omega^2 - A^2k^4)^2 + 4\omega^2 k^6 Y^2}. \qquad (28)$$

This reduces to Eq. (8) in the limit $Y \to 0$. Substituting Eq. (28) into Eq. (27) and then integrating, we finally obtain

$$\Pi_{ik} = \frac{T^2 L}{60\pi^2 Y}(k^2\delta_{ik} + 2k_i k_k + 10\theta k_i k_k + 15\theta^2 k_i k_k). \qquad (29)$$

Here,

$$L = \ln \frac{\Lambda}{\max[(\omega/Y)^{1/2}, (Ak/Y)^{1/2}]}, \qquad (30)$$

and Λ is the characteristic wave vector corresponding to the ultraviolet cutoff.

Using now the relationship (25) and comparing Eq. (29) with Eq. (26), we reach the conclusion that the presence of the fluctuation contribution to Σ_{ik} and Π_{ik} is equivalent to the appearance of the following fluctuation corrections in the expressions for the viscosities:

$$\eta_{fl} = \frac{TL}{60\pi^2 Y}, \quad \zeta_{fl} = \frac{TL}{36\pi^2 Y}(1 + 3\theta)^2. \qquad (31)$$

Substituting here Eq. (16) for Y, we obtain

$$\frac{\eta_{fl}}{\eta} = \frac{4}{15\pi^2} L. \qquad (32)$$

It should be stressed that in Eq. (16) there is a sum $\eta = \eta_0 + \eta_{fl}$ of the inherent and fluctuation viscosities. The ratio η_{fl}/η_0 is universal and it depends logarithmically on the scale. Clearly, in reality the ratio η_{fl}/η_0 is small because of the smallness of the numerical factor in Eq. (32).

4. SPECTRUM OF A LIQUID-CRYSTAL FERROMAGNET IN THE EXCHANGE APPROXIMATION

In this section we shall generalize the results obtained in the two preceding sections to the case of a liquid-crystal ferromagnet. A distinguishing feature of the liquid-crystal state is its anisotropy. It results, firstly, in an anisotropy of the exchange interaction, and secondly, in the appearance of relativistic (spin–orbit) terms. In this section we shall discuss only the consequences of the exchange interaction anisotropy. The relativistic effects will be dealt with in the next section.

In the case of a nematic, instead of the gradient energy of Eq. (1), we have to use the following expression

$$E_{ex} = \frac{T}{2g_1}(\nabla \mathbf{m})^2 + \frac{T}{2g_2}(\mathbf{n}\nabla \mathbf{m})^2, \qquad (33)$$

where $\mathbf{n}$ is the director. A similar expression applies also to type A smectics, where the role of the director $\mathbf{n}$ is played by the normal to the smectic layers. Equation (2) then becomes

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{v}\nabla \mathbf{m} + \nabla_i[A_1[\mathbf{m}, \nabla_i \mathbf{m}] + A_2 n_i n_k [\mathbf{m}, \nabla_k \mathbf{m}]]. \qquad (34)$$

Consequently, the dispersion law of spin waves becomes anisotropic:

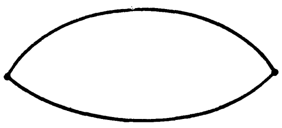

FIG. 2.

$$\omega = \pm (A_1 k^2 + A_2 k_z^2) \equiv A k^2, \qquad (35)$$

where

$$A = A_1 + A_2 \frac{k_z^2}{k^2},$$

and the z axis is directed along the equilibrium orientation of the director $\mathbf{n}$. In view of the strong anisotropy of real nematics, we have to assume that $g_1 \sim g_2$ and, consequently, $A_1 \sim A_2$.

We now consider the fluctuation damping of spin waves in the exchange approximation.

