Weak localization of waves in incoherent scattering in crystals

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The problem of weak localization of waves is solved using a model of a semi-infinite periodic system of fluctuating potential centers. The Green function of the transport equation for the density matrix is calculated without postulating diffuse propagation of radiation in a medium. It is shown that maxima of the incoherent scattering intensity due to weak localization of the waves may appear not only in the backward direction, but also along other directions, depending on the orientation of a crystal. The angular distribution of the intensity in these maxima is found analytically for the symmetric geometry of the two-wave Laue diffraction case. An additional enhancement of the peak representing weak localization in the backward direction is found for the Bragg diffraction case.

1. INTRODUCTION

Weak localization is manifested in experimental studies of the scattering of waves in randomly inhomogeneous media by a strong enhancement of the incoherent intensity of the waves scattered close to the backward direction.[1,2] It has been established reliably that the physical origin of the coherent backscattering peak reported in Refs. 1 and 2 is the interference of the waves transmitted by the same inhomogeneities of the medium and following coincident paths in the forward and backward directions.[3-12] A weak localization of light has been observed also in unoriented liquid crystals[13] and in quasi-two-dimensional systems.[14] The possibility of localization of light in semiconductors and of neutrons in disordered structures was considered in Refs. 15 and 16.

All these theoretical treatments describe the weak wave localization effect using a model of a random medium with a totally disordered spatial distribution of the scatterers. However, there is also a different type of disorder associated with fluctuations of the potentials of the interaction of a particle with an ideal periodic system of the scattering centers.[17,18] An example of such a system is a crystal with an irregular distribution of isotopes in the lattice or a liquid-crystal structure in which fluctuations of the permittivity are due to the thermal motion of molecules. A distinguishing feature of systems with a disorder of this type in the case when the wavelength λ of a particle is much less than the mean free path $l_{\rm mfp}$ of particles (photons) in the investigated medium and also much less than double the crystal lattice period a is the occurrence not only of incoherent scattering, but also of diffraction by the average periodic distribution of atoms.

Interference of the scattered waves in periodic structures may have effects other than a change in the incoherent collision cross section compared with that for a disordered substance.[3,4,7] The spatial symmetry in the distribution of the scattering centers together with a general invariance of the properties of the medium under time inversion[5,7,8] reduce the number of the additional paths of a particle along which amplification of the incoherent intensity can occur in the same way as in the backward direction.

We shall use the weak localization limit $\lambda \gg l_{\rm mfp}$ to solve the problem of multiple incoherent scattering of waves in a system of periodically distributed potential centers with a small radius $r_0 \ll \lambda$. The cross section for the scattering of a wave by an isolated potential considered in the limit $r_0 \ll \lambda$ is isotropic, which is typical of the incoherent neutron scattering in crystals or the scattering of light by nonideal diffraction gratings. In contrast to the methods developed elsewhere,[5,7,9,11] we shall solve the albedo problem of multiple scattering without postulating diffuse propagation of light in a medium. In particular, we shall find an analytic expression for the angular spectrum under weak localization conditions, which is valid for arbitrary angles between the wave vectors of the incident and scattered waves, and not only in a narrow angular range near the maximum along the backward direction. We shall show that in the Bragg diffraction case the intensity of the weak localization peak is enhanced compared with the incoherent background of backscattered particles. An analysis of the angular distribution of the scattered radiation in the symmetric Laue diffraction case will be used to show that this distribution can have a second weak localization peak differing from the backward scattering by a specular reflection transformation by the crystal lattice planes.

2. MULTIPLE INCOHERENT SCATTERING IN A CRYSTAL

The description of the angular spectrum of radiation scattered incoherently under weak localization conditions when $\lambda / l_{\rm mfp} \ll 1$ requires derivation of the Green function for the elastic scattering problem, solution of the transport equation for the density matrix (mutual coherence function), and calculation of the contribution of the fan diagrams describing interference waves which have crossed the same scattering inhomogeneities along mutually opposite directions.[3,7,11]

We shall consider the motion of a nonrelativistic particle in the field created by periodically distributed potential centers

$$U(\mathbf{r})=\sum_a u_a(\mathbf{r}-\mathbf{R}_a), \qquad a=1,2,3\ldots N, \tag{1}$$

where the radius of action r_0 of each of these centers is much less than the particle wavelength:

$$r_0\ll\lambda. \tag{2}$$

We shall assume that all the functions $u_a(\mathbf{r})$ satisfy the conditions of validity of the Born approximation. We shall rep-

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resent Eq. (1) in the form of a sum of the periodic potential averaged over the statistical realizations

$$U_0(\mathbf{r}) = \sum_a u_0(\mathbf{r}-\mathbf{R}_a) = \sum_a \langle u_a(\mathbf{r}-\mathbf{R}_a)\rangle \tag{3}$$

and a fluctuating component

$$\delta U(\mathbf{r}) \equiv U(\mathbf{r}) - U_0(\mathbf{r}) = \sum_a \delta u_a(\mathbf{r}-\mathbf{R}_a). \tag{4}$$

The Green function of the problem of the elastic scattering by the potential of Eq. (1) averaged over the random realizations of Eq. (4) and considered in the approximation which is quadratic in $\delta U(\mathbf{r})$ satisfies an equation[7] (Fig. 1) which has the differential form (here and later, we shall assume that $\hbar = 1$)

$$\left(E + \frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} - U_0(\mathbf{r})\right) G(\mathbf{r}, \mathbf{r}') - \sum_a \int d^3R \langle \delta u_a(\mathbf{r}-\mathbf{R}_a)\,\delta u_a(\mathbf{R}-\mathbf{R}_a)\rangle G_0(\mathbf{r}, \mathbf{R}) \times G(\mathbf{R}, \mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'), \tag{5}$$

where allowance is made for the condition of the absence of correlations of fluctuations of the potentials of different centers $\langle \delta u_a \delta u_b\rangle \sim \delta_{ab}$, and $E = p_0^2/2m$ is the particle energy.

The quantity $G_0(\mathbf{r}; \mathbf{R})$ in Eq. (5) describes the wave field of a point source with the potential (3), which at short distances $|\mathbf{r} - \mathbf{R}| \lesssim r_0$ becomes

$$G_0(\mathbf{r}; \mathbf{R}) \approx -\frac{m}{2\pi}\frac{1}{|\mathbf{r}-\mathbf{R}|}\exp\{ip_0|\mathbf{r}-\mathbf{R}|\}. \tag{6}$$

The inequality (2) allows us to take the function $G(\mathbf{R}, \mathbf{r})$ outside the integral on the left-hand side of Eq. (5) at the point $\mathbf{R} = \mathbf{r}$, so that the effective optical elastic-scattering potential becomes

$$V(\mathbf{r}) = \sum_a \left\{ u_0(\mathbf{r}-\mathbf{R}_a) + \int d^3R \langle \delta u_a(\mathbf{r}-\mathbf{R}_a)\,\delta u_a(\mathbf{R}-\mathbf{R}_a)\rangle G_0(\mathbf{r}; \mathbf{R}) \right\}. \tag{7}$$

The Fourier component of Eq. (7), corresponding to the Bragg diffraction reflection by one of the reciprocal-lattice vectors $\mathbf{G}$, is

$$V(\mathbf{G}) = \frac{1}{\Omega}\int d^3r\, e^{-i\mathbf{G}\mathbf{r}} V(\mathbf{r}) = \Lambda(\mathbf{G}) - \frac{i}{2}\Gamma(\mathbf{G}), \tag{8}$$

where Ω is the volume of the medium.

