Magnetohydrodynamic stability of force-free magnetic fields in a low-density plasma

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The stability of force-free magnetic fields in a low-density plasma is considered in connection with the problem of magnetic heating of the solar corona. The energy principle is used to demonstrate the important role played in this problem by the line-tying effect. The threshold conditions for the onset of tearing instability for a planar sheared force-free magnetic field are investigated in detail. It is shown that such a field can remain stable at an arbitrarily large shear localized near a perfectly conducting plasma boundary.

1. INTRODUCTION

Force-free magnetic configurations in which electric currents flow along the magnetic field lines (so that there are no volume forces) play an important role in many astrophysical problems, such as plasma heating in the solar corona. It is by now universally accepted that the corona is heated by the internal magnetic field.[1] The heat source is the kinetic energy of the photospheric motions that perturb the coronal magnetic field, in which they generate currents. The manner of heating is the following. The magnetic field lines in the solar corona begin and end on the surface of the photosphere (Fig. 1). This surface can be regarded approximately as an abrupt boundary between the low-density plasma of the solar atmosphere, where the magnetic field predominates (here $\beta \equiv 8\pi p/B^2 \ll 1$, p is the plasma pressure, and B is the magnetic induction) and the dense subphotosphere medium whose motion governs the displacements of the bases of the corona field lines on the photosphere surface.

In active regions with relatively strong magnetic fields, the photospheric perturbations are quasistatic. Indeed, at the typical values $B \sim 10^2$ G, plasma density $n \sim 10^9$ cm^{-3}, and magnetic-structure size $L \sim 10^9$ cm the perturbation propagation time $\tau_A \sim L/v_A \sim 1$ s [where $v_A = B/(4\pi n m_i)^{1/2} \sim 10^9$ cm/s is the Alfvén velocity] is much shorter than the characteristic period $\tau_v \sim l/v \sim 10^3$ s of the photospheric motions ($l \sim 10^8$ cm is the dimension of the photosphere convective cells and $v \sim 10^5$ cm/s is the pulsation velocity). The photospheric perturbations therefore cause the coronal magnetic configuration to go through a sequence of equilibrium states that are force-free if $\beta \ll 1$. The excess magnetic energy due to the longitudinal currents produced in the plasma is quite sufficient to account for the various manifestations of solar activity. Under solar-corona conditions, however, when the hot plasma has high conductivity, the usual Joule dissipation of the magnetic energy is extremely ineffective. The problem is thus to determine the mechanisms that produce the necessary rate of energy release.

A crucial role can be played here by magnetic reconnection. Originating in narrow current layers (local reconnection) due to the small but finite resistance of the plasma, it makes possible a rapid (compared with the global damping time of the currents) transition of the system into a lower-magnetic-energy state which is topologically forbidden in an ideally conducting medium by the magnetic-field line-tying. A distinction can be made here between two approaches. In the first, developed in Refs. 2 and 3, a force-free equilibrium is impossible in a sufficiently deformed corona magnetic field. Discontinuities (pinch sheets) are therefore produced in the magnetic field, and it is the magnetic reconnection in them which leads to effective dissipation of the excess magnetic energy. In the second approach resistive instabilities develop in the plasma.[4] The plasma is then heated in the corona by continuous multiple magnetic-reconnection processes.[5–7] These two approaches have been dubbed respectively forced and spontaneous reconnection of the magnetic field.

It follows that development of a consistent theory of magnetic solar-corona heating is impossible without a detailed investigation of the equilibria and stability of the force-free magnetic fields. The latter are described by the equations

$$\mathrm{rot}\,\mathbf{B} = \alpha \mathbf{B}, \quad \mathbf{B}\nabla\alpha = 0, \tag{1}$$

so that the function $\alpha(\mathbf{r})$ is constant along the magnetic field lines. In this context, Eqs. (1) must be solved assuming that the locations of the field line bases and the normal component of the magnetic field on the photosphere are specified. In the general case this is a complicated nonlinear problem. An analytic solution can usually be obtained only either when $\alpha = \text{const}$ and the equations become linear (linear force-free field), or if the magnetic configuration is independent of one of the coordinates (e.g., y). In the latter case the force-free magnetic field can be represented in the form