First of all, we have to discuss the contribution, described by the diagram in Fig. 1, made to the polarization operator $\hat{\Pi}$ introduced in Eq. (11). In the case of a liquid crystal the structure of the triple vertices (governed by the convective term in the equation for $\mathbf{m}$) is the same as for a liquid. The correlation function of Eq. (7) for a liquid crystal has the same structure [Eq. (8)], apart from the anisotropy, as in the case of a liquid. Finally, the correlation function (9) for any liquid-crystal phase includes contributions of the type described by Eq. (10) and associated with viscous modes describing the velocity relaxation process (such modes are absent only in the case of crystals).

An explicit calculation of $\Pi_{\mu\nu}$ is difficult because of the anisotropy. However, the law of proportionality $\Pi_{\mu\nu} \propto k\delta_{\mu\nu}$ remains valid. Therefore, the dispersion law of Eq. (17) applies also to a liquid crystal, but now Y is a function of the angle between the director and the wave vector. As an estimate of Y it is reasonable to use Eq. (16), which is justified for $A \lesssim \eta/\rho$. Clearly, this condition is satisfied by liquid crystals characterized by a high viscosity which usually exceeds the viscosity of the corresponding isotropic phase.

Let us now discuss in detail the nematic phase. This phase exhibits an additional fluctuation mechanism ensuring a contribution which competes with the contribution to the spin wave damping discussed above. This mechanism is associated with the anisotropy of the gradient energy of Eq. (33) and of the Landau–Lifshitz equation (34). In view of this anisotropy the effective action generalizing Eq. (5) now includes the following interaction term:

$$I_{int} = \int dt\, d^3r\, A_2 [\delta\mathbf{n}\nabla\mathbf{p}(\mathbf{m}, \mathbf{n}\nabla\mathbf{m}) + \mathbf{n}\nabla\mathbf{p}(\mathbf{m}, \delta\mathbf{n}\nabla\mathbf{m})]. \qquad (36)$$

Here, $\delta\mathbf{n}$ is the deviation of the director from its equilibrium value.

The interaction term of Eq. (36) gives rise to a contribution to the polarization operator $\Pi_{\mu\nu}$ described by the diagram shown in Fig. 3. In this diagram the continuous curve represents the correlation function of Eq. (7), the dashed curve represents the correlation function

$$d_{\alpha\beta}(t, \mathbf{r}) = \langle \delta n_\alpha(t, \mathbf{r}) \delta n_\beta(0, 0) \rangle, \qquad (37)$$

and the points represent a triple vertex governed by the contribution of Eq. (36) to the effective action. The Greek indices in Eq. (37) identify the components along the x and y axes (we recall that in equilibrium the director is assumed to be oriented along the z axis). The explicit expression for this contribution to $\Pi_{\mu\nu}$ is

$$\Pi_{\mu\nu}(\omega, \mathbf{k}) = \frac{A_2^2}{2} \int \frac{d\nu\, d^3q}{(2\pi)^4} D_{\mu\nu}(\nu, \mathbf{q})\, d_{\alpha\beta}(\omega+\nu, \mathbf{k}+\mathbf{q}) \times (k_\alpha q_z + k_z q_\alpha)(k_\beta q_z + k_z q_\beta). \qquad (38)$$

The correlation function $D_{\mu\nu}$ should be described by Eq. (8) when calculating the integral of Eq. (38). The correlation function $d_{\alpha\beta}$ is given by

$$d_{\alpha\beta}(\omega, \mathbf{k}) = \frac{2T\gamma_l}{\gamma_l^2\omega^2 + K_l^2 k^4} \frac{k_\alpha k_\beta}{k_\perp^2} + \frac{2T\gamma_t}{\gamma_t^2\omega^2 + K_t^2 k^4} \left(\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k_\perp^2} \right). \qquad (39)$$

Here, $k_\perp^2 = k^2 - k_z^2$; γ_l and γ_t are certain combinations of the viscosities of a nematic; K_l and K_t are combinations of the Frank elastic moduli. These quantities depend in a complex manner on the angle between the director and the wave vector.