In the case of the first strong reflections characterized by $Gr_0 \ll 1$, we can expand the Green function $G_0(\mathbf{r}, \mathbf{R})$ of Eq. (6) as a series in terms of a small parameter $p_0 r \ll 1$ and then the real and imaginary parts of Eq. (8) can be written in the form

$$\Lambda(\mathbf{G}) = \Lambda = n\int d^3r \left\{ u_0(\mathbf{r}) - \frac{m}{2\pi}\int d^3R \frac{\langle \delta u(\mathbf{r})\,\delta u(\mathbf{R})\rangle}{|\mathbf{r}-\mathbf{R}|} \right\},$$

$$\Gamma(\mathbf{G}) = \Gamma = nv\sigma_{in} = nmp_0\frac{1}{\pi}\int d^3r\, d^3R \langle \delta u(\mathbf{r})\,\delta u(\mathbf{R})\rangle, \tag{9}$$

where the choice of the phase of the potential corresponds to the case of a simple cubic lattice with one of its sites coinciding with the origin of the coordinate system; $n = N/\Omega$ is the number of the scattering centers per unit volume; $v = p_0/m$ is the particle velocity. An allowance for the additional inelastic channel of particle absorption results in replacement of the incoherent scattering cross section σ_{in} in the imaginary part of the potential (9) with the total cross section of the $\sigma_r = \sigma_{in} + \langle \sigma_a \rangle$, reactions, where $\langle \sigma_a \rangle$ is the average (over the realizations) cross section for the absorption of particles by individual scattering centers.[19]

The Green function $G(\mathbf{r}, \mathbf{r}')$ of Eq. (5) describes the propagation of a wave in the scattering medium from a point $\mathbf{r}'$ to a point $\mathbf{r}$. In some cases in addition to $G(\mathbf{r}, \mathbf{r}')$ we need to know the solution of the elastic scattering problem in the case of a unidirectional particle flux incident on a medium. The wave function $\Psi(\mathbf{r}, \mathbf{p}_0)$ for the problem of a plane wave with a momentum $\mathbf{p}_0$ incident from infinity

$$\psi(\mathbf{r}, \mathbf{p}_0)\,|_{\text{inc}} = \exp(i\mathbf{p}_0\mathbf{r})$$

is—in accordance with Eqs. (5) and (7)—the solution of the equation

$$\left(-\frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} + V(\mathbf{r})\right)\psi(\mathbf{r}, \mathbf{p}_0) = \frac{p_0^2}{2m}\psi(\mathbf{r}, \mathbf{p}_0). \tag{10}$$

We shall now turn to the relationships satisfied by bilinear combinations of the wave functions of a moving particle. The "kinetic" operator $K(\mathbf{r}, \mathbf{r}_1'; \mathbf{r}_2\, \mathbf{r}_2')$ (representing the Green function of the transport equation for the density matrix)[11] describes the evolution of the direct product of the wave functions in the incoherent scattering case and represents a sum of diagrams of the ladder type[3,7] (see Fig. 2) and satisfies the integral equation

$$K(\mathbf{r}_1, \mathbf{r}_1'; \mathbf{r}_2, \mathbf{r}_2') = G(\mathbf{r}_1, \mathbf{r}_1')\, G^*(\mathbf{r}_2, \mathbf{r}_2') + \sum_a \int d^3R\, d^3R'\, G(\mathbf{r}_1, \mathbf{R})\, G^*(\mathbf{r}_2, \mathbf{R}') \times \langle \delta u_a(\mathbf{R}-\mathbf{R}_a)\,\delta u_a(\mathbf{R}'-\mathbf{R}_a)\rangle K(\mathbf{R}, \mathbf{r}_1'; \mathbf{R}', \mathbf{r}_2'). \tag{11}$$

The action of the operator $\hat{K}$ of Eq. (11) on the particle source function $I(\mathbf{r}_1', \mathbf{r}_2')$ leads in Eq. (11) to the usual transport equation for the density matrix[19,20]

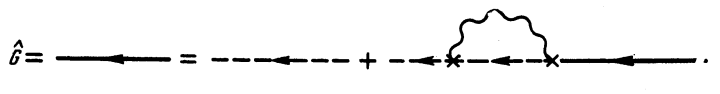

FIG. 1.

$$\int d^3r_1'\, d^3r_2'\, K(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')\, I(\mathbf{r}_1',\mathbf{r}_2')=\rho(\mathbf{r}_1,\mathbf{r}_2)=\rho_0(\mathbf{r}_1,\mathbf{r}_2)$$

$$+\sum_a \int d^3R\, d^3R'\, G(\mathbf{r}_1,\mathbf{R})\, G^*(\mathbf{r}_2,\mathbf{R}')$$

$$\times \langle \delta u_a(\mathbf{R}-\mathbf{R}_a)\, \delta u_a(\mathbf{R}'-\mathbf{R}_a)\rangle \rho(\mathbf{R};\mathbf{R}'). \qquad (12)$$

Allowance for the inequality (2) and for the relationship between the correlation function of the fluctuations characterized by the incoherent scattering cross section σ_{in} of Eq. (9) allows us to rewrite Eq. (11) in the form

$$K(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')=G(\mathbf{r}_1,\mathbf{r}_1')\, G^*(\mathbf{r}_2,\mathbf{r}_2')$$

$$+\frac{\pi\sigma_{in}}{m^2}\sum_a G(\mathbf{r}_1,\mathbf{R}_a)\, G^*(\mathbf{r}_2,\mathbf{R}_a)$$

$$\times K(\mathbf{R}_a,\mathbf{r}_1';\mathbf{R}_a,\mathbf{r}_2'). \qquad (13)$$

Since the diagram equation in Fig. 2 permits transposition of the wavy line from left to right relative to the operator $\hat{K}$, Eq. (13) is equivalent to

$$K(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')=G(\mathbf{r}_1,\mathbf{r}_1')\, G^*(\mathbf{r}_2,\mathbf{r}_2')$$

$$+\frac{\pi\sigma_{in}}{m^2}\sum_b K(\mathbf{r}_1,\mathbf{R}_b;\mathbf{r}_2,\mathbf{R}_b)$$

$$\times G(\mathbf{R}_b,\mathbf{r}_2)\, G^*(\mathbf{R}_b,\mathbf{r}_2'). \qquad (14)$$

A comparison of Eqs. (13) and (14) gives

$$K(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')=G(\mathbf{r}_1,\mathbf{r}_1')\, G(\mathbf{r}_2,\mathbf{r}_2')$$

$$+\frac{\pi\sigma_{in}}{m^2}\sum_a G(\mathbf{r}_1,\mathbf{R}_a)\, G^*(\mathbf{r}_2,\mathbf{R}_a)$$

$$\times G(\mathbf{R}_a,\mathbf{r}_1')\, G^*(\mathbf{R}_a,\mathbf{r}_2')$$

$$+\left(\frac{\pi\sigma_{in}}{m^2}\right)^2\sum_{a,b} G(\mathbf{r}_1,\mathbf{R}_a)\, G^*(\mathbf{r}_2,\mathbf{R}_a)\, k(\mathbf{R}_a,\mathbf{R}_b)$$

$$\times G(\mathbf{R}_b,\mathbf{r}_1')\, G^*(\mathbf{R}_b,\mathbf{r}_2'), \qquad (15)$$

where $k(\mathbf{R}_a,\mathbf{R}_b)=K(\mathbf{R}_a,\ \mathbf{R}_b;\ \mathbf{R}_a,\ \mathbf{R}_b)$.

