Dynamic Stark effect and optical-line satellites of atoms in a quasimonochromatic stochastic field

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A theory is developed for the line shapes of multiphoton satellites of allowed and forbidden transitions in atoms placed in a strong quasimonochromatic stochastic electromagnetic field in the form of a two-dimensional Gaussian random process. The optical-transition probabilities are given in terms of Fourier transforms of generating functions that are the averages of nonlinear functionals of the field strength, evaluated over all the possible realizations of the noise field, using the method of functional integration in an explicit analytic form. A unified approach is used to examine the evolution of the satellite line shapes for spectral widths of the noise field varying from very low values, for which the quasistatic approximation is valid, to asymptotically large values, for which multiphoton processes can be described in the language of perturbation theory.

One of the most important consequences of the interaction between high-intensity electromagnetic radiation and atoms is the change that this induces in their energy and, consequently, optical spectra. Because of the dynamic Stark effect, their strong external electromagnetic field gives rise to atomic level shifts that are functions of the field strength. In addition, the optical spectra of atoms acquire satellites of allowed and forbidden (in zero external field) dipole transitions that are due to multiphoton processes induced by the external field. These effects are of interest in connection with a variety of applications in nonlinear optical and radio-frequency spectroscopy,[1–4] the diagonostics of high-frequency electric fields in turbulent plasmas,[5–11] etc. Studies of these effects have largely been confined to monochromatic fields.[1–3,9,11–13] However, in practice, one frequently encounters situations in which atoms are subjected to quasimonochromatic fields with stochastically varying parameters. For example, the radiation produced by powerful random-phase multifrequency lasers, which has been used in experimental studies of multiphoton transitions in atoms, is satisfactorily described by a complex Gaussian random process.[1,2,14] High-frequency electric fields that arise in turbulent plasmas are also found to be stochastic,[10,15,16] strong stochastic radio-frequency fields are used in investigations of magnetic resonance in spin systems,[17,18] and so on.

The Stark broadening of allowed optical transitions in atoms subjected to quasimonochromatic stochastic fields was examined in detail in Refs. 19–26. In particular, a theory of Stark broadening in a Gaussian noise field was developed in Ref. 26. It is valid for arbitrary relationships between the spectral width of this field, the detuning from resonance, the widths of quantum transitions, and the mean Stark shifts of the atomic energy levels.

Stochastic modulation of optical-emission parameters in multiphoton processes has been investigated in detail for direct multiphoton ionization.[1,2,14,27] Multiphoton transitions between discrete energy levels in stochastic fields have attracted much less atttention. Laser resonance spectroscopy and the diagnostics of stochastic fields in plasmas, based on satellites of atomic transitions, have relied mostly on the quasistatic approximation.[1,2,14,21,22,28] The multiphoton transition probabilities are obtained in this approximation in the limit of a very narrow spectrum (infinite correlation time) of the strong stochastic field, which corresponds to exceedingly slow variations in the field parameters. This is achieved by simple averaging of the corresponding expressions for a strong monochromatic field with fixed parameters over the distribution of these parameters. A much more complicated problem arises in the situation important in practice, in which the correlation time of the random field, which characterizes the rate of fluctuations in its parameters, is comparable with the characteristic times governing the transition, i.e., the random-field spectrum cannot be regarded as having an infinitesimal width.

In this paper, we shall examine the typical basic features of this problem by considering a simple model of a three-level quantum-mechanical system subjected simultaneously to a high-intensity nonresonant noise radiation and a low-intensity light probe. We shall investigate the absorption of the low intensity light in resonance transitions, of which one is allowed in the dipole approximation and the other is forbidden. The two excited states of the system are assumed to be dipole-coupled, and the gap between each of them and the ground state is assumed to be large. The effect of the quasimonochromatic stochastic field on the chosen pair of levels, for which the level separation is significantly greater than the central frequency of the external field, can then be shown to reduce in the adiabatic approximation to field-dependent increments in the phases of the wave functions corresponding to these levels. This description is equivalent to the introduction of the dynamic Stark level shift that is a function of the instantaneous noise field strength.