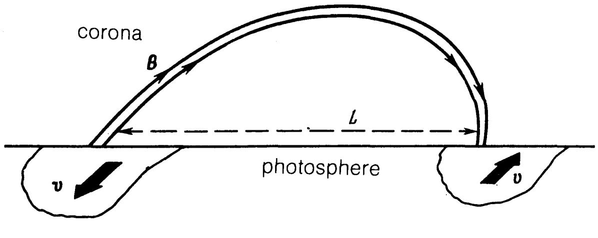

FIG. 1. Perturbation of the magnetic field due to a shift in the bases of the field lines at the photosphere surface.

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$$\mathbf{B}=[\nabla\psi(x,z)\mathbf{e}_y]+b_y(x,z)\mathbf{e}_y,$$

$$b_y=f(\psi),\quad \Delta\psi=-f\frac{df}{d\psi},\quad \alpha=\frac{df}{d\psi}, \tag{2}$$

where ψ is the stream function of the poloidal field, and the photosphere surface is identified with the plane $z = 0$.

Some results pertaining to the existence of solutions of Eqs. (1) and (2) are contained in Refs. 8 and 9. As to MHD instability, according to Ref. 9 force-free fields such as (2) are stable in ideal magnetohydrodynamics for a large class of functions $f(\psi)$ in the two-dimensional approximation, when both the initial field **B** and the perturbation are independent of the coordinate y. Since the displacement of the bases of the magnetic force lines is governed by the photosphere motions, the corona field can be checked for stability only against perturbations that do not change the locations of the bases on the photosphere. This means that the magnetic field is line-tied to the photosphere and is made additionally stable thereby.[10] Thus, in Refs. 11 and 12 examples are cited of force-free magnetic fields that are MHD-stable also in the three-dimensional case when the line-tying is taken into account. It was recently[13] proved that in ideal magnetohydrodynamics, line-tying in the bases of the force lines leads to stability of a large class of force-free fields.

The organization of this paper is the following. In Sec. 2 a general criterion is given of stability of nonlinear force-free fields, and line-tying at the bases is discussed. Section 3 deals with the stability of a plane force-free magnetic field.

2. ENERGY PRINCIPLE FOR FORCE-FREE FIELDS

In ideal magnetohydrodynamics, the stability of the static equilibrium of a plasma is determined, in accordance with the energy principle,[14] by the sign of the potential energy W of the perturbations. For the low-density solar-corona plasma of interest to us we neglect all plasma-pressure effects, so that the expression for W becomes

$$W=\frac{1}{8\pi}\int dV\{(\mathrm{rot}[\xi\mathbf{B}])^2-[\xi\,\mathrm{rot}\,\mathbf{B}]\cdot\mathrm{rot}[\xi\mathbf{B}]\}, \tag{3}$$

where **B** is the initial magnetic field and ξ is the displacement from the equilibrium position. With allowance for (1), it follows hence that the value of W for a force-free field is determined by a single function $\mathbf{A} = [\xi\times\mathbf{B}]$:

$$W=\frac{1}{8\pi}\int dV\{(\mathrm{rot}\,\mathbf{A})^2-\alpha\mathbf{A}\,\mathrm{rot}\,\mathbf{A}\}. \tag{4}$$

This yields a simple stability criterion in the case of a linear force-free field, when $\alpha(\mathbf{r}) = \alpha_0 = \mathrm{const}$.[15] Let the plasma occupy a certain volume V bounded by an ideally conducting surface. The field **B** has in general a nonzero normal component on the boundary: $B_{n|S}\neq 0$. Therefore, besides the constraint $\mathbf{A}\cdot\mathbf{B} = 0$ that is obvious from the definition of **A**, the allowed perturbations must satisfy also the line-tying condition on the boundary: $\xi|_S = 0$, i.e., $\mathbf{A}|_S = 0$. We now minimize Eq. (4) for W, using the normalization

$$\int dV(\mathrm{rot}\,\mathbf{A})^2=\mathrm{const}. \tag{5}$$

The extremum condition

$$\delta\int dV\{\mathbf{A}\,\mathrm{rot}\,\mathbf{A}-\mu(\mathrm{rot}\,\mathbf{A})^2\}=0 \tag{6}$$