Representation of the operator $\hat{K}$ in the form of the sum (15) has a simple meaning: the first, second, and third terms in Eq. (15) represent the operators of the elastic scattering, single coherent scattering, and total contribution of all the processes involving two or more incoherent collisions.

As pointed out already, the kinetic operator $K(\mathbf{r}_1,\ \mathbf{r}_1';\ \mathbf{r}_a,\mathbf{r}_2')$ describes the wave field created by a series of consecutive independent incoherent scattering events. The main interference correction to Eq. (15) represents the contribution of the fan diagrams for waves which cross the same scattering centers in opposite directions[3,7] (see Fig. 3).

A direct comparison of the diagrams shown in Figs. 2 and 3 demonstrates that the operator $C(\mathbf{r}_1,\ \mathbf{r}_1';\ \mathbf{r}_2,\ \mathbf{r}_2')$ can be expressed in terms of the "multiple" part of the kinetic operator $K(\mathbf{r}_1,\ \mathbf{r}_1';\ \mathbf{r}_2,\ \mathbf{r}_2')$ [third term in Eq. (15)], where the first pair of the arguments is transposed:

$$C(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2;\mathbf{r}_2')=K_{\geqslant 2}(\mathbf{r}_1',\ \mathbf{r}_1;\ \mathbf{r}_2,\ \mathbf{r}_2'). \qquad (16)$$

Comparing Eqs. (16) and (15), we find that

$$C(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')=\left(\frac{\pi\sigma_{in}}{m^2}\right)^2\sum_{a,b} G(\mathbf{r}_1';\mathbf{R}_a)\, G^*(\mathbf{r}_2,\mathbf{R}_a)\, k(\mathbf{R}_a,\mathbf{R}_b)$$

$$\times G(\mathbf{R}_b,\mathbf{r}_1)\, G^*(\mathbf{R}_b,\mathbf{r}_2'). \qquad (17)$$

The equation for the quantity $k(\mathbf{R}_a,\ \mathbf{R}_b)$ ocurring in Eqs. (15) and (17) can easily be obtained by selecting suitable values of the arguments in the relationships (13) and (14):

$$k(\mathbf{R}_a,\mathbf{R}_b)=|G(\mathbf{R}_a,\mathbf{R}_b)|^2+\frac{\pi\sigma_{in}}{m^2}\sum_c |G(\mathbf{R}_a,\mathbf{R}_c)|^2 k(\mathbf{R}_c,\mathbf{R}_b). \qquad (18)$$

Therefore, calculation of both the intensity operator and the main interference correction $\hat{C}$ (which allows for the weak localization effects) for periodic structures reduces to a solution of Eq. (18), which describes the distribution (over the sites of a crystal lattice) of the density of the particles emitted by a point source located at $\mathbf{R}_b$. It is shown below that in some cases an equation of type (18) can be solved in quadratures.

3. ENHANCEMENT OF THE BACKSCATTERING IN THE CASE OF WEAK LOCALIZATION OF WAVES IN A CRYSTAL

In quantum-mechanical calculation of any observable quantity it is sufficient to know the density matrix $\rho(\mathbf{r}_1,\mathbf{r}_2)$ of a particle.[21] For arbitrary values of $\mathbf{r}_1$ and $\mathbf{r}_2$ the matrix in question can be found by applying the sum of the operators $\hat{K}$ and $\hat{C}$ from Eqs. (15) and (17) to the particle source function $I(\mathbf{r}_1',\ \mathbf{r}_2')$. Since application of the product of the Green functions of Eq. (5) to $I(\mathbf{r}_1',\ \mathbf{r}_2')$ gives the density matrix for a coherent field, equal to a bilinear combination of wave functions of type (10)

$$\int d^3r_1'\, d^3r_2'\, G(\mathbf{r}_1,\mathbf{r}_1')\, G^*(\mathbf{r}_2,\mathbf{r}_2')\, I(\mathbf{r}_1',\mathbf{r}_2')$$

$$=\rho_0(\mathbf{r}_1,\mathbf{r}_2)=\psi(\mathbf{r}_1,\mathbf{p}_0)\,\psi^*(\mathbf{r}_2,\mathbf{p}_0), \qquad (19)$$

we find that the density matrix of a particle can be described by the following easily derived expression

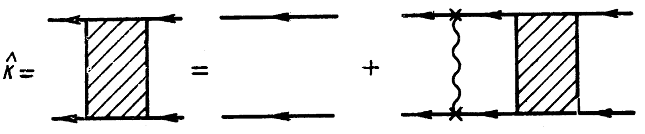

FIG. 2.

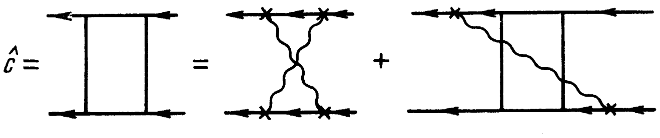

FIG. 3.

$$\rho(\mathbf{r}_1,\mathbf{r}_2)=\rho_0(\mathbf{r}_1,\mathbf{r}_2)+\frac{\pi\sigma_{in}}{m^2}\sum_a G(\mathbf{r}_1,\mathbf{R}_a)G^*(\mathbf{r}_2,\mathbf{R}_a)\rho_0(\mathbf{R}_a,\mathbf{R}_a)$$

$$+\left(\frac{\pi\sigma_{in}}{m^2}\right)^2\sum_{a,b}\{G(\mathbf{r}_1,\mathbf{R}_a)G^*(\mathbf{r}_2,\mathbf{R}_a)k(\mathbf{R}_a,\mathbf{R}_b)\rho_0(\mathbf{R}_b,\mathbf{R}_b)$$

$$+G(\mathbf{r}_1,\mathbf{R}_b)G^*(\mathbf{r}_2,\mathbf{R}_a)$$

$$\times k(\mathbf{R}_a,\mathbf{R}_b)\rho_0(\mathbf{R}_a,\mathbf{R}_b)\}. \tag{20}$$

