Dissipation of magnetic energy in the merging of current filaments

A. T. Altyntsev, V. I. Krasov, N. V. Lebedev, V. L. Papernyĭ, and N. A. Strokin

Siberian Institute of Terrestrial Magnetism, the Ionosphere, and Radio Wave Propagation, Siberian Branch of the Academy of Sciences of the USSR

(Submitted 20 June 1988; resubmitted 1 February 1989)
Zh. Eksp. Teor. Fiz. **96**, 1243–1251 (October 1989)

The merging of current filaments formed as a result of the tearing-mode instability of the current sheet in the plasma of a theta pinch has been studied experimentally. Ohmic dissipation of the current in the filament is observed to increase during the merging. The merging rate is controlled by the level of small-scale turbulence in the sheet, which determines the necessary values of the anomalous conductivity and thermal conductivity.

1. INTRODUCTION

The magnetic field energy near a current sheet separating plasma regions with antiparallel magnetic fields can dissipate in a variety of ways. One effective dissipation mechanism is the Joule heating of the electrons as a result of current dissipation.[1,2] In this connection, we know that the magnetic structure of a current sheet is unstable with respect to reconnection of the field lines. The spontaneous reconnection which is caused by the onset of a tearing-mode instability leads to the formation of closed magnetic configurations (islands) in an extended sheet. These islands correspond to regions where pinching occurs: current filaments.[3] The breakup of the originally uniform structure of the sheet into distinct filaments results in a subsequent sharp decrease in the Joule dissipation power in them.[2]

The breakup of the sheet structure may be accompanied by an inverse process, seen experimentally,[4,5] in which neighboring islands which are linked by common field lines move closer together, and one of them eventually disappears. This restructuring of the magnetic topology of a sheet, called an "island coalescence" in Ref. 5, occurs at a velocity below the Alfvén velocity at the given parameter values of the plasma in the sheet. There has been no systematic study of this process, in particular, of how its characteristics depend on the plasma parameters. Some preliminary results in this direction were reported in Ref. 6, where it was shown that the merging occurs only after a certain threshold is reached. Our purpose in the present study is to determine the conditions required for merging, the parameters which control the merging rate, and how the restructuring of the magnetic topology during merging is related to energy dissipation processes.

2. DIAGNOSTIC METHODS AND EXPERIMENTAL RESULTS

Experiments have been carried out on the UN-Phoenix theta pinch. The initial plasma was produced by ionizing a neutral gas (hydrogen or nitrogen) in a quasisteady magnetic field B_0 ($B_0 = 310$–450 G), which was directed along the z axis of the working volume. The working volume was 100 cm long and 18 cm in diameter. The plasma parameters were $n_0 \approx (2\text{–}7)\cdot 10^{11}$ cm^{-3} for nitrogen, $n_0 \approx 2\cdot 10^{12}\text{–}1.5\cdot 10^{13}$ cm^{-3} for hydrogen, and $T_{e0} \approx T_{i0} \approx 1$–$5$ eV. The plasma was compressed by a cylindrical magnetic piston formed by the current of a shock coil $L = 30$ cm wide looping the volume. The field of this turn increased aperiodically to an amplitude $B_{1\max} = 1100$ G in a time of 0.45 μs and then decayed to half the amplitude in about 3.5 μs. This field was directed antiparallel to the initial field B_0.

To study the magnetic structure of the sheet we used a system of six magnetic probes at steps $\Delta r = 1$ cm along the radius. These probes measured the field component B_z. The probes were moved parallel to the axis at steps of $\Delta z = 2$ cm. The time resolution was ~ 10 ns, set by the accuracy with which the various channels of the measurement apparatus were synchronized. The value of the field at each probe position was averaged over three operating cycles ("shots") of the apparatus. The error in the measurement of B_z was $\sim 10\%$. The applicability and errors of this method are discussed in Ref. 7.

The measurement revealed that the magnetic field is of axial symmetry. In this case the contour lines of the magnetic flux,

$$\Phi(r, z) \equiv \int_0^r B_z(\rho, z)\,\rho\, d\rho = \text{const},$$

coincide with magnetic field lines. From the measured spatial distribution $B_z(r,z)$ we calculated the values of the radial field component

$$B_r(r_0, z) = (-1/r_0)\,(\partial\Phi/\partial r)\,|_{r=r_0}$$

on the null line of the axial field component, with $B_z(r_0,z) \equiv 0$.