(μ is a Lagrangian multiplier) and the requirement $\delta\mathbf{A}|_S = 0$ yield, after straightforward transformations, the following equation for **A**:

$$\mathrm{rot}\,\mathrm{rot}\,\mathbf{A}=\mu\,\mathrm{rot}\,\mathbf{A}. \tag{7}$$

Together with the boundary condition $\mathbf{A}|_S = 0$ this equation determines the set of eigenfunctions $\mathbf{A}_i$ and their corresponding eigenvalues μ_i. For a given $\mathbf{A}_i$, the potential energy is

$$W(\mathbf{A}_i)=\frac{1}{8\pi}\int dV\{(\mathrm{rot}\,\mathbf{A}_i)^2-\alpha_0\mathbf{A}_i\,\mathrm{rot}\,\mathbf{A}_i\}$$

$$=\frac{1}{8\pi}\int dV\left\{(\mathrm{rot}\,\mathbf{A}_i)^2-\frac{\alpha_0}{\mu_i}\mathbf{A}_i\,\mathrm{rot}\,\mathrm{rot}\,\mathbf{A}_i\right\}$$

$$=\frac{1}{8\pi}(1-\alpha_0/\mu_i)\int(\mathrm{rot}\,\mathbf{A}_i)^2\,dV. \tag{8}$$

Since the constants α_0 and μ_i are pseudoscalars, they can always be regarded as positive. It follows therefore from (8) that the minimum of W is reached when **A** is equal to the eigenfunction $\mathbf{A}_1$ having the smallest eigenvalue μ_1. We have then $W(\mathbf{A}_1)>0$ if $\alpha_0<\mu_1$ and conversely $W(\mathbf{A}_1)<0$ if $\alpha_0>\mu_1$.

It does not follow, however, that a linear force-free magnetic field is unstable if $\alpha_0>\mu_1$, since it is also required that the perturbations be transverse, i.e., $\mathbf{A}\cdot\mathbf{B} = 0$. The possibility of meeting this condition is connected with the degeneracy in (7), viz, one and the same eigenvalue μ_i corresponds to an infinite set of eigenfunctions $\mathbf{A}_i$ with different gradients of the scalar function $\varphi(\mathbf{r})$:

$$\mathbf{A}_i=\mathbf{A}_i'-\nabla\varphi, \tag{9}$$

where the condition $\nabla\varphi|_S = 0$ is satisfied (since $\mathbf{A}_i|_S = \mathbf{A}_i'|_S = 0$). Therefore, if a certain solution $\mathbf{A}_i'$ is known, satisfaction of the transversality condition calls for

$$\nabla\varphi\mathbf{B}=\mathbf{A}_i'\mathbf{B}. \tag{10}$$

This equation specifies the change of $\varphi(\mathbf{r})$ along the lines of the initial magnetic field **B**:

$$\partial\varphi/\partial s=(\mathbf{A}_i'\mathbf{B})/B. \tag{11}$$

The possibility of determining, by starting from (11), a single-valued function $\vartheta(\mathbf{r})$ depends substantially on the geometry of the magnetic field **B**. A solution is impossible if the field lines begin and end (within the volume V) on the boundary surface S (Fig. 2). Indeed, integrating (11) along a field line from one base to another (Fig. 2a) we obtain a

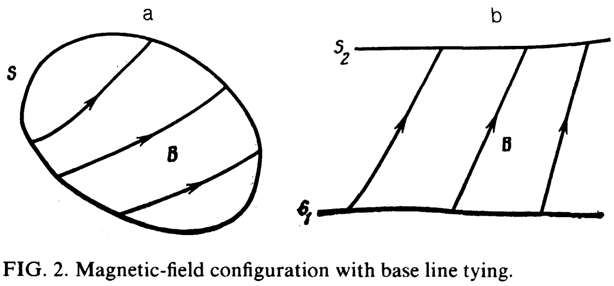

FIG. 2. Magnetic-field configuration with base line tying.

finite change of φ, whereas from the boundary condition $\nabla\varphi|_S = 0$ it follows that $\varphi|_S = \text{const}$. A similar contradiction occurs also in the case shown in Fig. 2b. The change of φ should be the same here for all lines beginning on S_1 and ending on S_2.