The density of the flux $J(\vartheta_1,\varphi_1)$ of particles scattered along the direction $\mathbf{n}_1$ and normalized to the velocity $v=p_0/m$ is related to the Fourier component of the density matrix of Eq. (20) taken over the transverse coordinates (x,y) on the $z=-0$ surface of the medium[19,20]

$$J(\vartheta_1,\varphi_1)=(p_0\cos\vartheta_1/2\pi)^2(1/\Sigma)\rho(\mathbf{q},z=-0;\mathbf{q},z=-0), \tag{21}$$

where $q_x=p_0\sin\vartheta_1\cos\varphi_1$; $q_y=p_0\sin\vartheta_1\sin\varphi_1$; Σ is the surface area of the investigated crystal; $\vartheta_1>\pi/2$ is the angle between the direction of escape of a particle and the z axis. Equation (21) is derived on the assumption that the scattering medium occupies the half-space $z>0$. Substitution of Eq. (20) into Eq. (21) gives

$$J(\vartheta_1,\varphi_1)=\frac{n\sigma_{in}}{4\pi\Sigma}\Big\{\frac{1}{n}\sum_{a,b}|\psi(\mathbf{R}_a,\mathbf{p}_0)|^2|\psi(\mathbf{R}_a,-\mathbf{p}_1)|^2$$

$$+\frac{1}{n^2}\sum_{a,b}|\psi(\mathbf{R}_a,\mathbf{p}_0)|^2|\psi(\mathbf{R}_b,-\mathbf{p}_1)|^2Q(\mathbf{R}_a,\mathbf{R}_b)$$

$$+\frac{1}{n^2}\sum_{a,b}[\psi(\mathbf{R}_a,\mathbf{p}_0)\psi^*(\mathbf{R}_a,-\mathbf{p}_1)]$$

$$\times[\psi^*(\mathbf{R}_b,\mathbf{p}_0)\psi(\mathbf{R}_b,-\mathbf{p}_1)]Q(\mathbf{R}_a,\mathbf{R}_b)\Big\}, \tag{22}$$

where we introduced $Q(\mathbf{R}_a,\mathbf{R}_b)=(\pi n\sigma_{\rm in}/m^2)\times k(\mathbf{R}_a,\mathbf{R}_b)$ and used a consequence of the reciprocity theorem[22]

$$\int dx\,dy\,G(x,y,z=-0,\mathbf{R})\exp(-iq_xx-iq_yy)$$

$$=-\frac{i}{v|\cos\vartheta_1|}\psi(\mathbf{R},-\mathbf{p}_1), \tag{23}$$

where $(\mathbf{p}_1)_{x,y}=(\mathbf{q})_{x,y}$; and $(\mathbf{p}_1)_z=p_0\cos\vartheta_1<0$.

The first two terms of Eq. (22) represent the kinetic part of the scattered intensity, whereas the third term is the contribution of the weak localization effect. Clearly, if $\mathbf{p}_1=-\mathbf{p}_0$ (i.e., in the forward direction), the second and third terms of Eq. (22) are identical. This result generalizes the conclusion, proved using a model of a disordered medium in Ref. 3, that the multiple part of the incoherent intensity is doubled in the backward direction in the case when the scatterers have a periodic distribution.

On the other hand, the results obtained using the two models discussed above are very different in other respects. This difference is revealed most easily if we compare the expression for the current density in Eq. (22) with that found using a model of a disordered medium (see, for example, Refs. 9 and 23):

$$J_r(\vartheta_1,\varphi_1)=\frac{n\sigma_{el}}{4\pi\Sigma}\Big\{\int_\Omega d^3R|\psi(\mathbf{R},\mathbf{p}_0)|^2\psi(\mathbf{R},-\mathbf{p}_1)|^2$$

$$+\int_\Omega d^3R\,d^3R'|\psi(\mathbf{R},\mathbf{p}_0)|^2$$

$$\times|\psi(\mathbf{R}',-\mathbf{p}_1)|^2Q(\mathbf{R},\mathbf{R}')+\int_\Omega d^3R\,d^3R'[\psi(\mathbf{R},\mathbf{p}_0)\psi^*(\mathbf{R},-\mathbf{p}_1)]$$

$$\times[\psi^*(\mathbf{R}',\mathbf{p}_0)\psi(\mathbf{R}')-\mathbf{p}_1)]Q(\mathbf{R},\mathbf{R}')\Big\}. \tag{24}$$

where σ_{el} is the elastic scattering cross section for a single scattering center and $Q(\mathbf{R},\mathbf{R}')$ is the solution of the equation

$$Q(\mathbf{R},\mathbf{R}')=\frac{\pi n\sigma_{el}}{m^2}|G(\mathbf{R},\mathbf{R}')|^2$$

$$+\frac{\pi n\sigma_{el}}{m^2}\int d^3R''|G(\mathbf{R},\mathbf{R}'')|^2Q(\mathbf{R}'',\mathbf{R}'). \tag{25}$$

It is clear from Eq. (24) that the condition ensuring equality of the second and third terms in the above expression is the identity of the wave functions $\psi(\mathbf{R},\mathbf{p}_0)$ and $\psi(\mathbf{R},-\mathbf{p}_1)$ at each point $\mathbf{R}$ within the volume Ω occupied by the investigated medium. Such a stringent condition, supplemented by the energy conservation law $p_1^2=p_0^2$, is obviously satisfied at a single point in the momentum space $\mathbf{p}_1=-\mathbf{p}_0$ (for the strictly backward direction). A periodic system of fluctuating potential centers can give rise to an additional incoherent scattering intensity peak when a much less stringent requirement is satisfied: the wave functions of the scattering problem (10) corresponding to the initial momenta $\mathbf{p}_0$ and $-\mathbf{p}_1$ must be equal in a denumerable set of the crystal lattice sites $\{\mathbf{R}_a\}$, i.e.,

$$\psi(\mathbf{R}_a,\mathbf{p}_0)=\psi(\mathbf{R}_a,-\mathbf{p}_1),\quad \forall\mathbf{R}_a. \tag{26}$$

Clearly, Eq. (26) can be satisfied by several functions differing from one another by multiplication by a factor which is periodic over the lattice and is equal to unity for the set $\{\mathbf{R}_a\}$. As shown below, it is this situation that occurs in the symmetric Laue diffraction geometry when a distribution of the incoherently scattered intensity includes, in addition to a peak of weak backward localization, an additional maximum in a direction which is a mirror reflection of the former in the crystal lattice planes.

4. ANGULAR DISTRIBUTION OF THE INTENSITY UNDER NONCOHERENT SCATTERING CONDITIONS

It follows from Eq. (22) that calculation of the angular distribution of incoherently scattered particles reduces to calculation of a sum (over the lattice sites) of products of the wave functions in the diffraction problem using the periodic potential (10) weighted by $Q(\mathbf{R}_a,\mathbf{R}_b)$.