The evolution of the magnetic structure of the sheet was studied by means of contour maps of the magnetic flux, $\Phi(r,z) = \text{const}$, plotted for various times. These maps were constructed in the (r,z) plane, bounded along the radius by the axis of the plasma volume ($r = 0$) and by the boundary of this volume ($r = 9$ cm). The boundaries along the z axis were the boundaries of the shock coil ($z = 0$ and 30 cm). The contour lines on the maps were plotted at steps of $5\cdot 10^2$ or 10^3 G$\cdot$cm^2 and were bounded by a separatrix $\Phi(r,z) \equiv 0$. It can be seen from the contour maps that a cylindrical current sheet 2–3 cm thick, with a width close to that of the shock turn, is formed in the plasma. At a time $t \lesssim 80$ ns after the appearance of a null line at the plasma boundary, the electron temperature in the sheet increases to 0.5–1.0 keV (Ref. 8), as a result of the heating caused by the ohmic dissipation of the current in the anomalous resistance:

$\sigma_{\rm eff}^{-1} = m_e\nu_{\rm eff}/ne^2$, where the effective collision rate $\nu_{\rm eff}$ is a consequence of the creation of small-scale (ion acoustic) turbulence by the current. A quantitative measure of the local dissipation rate is the azimuthal electric field on the null line:

$$E_\varphi(r_0, z, t)\approx-\frac{1}{cr_0}\frac{\delta\Phi}{\delta t}$$

where

$$\delta\Phi=\Phi(r_0(t_2))-\Phi(r_0(t_1)),\quad \delta t=t_2-t_1.$$

The formation of a current sheet is accompanied by a simultaneous restructuring of the magnetic structure of the sheet. In the initial stage of this process, the B_0 field lines are slightly curved only at the boundary of the plasma column, due to the initial perturbations which have a component B_r. Correspondingly, the component $E_\varphi(r_0,z)$, which is initially the strongest component, is nearly uniform along the sheet. Some 40–80 ns after the null line appears, a reconnection process is observed; i.e., closed island loops of various scales l, with various levels of the captured magnetic flux $\Phi^{(1,2)}$, and with a transverse field component $B_r(r_0,z)\leqslant B_r^{\max}$ on the null line form from initial perturbations (Figs. 1a, 1b, and 1d). The scatter in the positions and sizes of the islands constructed from various series of shots falls off sharply as time elapses. The structure of the sheet thus becomes deterministic, providing experimental justification for our procedure for plotting field-line maps from a series of shots. This question is discussed in detail in Ref. 9.

The initial stage of the rapid growth of the magnetic fluxes captured in islands, corresponding to closed field lines, can be attributed to the onset of a tearing-mode instability in the current sheet, which leads to the formation of a regular structure: chains of islands corresponding to regions in which pinching occurs (current filaments).[7] The rapid decrease in the dissipation rate at the O points, $E^{(O)}$, in the process, while the dissipation at the X points, $E^{(X)}$, remains strong, causes a further, relatively slow growth of the magnetic fluxes captured in islands.[2] The nature of the motions of the islands along the sheet observed in this case is determined for each pair of islands by the value of the parameter $A\equiv\Phi^{(1)}\Phi^{(2)}/\lambda_{12}$, as was shown previously.[6] If A is below a certain A_0, the islands move away from each other (Fig. 2); in the opposite case, they approach each other (Fig. 1a; maps 1 and 2). Since the fluxes $\Phi^{(1,2)}$ are measured from the outer X point, the value of the parameter A_0 is determined primarily by the magnetic flux which is common to the given pair of islands; here we have a measure of the interaction of the corresponding current filaments. The threshold value A_0, averaged over all pairs of interacting islands at $t\lesssim 500$ ns (at which times the sheet is being compressed by the pressure of the magnetic piston, which is increasing at the boundary), is $A_0\approx 5\cdot 10^5$ erg (Ref. 6).