The expressions for the probability of absorption of the probe beam by the atoms will be given in the form of Fourier transforms of generating functions that are the averages of nonlinear functionals of the field strength, evaluated over all the possible realizations of the noise field. These averages are found by the method of functional integration in an explicit analytic form for a field in the form of a two-dimensional Gaussian stochastic process with a Lorentz spectrum.

A theory of the shape of multiphoton satellites of the optical lines of atoms is developed, providing a unified approach to the evolution of atomic optical spectra when the spectral width of the noise field is varied from the limit of

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very small values, in which the quasistatic approximation is valid, to asymptotically large values, for which multiphoton processes can be described in terms of perturbation theory. Radical changes in the optical spectra of atoms are demonstrated in the intermediate region in which the parameter values lie well outside the ranges of validity of the above two opposite limiting cases.

1. GENERAL EXPRESSIONS FOR TRANSITION PROBABILITIES IN A STOCHASTIC FIELD

We shall consider a three-level quantum-mechanical system with nondegenerate levels 1,2,3 (energies $\varepsilon_1 < \varepsilon_2 < \varepsilon_3$) on which two electromagnetic fields are acting (see Fig. 1). The strong stochastic field $F(t)$ has the carrier frequency $\omega \ll \omega_{32}$ ($\omega_{ij} = (\varepsilon_i - \varepsilon_j)/\hbar$), and is a two-dimensional stationary Gaussian process (with random amplitude and phase). Its correlation function is

$$\langle F(0)F(t)\rangle = B(t) = B_0 \cos \omega t \exp(-\gamma|t|), \tag{1}$$

where B_0 is the mean intensity of the noise field and γ^{-1} is the time constant of the correlation function (1), which determines the width of the Lorentz profile of the emission spectrum $F(t)$.

We shall investigate the absorption-line profile for the low-intensity probe beam $\mathscr{E}(t) = \mathrm{Re}[\mathscr{E}_0 \exp(-i\Omega t)]$ whose frequency Ω is close to the $1\to 2$ transition frequency (allowed transition) or the $1 \to 3$ frequency (forbidden transition). We suppose that $\omega_{21} \gg \omega_{32}$ and that the perturbation of the ground state 1 by the field $F(t)$ can be ignored.

Since $F(t)$ is assumed to be a low-frequency field $\omega/\omega_{32} \ll 1$, the adiabatic functions for the pair of excited states of the system in this field are given by

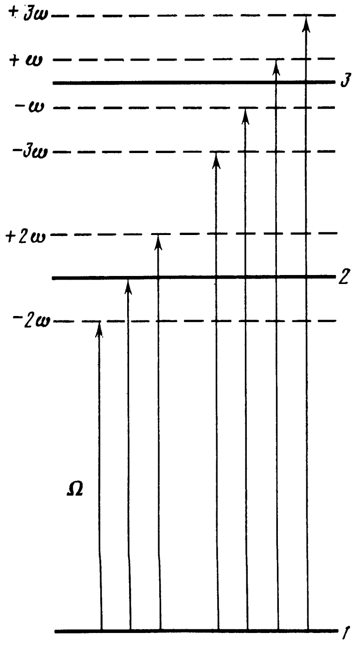

FIG. 1. Three-level system in a low-frequency quasimonochromatic noise field (ω) and a probe field (Ω). Arrows show the satellites of allowed ($1\to 2$) and forbiden ($1 \to 3$) transitions.

