Electron stochasticity in semiconductors with nonparabolic dispersion law

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We consider the chaotic behavior of an electron plasma in a one-dimensional superlattice and in a semiconductor with a Kane dispersion law, located in an external magnetic field and excited by an electromagnetic wave. The cause of the chaotization is the nonparabolicity of the carrier dispersion law. The stochasticity conditions are obtained. The ensuing stationary state and the possible experimental manifestations are described. A general computational scheme is presented for semiconductors with arbitrary nonparabolicity.

INTRODUCTION

Various nonlinear dynamic systems exhibit a chaotic behavior when taken out of equilibrium. The direct cause is formation, due to nonlinearity, of closed phase-space regions filled with unstable equilibrium positions. It is just the internal instability of these fully determined systems which leads to an empirical non-repetition of the trajectories.

Each unstable point near which the system trajectory actually passes increases the sensitivity of the system to the initial data. It was assumed for a long time that this leads only to a technical complication of the determination of the initial conditions. However, to determine uniquely the trajectories in a region which is everywhere closed it is necessary to identify them with absolute accuracy. In other words, an infinite volume of information is necessary, a most important problem. By now, the understanding of the impossibility in principle of determining with absolute accuracy the initial data, as an additional fundamental law of nature, has led to the development of constructive concepts[1] and permits these phenomena to be described in practically any medium.

The present paper is devoted to an investigation of the above phenomenon in the electron plasma of a semiconductor. The nonlinearity of this system has many physical causes: nonparabolicity (non-quadratic dependence) of the carrier dispersion law[2]; inconstancy of the interaction parameters of various subsystems (electrons, phonons, and others) with one another and with external fields[3,4]; dependence of the kinetic coefficients of the medium on the intensities of the macroscopic field,[5] as well as feedback in the specific semiconductor device.[6] Many experimental[3–6] and theoretical (see, e.g., Ref. 7) publications deal with various aspects of the problem. We confine ourselves, however, to the cause which in our opinion has been least investigated—the nonparabolicity of the dispersion law. In this case chaotization is observed on a microscopic level,[2] in the motion of an individual electron, and can be considered by a Hamiltonian approach.[8]

In Sec. 1 we develop and discuss a general scheme that is subsequently (Sec. 2) specifically formulated for one-dimensional superlattices (SL) and semiconductors with Kane dispersion (K). The ensuing stationary state is analyzed in Sec. 3, where a method of calculating the microscopic properties of a semiconductor is discussed. In Sec. 4, the proposed method is used to obtain the magnetic-bremsstrahlung spectra of the considered specific semiconductors. The possibility of diagnosing a chaotic plasma is discussed.

1. GENERAL SCHEME

It is usually assumed[1] that a regular behavior is the consequence of the explicit or implicit presence in the system of a symmetry of special type. Chaotization of motion requires both nonlinearity of the dynamic system and impossibility, in principle, of reducing it by some canonical change of variables to an aggregate of noninteracting subsystems with one degree of freedom and with a dynamics known[9] to be deterministic by virtue of the Liouville theorem. These conditions are realized in a semiconductor[2] placed in an external constant magnetic field and irradiated by the electromagnetic wave.

We consider the motion of an individual electron. We describe the corresponding dynamic system by a Hamiltonian of the form

$$\mathcal{H}=P_X^2/2m+[P_Y+(q/c)A]^2/2m+\Delta_0 U(P_Z) \qquad (1)$$

with the following gauge of the vector potential:

$$A=-H_{ext}Z+A_{exc}=-H_{ext}Z+\varepsilon\sum_i \frac{c}{\omega_i}E_i\cos(\omega_i t-k_i Z).$$

Here $\mathbf{P}=(P_X,P_Y,P_Z)$ is the quasimomentum vector, $\mathbf{r}=(X,Y,Z)$ is the radius vector of the conduction electron with charge q and effective mass m; $H_{\rm ext}$ is the intensity of the external constant magnetic field with direction chosen along the X axis; E_i, ω_i, and k_i are respectively the amplitude, frequency, and wave vector of the ith harmonic of the electromagnetic wave; ε is a dimensionless parameter introduced for convenience. The term $\Delta_0 U(P_Z)$ specifies the explicit form of the dispersion law along the Z axis. A more general form of the dispersion law is unnecessary in the present paper. The law can, however, be included in the proposed scheme by standard nonlinear-mechanics methods[9] without affecting the method used to analyze the system-chaotization conditions.