In view of the weak spin wave damping, we can replace ω in Eq. (38) with the value given by Eq. (35). Moreover, we shall assume that the condition $A \gtrsim K/\gamma$, is satisfied and this condition is clearly obeyed by a nematic because of the high viscosity (including the torsional viscosity). It is difficult to calculate the integral in Eq. (38) explicitly because of the anisotropy of the quantities occurring in the integrand. An estimate of $\Pi_{\mu\nu}$ taking into consideration all the above comments yields the following result:

$$\Pi_{\mu\nu} \sim 0.1 \frac{gTA}{K} k\delta_{\mu\nu}. \qquad (40)$$

Allowing now for the fluctuation–dissipation theorem of Eq. (12) (which in the case of a nematic must be generalized to allow for the anisotropy), we reach the conclusion that the mechanism in question (it is natural to call it the anisotropy mechanism) ensures a spin wave damping which is again proportional to k^3. Therefore, in the case of nematics we must take into account not only the isotropic but also the anisotropic mechanisms of the fluctuation-induced spin wave damping.

We thus find that in the case of nematics the same dispersion law (17) applies to spin waves and in this law we have now $Y = Y_1 + Y_2$. The quantity Y_1 is given by the approximate expression in Eq. (16), whereas in the case of Y_2, because of Eq. (49), the corresponding estimate is

$$Y_2 \sim 0.1 TA/K. \qquad (41)$$

The two quantities Y_1 and Y_2 exhibit a nontrivial dependence on the angle between the wave vector and the director, which dependence can be found analytically. We can expect Y_2 to exceed Y_1 because of the high values of the viscosities of a nematic.

The proposed anisotropy mechanism is unimportant in smectics. In smectics A and B we find that instead of fluctuations of the director we must consider fluctuations of the normal to the smectic layers. The corresponding correlation function is even more stringent. In the case of a smectic C there is a correlation of the same type [Eq. (37)] as for a

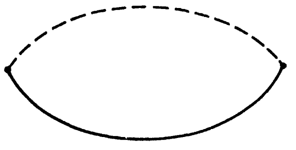

FIG. 3.

nematic. However, the corresponding contribution of Y should be small because of the weak anisotropy of a smectic layer in real smectics C.

The fluctuation contribution to the viscosity of a liquid crystal does not differ qualitatively from the situation in a liquid ferromagnet. Fluctuations of the vector **m** give rise to logarithmic corrections to the viscosities (the number of which is governed by the symmetry of the liquid crystal and is greater than for an isotropic liquid[11]). These fluctuation contributions are described by Eq. (31).

It is known that in the case of a nematic there are not only the usual viscosities governing the dissipation associated with an inhomogeneous flow of a liquid crystal, but there is also a torsional viscosity γ_1 governing relaxation of the director. In a study of the fluctuation contribution to γ_1 we have to consider the contribution to the effective action associated with the director dynamics. This contribution is

$$\int dt\, d^3r \left(\gamma_1 \mathbf{y} \frac{\partial \mathbf{n}}{\partial t} + \mathbf{y} \frac{\delta E}{\delta \mathbf{n}} + i\gamma_1 T \mathbf{y}^2 \right). \tag{42}$$

Here, **y** is an auxiliary Bose field which must satisfy the condition $\mathbf{y}\cdot\mathbf{n} = 0$.

The gradient energy of Eq. (33) creates the following contribution to the effective action:

$$\int dt\, d^3r \frac{T}{g_2} \mathbf{y} \nabla \mathbf{m} \mathbf{n} \nabla \mathbf{m}, \tag{43}$$

which is obtained from the second term in Eq. (42). The expression (43) governs the third-order vertex used to find the contribution to γ_1 given by the diagram shown in Fig. 2. The explicit expression for the correction γ_1 is

$$\gamma_{1fl} = \frac{T}{2g_2^2} \int \frac{d\nu\, d^3q}{(2\pi)^4} q_\alpha^2 q_z^2 D_{\mu\nu}^2(\nu, \mathbf{q}). \tag{44}$$

The integral in the above expression, like that in Eq. (28), is logarithmic. An analysis of Eq. (44) gives an estimate of γ_{1fl} described by the same expression (31) as in the case of the conventional viscosities.