The longitudinal motions of islands may be accompanied by restructuring of the magnetic structure of the sheet. This restructuring does not occur when the islands move away from each other. The reconnection process, continuing in the manner described above (i.e., because of the relation $E^{(X)} > E^{(O)}$), leads to a subsequent increase in the magnetic fluxes closed in islands. This process culminates in the breakup of the current sheet into distinct islands (current filaments; Fig. 2). The closing of islands on each other, in contrast, leads to a change in the topology of the magnetic structure: The field lines of an island with relatively small values of $B_r^{(\max)}$ and l disappear, leaving only a single O point (Fig. 1a; maps 2 and 3). This simplification of the current topology during the closing of islands on each other is naturally called "merging." The process by which the islands approach each other is accompanied by a characteristic deformation—"peaking"—of the current profile in a filament

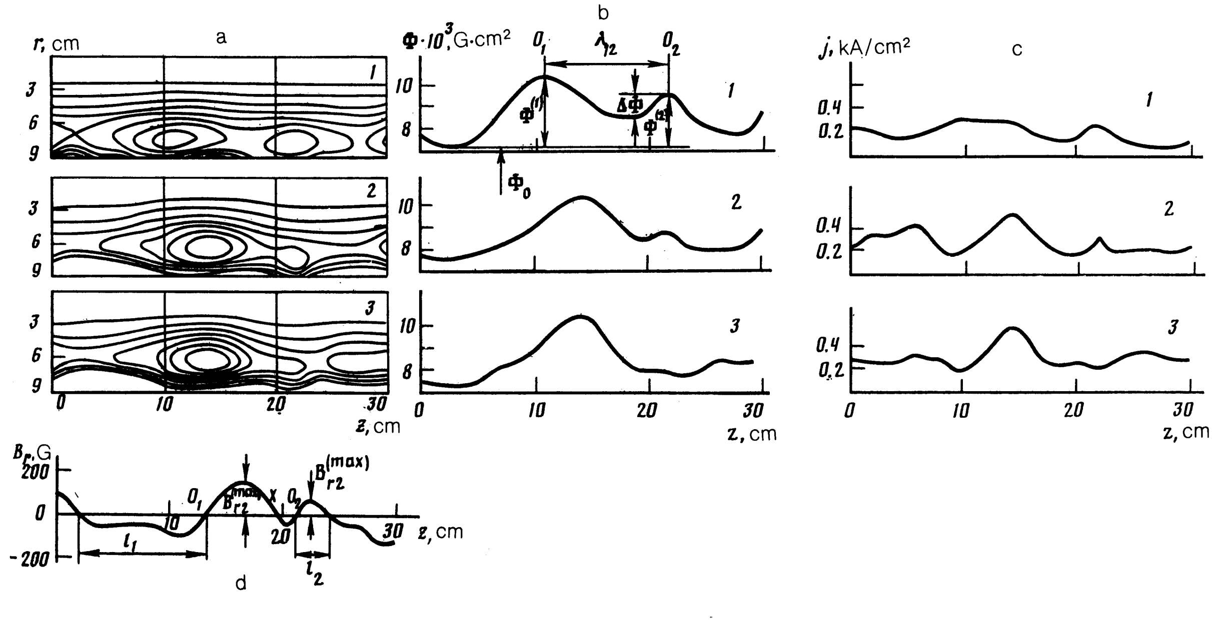

FIG. 1. Evolution of the magnetic structure of a sheet during the merging of islands. 1—t = 80 ns; 2—140 ns; 3—160 ns. a) Maps of magnetic field lines at steps of 10^3 G·cm²; b) distribution of magnetic flux along the null line; c) distribution of current density on the null line; d) distribution of transverse component of magnetic field at t = 140 ns.

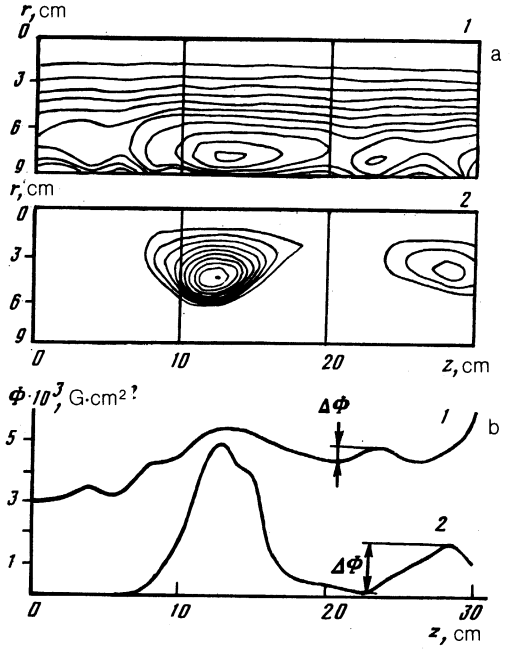

FIG. 2. Evolution of magnetic structure during the dispersal of islands. 1—$t = 80$ ns; 2—320 ns. a) Maps of field lines with a step of $5\cdot 10^2$ G·cm²; b) distribution of magnetic flux along the null line.