At $\varepsilon=0$ the equations of motion with the Hamiltonian (1) have an exact regular solution. For $\varepsilon\neq 0$ one can count on being able to construct approximate solutions that are analytic in the parameter ε; this corresponds to preservation of the regularity of the motion. The conditions for the parameters of a problem in which such an approximation is impossible in principle are obtained by a scheme proposed by Chirikov[10] and used in the present paper. It is just these conditions which determine the boundaries of the chaotic regions.

We regard the system (1) as weakly perturbed, i.e., we put $\varepsilon\ll 1$. All the calculations here and henceforth are car-

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ried out accurate to first order in ε inclusive. If $\varepsilon = 0$ and the Hamiltonian is unperturbed, a canonical replacement of the variables permits their complete separation (see the Appendix). The equations of motion can then be integrated exactly.

Since the chaotization is due only to the nonlinearity of the oscillations in the (P_Z, Z) plane, to analyze the conditions for its onset we consider a Hamiltonian of general form in terms of those variables in which the substitutions indicated in the Appendix have already been made:

$$\mathscr{H}=H(J)+\varepsilon\sum_{n,k} H_{n,k}(J)\exp[i(\omega_k t+n\Theta)],$$
$$\omega(J)=dH/dJ. \tag{2}$$

Here (J, Θ) are the "action–angle" variables of the unperturbed system, and $\omega(J)$ is the natural-oscillation frequency $[d\Theta/dt = \omega(J)]$. In the nonresonance case, i.e., for $\omega_k \pm n\omega(J) \neq 0$, an analytic series in ε can always be constructed and takes, to first order in ε, the form

$$J=J_0-\varepsilon\sum_{n,k\neq 0} nH_{n,k}(J_0)\,(n\omega(J_0)+\omega_k)^{-1}\exp[i(\omega_k t+n\Theta)],$$
$$\Theta=\Theta_0-\varepsilon i\sum_{n,k\neq 0}\frac{dH_{n,k}(J_0)}{dJ_0}(n\omega(J_0)+\omega_k)^{-1}\exp[i(\omega_k t+n\Theta)], \tag{3}$$

which excludes chaotization.

It follows from (3) that the solution ceases to be analytic near the resonances $r\omega(J_0) \pm \omega j = 0$, where r are integers. Following a procedure that has become traditional,[10] we introduce near resonance in place of Θ the slow phase

$$\Psi=\omega_j t\pm r\Theta,\quad d\Psi/dt\ll\{\omega_j,\, r\omega(J_0)\}.$$

Averaging over the fast variable $\omega_j t$ with allowance for $\varepsilon \ll 1$, and assuming a rapid decrease of the amplitudes $H_{n,k}$ of the harmonics in (2) with increase of their number, we have

$$\mathscr{H}-H(J_0)=\omega(H)\left(\frac{d\omega(H)}{dH}\right)^2\left|\frac{d^2H}{d\omega^2}\right| P_\Psi^2+2|H_{r,j}(J_0)|\cos\Psi,$$

where P_Ψ is the canonical momentum conjugate to the new phase Ψ.

It is convenient for our purposes to choose the natural-oscillation frequency as the new canonical momentum: $P_\Phi = \omega(J)$, corresponding to a generating function $F = -\omega(J)\Phi$. We obtain ultimately for the maximum frequency change

$$\delta P_\Phi\equiv\delta_j=2[2|H_{r,j}(J_0)/(d^2H/d\omega^2)|]^{1/2}. \tag{4}$$

Two variants are possible.[1] If the closest resonances are far enough apart, $\delta_j + \delta_{j+1} < |\omega_j - \omega_{j+1}|$, the considered averaging method is correct and leads to series that are analytic in ε. Otherwise the resonance regions overlap and two conditions, incompatible in general, should be simultaneously satisfied in their common part. This means loss of the perturbation-theory series analyticity in ε. The realization of the second alternative is in fact fixed by the Chirikov criterion:

$$\left|\frac{H_{j,r}(J_0)}{d^2H/d\omega^2}\right|_j^{1/2}+\left|\frac{H_{j+1,r}(J_0)}{d^2H/d\omega^2}\right|_{j+1}^{1/2}\geqslant\frac{1}{3\sqrt{2}}|\omega_j-\omega_{j+1}|. \tag{5}$$

The factor 2/3 takes into account the influence of higher-order resonances.

We emphasize that actually there are no "overlapping" resonances whatever in the chaotic region (5). They have been used only to derive analytically a criterion based on the fact that the assumption of their existence in the chaotic region leads to a contradiction.