5. ROLE OF RELATIVISTIC EFFECTS IN A NEMATIC

The anisotropy of liquid crystals means that in the ferromagnetic state at sufficiently low frequencies an important role should be played by the relativistic effects. For a nematic the main relativistic (spin–orbit) term in the energy can be written in the following way:

$$E_{rel} = \mp \frac{T}{2g_1} q_{rel}^2 (\mathbf{nm})^2. \tag{45}$$

The upper-sign in Eq. (45) applies to the easy-axis anisotropy, whereas the lower sign corresponds to the easy-plane anisotropy. In the former case under equilibrium conditions both **m** and **n** are collinear, whereas in the latter case they are perpendicular. The quantity $q_{\rm rel}$ in Eq. (45) plays the role of a characteristic wave vector because the relativistic effects become important beginning from this vector.

The relativistic contribution (45) to the ferromagnetic energy of a nematic means that the coupling between orientational and spin modes exists even in the linear approximation. Therefore, in a study of the lf spectrum we have to consider simultaneously the dynamic equations for **m** and **n**.

We shall show below that in the case of **m** it is sufficient to consider only the nondissipative equation which, subject to Eq. (45), is

$$\frac{\partial \mathbf{m}}{\partial t} = -S_0 \left[\mathbf{m}, \frac{\delta E}{\delta \mathbf{m}} \right] = A [\mathbf{m}, \nabla^2 \mathbf{m}] \pm \Xi [\mathbf{m}, \mathbf{n}] \mathbf{mn}. \tag{46}$$

The convective term of Eq. (2) is omitted above, because it makes no contribution to the linear equations. The first term on the right-hand side of Eq. (46) is due to the gradient term of Eq. (33), whereas the coefficient A contains the dependence on the angle between the wave vector and the director given by Eq. (35). The second term on the right-hand side of Eq. (46) is due to the contribution of Eq. (45) to the energy and the coefficient Ξ is now given by

$$\Xi = S_0 \frac{T}{g_1} q_{rel}^2.$$

The equation describing relaxation of the director **n** can be written in the form[11]

$$\gamma \frac{\partial n_i}{\partial t} = -\frac{\delta E}{\delta n_i} = (\delta_{ik} - n_i n_k) \left[K \nabla^2 n_k \pm \frac{T}{g_1} q_{rel}^2 m_k (\mathbf{mn}) \right]. \tag{47}$$

However, we must bear in mind that the coefficients γ and K in this equation for nematics are functions of the angle between the orientations of the wave vector and the director, and they also differ for two independent components of the fluctuations of the director.[11]

We first consider the easy-axis anisotropy corresponding to the upper signs in Eqs. (45)–(47). In this case the vectors **m** and **n** are collinear under equilibrium conditions. We assume that they are directed along the z axis. The deviations from equilibrium are then described by the components m_α and n_α (where the Greek index identifies the components along the x and y axes). Linearization of Eqs. (46) and (47) with respect to m_α and n_α yields a system of four linear scalar equations. The solution of this system allows us to find the dispersion law for all the eigenmodes of the system.

In the long-wavelength limit, i.e., when $k \ll q_{\rm rel}$, this system of equations describes two zero-gap modes and two modes with a gap. The appearance of the gap is due to the anisotropic relativistic contribution of Eq. (45) to the energy of a ferromagnet. The dispersion law of these modes is very special:

$$\omega = \pm \Xi - i \frac{T}{g_1 \gamma_1} q_{rel}^2, \tag{48}$$

where γ_1 is the torsional viscosity. Therefore, the frequency of these modes has a real part, associated with the spin dynamics, as well as an imaginary part, associated with the director dynamics.