(Fig. 1c). This effect does not occur during separation.

Let us examine the merging process in more detail. In principle, it could occur in two ways: through joining of the field lines of neighboring islands to form common field lines, by virtue of reconnection proceeding in the opposite direction, or through rapid dissipation of the magnetic flux of the "small" island and the disappearance of the corresponding field lines. In any case, the changes in the magnetic flux are opposite those which occur during the formation of the islands, so the relation between the rates of dissipation at the X and O points (a small island) also reverses: $E^{(X)} < E^{(O)}$. In the former case, the reversal results from a decrease in $E^{(X)}$, and in the latter case from an increase in $E^{(O)}$. Figure 3 shows the distribution along the null lines of the difference between the values of $E(r_0,t)$ at the times t_1 (before the beginning of merging) and t_2 (during the merging process). We see from this figure that $E^{(O)}$ grows rapidly in the small island; i.e., the merging occurs by the second mechanism. Consequently, in this study, as in the experiments by other investigators,[4,5] the merging involves enhanced dissipation of flux in the small island which accompanies the islands closing in on each other; the flux in the large island again increases (Fig. 1b). The longitudinal motion of an island during merging at a velocity $v_z \lesssim 3\cdot 10^7$ cm/s leads to a variation in the component B_r ($\delta B_r \lesssim 50$ G) at the point on the null line with the coordinate $z = 22$ cm, which is the position at time $t = 140$ ns of the O point of the small island [i.e., $B_r(t = 140\text{ ns}) = 0$]. The result is a corresponding variation in the electric field,

$$|\delta E_\varphi| \approx (1/c)\, v_z \delta B_r \lesssim 5\cdot 10^{-2}\ \text{esu},$$

as shown by the vertical bar in Fig. 3.

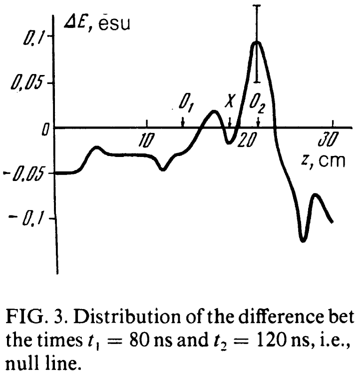

FIG. 3. Distribution of the difference between the values of $E_\varphi(r_0,z,t)$ at the times $t_1 = 80$ ns and $t_2 = 120$ ns, i.e., $\Delta E = E(t_2) - E(t_1)$, along the null line.

A quantitative analysis of the relationship between the longitudinal motions of the islands on the one hand, and the nature of the change in the magnetic fluxes captured in them on the other, can be carried out for the various cases with the help of the parameter $E^* \equiv (-1/cr_0)\Delta\Phi/\Delta t)$, where $\Delta\Phi$ is the change over the time Δt in the flux in an island, measured from the inner X point in the given pair (Fig. 1b). If the flux in an island decreases during the merging, i.e., if $E^* > 0$, when the islands move away from each other, and the closed magnetic flux in an island increases, then we have $E^* < 0$. Figure 4 shows values of E^* and of the velocity of the relative motion, v, averaged over the interval of approach (or withdrawal), for pairs of islands. The values shown here were observed in various regimes and in various stages of the evolution of the sheet. The data in this figure confirm the result found above: the dispersal of islands (the region $v < 0$) is accompanied by growth of the magnetic fluxes captured in the islands (the corresponding region $E^* < 0$), while the approach of islands toward one another ($v > 0$) is accompanied by a dissipation of the closed magnetic flux ($E^* > 0$). This result can thus be regarded as statistically grounded.