2. REGION OF DYNAMIC CHAOS IN SEMICONDUCTORS

As examples of the realization of the described scheme, we consider the two most widely used models of nonlinear semiconductors: one-dimensional superlattice[8] and semiconductor with a Kane dispersion law.[11]

Superlattice. The dispersion law for the electrons in the lower miniband can be approximated with sufficient accuracy by the expression

$$\Delta_0 U(P_Z)=-\Delta_0\cos(P_Z d/\hbar),$$

where d is the superlattice period and Δ_0 is the miniband half-width. The corresponding unperturbed motion (see the Appendix) is described by the expressions[8]

$$J(H)=\frac{8}{\pi}\left(\frac{m\Delta_0}{W^2}\right)^{1/2}\frac{\hbar}{d}[\mathrm{E}(k)+(k^2-1)\mathrm{K}(k)],$$
$$k^2=(\Delta_0+H)/2\Delta_0,$$
$$P_\xi=2\frac{\Delta_0}{\omega_0}\frac{d}{\hbar}k\,\mathrm{cn}\left(\frac{\omega_0}{\omega(H)}\Theta,\ k\right),$$
$$\xi=2\frac{\hbar}{d}\arcsin\left[k\,\mathrm{sn}\left(\frac{\omega_0}{\omega(H)}\Theta,\ k\right)\right],$$
$$\omega(H)=\frac{\pi\omega_0}{2\mathrm{K}(k)},\quad \omega_0=\left(\frac{\Delta_0}{m}\right)^{1/2}\frac{qH_{ext}}{c\hbar}d,$$
$$\left|\frac{d^2H}{d\omega^2}\right|\approx\frac{1}{\pi^2}\frac{\Delta_0-H}{\omega_0^2}\ln^4\left(\frac{\Delta_0-H}{2\Delta_0}\right)\quad\text{as}\quad H\to\Delta_0. \tag{6}$$

Here $\mathrm{K}(k)$ and $\mathrm{E}(k)$ are complete elliptic integrals of first and second kind, respectively. In these variables, the Hamiltonian of the considered system takes the form

$$\mathscr{H}_{\mathrm{SL}}=\frac{P_x^2}{2m}+H(J)+2\varepsilon k\,\mathrm{cn}\left(\frac{\omega_0}{\omega(H)}\Theta,k\right)$$
$$\times\sum\left(\frac{\Delta_0}{m}\right)^{1/2}\frac{qE_i}{\omega_i}\cos(\omega_i t+k_i Z). \tag{7}$$

Note that the solution (6) is strongly nonlinear and contains many harmonics near the separatrix $k \to 1$. A monochromatic perturbation produces therefore in the system more than one resonance. The separatrix of the Hamiltonian (7) corresponds in the superlattice to the top of the miniband ($k \to 1$, $H \to \Delta_0$, see Fig. 1a). The resonances condense therefore near Δ_0 and it is there that the chaotization region is located. One can point to an energy boundary with chaotized electrons above but not below. Figure 1a shows sche-

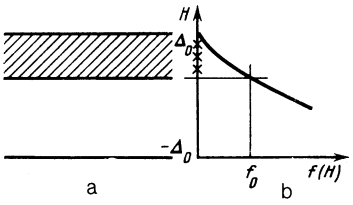

FIG. 1. Electron chaotization region in superlattice (crosses): a—arrangement in miniband, b—Chirikov criterion; $f(H)$ and f_0—left and right sides of (9), respectively.

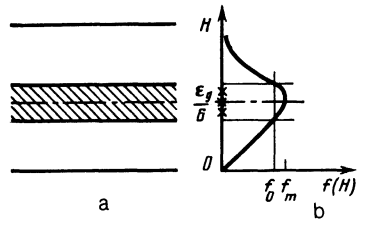

FIG. 2. Electron chaotization region of narrow-gap semiconductor (crosses): a—location in conduction band; b—Chirikov criterion, $f(H)$ and f_0—left and right sides of (14), respectively.

matically the location, in the miniband, of the "new electronic phase," which is the region of dynamically chaotized particles.