The occurrence of zero-gap modes in the spectrum is related simply to the total rotational invariance of the system When these modes are excited the vectors **m** and **n** become collinear, i.e., $m_\alpha = n_\alpha$. The dispersion equation for the modes in question is cumbersome, but we can assume that $\omega \propto k^2$. We give the dispersion laws of these modes on the assumption (in accordance with the reality) that the torsional viscosity is high:

$$\omega_{l,t}=-\frac{i}{\gamma_{l,t}}\left(K_{l,t}+\frac{TAq_{rel}^{2}}{\Xi g_1}\right)k^2. \qquad (49)$$

Here, the indices l and t represent the components n_α directed along and across the wave vector **k** [compare with Eq. (39) for the correlation function]. Therefore, the modes in question are very similar to the orientational modes and their dispersion laws are of the same diffusion nature. The spin degrees of freedom increase the effective Frank constant of a ferromagnetic nematic.

We now consider the easy-plane anisotropy which corresponds to the lower signs in Eqs. (45)–(47). In this case the vectors **m** and **n** are perpendicular to one another in equilibrium. Linearization of Eqs. (46) and (47) in terms of deviations of **m** and **n** from equilibrium yields a system of four scalar equations from which we can find the required mode dispersion laws. Among these modes three are zero-gap and associated with inhomogeneous rotations of **m** and **n** conserving their perpendicular mutual orientation. One mode associated with the change in the angle between **m** and **n** has a gap.

The dispersion law of the latter mode considered in the long-wavelength limit is

$$\omega=-i\frac{T}{g_1\gamma}q_{rel}^{2}. \qquad (50)$$

In contrast to Eq. (48), this dispersion law describes simple attenuation. Out of three zero-gap modes, two describe simultaneous relaxation of **n** and **m**. If we allow for the large value of γ, we find that the dispersion law of these modes is given by an expression of the type described by Eq. (49).

Finally, the last zero-gap mode is related to rotation of **m** by **n**. For the wave vectors

$$q_{rel}\gg k\gg Tq_{rel}/g_1\gamma A$$

the dispersion law of this mode is acoustic:

$$\mathrm{Re}\,\omega=\pm(A\Xi)^{1/2}k. \qquad (51)$$

The damping is then

$$\mathrm{Im}\,\omega\sim Tq_{rel}/g_1\gamma. \qquad (52)$$

For $k\ll Tq_{\rm rel}/g_1\gamma A$ this mode becomes diffusive type and we have

$$\omega\sim-i\frac{A^2g_1\gamma}{T}k^2. \qquad (53)$$

An analysis of the dispersion laws of these modes is difficult when the wave vectors satisfy $k\sim q_{\rm rel}$. In this case an increase in the wave vector results in a smooth conversion of the modes discussed in the present section into a pair of orientational modes and into spin waves which in the $k\gg q_{\rm rel}$ case can be considered in the exchange approximation.

All the modes discussed in the present section exhibit the damping associated with relaxation of the director even in the linear approximation. A fairly cumbersome analysis shows that the fluctuation-induced damping of spin waves discussed in the preceding sections plays no significant role in the case of the wave vectors $k\lesssim q_{\rm rel}$. On the other hand, the linear damping (associated with relaxation of **n**) becomes unimportant at short wavelengths. We shall now estimate the limiting wave vector at which these two types of damping become equal.

In the limit $k\gg q_{\rm rel}$ the damping of spin waves associated with relaxation of **n** is described by the following expression if $\gamma A\gtrsim K$:

$$\mathrm{Im}\,\omega\sim Tq_{rel}^{4}/gk^2\gamma. \qquad (54)$$

Comparing Eqs. (54) and (17), we obtain the following estimate for the limiting wave vector:

$$k^{*}\sim\left(\frac{Tq_{rel}}{gY\gamma}\right)^{1/5}. \qquad (55)$$

We recall that in the case of a nematic we have $Y=Y_1+Y_2$, where Y_1 and Y_2 are given by Eqs. (16) and (41).