Selection of the solution method of the diffraction problem (10) is known to depend on the parameter $\mathbf{G}_1^2/2m|\Lambda|$ (Refs. 24 and 25), where $\mathbf{G}_1$ is the first reciprocal lattice vector. In the "weak coupling" limit characterized by $\mathbf{G}_1^2/2m|\Lambda|\gg1$, we can solve Eq. (10) using a two-wave approximation of the dynamic theory of diffraction. Using the inequality (2) and the validity of the Born approximation[26]

$$|u(0)| \ll 1/mr_0^2, \qquad (27)$$

we can readily obtain an estimate

$$\frac{G_1^2}{2m|\Lambda|} \sim \frac{|u_0|^{-1}}{mr_0^2}\frac{1}{G_1 r_0} \gg 1, \qquad (27a)$$

which demonstrates the feasibility of replacing the exact expression for the potential $V(\mathbf{r})$ in Eq. (10) by

$$V(\mathbf{r}) = \theta(z) V(1+2\cos(\mathbf{Gr})), \qquad (28)$$

where $\mathbf{G}$ is the reciprocal lattice vector $V = \Lambda - 1/2i\Gamma$, closest to satisfying the Bragg condition $\mathbf{p}_0^2 = (\mathbf{p}_0 + \mathbf{G})^2$, whereas the quantities Λ and Γ are described by the two expressions in the system (9). Depending on the orientation of the particle momentum $\mathbf{p}_0$ and the diffraction reflection vector $\mathbf{G}$ relative to the $z = 0$ surface of the investigated crystal, we can distinguish two diffraction geometries: the Laue case for $(\mathbf{p}_0 + \mathbf{G})_z > 0$ and the Bragg case for $(\mathbf{p}_0 + \mathbf{G})_z < 0$. We shall find the solution of Eq. (10) in the Laue case, when the vector $\mathbf{G}$ is parallel to the crystal surface, i.e., $(\mathbf{G})_z = 0$, as well as in the Bragg case, when the vector $\mathbf{G}$ is antiparallel to the z axis, i.e., $(\mathbf{G})_z = -G$. In both these cases the wave function of the problem described by Eq. (10) is

$$\psi(\mathbf{r}, \mathbf{p}_0) = \alpha(z, \mathbf{p}_0)\exp(i\mathbf{p}_0\mathbf{r}) + \beta(z, \mathbf{p}_0)\exp[i(\mathbf{p}_0+\mathbf{G})\mathbf{r}], \qquad (29)$$

where the characteristic spatial scale of variation of the quantities $\alpha(z, \mathbf{p}_0)$ and $\beta(z, \mathbf{p}_0)$ is many times larger than the lattice constant $2\pi/G_1$.

Substituting Eq. (29) into Eq. (10), we find that in the Laue geometry we can obtain

$$\psi(\mathbf{R}_a, \mathbf{p}_0) = \exp(i\mathbf{p}_0\mathbf{R}_a)\frac{1}{2(4V^2+\varepsilon_0^2)^{1/2}}\Big\{[2V-\varepsilon_0+(4V^2+\varepsilon_0^2)^{1/2}]$$
$$\times\exp\left[-\frac{i}{2v\cos\vartheta_0}(\varepsilon_0+2V+(4V^2+\varepsilon_0^2)^{1/2})z_a\right]$$
$$+[-2V+\varepsilon_0+(4V^2+\varepsilon_0^2)^{1/2}]$$
$$\times\exp\left[-\frac{i}{2v\cos\vartheta_0}(\varepsilon_0+2V-(4V^2+\varepsilon_0^2)^{1/2})z_a\right]\Big\}. \qquad (30)$$

In the Bragg case the results of these calculations give

$$\psi(\mathbf{R}_a; \mathbf{p}_0) = \exp(i\mathbf{p}_0\mathbf{R}_a)\left\{\frac{\varepsilon_0}{2V}\left[\left(1+\frac{4V}{\varepsilon_0}\right)^{1/2}-1\right]\right\}$$
$$\times\exp\left\{\frac{i\varepsilon_0 z_a}{2v\cos\vartheta_0}\left(1-\left(1+\frac{4V}{\varepsilon_0}\right)^{1/2}\right)\right\}. \qquad (31)$$

In Eqs. (30) and (31) we shall allow for the equality $\exp(i\mathbf{GR}_a) = 1$ and use the notation $\varepsilon_0 = [(\mathbf{p}_0 + \mathbf{G})^2 - \mathbf{p}_0^2]/2m$. A distinguishing feature of Eqs. (30) and (31) is an exponential dependence of the wave functions $\psi(\mathbf{R}_a, \mathbf{p})$ on the depth of penetration of a particle z_a into the investigated crystal. Consequently, a calculation of the current density of Eq. (22) can be reduced, without any loss of generality, to calculation of the sum

$$\mathscr{L}(s, s'|\mathbf{f}) = \frac{1}{n^2}\sum_{a,b}\exp[i\mathbf{f}(\boldsymbol{\rho}_a-\boldsymbol{\rho}_b) - sz_a - s'z_b]\,Q(\mathbf{R}_a, \mathbf{R}_b), \qquad (32)$$

where $Q(\mathbf{R}_a, \mathbf{R}_b)$ satisfies an inhomogeneous equation [compare with Eq. (18)]

$$Q(\mathbf{R}_a, \mathbf{R}_b) = \frac{\pi n\sigma_{in}}{m^2}|G(\mathbf{R}_a, \mathbf{R}_b)|^2 + \frac{\pi\sigma_{in}}{m^2}\sum_c |G(\mathbf{R}_a, \mathbf{R}_c)|^2 Q(\mathbf{R}_c, \mathbf{R}_b). \qquad (33)$$

The quantity $Q(\mathbf{R}_a, \mathbf{R}_b)$ in Eq. (33) is proportional to the probability of detection at a point $\mathbf{R}_a$ of a particle emitted by an atom located at $\mathbf{R}_b$ subject to allowance for all possible scattering processes in the bulk of the medium. The condition of smallness of the angular width of the Bragg diffraction regions $\Delta\vartheta_D \sim m|\Lambda|/G_1 p_0 \ll 1$ means that in the majority of the paths of propagation of a wave from $\mathbf{R}_a$ to $\mathbf{R}_b$ we can ignore the influence of the Fourier components of the potential (7) with $\mathbf{G} \neq 0$. An allowance for this circumstance makes it possible to describe the square of the modulus of the Green function $|G(\mathbf{R}_a, \mathbf{R}_b)|^2$ in Eq. (33) by

$$|G(\mathbf{R}_a, \mathbf{R}_b)|^2 = \left(\frac{m}{2\pi}\right)^2\frac{1}{|\mathbf{R}_a-\mathbf{R}_b|^2}\exp(-n\sigma_r|\mathbf{R}_a-\mathbf{R}_b|). \qquad (34)$$

Substituting the above expression in Eqs. (32) and (33) subject to the conditions $|\mathbf{f}|, |s|, |s'| \ll G_1$ and replacing summation over the lattice sites with integration, we obtain

$$\mathscr{L}(s, s'|\mathbf{f}) = \Sigma\iint_0^\infty dz\,dz'\exp(-sz-s'z')\,\Pi(z, z'|\mathbf{f}), \qquad (35)$$

where the function

$$\Pi(z, z'|\mathbf{f}) = \int d^2\rho\, Q(\boldsymbol{\rho}; z; 0; z')\exp(i\mathbf{f}\boldsymbol{\rho})$$

satisfies the integral relationship

$$\Pi(z, z'|\mathbf{f}) = A_f(|z-z'|) + \int_0^\infty dz'' A_f(|z-z''|)\,\Pi(z'', z'|\mathbf{f}). \qquad (36)$$