At the top of the miniband the Hamiltonian (7) takes the form

$$\mathscr{H}_{SL}=H(J)+2\varepsilon\frac{E}{H_{ext}}(mc^2\Delta_0)^{1/2}\frac{\omega_L}{\omega}k\,\mathrm{cn}\left(\frac{\omega_0}{\omega(H)}\Theta,\ k\right)\cos\omega t, \tag{8}$$

where $\omega_L = qH_{ext}/mc$ is the Larmor frequency and it is taken into account that $kZ/\omega t \lesssim \Delta_0 d/\hbar c \sim 10^{-2}$. Near the resonance $a\omega(H) = \omega$ the leading term of the Hamiltonian (8) is

$$|\mathscr{H}_{a,1}(J)|=2\varepsilon\frac{E}{H_{ext}}(mc^2\Delta_0)^{1/2}\frac{\omega_L}{\omega}\frac{\pi}{K(k)}$$
$$\times\,\mathrm{sech}\left[\frac{\pi\omega}{2\omega_0}K(\sqrt{1-k^2})\right].$$

The Chirikov criterion (5) takes near the separatrix ($k\to1$) the final form

$$f(H)\equiv\varkappa\ln^3\varkappa\leqslant72\pi\varepsilon\frac{E}{H_{ext}}\frac{\omega_L}{\omega}\left(\frac{mc^2}{\Delta_0}\right)^{1/2}\mathrm{sech}\left(\frac{\pi^2}{4}\frac{\omega}{\omega_0}\right), \tag{9}$$

where $\varkappa = (\Delta_0 - H)/\Delta_0$ is the "distance" from the separatrix (see Fig. 1).

It follows from (9) and from Fig. 1 that a region of irregular motion exists near the separatrix as $H\to\Delta_0$ for any amplitude of the external field. The reason is that the difference between the frequencies of the unperturbed oscillations tends to zero at the separatrix. The presence of overlap in this region is therefore determined not by the amplitude of the forced oscillations but by the presence of resonances, i.e., by the perturbation frequency. The amplitude f_0 determines only the width of the chaotization region (see Fig. 1b) and by the same token the possibility of its observation in experiment.

Narrow-gap semiconductor. A characteristic feature of the Kane dispersion law

$$\mathscr{H}_K=\frac{\varepsilon_g}{2}[(1+2|\mathbf{P}|^2/m\varepsilon_g)^{1/2}-1]$$

(ε_g is the band gap) is the interrelation between the motions in all the phase-space planes. However, the expression for the particle velocity

$$\mathbf{V}=\mathbf{P}/m(1+2|\mathbf{P}|^2/m\varepsilon_g)^{1/2} \tag{10}$$

is similar to that in the case of a quadratic dispersion law if the denominator is regarded as a new effective mass M. In the absence of a perturbation M is an integral of the motion, but varies from one trajectory to another. This is what makes this system nonlinear.

At the assumed accuracy, the total Hamiltonian can be represented in the form

$$\mathscr{H}_K=\left[P_X^2+\left(P_Y-\frac{q}{c}H_{ext}Z\right)^2+P_Z^2\right]\frac{1}{2M}[1+\varepsilon\hat{\mathscr{H}}(t)]$$
$$+\varepsilon\frac{q}{M}\left(P_Y-\frac{q}{c}H_{ext}Z\right)\sum_i\frac{E_i}{\omega_i}\cos(\omega_i t+k_iZ),$$
$$\hat{\mathscr{H}}(t)=\sum_i\left(\frac{\omega_L}{\omega_i}Z-\frac{P_Y}{m\omega_i}\right)\frac{qE_i}{h+\varepsilon_g/2}\cos(\omega_i t+k_iZ). \tag{11}$$

The energy of motion along the magnetic field, the energy of the Larmor oscillations, and the total energy are respectively

$$\Pi=P_X^2/2M,\quad H=[P_Y-(q/c)H_{ext}Z]^2/2M,\quad h=\Pi+H.$$

Thus, for a Kane dispersion law we have $U(P_Z) = P_Z^2$, $\Delta_0 = 1/2M$, and the unperturbed motion is described by the expressions (see the Appendix)

$$J(H)=\frac{H}{\omega(H)},\quad \omega(H)=\frac{\omega_0}{1+2h/\varepsilon_g},\quad \omega_0=\omega_L=\frac{qH_{ext}}{mc};$$
$$\xi=(2HM)^{1/2}\sin\Theta,\quad P_\xi=[2H/M\omega^2(H)]^{1/2}\cos\Theta,$$
$$|d^2H/d\omega^2|=\omega_0\varepsilon_g/\omega^3(H). \tag{12}$$

The natural oscillations in this system are linear. The perturbation needed to organize the interacting resonance must be at least biharmonic.