We thus find that for $k<k^*$, the spin wave damping is governed by relaxation of the director which is coupled linearly to spin waves because of the relativistic effects. In the limit $k\gg k^*$ these effects are unimportant and the spin wave damping is of purely fluctuation origin.

6. ROLE OF RELATIVISTIC EFFECTS IN A SMECTIC

A smectic is a much more rigid system than a nematic. Therefore, in the case of a smectic the relativistic (spin–orbit) terms affect much less the spin wave spectrum (the effect is approximately the same as in the case of solid uniaxial crystals). Nevertheless, the spin wave spectrum of a smectic exhibits a number of special features, which make it necessary to consider this spectrum in greater detail.

The contribution made by the relativistic effects to the energy of a ferromagnetic smectic is described by Eq. (45) derived for a nematic (subject to the substitution $\mathbf{n}\to\mathbf{l}$, where **l** is a unit vector along the normal to the smectic layers). We assume that under equilibrium conditions the smectic layers are perpendicular to the z axis, so that the vector **l** is directed along this axis. In the case of a slight deviation from equilibrium the variation of the vector **l** is related in the following way to the displacement vector **u** of the smectic layers along the z axis:

$$l_\alpha=-\nabla_\alpha u. \qquad (56)$$

Here, the Greek index means the components along the x and y axes.

The task in this section will be to investigate the combined system of dynamic equations for the magnetic vector **m** and the variable u describing the displacement of the smectic layers. The equation for the vector **m** is of the same form as Eq. (46), derived for a nematic (provided the substitution $\mathbf{n}\to\mathbf{l}$ is made). The equation for the vector **u** (obtained after eliminating the velocity and other hydrodynamic variables of a smectic A) can be written as follows[11]

$$\frac{\nabla^2}{\nabla_\perp^2}\left(\rho\frac{\partial^2u}{\partial t^2}-\eta_{sm}\nabla^2\frac{\partial u}{\partial t}\right)$$

$$=-\frac{\delta E}{\delta u}=B\nabla_z^2u\pm\frac{T}{g_1}q_{rel}^{2}\nabla[(\mathbf{ml})(\mathbf{m}-\mathbf{l}(\mathbf{ml}))]. \qquad (57)$$

Here, B is the bulk modulus of the smectic layers and $\nabla_\perp^2=\nabla^2-\nabla_z^2$. The symbol $\eta_{\rm sm}$ is used to denote a combination of the viscosities characterized by a dependence on the angle between the wave vector and the z axis.

Equation (57) describes propagation of the second sound in a ferromagnetic smectic. The last term on the right-hand side of Eq. (57) modifies the dispersion law of the second sound (compared with the case of a nonmagnetic smectic A), which becomes

$$\omega=\pm\left(Bk_z^2+\frac{T}{g_1}q_{rel}^2k_\perp^2\right)^{1/2}\frac{k_\perp}{k}-\frac{i}{2}\eta_{sm}k^2. \quad (58)$$

This dispersion law is derived for the easy-axis anisotropy whereas in the case of the easy-plane anisotropy the quantity $k_\perp$ in the radicand should be replaced with k_x, where the x axis is selected along the equilibrium direction of the vector **m**. Equation (58) is derived on the implicit assumption that the damping is much less than the real part of the spectrum.

We can thus see that there is an effective shear modulus of the smectic layers, which affects the form of the dispersion law of Eq. (58). However, this is true only if the frequency is sufficiently high. The point is this: Eq. (58) is derived assuming that the vector **m** is fixed, which is true only for $\omega \gg \Xi$ in the case of the easy-axis anisotropy and for $\omega \gg \Xi k/q_{rel}$ for the easy-plane anisotropy. In the opposite limiting case we return to the usual dispersion law for the second sound in smectic liquid crystals[11] (we recall that $\Xi = S_0Tq_{rel}^2/g_1$).