The kernel of Eq. (36) can be represented by an integral

$$A_f(|z|) = \frac{n\sigma_{in}}{4\pi}\int d^2\rho\frac{1}{z^2+\rho^2}\exp[-n\sigma_r(z^2+\rho^2)^{1/2} + i\mathbf{f}\boldsymbol{\rho}]$$
$$= \frac{n\sigma_{in}}{2\pi}\int_{-\infty}^{+\infty} dk\exp(ikz)\frac{1}{(k^2+\mathbf{f}^2)^{1/2}}\operatorname{arctg}\left[\frac{(k^2+\mathbf{f}^2)^{1/2}}{n\sigma_r}\right], \qquad (37)$$

which has the following exponential asymptotic form in the case when $n\sigma_r|z| \gg 1$:

$$A_f(|z|) \sim \frac{\sigma_{in}}{2\sigma_r}\frac{1}{|z|}\exp\left(-n\sigma_r|z| - \frac{|z|\mathbf{f}^2}{n\sigma_r}\right). \qquad (38)$$

We can calculate the sum $\mathscr{L}(s, s'|\mathbf{f})$ in Eq. (32) by noting that the quantity $\Pi(z, z'|\mathbf{f})$ satisfies a differential relationship[27]

$$(\partial/\partial z + \partial/\partial z')\,\Pi(z, z'|\mathbf{f}) = \Pi(z, 0|\mathbf{f})\,\Pi(0, z'|\mathbf{f}), \qquad (39)$$

which can be used to go over from Eq. (36) to an equation containing a function of one variable:

$$\Pi(z,0|\mathbf{f})\equiv\Phi(z|\mathbf{f})=A_t(|z|)+\int_0^\infty dz''A_t(|z-z''|)\Phi(z''|\mathbf{f}). \quad (40)$$

We can solve Eq. (40) by the Wiener–Hopf method. After the usual transformations describing detail in Ref. 28, we find that the Laplace transform $\Phi(z|\mathbf{f})$ is described by

$$\int_0^\infty dze^{-sz}\Phi(z|\mathbf{f})=H\left(\frac{n\sigma_r}{s},\frac{\sigma_{in}}{\sigma_r}\Big|\frac{f}{n\sigma_r}\right)-1, \quad (41)$$

where the function $H(\mu,\omega|\beta)$ can be described by an integral:

$$H(\mu,\omega|\beta)=\exp\left\{-\frac{1}{\pi}\int_0^\infty d\xi\frac{\mu}{1+\mu^2\xi^2}\right.$$
$$\left.\times\ln\left[1-\frac{\omega}{(\xi^2+\beta^2)^{1/2}}\operatorname{arctg}(\xi^2+\beta^2)^{1/2}\right]\right\}, \quad (42)$$

where $\operatorname{Re}\mu>0$, $|\omega|\leqslant 1$. If $\beta=0$, the quantity $H(\mu,\omega|\beta)$ is identical with the usual Chandrasekhar function $H(\mu,\omega)=H(\mu,\omega|0)$ (Refs. 29 and 30). Substitution of Eq. (41) into Eqs. (39) and (35) allows us to write down the result of summation of Eq. (32) in the form

$$\mathscr{L}(s,s'|\mathbf{f})=\Sigma\left[H\left(\frac{n\sigma_r}{s},\frac{\sigma_{in}}{\sigma_r}\Big|\frac{f}{n\sigma_r}\right)H\left(\frac{n\sigma_r}{s'},\frac{\sigma_{in}}{\sigma_r}\Big|\frac{f}{n\sigma_r}\right)-1\right]\Big/(s+s'). \quad (43)$$

It is interesting to note that Eq. (43) is in fact the exact solution of the problem of the angular distribution of particles reflected by a semi-infinite random medium containing isotropically scattering centers.[31] In fact, if we substitute Eq. (43) into Eq. (24) and use the explicit form of the wave functions describing the problem of the scattering in a disordered substance

$$\psi(\mathbf{R},\mathbf{p})\approx\exp(i\mathbf{pR}-n\sigma_t z/2|\mu|),$$

where $\mu=\cos\vartheta$, $|\mu|\leqslant 1$, we readily find that[31]

$$J_r(\vartheta_1,\varphi_1)=\frac{\sigma_{el}}{4\pi\sigma_t}\frac{|\mu_1|\mu_0}{|\mu_1|+\mu_0}\left[H\left(|\mu_1|,\frac{\sigma_{el}}{\sigma_t}\Big|0\right)H\left(\mu_0,\frac{\sigma_{el}}{\sigma_t}\Big|0\right)+\left|H\left(\frac{n\sigma_t}{i(\varkappa_1^*-\varkappa_0)},\frac{\sigma_{el}}{\sigma_t}\Big|\frac{f}{n\sigma_t}\right)\right|^2-1\right], \quad (44)$$

where $\varkappa_j=p_0|\mu_j|+in\sigma_t/2|\mu_j|$, σ_t is the total single scattering cross section, and the vector $\mathbf{f}$ is a sum of two-dimensional projections of the momenta $\mathbf{p}_0$ and $\mathbf{p}_1$ onto the (x,y) plane: $\mathbf{f}=(\mathbf{p}_0+\mathbf{p}_1)_{x,y}$. Equation (44) has a clear maximum at $|\mu_1|=\mu_0$ and it falls rapidly to the usual background of incoherently scattered radiation[27,29]

$$J_r(\vartheta_1,\varphi_1)=\frac{\sigma_{el}}{4\pi\sigma_t}\frac{|\mu_1|\mu_0}{|\mu_1|+\mu_0}H\left(|\mu_1|,\frac{\sigma_{el}}{\sigma_t}\Big|0\right)H\left(\mu_0,\frac{\sigma_{el}}{\sigma_t}\Big|0\right) \quad (45)$$

if $||\mu_1|-\mu_0|\gg n\sigma_t/p_0=\lambda/2\pi l_{\mathrm{mfp}}$.