Generally speaking, strictly linear systems can also be chaotized in this manner.[12] Their oscillation frequency is independent of the amplitude, and "resonance overlap" takes place simultaneously in the entire region of motion when the excited oscillations (5) reach a sufficient amplitude. The case considered is intermediate between linear and nonlinear systems. On the one hand, motion on each Larmor orbit is linear and chaotization requires correspondingly a polyharmonic perturbation. On the other hand, by virtue of the relativistic change of the mass M in (10) the oscillation frequency varies from orbit to orbit and depends on the amplitude of the oscillations. The chaotization criterion is therefore met only in a limited layer of the conduction band, in analogy with the superlattice considered above.

We confine ourselves to the simplest case of a two-frequency perturbation:

$$\mathscr{H}_K=\frac{P_X^2}{2M}+J\omega(h)+\varepsilon q\left(\frac{2H}{M}\right)^{1/2}\left(1+\frac{2h}{\varepsilon_g}\right)$$
$$\times\cos\Theta\sum_i\frac{E_i}{\omega_i}\cos(\omega_i t). \tag{13}$$

Integrating near both resonances $\omega_{1,2} = \omega(H_{1,2})$ with respect to the fast phase $\omega_i t$, we separate the principal terms in the Hamiltonian (13):

$$|\mathscr{H}_{1,2}-\mathscr{H}(J_0)|=(qE_i/2\omega_i)(2H_i/M_i)^{1/2}(1+2h/\varepsilon_g).$$

Here $i = 1, 2$ marks a resonance with the first and second harmonics of the perturbation. Assuming for simplicity that the amplitudes of the harmonics are equal, $E_1 = E_2 = E$, and introducing the notation $\omega = \omega_1$ and $\Delta = \omega_2 - \omega_1$, we obtain the Chirikov criterion in final form:

$$f(H)=H^{1/2}\left(1+\frac{2h}{\varepsilon_g}\right)^{-2}\geqslant\frac{1}{9}\left(\frac{\Delta}{\omega_0}\right)^2\omega\left(\frac{M}{2}\right)^{1/2}\frac{\varepsilon_g}{\varepsilon qE}. \tag{14}$$

The chaotization region is shown schematically in Fig.

2. It can be seen that chaotization is possible only under the condition $f_0 \lesssim f_m$, i.e., if

$$\varepsilon q E \geqslant {}^{8}/_{27}(\Delta/\omega_0)^2\omega(m\varepsilon_g)^{1/2}. \tag{15}$$

Figure 2 illustrates the difference between the considered system and either the superlattice or a purely linear system. In the former case, owing to the presence of a separatrix, the chaotization takes place at any amplitude of the external fields. In the opposite case the chaotization, having a threshold, takes place in all of phase space.

3. STATISTICAL PROPERTIES OF STATIONARY STATE

We develop here a meaningful description of the electron dynamics in the regions obtained above. Formation of phase-space layers filled with unstable equilibrium positions leads to mixing.[1] This means that the initial correlations in the Hamiltonian system vanish with time, a property stronger than ergodicity. The Birkhoff fundamental ergodic theorem[13] guarantees here satisfaction of the ergodic hypothesis. Mixing leads thus to equipartition over the canonical variables J and Θ in the entire chaotized region.

It follows from the foregoing that in the steady chaotic regime the distribution function takes the following form: in "action–angle" variables

$$df(J,\ \Theta) = NdJd\Theta/V$$

or in "energy–angle" variables

$$df(H,\ \Theta) = NdHd\Theta/V\omega(H), \tag{16}$$

where V is the phase volume of the irregular region and N is the number of chaotized electrons. Since the considered system is Hamiltonian, the boundaries of the chaotic regions are in themselves regular trajectories "impermeable" to the particles. Their number can therefore be calculated with the aid of a Fermi distribution function, since the boundary of the chaotic region locks in all the electrons that are actualy contained in it.

It is important to note that the distribution (16) of a chaotized group of electrons is not a Fermi function and is determined not by thermodynamic parameters but by purely dynamic ones. The reason is that the energy H is no longer an integral of the motion in view of the new internal symmetry of the system (1), (7), and (11).

The state of a system with mixing is "coarse" (Ref. 14). This means that small additional perturbutations do not disturb the established motion and its statistical properties. Since equipartition in phase is equivalent to equipartition in the quasimomentum direction, it is natural to expect, for example, no drift of chaotized electrons in external weak magnetic fields. A chaotic electronic phase that is equipartitioned with respect to the quasimomentum is a dielectric even in the presence of external weak fields.[2]

The change of the macroscopic properties of a semiconductor with a nonquadratic dispersion law through chaotization of the electron dynamics can be correctly calculated in two cases:

1. For setups operating in a ballistic regime. Their macroscopic manifestations coincide, roughly speaking, with the microscopic ones.[15] Therefore calculation of any physical quantity reduces to averaging it with the aid of the distribution function (16) in the chaotic and the Fermi function in the region of the regular motion.