In an investigation of the spectrum associated with the spin degrees of freedom we can in most cases assume that the vector **l** is fixed. It then follows from Eq. (46) for **m** (where we have to replace **n** with **l** = const) that we obtain again the familiar dispersion laws

$$\omega=\pm(\Xi+Ak^2), \quad (59)$$

$$\omega=\pm A^{1/2}k(\Xi+Ak^2)^{1/2}. \quad (60)$$

The dispersion law of Eq. (59) applies to the easy-axis anisotropy, whereas Eq. (60) applies to the easy-plane anisotropy.

Now we are interested in the contribution made to the dispersion law of spin waves by relaxation of the vector **u**. In calculating this contribution we shall need Eq. (57) for the vector **u**. We assume from now on that

$$\left(\frac{B}{\rho}\right)^{1/2}\frac{k_zk_\perp}{k}\gg\omega, \quad (61)$$

which allows us to ignore the term with the second derivative with respect to time in Eq. (57). It should be noted that ω in Eq. (61) is described by Eqs. (59) or (60), depending on the nature of the anisotropy of a ferromagnetic smectic.

Usng Eq. (56) and going over to the Fourier components, we find that the following relationship applies in the case of the easy-axis ansotropy:

$$l_\alpha=\frac{Tq_{rel}^2}{g_1B^2}\frac{1}{k_z^4}\left(Bk_z^2+i\eta_{sm}\frac{k^4}{k_\perp^2}\omega\right)k_\alpha k_\beta m_\beta. \quad (62)$$

In the derivation of Eq. (62) we assume that the imaginary part of l_α is much less than the real part. Moreover, it is postulated that $l_\alpha \ll m_\alpha$. Both assumptions are justified only if k_z and $k_\perp$ are not very small, so that the inequality (61) is satisfied and the second sound does not split into two diffusion modes.

Now substituting Eq. (62) into Eq. (46) for **m** (and replacing **n** with **l**), we find the correction to the dispersion law of Eq. (59). The real part of Eq. (62) gives rise to an unimportant correction to the real part of the dispersion law, whereas the imaginary part is responsible for the damping Im ω of spin waves in a ferromagnetic smectic:

$$\mathrm{Im}\,\omega=-\frac{\eta_{sm}}{2}\frac{T\Xi}{g_1B^2}q_{rel}^2\frac{k^4}{k_z^4}(Ak^2+\Xi). \quad (63)$$

Similarly, in the case of the easy-plane anisotropy, we obtain

$$\mathrm{Im}\,\omega=-\frac{\Xi Ak^2q_{rel}^2}{2g_1B^2}\eta_{sm}\frac{k^4}{k_z^4}\frac{k_x^2}{k_\perp^2}. \quad (64)$$

Here, the x axis is selected along the equilibrium direction of **m**.

The damping described by Eqs. (63) and (64) should be considered together with the fluctuation-induced damping discussed in Secs. 2 and 3. Consequently, it is necessary to generalize the expression obtained in the exchange approximation to the case when the relativistic effects are important. The corresponding contribution to $\Pi_{\mu\nu}$ is still described by the diagram shown in Fig. 1. In the case of the velocity correlation function we have an expression of the form (10). The correlation function of Eq. (7) is modified because instead of Eq. (8) we now have a similar expression where the argument of the δ function is governed by the dispersion law of Eq. (59) or (60).

An analysis shows that if $k \lesssim q_{rel}$, the integral of Eq. (14) is governed by the wave vectors $q \sim q_{rel}$. Therefore, instead of Eq. (15), we obtain

$$\Pi_{\mu\nu}\sim(gYk^2/q_{rel})\delta_{\mu\nu}, \quad (65)$$

where the quantity Y is described by Eq. (16). If the relativistic terms are present, then instead of Eq. (12) we obtain

$$\mathrm{Im}\,\Sigma_{\mu\nu}\sim-\frac{q_{rel}^2}{g}\Pi_{\mu\nu}\sim(Yq_{rel}k^2)\delta_{\mu\nu}. \quad (66)$$

It is this expression that describes the fluctuation-induced spin wave damping when $k \lesssim q_{rel}$ and it applies to both types of anisotropy. Such damping should be added to the damping deduced from the linear theory and described by Eqs. (63) and (64).