In the case of diffraction of incident and scattered particles in the Bragg geometry, the substitution of the wave function of the (31) type into Eq. (22) and allowance for the results of summation in Eq. (43) gives

$$J(\vartheta_1,\varphi_1)=\frac{\sigma_{in}}{4\pi\sigma_r}\left|\frac{\varepsilon_0}{2V}\left[\left(1+\frac{4V}{\varepsilon_0}\right)^{1/2}-1\right]\right|^2\times\left|\frac{\varepsilon_1}{2V}\left[\left(1+\frac{4V}{\varepsilon_1}\right)^{1/2}-1\right]\right|^2$$
$$\times\frac{\cos\vartheta_0|\cos\vartheta_1|\operatorname{Re}(1+4V/\varepsilon_0)^{1/2}\operatorname{Re}(1+4V/\varepsilon_1)^{1/2}}{\cos\vartheta_0\operatorname{Re}(1+4V/\varepsilon_0)^{1/2}+|\cos\vartheta_1|\operatorname{Re}(1+4V/\varepsilon_1)^{1/2}}$$
$$\times\left[H\left(\cos\vartheta_0\operatorname{Re}\left(1+\frac{4V}{\varepsilon_0}\right)^{1/2};\frac{\sigma_{in}}{\sigma_r}\Big|0\right)\times H\left(|\cos\vartheta_1|\operatorname{Re}\left(1+\frac{4V}{\varepsilon_1}\right)^{1/2};\frac{\sigma_{in}}{\sigma_r}\Big|0\right)\right.$$
$$\left.+\left\{\left|H\left(\frac{n\sigma_r}{i(\varkappa_1^*-\varkappa_0)};\frac{\sigma_{in}}{\sigma_r}\Big|\frac{f}{n\sigma_r}\right)\right|^2-1\right\}\right]$$
$$=J_k(\vartheta_1,\varphi_1)+J_c(\vartheta_1,\varphi_1), \quad (46)$$

where

$$\varkappa_j=|(p_j)_z|+\frac{\varepsilon_j}{2v|\cos\vartheta_j|}[1-(1+4V/\varepsilon_j)^{1/2}].$$

The quantity ε_1 used in Eq. (46) is identical with ε_0 from Eq. (30) apart from the substitution $\mathbf{p}_0\to-\mathbf{p}_1$.

An expression valid in the Laue geometry case is obtained for the current density by substituting the wave function (30) into Eq. (22), but is very cumbersome. Therefore, we shall simply analyze the case of the Laue diffraction only near the exact Bragg condition $|\varepsilon_j|\ll 2V$.

5. DISCUSSION OF RESULTS

An analysis of the characteristic features of the angular distribution of particles scattered incoherently by periodic structures is best started from the case of the Bragg diffraction described by Eq. (46). It follows from the general expression (22) that the first term in the square brackets of Eq. (46) represents the "kinetic" part of the scattered intensity, whereas the second represents the effect of weak localization of particles in the fluctuating potential of Eq. (4). The profile of the weak localization peak in the potential scattering case $|\Lambda|\gg\Gamma$ considered here is characterized by two angular scales. The first scale is of the order of the angular width of the region of diffraction by reflection (see, for example, Ref. 25)

$$\Delta\vartheta_D\sim m|\Lambda|/Gp_0\ll 1. \quad (47)$$

The second scale, governed by the condition $\beta=f/n\sigma_r\lesssim 1$, is obtained from an estimate of the angular width of the region of constructive interference between the waves that have traveled the same paths in the medium in the forward and reverse directions[3,7,9-12]

$$\Delta\vartheta_{int}\sim\Gamma/vp_0\sim\lambda/l_{\mathrm{mfp}}. \quad (48)$$

If the condition (27) is satisfied, then $\Delta\vartheta_D$ and $\Delta\vartheta_{\rm int}$ are related by

$$\Delta\vartheta_{int}\ll\Delta\vartheta_D. \tag{49}$$

The inequality (49) simplifies greatly an analysis of the angular distribution of the scattered particles. In fact, this condition allows us to neglect the change in the first argument of the Chandrasekhar function (46) within the angular width of the weak localization peak and to write down

$$\frac{J_c(\vartheta_1,\varphi_1)}{J_k(\vartheta_1,\varphi_1)}=\frac{|H(\cos\vartheta_0\,\mathrm{Re}(1+4V/\varepsilon_0)^{1/2};\,(\sigma_{in}/\sigma_r)\,|\,|(\mathbf{p}_0+\mathbf{p}_1)_{x,y}/n\sigma_r|)\,|^2-1}{H^2(\cos\vartheta_0\,\mathrm{Re}(1+4V/\varepsilon_0)^{1/2};\,(\sigma_{in}/\sigma_r)\,|\,0)}. \tag{50}$$

The expression for the "wings" of the angular spectrum of Eq. (50) is readily obtained by expanding Eq. (42) as a series in terms of a small parameter $\beta^{-1}\ll\min\{|\mu|,1\}$:

$$\frac{J_c(\vartheta_1;\varphi_1)}{J_k(\vartheta_1;\varphi_1)}\approx\frac{\pi}{2}n\sigma_{in}|(\mathbf{p}_0+\mathbf{p}_1)_{x,y}|^{-1}H^{-2}$$
$$\times\left(\cos\vartheta_0\,\mathrm{Re}\left(1+\frac{4V}{\varepsilon_0}\right)^{1/2};\left(\frac{\sigma_{in}}{\sigma_r}\right)\bigg|\,0\right). \tag{51}$$

It should be noted that the relationship (51) describes a quantity which is always positive and which tends to zero when the direction of observation n_1 tilts away from the backward direction.

Interesting behavior as a function of the particle diffraction conditions is exhibited by the ratio of the maximum of the weak localization peak in the backward direction to the background in the same direction:

$$\frac{J_c(-\mathbf{p}_0)}{J_k(-\mathbf{p}_0)}=1-H^{-2}\left(\cos\vartheta_0\,\mathrm{Re}\left(1+\frac{4V}{\varepsilon_0}\right)^{1/2};\frac{\sigma_{in}}{\sigma_r}\bigg|\,0\right). \tag{52}$$

The distribution calculated using Eq. (52) is shown in Fig. 4. A special feature of the function in this figure is a sharp maximum near the edge of the Bragg region of "total" diffraction by reflection. The nature of this maximum is related directly to the physical nature of the weak localization effect. In accordance with the general expression (16), the peak in the incoherent intensity distribution in the backward direction is due to double, triple, or higher orders of the scattering of a particle by fluctuating potential centers. Hence, we can expect a relative enhancement of the weak localization effect in all those cases when there is a relative suppression of the processes of single incoherent collisions compared with multiple collisions. We can easily see that this is exactly the situation when the momenta of the incident and backscattered particles lie close to the left-hand edge of the Bragg "total" reflection region (Fig. 4). Under these conditions the structure of the Bloch wave field, which appears due to diffraction by the periodic potential (7) and (8), is characterized by minima of the density of the particles at the lattice sites (see, for example, Ref. 32). Suppression of the incoherent scattering channels, however, has less effect on the collision processes of high scattering orders. In fact, when the inequality (47) is satisfied, a large proportion of the intermediate paths of the waves from $\mathbf{R}_a$ to $\mathbf{R}_b$ lies far from the Bragg directions and the frequency of incoherent collisions in these intermediate states is higher in the incident or backscattered waves. The relative enhancement of the contribution of the processes of multiple incoherent collisions then gives rise to a maximum shown in Fig. 4.

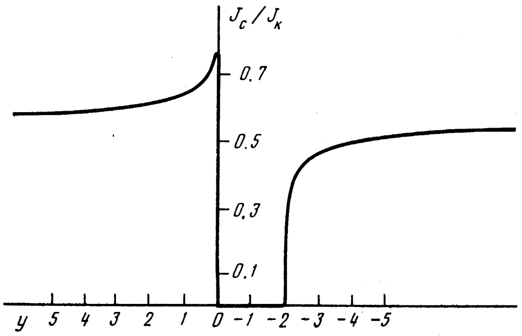

FIG. 4. Ratio of the current density $J_c(\vartheta_1;\varphi_1)$ to the kinetic part of the incoherent scattering intensity $J_k(\vartheta_1;\varphi_1)$ along the backward direction [Eq. (52)] plotted as a function of the parameters of the deviation of the particle momentum $\mathbf{p}_0$ from the exact Bragg condition $y=\varepsilon_0/2|\Lambda|$. The ratio $\Gamma/2|\Lambda|$ is equal to 0.01; $\cos\vartheta_0=0.7$; $\sigma_{\rm in}/\sigma_r=0.8$.