2. To analyze the properties of semiconductors described by a kinetic equation in the τ approximation it is convenient to use a chaotized distribution function. The kinetic equation is then linear, of first order, and can be solved by the method of characteristics. The dynamics of the characteristics themselves is then completely separated from the establishement of an equilibrium state along them under the influence of the collision integral. The equation for the characteristics coincides with the Hamilton equations for an electron moved by the action of the field actually existing in the system, but without allowance for particle collisions. This is precisely the situation considered above.

All the arguments advanced above concerning the dynamics of an individual electron are fully applicable to the characteristics of the kinetic equation, which are chaotized under the same conditions and for the same initial data (9), (14), and (15). This leads to corresponding changes of the total distribution function.

4. DIAGNOSTICS OF CHAOTIZED PLASMA

Let us examine the macroscopic manifestations of the change of the character of the carrier motion. It was shown above that in the case of a semiconductor this alters the statistics of the electrons, and can be manifested by changes in the conductivity of the medium,[2] of their emission and absorption spectra, of their thermodynamic properties, etc. Let us calculate by way of example the magnetic bremsstrahlung of a superlattice and a Kane semiconductor. Generally speaking, when a magnetized chaotized electron plasma of a semiconductor is irradiated by an electromagnetic wave, two possible radiophysical effects are possible: magnetic bremsstrahlung of the electrons chaotized by the incident wave, and Raman scattering of the incident wave by the electrons it has chaotized. Both phenomena alter the corresponding spectra and make it possible in principle to study the chaotization processes in a semiconductor. Since, however, the perturbation is assumed small, the relative magnitude of the Raman scattering is small and will therefore not be considered here.

For the purpose of demonstration, let us consider the simplest experimental situation. A semiconductor film with a superlattice is placed in an external constant magnetic field perpendicular to the superlattice axis. For a narrow-gap semiconductor, the magnetic-field direction is immaterial. The object is irradiated by an electromagnetic wave normally incident on the sample. The radiation receiver is placed

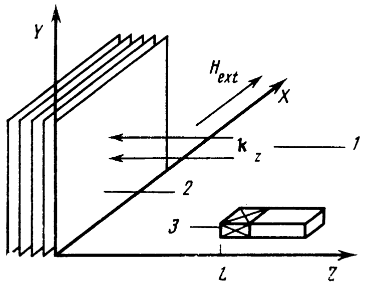

FIG. 3. Geometry of problem: *1*—incident wave, *2*—sample, *3*—radiation receiver.

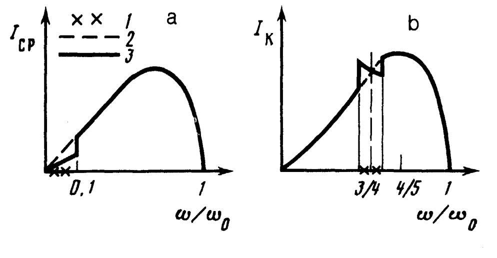

FIG. 4. Schematic frequency dependence of cyclotron-radiation intensity of a superlattice (a) and of a narrow-gap semiconductor (b): *1*—irregular-motion region, *2*—plot of regular motion in the entire band, *3*—plot with allowance for dynamic chaotization.

perpendicular to the plane of the film at a distance L (Fig. 3).

The intensity of the electromagnetic field emitted in the receiver by one electron is equal to

$$\mathbf{E}(t)=(0,E_Y,0),\qquad E_Y=-q\frac{d^2Y(t-L/c)}{dt^2}\frac{1}{4\pi\varepsilon_0c^2L}.$$

Or, using (6) and (12), we get

$$E_{Y,\mathrm{SL}}=\frac{q^2H_{ext}\,d\Delta_0}{c^3\hbar m\varepsilon_0L}\frac{\pi}{4\mathrm{K}^2(k)}\sum_{n=0}^{\infty}(2n+1)\,\mathrm{sech}\Big[\Big(n+\frac{1}{2}\Big)\frac{\pi\mathrm{K}(\sqrt{1-k^2})}{\mathrm{K}(k)}\Big]\sin[(2n+1)\Theta],\tag{17}$$

$$E_{Y,\mathrm{K}}=\frac{q\omega(H)}{4\pi\varepsilon_0c^2L}\Big(\frac{2H}{M}\Big)^{1/2}\cos\Theta.\tag{18}$$

The expressions obtained must be averaged with the distribution function in the chaotic region and with the Fermi function in the regular region. Both cases include an averaging over the phase Θ, so that the mean values of the intensities are zero.