Some comment should be made about the cases outside the framework of the discussion in the present section. If the condition (61) is disobeyed, the damping described by Eqs. (63) and (64) of the linear theory is suppressed and it clearly should be ignored.

It is known[11] that in the case of sufficiently small values of k_z and $k_\perp$ we find that two diffusion modes appear in the smectic instead of the second sound. This case requires special analysis, but in our opinion it is not relevant to the situation under discussion. Some idea of the resultant effects can be obtained from the analysis given in the preceding section for nematics, because the situation in nematics and smectics is similar in this range of wave vectors.

7. CONCLUSIONS

We now draw some conclusions. The main effect demonstrated above is the leading role of the fluctuation-induced spin wave damping in a liquid-crystal ferromagnet. In the exchange approximation this damping is proportional to k^3. This dependence of the spin wave damping on the wave vector gives rise to logarithmic corrections to the viscosities.

In the lf range the relativistic terms play an important role in a liquid-crystal ferromagnet. In a nematic the presence of these effects results in a drastic modification of the lf spectrum and this is due to the softness of the director dynamics. In a ferromagnetic smectic relativistic effects alter the real part of the dispersion law of spin waves [see Eqs. (59) and (60)] and the damping of these spin waves is clearly governed mainly by fluctuations and is proportional to k^2 in the long-wavelength limit. When spin waves propagate along the layers or along the normal to them, we can expect a modification of the spin wave spectrum because of the linear coupling to a mode associated with relaxation of displacements of the smectic layers.

Some comments are due about the effects ignored above. As shown in Ref. 7, fluctuations of the vector **m** are responsible for logarithmic corrections to the inherent (proportional to k^4) spin wave damping considered in the exchange approximation. This effect is negligible compared with the fluctuation-induced (proportional to k^3) damping considered above. Moreover, it is demonstrated in Ref. 7 that the fluctuation contribution to the spectrum of a mode associated with relaxation of the magnitude of the spin smears out the inherent diffusion pole, which should be obtained in the exchange approximation. Such smearing is associated with a weak (proportional to k^4) spin wave damping. This smearing does not occur when the spin wave damping is proportional to k^3. Therefore, in the case of a liquid-crystal ferromagnet considered in the exchange approximation we should observe a diffusion pole associated with relaxation of the magnitude of the spin. Antiferromagnetic liquid crystals should be analyzed separately. We simply mention that because the spin wave spectrum is harder in antiferromagnets than in ferromagnets, the fluctuation effects are unimportant.

All that we have said so far about ferromagnetic smectics applies directly to smectics A and hexatic smectics B (hexatics). Smectics C exhibit an orientational (in fact, nematic) order in a layer. The role of the corresponding soft degree of freedom in the formation of the lf spectrum may be important. However, this is unimportant at the present stage and is therefore ignored here.

The spectrum of spin waves is modified by a magnetic field H. If the Larmor frequency exceeds the characteristic relativistic frequency $Aq^2_{\rm rel}$, it is the Larmor frequency that determines the lower cutoff of the integral governing the fluctuation-induced spin wave damping. It should be stressed that in this situation it is the fluctuation mechanism of the damping that predominates. In the long-wavelength limit the spin wave damping obeys the proportionality law $\operatorname{Im}\omega \propto H^{1/2}k^2$. It should also be noted that formally all the results of the present paper apply also to colloidal solutions of ferromagnetic particles in isotropic liquids and liquid crystals. However, the corresponding contributions to the mode spectrum associated with the interaction of the ferromagnetic and "liquid" degrees of freedom are small because the concentration of ferromagnetic particles in such liquids is usually low.

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