The approach described can be used to investigate the MHD instability of only those linear force-free lines for which $B_n|_S = 0$. In this case, as follows from (8), the necessary and sufficient condition for stability of these lines is the inequality $\alpha_0 < \mu_1$. This criterion was applied in Ref. 16 to the so-called Lundquist instability in an infinitely long cylinder, for which

$$B_r=0, \quad B_\varphi=B_0J_1(\alpha_0 r), \quad B_z=B_0J_0(\alpha_0 r). \tag{12}$$

Another example is the planar force-free field considered below.

Since the eigenvalue that determines the field stability is $\mu_1 \sim d^{-1}$, where d is the characteristic dimension of the region V, there is no instability in these cases so long as $\alpha_0 < \alpha_{cr} \sim d^{-1}$. It should be noted here that in expression (4) for the perturbation potential energy W, expressed in terms of the function $\mathbf{A}$, it is not assumed that the displacements ξ are bounded. This method can therefore be used to investigate the stability of force-free magnetic fields not only in the context of ideal MHD, but also for the tearing instability.[15] This circumstance becomes particularly clear in the case of a planar force-free field (see the next section).

Thus, a magnetic field with line-tied bases of the field lines is more stable than a force-free field with the same value of α_0 but with $B_n|_S = 0$. Moreover, there exist linear force-free line combinations for which stability is preserved at arbitrarily large values of the parameter $\alpha_0 d$. Examples are the laminar force-free fields considered in Ref. 13, with the following construction. Assume a two-dimensional potential magnetic field $\mathbf{B} = (0, B_y(y,z), B_z(y,z))$. In complex notation it can be expressed in the form $B_y - iB_z = F(\omega)$, $\omega = y + iz$, where $F(\omega)$ is an analytic function in the complex ω plane [the equations div $\mathbf{B} = 0$ and curl $\mathbf{B} = 0$ are in this case equivalent to the Cauchy-Riemann conditions for the function $F(\omega)$]. If we now rotate in each plane $x = \text{const}$ the vector $\mathbf{B}$ through one and the same angle φ that depends only on x, the resultant magnetic field

$$B_y - iB_z = F(\omega)\, e^{i\varphi(x)} \tag{13}$$

is force-free, with $\alpha = d\varphi/dx$. To investigate its stability in ideal magnetohydrodynamics, we can reduce expression (3) for the potential energy W of the perturbations, by simple transformations, to the form

$$W=\frac{1}{8\pi}\int dV\left\{\left(\frac{\partial D}{\partial z}-B_y\frac{\partial \xi_x}{\partial x}\right)^2+\left(\frac{\partial D}{\partial y}+B_z\frac{\partial \xi_x}{\partial x}\right)^2 + (\mathbf{B}\nabla\xi_x)^2-\operatorname{div}(\alpha D\xi_x\mathbf{B})\right\},$$

$$D\equiv\xi_y B_z-\xi_z B_y. \tag{14}$$

If there is no line-tying in the bases ($D|_S = \xi_x|_S = 0$), the last term of (14), which reduces to an integral over the boundary surface, makes no contribution to W. It follows then from (14) that $W > 0$, so that such force-free fields (including the linear ones with $\alpha = \alpha_0 = \text{const}$), which occupy an arbitrary closed volume, are always stable in the ideal-magnetohydrodynamics approximation. To be sure, the question of their tearing-mode stability remains open.