The quantities finally calculated are the intensity spectra of the magnetic bremsstrahlung:

$$I_{\mathrm{SL}}(\omega)=\frac{q^2\omega^4}{8\varepsilon_0{}^2c^2L^2\pi^2\omega_0{}^2}\,\mathrm{sech}^2\Big[\frac{\pi\omega_0}{2\omega}\mathrm{K}(\sqrt{1-k^2})\Big]\times\sum_{n=0}^{\infty}\frac{\delta(\omega-(2n+1)\omega(H))}{(2n+1)^2}$$

$$I_{\mathrm{K}}(\omega)=\frac{q^2\omega^2H}{16\pi^2\varepsilon_0{}^2c^4L^2M}\delta(\omega-\omega(H)).\tag{19}$$

Averaging (19) with the aid of the Fermi function and expression (16), we obtain for the superlattice:

$$I_{\mathrm{SL}}{}^{\mathrm{R}}=A\frac{\omega}{\omega_0}\sum_{n=0}^{\infty}\mathrm{sech}^2\Big[\frac{\pi\omega_0}{2\omega}\mathrm{K}\Big(\sqrt{\frac{\Delta_0-H(\omega/(2n+1))}{2\Delta_0}}\Big)\Big]$$
$$\times\exp\Big[-\pi\frac{\omega_0}{\omega}(2n+1)\Big]$$
$$\times\exp\Big\{\frac{\Delta_0}{kT}\Big[2\exp\Big(-\pi\frac{\omega_0}{\omega}(2n+1)\Big)+1\Big]\Big\},$$
$$I_{\mathrm{SL}}{}^{\mathrm{S}}=A\frac{\omega}{\omega_0}\mathrm{sech}^2\Big(\frac{\pi^2\omega_0}{4\omega}\Big)\frac{\mathrm{cosech}(-\pi\omega_0/\omega)}{2},$$
$$A=\frac{q^2\omega_0nSl}{8\pi\varepsilon_0{}^2c^2L^2}e_2F_2{}^{-1}(1;1;{}^3/_2;{}^3/_2;2\Delta_0/kT),\tag{20}$$

where n is the electron density, S is the sample area, l is its thickness, ${}_2F_2$ is a hypergeometric function, and the superscripts R and S label respectively the regular and stochastic cases.

For a narrow-gap semiconductor we have

$$I_{\mathrm{K}}{}^{\mathrm{R}}=A\pi^{-1/2}\Big(\frac{\omega}{\omega_0}\Big)^{1/2}\Big(\frac{\varepsilon_g}{kT}\Big)^{1/2}\Big(\frac{\omega_0}{\omega}-1\Big)^{1/2}\times\exp\Big[-\Big(\frac{\omega_0}{\omega}-1\Big)\frac{\varepsilon_g}{2kT}\Big],$$
$$I_{\mathrm{K}}{}^{\mathrm{S}}=A\Big(\frac{\omega_0}{\omega}\Big)^{1/2}\Big(\frac{\omega_0}{\omega}-1\Big)\exp\Big(-\frac{\varepsilon_g}{6kT}\Big),$$
$$A=\frac{q^2\omega_0\varepsilon_gnSl}{12\varepsilon_0{}^2c^4\pi^2L^2m}\frac{\varepsilon_g}{2kT}.\tag{21}$$

A comparison of (20) and (21) (see Fig. 4) shows that both the fact that the electron plasma is chaotized and the boundaries of the chaotized region can be established by measuring the magnetic-bremsstrahlung spectrum.

To obtain the specific values shown in Fig. 4b we used for InSb a band gap $\varepsilon_g=0.18$ eV. It can also be seen that in the case of a narrow-gap semiconductor the onset of electron chaotization, namely the change of their statistics in the form of a singularity on the spectrum profile, occurs near the maximum of the curve. This case should therefore be regarded as most suitable for experiments.

The real shape of the curve is made complicated by the presence of regular-behavior islands in the chaotic region. In fact,[1] the chaotized layer is generally speaking not homogeneous. The presence of the stability islands and of the impermeable boundaries between them introduces also some order in the motion of the chaotized phase of the electrons. Our analysis was made for a negligible relative volume of the islands in the chaotized layer of the conduction (mini)band of the semiconductor.