It also follows from this figure that the merging occurs at different velocities in different stages of the evolution of the sheet. At $t \lesssim 200$ ns the merging occurs at a velocity $v \sim v_{A0}$; note the very high merging velocity in the plasma of the heavy gas (nitrogen), $v > 3v_{A0}$. In a regime with a high initial density, the merging velocity observed for $t > 0.5\ \mu$s, i.e., when the magnetic structure is almost entirely reconnected ($\Phi_0/\Phi^{(1,2)} \lesssim 0.2$), falls off substantially, $v < 0.2v_{A0}$. This decrease occurs despite the high level of the interaction between current filaments: The value of the parameter A

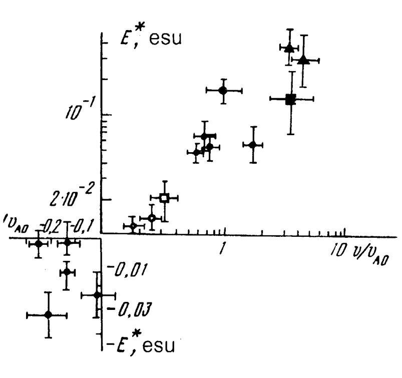

FIG. 4. Characteristics of the magnetic structure of islands during merging ($v > 0, E^* > 0$) and during dispersal ($v < 0, E^* < 0$); $v_{A0} = B_0/(4\pi n_0 m_i)^{1/2}$. ●—in the interval $t \lesssim 200$ ns; ○—in the interval $t \approx 0.5$–$2.0\ \mu$s; △—nitrogen plasma; ■—data of Ref. 4, $t \approx 0.6$–$0.76\ \mu$s; □—data of Ref. 5, $t = 1.5$–$4.0\ \mu$s.

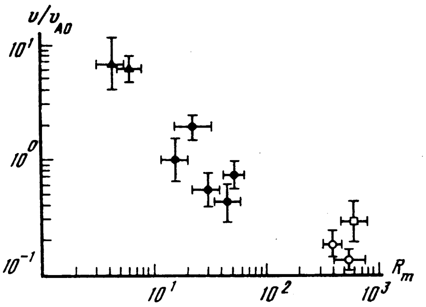

FIG. 5. Merging velocity as a function of the magnetic Reynolds number (the notation is the same as in Fig. 4).

here is greater than $5\cdot 10^6$ erg. Note also that the $E^*(v)$ dependence found in this study does not contradict the data found by other investigators,[4,5] also shown here. To compare the parameter values of the merging processes which occur under different experimental conditions, it is convenient to convert the data in Fig. 4 to the form in Fig. 5, where $R_m = 4\pi\sigma v_A l/s^2$ is the magnetic Reynolds number ($\sigma = j^{(O_2)}/E^*$, where $j^{(O_2)}$ is the average current density in the small island). It can be seen from this figure that as R_m is increased, i.e., as the role of dissipation processes decreases, the merging velocity decreases.

3. DISCUSSION OF RESULTS

The experimental results presented here suggest the following model for the structural evolution of the current sheet. The onset of a tearing-mode instability leads to the formation of magnetic islands. The threshold which is required for islands to merge in the stage in which the sheet is compressed by the increasing magnetic-piston pressure at the boundary (at $t \lesssim 0.5\,\mu$s) was attributed in Ref. 6 to competition between the attractive forces between parallel currents, corresponding to islands of current filaments, and the repulsive forces due to the antiparallel current of the shock coil which creates the piston. Because of the longitudinal inhomogeneity of the system, islands are squeezed out of the sheet.

How are the longitudinal motions of the islands related to the dissipative boundaries in a sheet? It has been shown previously[2] that these processes occur at a high rate only if the ratio u_e/v_{Te} is large, 0.2–0.3 [$v_{Te} = (2T_e/m_e)^{1/2}$, and $u_e = j/en$ is the electron current velocity]. Such values indicate a high level of ion acoustic turbulence in the sheet, which provides the anomalously low conductivity σ_{eff} required for effective ohmic dissipation. The dissipation rate decreases sharply when this ratio decreases, due to either a decrease in the value of u_e caused by an increase in the density (as is observed during the formation of islands[2]) or an increase in v_{Te} due to Joule heating of electrons caused by the dissipation. Consequently, effective removal of the evolved heat from the heating region is necessary in order to sustain a high dissipation rate. In a sheet with a reconnected magnetic structure, this situation can be arranged near the X points, from which heat is removed along open field lines. To demonstrate the point, we note that an estimate of the time scale τ_E for the heat loss from the sheet due to the longitudinal electron thermal conductivity with the anomalous value $\varkappa_0 = 3.6 n_e T_e/m_e \nu_{\text{eff}}$, in accordance with