At the same time, line-tying in the bases does not guarantee MHD stability if the boundary surface has regions where $B_n = 0$, so that "slippage" of the magnetic force lines is possible. This can be verified with the force-free configuration (12) as an example. It is known[15] that in an infinitely long cylinder of radius R such a field is unstable in the context of ideal magnetohydrodynamics if $\alpha_0 R > 3.18$. Let the field now occupy a region of finite length, so that $B_z \neq 0$ on the end faces and the field lines are frozen in the bases. We choose the displacement field ξ in the plasma so that it coincides in the main volume with the displacements that lead to the indicated instability, and then let ξ fall off smoothly and vanish on the end faces. Obviously, if the cylinder is long enough the positive contribution of the main volume becomes predominant, so that the instability is preserved.

3. PLANE FORCE-FREE FIELD

Consider the stability of one of the simplest force-free configurations—a plane sheared field:

$$\mathbf{B}=(B_0\cos\theta(z),\ -B_0\sin\theta(z),\ 0), \tag{15}$$

for which $\alpha = d\theta/dz$ [this is particular case of the laminar field (13) when $F(\omega) = \text{const} = B_0$]. Let the region occupied by this field be bounded by two ideally conducting planes at $z = 0$ and $z = a$. The question of the stability of a field with constant shear, when $\alpha = \alpha_0 = \text{const}$, reduces then to a determination of the eigenvalues of Eq. (7) for this region. In view of the homogeneity with respect to the transverse coordinates $\mathbf{r}_\perp = (x,y)$ we can seek the solution in the form $\mathbf{A}(\mathbf{r}) = \mathbf{A}(z) = \exp(i\mathbf{k}_\perp\mathbf{r}_\perp)$. Choosing a coordinate frame in which $\mathbf{k}_\perp = (k,0)$, we obtain from (7)

$$\begin{aligned} d^2A_y/dz^2-k^2A_y&=-\mu(dA_x/dz-ikA_z),\\ dA_x/dz-ikA_z&=\mu A_y, \end{aligned} \tag{16}$$

from which it follows that

$$d^2A_y/dz^2=(k^2-\mu^2)A_y. \tag{17}$$

A solution of this equation that vanishes for $z = 0$ is possible only if $\varkappa^2 \equiv \mu^2 - k^2 > 0$ and takes the form $A_y = A_0 \sin\varkappa z$, with $\varkappa a = n\pi$, where $n = 1,2\ldots$. The smallest eigenvalue is thus $\mu_1 = \pi/a$ and corresponds to $n = 1$ and $k \to 0$. Therefore, in accord with the results of the preceding section, a planar force-free field with constant shear becomes unstable at $\alpha_0 > \pi/a$, when the total rotation angle of the magnetic-field vector exceeds π.

To assess the nature of this instability we consider first perturbations in ideal magnetohydrodynamics. We get then from (3) for the field (15)

$$W=\frac{1}{8\pi}\int dV\left\{\left[\frac{\partial}{\partial z}(\xi_z B_x)\right]^2+\left[\frac{\partial}{\partial x}(\xi_z B_x)\right]^2+\left[\frac{\partial}{\partial z}(\xi_z B_y)+\frac{\partial}{\partial x}(\xi_x B_y-\xi_y B_x)\right]^2-\alpha\left[\xi_z B_y\frac{\partial}{\partial z}(\xi_z B_x)-\xi_z B_x\frac{\partial}{\partial z}\times(\xi_z B_y)-\xi_z B_x\frac{\partial}{\partial x}(\xi_x B_y-\xi_y B_x)+(\xi_x B_y-\xi_y B_x)\frac{\partial}{\partial x}(\xi_z B_x)\right]\right\}. \tag{18}$$

For displacements $\xi \propto \exp(i\mathbf{k}_\perp \mathbf{r}_\perp)$, where $\mathbf{k}_\perp = (k,0)$, we introduce

$$\xi_z(x,z)=\xi_1(z)\cos kx, \qquad (19)$$
$$\xi_x B_y - \xi_y B_x = B_0[\xi_2(z)\sin kx + \xi_3(z)\cos kx].$$