We have also disregarded here the interaction of the magnetic bremsstrahlung with the plasma. In the general case it can lead to spectrum distortion by wave absorption. In particular, if the radiation manages to reach thermodynamic equilibrium with the plasma before it reaches its surface, the resultant spectrum should correspond to absolute blackbody equilibrium radiation. The characteristic length at which this phenomenon is substantially manifested, meaning the skin-layer depth, is assumed thus to be much larger than the film thickness l. For the same reason, the damping of the incident wave in the semiconductor plasma is also disregarded. On the other hand, the electron and phonon subsystems are assumed to be in equilibrium, so that the film thickness is much larger than the mean free path. For the most popular semiconductors we have:

	m	n, cm^{-3}	μ, cm$^2\cdot$V$^{-1}\cdot$s^{-1}	l, cm
For InSb:	$\sim10^{-2}\ m_e$	$\sim10^{6}$	$\sim10^{5}$	$\sim10^{-3}$–10^{-2}
For Ge:	$\sim10^{-1}\ m_e$	$\sim10^{13}$	$\sim10^{3}$	$\sim10^{-3}$–10^{-1}

CONCLUSION

Mixing, or "deterministic chaos," constitutes motion with a principally new topology. This form of dynamics is typical in nature, while regularity corresponds to a degenerate case of explicit or implicit linearity of the system, or to smallness of the deviation from an equilibrium position. It is natural to expect in a medium substantial changes of most physical properties making up a subsystem that has been chaotized.

The electron dispersion law can actually be much more complicated than those considered above. In our opinion, the scheme described permits effective calculation of the conditions for the existence of chaotized regions and of their boundaries in an arbitrary case. Such calculations, however, can be made only with a computer.

We have considered only one of the possible macroscopic manifestations of a chaotized electron plasma of a semiconductor. We assume, however, that the results of Sec. 3 are effective in calculations of arbitrary macroscopic properties in a chaotic regime if the regular-behavior islands can be neglected.

APPENDIX

We rewrite the initial Hamiltonian (1)

$$\mathcal{H}=P_X^2/2m+[P_Y-(q/c)H_{ext}Z+(q/c)A_{exc}]^2/2m+\Delta_0 U(P_Z)$$

with the aid of the generating function

$$F=-P_Y(\xi+\eta)/W+Z\xi,\quad W=qH_{ext}/c$$

in the more convenient form

$$\mathcal{H}=\frac{P_X^2}{2m}+W^2\frac{P_\xi^2}{2m}+\Delta_0U(\xi)+\varepsilon qWP_\xi\sum_i\frac{E_i}{m\omega_i}\cos[\omega_i t+k_i(P_\eta-P_\xi)],$$

$$P_\xi=P_Y/W-Z,\ P_\eta=P_Y/W,\ \xi=P_Z,\ \eta=WY-P_Z,$$

where only terms of first order of smallness in ε are retained.

Since the variables have been separated, a generating solution can be obtained merely by changing to the "action–angle" variables corresponding to the coordinates $(P_\xi;\xi)$:

$$J=J(H)=\frac{2}{\pi}\left(\frac{2mH}{W^2}\right)^{1/2}\int_0^{\xi_{max}}\left[1-\frac{\Delta_0}{H}U(\xi)\right]^{1/2}d\xi,$$

$$P_\xi=P_\xi(J,\xi)=W^{-1}[2m(H(J)-\Delta_0U(P_Z))]^{1/2},$$

$$\Theta=\Theta(J,\xi)=\frac{dH}{dJ}\left[\frac{m}{2H(J)W^2}\right]^{1/2}\int_0^{\xi}\left[1-\frac{\Delta_0}{H}U(\xi)\right]^{-1/2}d\xi.$$

In the new variables, the Hamiltonian takes the form

$$\mathcal{H}=\frac{P_X^2}{2m}+H(J)+\varepsilon qP_\xi(J,\Theta)\times\sum_i\frac{WE_i}{m\omega_i}\cos[\omega_i t+k_i(P_\eta-P_\xi(J,\Theta))],$$

where H is the total energy of particle motion in the (Y,Z) plane and is an integral of the motion if $\varepsilon=0$.

The solution of the transformed system at $\varepsilon=0$ is obvious:

$$P_X=P_{X0}=\text{const},\ P_\eta=P_{\eta0}=\text{const},$$
$$J=J_0=\text{const},\quad X=P_{X0}t/m,\quad \eta=\eta_0=\text{const},$$
$$\Theta=\omega(H)t,\ \omega(H)=dH(J)/dJ.$$

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