$$\frac{d}{dt}(nT_e) \approx \varkappa_0 \frac{d^2 T_e}{d^2 z^2} \quad \text{or} \quad \frac{nT_e}{\tau_E} \approx \varkappa_0 T_e \left(\frac{L}{2}\right)^{-2},$$

yields

$$\tau_E \approx \left(\frac{L}{2}\right)^2 \frac{n}{\varkappa_0} \approx 0.3 \left(\frac{eL}{2}\right)^2 \frac{n}{\sigma T_e} \approx (1 \div 3) \cdot 10^{-8} \text{ c.}$$

Here we have used typical values of the plasma parameters in the interval $t \lesssim 200$ ns, during which the processes occur most intensely in the sheet: $T_e \lesssim 0.5$ keV, $n_e \approx (5\text{–}10)\cdot 10^{12}$ cm^{-3}, and $\sigma \approx (5\text{–}10)\cdot 10^{12}$ s^{-1}. The time scale observed experimentally for the dissipation of magnetic flux is $\sim (4\text{–}10)\cdot 10^{-8}$ s $> \tau_E$, so the heat can be removed from the vicinity of the X point, and the dissipation rate is therefore not limited here. In other words, $E^{(X)}$ remains large.

A different situation develops near the O point. The transverse component of the magnetic field on the null line, $B_r(r_0,z)$, which increases during the reconnection process, gives rise to a magnetic "barrier," which limits heat removal from an island. This limitation sets in when the condition $\nu_{\text{eff}}/\omega_{\text{Br}} \ll 1$ is satisfied ($\omega_{\text{Br}} = e\overline{B}_r/m_e c$, $\overline{B}_r \approx B_r^{\max}/2$ is the average value of B_r in an island). This limitation sharply reduces the thermal conductivity, $\simeq \varkappa_0(\nu_{\text{eff}}/\omega_{\text{Br}})^2 \ll \varkappa_0$, and correspondingly increases the plasma cooling time in the island, τ_E. Consequently, hot electrons accumulate in the island; this effect may also be the reason for the decrease in the dissipation rate $E^{(O)}$, as stated above. Consequently, as long as the MHD forces of the interaction between current filaments are small, the inequality $E^{(X)} > E^{(O)}$ will hold, and flux will continue to concentrate in islands.

If these forces instead become predominant, and the filaments (islands) are driven closer to each other by these forces, the sharp accompanying decrease in the flux of the small island can be explained under the assumption that it is the interaction between filaments that causes the redistribution (peaking) of the current in a filament. This redistribution intensifies the dissipation processes here and leads to a relation $E^{(X)} < E^{(O)}$. According to (1), the electron cooling time in an island satisfies $\tau_E \propto l^2/\varkappa \propto (B_r^{\max} l)^2$, so a high rate of heat removal and dissipation can be achieved far more easily in the small island. As a result, this small island disappears as a result of the merging. Early in the process ($t \lesssim 200$ ns), before the magnetic barrier becomes effective in limiting the heat removal from the small island, according to estimates, the merging velocity set by the MHD forces is correspondingly high, $v \sim v_{A0}$.

The increase in B_r during the reconnection, which terminates at $t > 500$ ns in the breakup of the current sheet into distinct current filaments (in the cylindrical geometry of these experiments, these are plasma configurations of the "compact torus" type), leads to degradation of the conditions for heat removal from them and thus a decrease in the rate of dissipation and merging, as shown above. Consequently, this model can also successfully explain the low island merging velocity which is observed in the later stages.