After simple calculations and averaging over the transverse coordinates, the potential energy W takes the form

$$W=\frac{B_0^2}{16\pi}\int_0^a dz\left\{\left(\frac{d\xi_1}{dz}\right)^2 + k^2\xi_1^2\cos^2\theta + k^2\xi_2^2 + 2k\xi_2\sin\theta\frac{d\xi_1}{dz} + k^2\xi_3^2\right\}. \qquad (20)$$

Now minimizing this expression with respect to ξ_2 and ξ_3 we find that

$$\xi_2=-\frac{\sin\theta}{k}\frac{d\xi_1}{dz}, \qquad \xi_3=0, \qquad (21)$$

and obtain for W:

$$W=\frac{B_0^2}{16\pi}\int_0^a dz\cos^2\theta(z)\left[\left(\frac{d\xi_1}{dz}\right)^2 + k^2\xi_1^2\right]\geqslant 0. \qquad (22)$$

It follows that the planar force-free field (15) is stable in the context of ideal magnetohydrodynamics for an arbitrary $\theta(z)$ dependence.

The instability revealed above can thus be due only to the development of a tearing mode in the plasma. The very presence (or absence) of this instability is governed entirely by the solution of the magnetohydrodynamics equation in the "outer" region, where the inertia and finite conductivity of the plasma can be disregarded.[4] Consequently we have here, accurate to first-order quantities

$$\mathbf{b}=\mathrm{rot}[\xi\mathbf{B}], \quad [\mathrm{rot}\mathbf{B}\mathbf{b}]+[\mathrm{rot}\mathbf{b}\mathbf{B}]=0, \qquad (23)$$

where $\mathbf{b}$ is the magnetic-field perturbation. Assuming again that ξ and $\mathbf{b}$ are proportional to e^{ikx}, we find from (23) that $b_y = -i\alpha b_z/k$, $b_x = (i/k)\cdot(db_z/dz)$, and that the magnetic-field component b_z satisfies the equation

$$\frac{d^2b_z}{dz^2}+\left(\alpha^2-k^2-\frac{d\alpha}{dz}\frac{B_y}{B_x}\right)b_z=0 \qquad (24)$$

with boundary condition $b_z(0) = b_z(a) = 0$.

We consider first the case of constant shear, when $\alpha = \mathrm{const} = \alpha_0$. We have then in place of (24)

$$\frac{d^2b_z}{dz^2}+(\alpha_0^2-k^2)b_z=0. \qquad (25)$$

Given α_0 and k, this equation has no regular solution with the required boundary conditions, and the existence of the tearing instability is determined by the sign of the discontinuity Δ' of the logarithmic derivative of b_z at the singular point $z = z_0$, where $B_x(z_0) = 0$ (Ref. 4):

$$\Delta'=\frac{b_z'(z_0^+)-b_z'(z_0^-)}{b_z(z_0)} \qquad (26)$$

and for instability we must have $\Delta' > 0$. From the solution of Eq. (25) it follows readily that if $\alpha_0 < \pi/a$ then $\Delta' < 0$ for any z_0 in the interval $(0,a)$, and $\Delta' > 0$ for $\alpha_0 > \pi/a$ and for small enough k [for $k < (\alpha_0^2 - \pi^2/a^2)^{1/2}$] if z_0 is not very close to $z = 0$ and $z = a$. The threshold for the onset of tearing instability is therefore the value $\alpha_0 = \pi/a$ obtained above from the energy principle.

It is of interest in this connection to ascertain whether a field with an inhomogeneous shear whose local values exceed this threshold can be stable. The following quantum mechanical analogy can be useful in the investigation of Eq. (24). At the threshold of the onset of the tearing instability we have $\Delta' = 0$, so that Eq. (24) has a regular solution. This means that the perturbation with wave vector $\mathbf{k}_\perp = (k,0)$ is at the stability threshold if the corresponding Schrödinger equation

$$-\frac{d^2\psi}{dz^2}+U(z)\psi=E\psi, \qquad (27)$$

where

$$U(z)=-\alpha^2(z)+\frac{B_y(z)}{B_x(z)}\frac{d\alpha}{dz}$$

and the wave function is localized in the interval $(0,a)$, has a ground-state energy $E = -k^2 < 0$. Since the most unstable modes are those with $k\to 0$, the instability threshold corresponds to the appearance of a zero-energy level in Eq. (27). Let now

$$\alpha(z)=\frac{\pi}{a}[1+\varepsilon g(z)], \qquad \varepsilon\ll 1. \qquad (28)$$