We shall now consider the weak localization effect in the Laue diffraction geometry [Eq. (30)]. The most interesting is the symmetric diffraction case characterized by $|\varepsilon_0/2\Lambda|\ll1$, when the angular distribution of the incoherent intensity do not have resonant singularities associated with rocking oscillations of the square of the modulus of the wave function $|\psi(\mathbf{r},\mathbf{p}_0)|^2$ at the lattice sites.[19] In the symmetric diffraction case the function $\psi(\mathbf{r},\mathbf{p}_0)$ is [see Eq. (30)]

$$\psi(\mathbf{R}_a,\mathbf{p}_0)=\exp\{ip_0\mathbf{R}_a-2iVz_a/v\cos\vartheta_0\}. \tag{53}$$

Substitution of Eq. (53) into Eqs. (22) and (43) gives

$$J_c(\vartheta_1,\varphi_1)$$
$$=\frac{\sigma_{in}}{16\pi\sigma_r}\cos\vartheta_0\left\{H^2\left(\frac{\cos\vartheta_0}{2},\frac{\sigma_{in}}{\sigma_r}\bigg|\frac{|(\mathbf{p}_0+\mathbf{p}_1)_{x,y}|}{n\sigma_r}\right)-1\right\}. \tag{54}$$

A special feature of the weak localization of waves in the symmetric two-wave Laue diffraction geometry is enhancement of the incoherent scattering not only in the backward direction, but also along a second direction $\mathbf{n}_1$ which is related to the backward direction $\mathbf{n}_0$ by a specular (mirror) reflection transformation relative to the system of the atomic planes of the crystal. The nature of this effect is easily understood on the basis of Eq. (30), by comparing the solutions of the wave diffraction problems [Eq. (10)] corresponding to the initial momenta $\mathbf{p}_0$ and $\mathbf{p}'=\mathbf{p}_0+\mathbf{G}$. If $\varepsilon_0=0$, the wave function $\psi(\mathbf{r},\mathbf{p}')$ at the lattice sites is

$$\psi(\mathbf{R}_a,\mathbf{p}')=\exp(ip'\mathbf{R}_a-2iVz_a/v\cos\vartheta_0). \tag{55}$$

Since the difference $\mathbf{p}' - \mathbf{p}_0 = \mathbf{G}$ contributes to the wave function phase of Eq. (55), the contribution is a multiple of 2π, Eqs. (53) and (55) become identical. In full agreement with the criterion of Eq. (26), the result of such identity is the appearance of a second peak in the distribution of the incoherently scattered intensity:

$$J_c^{(2)}(\vartheta_1, \varphi_1) = \frac{\sigma_{in}}{16\pi\sigma_r} \times \cos\vartheta_0 \left\{ H^2\left(\frac{\cos\vartheta_0}{2}; \frac{\sigma_{in}}{\sigma_r} \middle| \frac{|(\mathbf{p}_0+\mathbf{G}+\mathbf{p}_1)_{x,y}|}{n\sigma_r}\right) - 1 \right\} \tag{56}$$

with a maximum along the $\mathbf{p}_1 = -(\mathbf{p}_0 + \mathbf{G})$ direction. The physical origin of the additional intensity peak of Eq. (56) is the return of the wave along a "degenerate" path which is a mirror reflection of the "backward" path relative to the system of the crystallographic planes. An example of such a degenerate path is shown in Fig. 5. It should be noted that a transition from the two-wave to the multiwave Laue diffraction case may reduce the number of degenerate paths, so that the number of additional peaks in the angular distribution of the backscattered radiation may increase.

When the orientation of the momentum $\mathbf{p}_0$ differs from the symmetric diffraction position $\mathbf{p}_0^2 = (\mathbf{p}_0 + \mathbf{G})^2$, the condition $\mathbf{p}_1 = -(\mathbf{p}_0 + \mathbf{G})$ is incompatible with the energy conservation law $\mathbf{p}_1^2 = \mathbf{p}_0^2$. The intensity of the second weak localization peak of Eq. (56) then falls proportionally to the square of the parameter representing the detuning from the Bragg diffraction resonance $J_c^{(2)} \sim (2|\Lambda|/\varepsilon_0)^2$ and only one maximum in the backward direction remains in the distribution of the incoherently scattered particles. Clearly, this case of large values of the diffraction offset parameter $|\varepsilon_0/2\Lambda| \gg 1$ describes the results of experimental observations of the weak localization of light in liquid crystals reported in Ref. 13. In the case of highly symmetric orientations of a crystal we can expect either a relative enhancement of the weak localization peak in the backward direction within the Bragg geometry [Eq. (52)] or the appearance of an additional incoherent intensity maximum in the Laue geometry [Eqs. (54) and (56)]. These features of the weak localization of waves in periodic structures can be observed by an investigation of the backscattering of low- and moderate-energy electrons by crystalline targets under conditions of strong thermal motion of the atoms in the medium[33] or spin and isotropic incoherent scattering of neutrons.[34]

An allowance for the periodicity of the distribution of atoms in a solid can also modify the interference correction to the magnetoresistance, compared with that discussed in Refs. 5 and 6 for the case of a random distribution of the scattering centers.

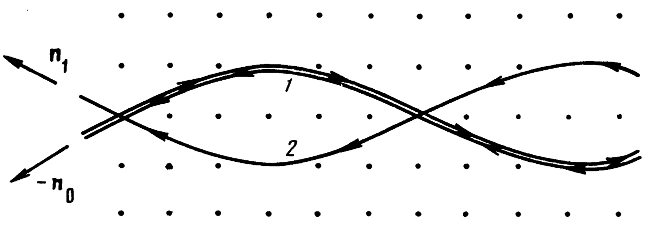

FIG. 5. 1) Forward and backward paths; 2) "degenerate" path. Interference enhancement of the incoherent scattering intensity occurs along the direction $\mathbf{n}_1$ and in the direction $-\mathbf{n}_0$.

CONCLUSIONS

The above solution of the problem of the weak localization of waves[1–16] was obtained using a periodic model of fluctuating potential centers. It is shown that the criterion of the weak localization effect in such a model of disorder [Eq. (26)] differs from the usual condition for enhancement of the incoherent scattering intensity in the backward direction in the case of a randomly inhomogeneous medium.[3,7,9–12] The angular distribution of the backscattering was calculated without assuming a diffuse propagation of waves in the bulk of a sample. It was found that under the symmetric Laue diffraction conditions we can expect additional peaks in the distribution of incoherently scattered particles. In the case of the Bragg geometry a relative enhancement of the intensity maximum in the backward direction was observed.

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