An important consequence of this model is an increase in the dissipation rate and thus in the island merging velocity upon the appearance of additional mechanisms for energy loss. In the plasma of a heavy gas, inelastic collisions of electrons with neutral atoms and ions might qualify as such a mechanism. An estimate of the time scale for the cooling of electrons in experiments in nitrogen in the interval $t > 100$ ns in which the electron temperature decreases to[8] 0.1–0.3 keV, yields $\tau_E^{(N_2)} \approx T_e / v_{Te} n_a S_E \approx (1-5) \cdot 10^{-7}$ s. This time scale is comparable to the time scale for the cooling due to longitudinal heat removal [$n_a \approx 10^{13}$–$10^{14}\,\text{cm}^{-3}$ is a typical density of neutral atoms, and $S_E \approx (0.7\text{–}1.0) \cdot 10^{14}\ \text{eV} \cdot \text{cm}^2$ is the total cross section for the energy loss of electrons in nitrogen[10]]. Since corresponding estimates for hydrogen show that this loss is negligible in that case, we could expect these processes to occur at a higher rate in a nitrogen plasma, in agreement with the experimental results.

4. CONCLUSION

Analysis of the experimental results reported here, those of our previous studies, and those of other investigators suggests a model for the merging of the current filaments (or of the magnetic islands which correspond to these filaments) which form as a result of the tearing-mode instability in a current sheet. An analysis of this model leads to the following conclusions.

1. The approach of islands toward one another due to attractive MHD forces between current filaments causes rapid ohmic dissipation of the magnetic flux corresponding to the small island, which leads in turn to restructuring of the topology of the magnetic structure of the sheet, which is called merging.

2. The merging velocity is determined by the dissipation rate, which is in turn controlled by the rate of heat removal from the region as a result of the dissipation of the Joule heating of electrons. In the early stage of the reconnection, the rate of heat removal is high, and the merging occurs with a maximum velocity close to the Alfvén velocity v_{A0}. In the late stage, heat removal from the current filaments is suppressed, and the merging occurs at a low velocity $v \ll v_{A0}$, which is limited by the low rate of dissipation in the filaments. In general, the merging velocity decreases as dissipative processes play a lesser role.

3. The operation of additional energy-loss mechanisms, in particular, electron cooling due to inelastic collisions with neutral atoms and ions in the plasma of a heavy gas, may be responsible for the enhancement of dissipation and therefore the increase in the merging velocity.

In summary, while the breakup of the current sheet into distinct current filaments caused by the tearing-mode instability decreases the rate of ohmic dissipation in these filaments, their merging works in the opposite direction, leading to an increase in the current dissipation in the filaments. Although the ultimate cause of the merging process is the operation of MHD forces, the velocity of the merging is set by the level of the small-scale turbulence, which controls both the conductivity value required for dissipation, σ_{eff}, and the corresponding value of the thermal conductivity, $\varkappa_{\text{eff}}$, which is responsible for the heat removal from the region of energy dissipation.

[1] A. T. Altyntsev, V. M. Bardakov, and V. I. Krasov, Zh. Eksp. Teor. Fiz. **81**, 901 (1981) [Sov. Phys. JETP **54**, 480 (1981)].

[2] A. T. Altyntsev, V. I. Krasov, N. V. Lebedev, and V. L. Papernyĭ, Zh. Eksp. Teor. Fiz. **94**(9), 75 (1988) [Sov. Phys. JETP **67**, 1777 (1988)].

[3] A. T. Altyntsev and V. I. Krasov, Zh. Tekh. Fiz. **44**, 2629 (1974) [Sov. Phys. Tech. Phys. **19**, 1639 (1975)].

[4] J. H. Irby, J. F. Drake, and H. R. Griem, Phys. Rev. Lett. **42**, 228 (1979).

[5] A. Sevilliano and F. L. Ribe, Phys. Fluids **28**, 3142 (1985).

[6] A. T. Altyntsev, V. I. Krasov, N. V. Lebedev, and V. L. Papernyĭ, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 360 (1985) [JETP Lett. **42**, 444 (1985)].

[7] A. T. Altyntsev, V. I. Krasov, N. V. Lebedev, and V. L. Papernyĭ, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 17 (1987) [JETP Lett. **45**, 20 (1987)].

[8] A. T. Altyntsev, V. I. Krasov, N. V. Lebedev, and V. L. Papernyĭ, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 360 (1981) [JETP Lett. **42**, 444 (1985)].

[9] A. T. Altyntsev, V. I. Krasov, N. V. Lebedev, and V. L. Papernyĭ, Fiz. Plazmy **14**, 972 (1988) [Sov. J. Plasma Phys. **14**, 571 (1988)].

[10] I. A. Krinberg, *Kinetics of Electrons in the Earth's Ionosphere* [in Russian], Nauka, Moscow, 1979.

Translated by Dave Parsons