Since the field is already at the stability threshold when $\alpha = \alpha_0 = \pi/a$, it remains stable also if the shear is not uniform, provided that the correction ΔE to the energy level is non-negative. We know[16] that

$$\Delta E=\int_0^a \psi_0^2 V\,dz, \qquad (29)$$

where the perturbation of the potential V is in this case

$$V=V_1+V_2, \qquad V_1=-\varepsilon\frac{2\pi^2}{a^2}g(z), \qquad V_2=\varepsilon\frac{\pi}{a}\frac{B_y(z)}{B_x(z)}\frac{dg}{dz}, \qquad (30)$$

and $\psi_0 = (2/a)^{1/2}\sin(\pi z/a)$ is the normalized unperturbed wave function of the ground state. Since simply increasing the shear uniformly in the layer upsets the field stability, it is useful, as seen from (29), to increase the shear ($g > 0$) near the edges, where ψ_0 is small, at the expense of decreasing the shear ($g < 0$) in the interior. We correspondingly choose the function $\alpha(z)$ in the form shown in Fig. 3, with $g = -1$ everywhere except in a thin layer of thickness $\delta \ll a$ next to

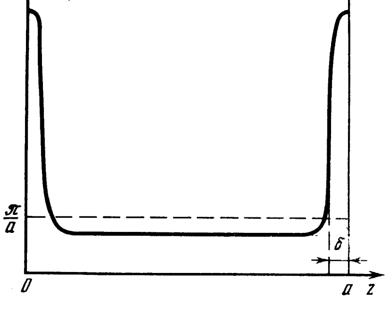

FIG. 3. Shear profile for a stable magnetic field.

the wall, where $g(z)$ becomes large and positive ($g \sim a/\delta \gg 1$). The correction to the level, necessitated by the perturbation V_1, is then positive and equal to $\Delta E_1 \approx \epsilon 2\pi^2/a^2$. As to the correction ΔE_2, it depends on the wave-vector direction, i.e., on the location of the singular surface where $B_x(z_0) = 0$. If z_0 is in the main volume, then $\Delta E_2 \sim \Delta E_1 \delta/a \ll \Delta E_1$. The contribution from the perturbation V_2 to the correction to the energy level becomes substantial when the point z_0 is close to an edge ($z_0 \sim \delta$, $a - z_0 \sim \delta$), and a simple estimate shows in this case that $\Delta E_2 \sim \Delta E_1$. It therefore follows that for this choice of the perturbation of $g(z)$ it is always possible to obtain the total correction for the level $\Delta E > 0$, i.e., make the field stable. A planar force-free field with nonuniform shear thus remains stable when the local shear is increased without limit in a thin layer near an ideally conducting boundary. The rotation angle of the magnetic-field vector on going through this layer can be of order unity.

The question of the stability of a planar force-free field with nonuniform shear is of interest also from another viewpoint. We know[15] that a linear force-free field has a minimum magnetic energy under the additional condition that the global helicity of the system $K = \int \mathbf{AB} dV$ is conserved (here $\mathbf{A}$ is the vector potential of the field $\mathbf{B}$). Since K remains almost constant upon development of a tearing mode,[1] this instability can be regarded as a possible mechanism for relaxation of the magnetic field to the lowest-energy state. However, the demonstrated stability of a force-free field with variable α shows that no such mechanism is realized here.

[1)]In ideal magnetohydrodynamics, the helicity K is an exact integral of the motion of the system (see, e.g., Ref. 1). Under conditions of tearing instability, the finite conductivity of the medium is significant only in a narrow region near the singular surface,[4] so that the changes of K are small.

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