$$\varphi_+(t) = \left[\,|2\rangle \sin\frac{\xi}{2} + |3\rangle\cos\frac{\xi}{2}\right]\exp\left[-\frac{i}{\hbar}\int E_+(t)\,dt\right],$$

$$\varphi_-(t) = \left[\,|2\rangle \cos\frac{\xi}{2} - |3\rangle\sin\frac{\xi}{2}\right]\exp\left[-\frac{i}{\hbar}\int E_-(t)\,dt\right],$$

$$\xi = \mathrm{arctg}\frac{2d_{23}F(t)}{\hbar\omega_{32}}, \tag{2}$$

$$E_\pm(t) = \frac{1}{2}(\varepsilon_2+\varepsilon_3) \pm \tilde{E}(t),$$

$$\tilde{E}(t) = [\hbar^2\omega_{32}{}^2 + 4d_{23}{}^2F^2(t)]^{1/2}/2$$

where $|i\rangle$ is the eigenfunction of the level ε_i and d_{ij} is the dipole matrix element. The adiabatic functions (2) satisfy the equation

$$H_0(t)\varphi_\pm(t) = E_\pm(t)\varphi_\pm(t), \tag{3}$$

where $H_0(t)$ is the Hamiltonian with the matrix representation

$$H_0(t) = \left|\begin{matrix} \varepsilon_2, & d_{23}F(t) \\ d_{32}F(t) & \varepsilon_3 \end{matrix}\right|. \tag{4}$$

We assume that the field $F(t)$ is restricted in magnitude by the condition

$$d_{23}{}^2 B_0 \ll \hbar^2\omega_{32}{}^2, \tag{5}$$

which means that we can take the above functions in the approximate form

$$\begin{aligned} \varphi_+(t) &\approx \left[\,|2\rangle\left(\frac{QF^2(t)}{\omega_{32}}\right)^{1/2} + |3\rangle\right] \\ &\quad\times\exp\left[-\frac{i}{\hbar}\varepsilon_3 t - iQ\int F^2(t)\,dt\right], \\ \varphi_-(t) &\approx \left[\,|2\rangle - |3\rangle\left(\frac{QF^2(t)}{\omega_{32}}\right)^{1/2}\right] \\ &\quad\times\exp\left[-\frac{i}{\hbar}\varepsilon_2 t + iQ\int F^2(t)\,dt\right], \end{aligned} \tag{6}$$

where $Q = d_{23}{}^2/\hbar^2\omega_{32}$.

We seek the wave function $\Psi(t)$ of the three-level system under investigation in the form

$$\Psi(t) = c_1|1\rangle + c_+|\varphi_+\rangle + c_-|\varphi_-\rangle. \tag{7}$$

This function satisfies the time-dependent Schroedinger equation with the total Hamiltonian of the three-level system in fields $F(t)$ and $\mathscr{E}(t)$ and the initial conditions $|c_1(t_0)| = 1$, $c_\pm(t_0) = 0$. In first-order perturation theory in the interaction between the system and the weak probe field $\mathscr{E}(t)$, we find that

$$c_- = -\frac{i}{2\hbar}\mathscr{E}_0 d_{12}\int_{t_0}^{t} dt' \exp\left[i(\omega_{21}-\Omega)t' - iQ\int_{t_0}^{t'} F^2(t'')\,dt''\right], \tag{8}$$

$$c_+ = -\frac{id_{23}d_{12}}{2\hbar^2\omega_{32}}\mathscr{E}_0\int_{t_0}^{t} dt' F(t')\exp\left[i(\omega_{31}-\Omega)t' + iQ\int_{t_0}^{t'} F^2(t'')\,dt''\right]. \tag{9}$$

The probabilities W_a and W_f of the allowed and forbidden transitions (per unit time) are given by

$$W_a = 2\,\mathrm{Re}\langle\dot{c}_- c_-^*\rangle,\quad W_f = 2\,\mathrm{Re}\langle\dot{c}_+ c_+^*\rangle. \tag{10}$$

The angle brackets in these expressions represent averages over all the possible realizations of the random process $F(t)$. It is readily shown from (8)–(10) that

$$W_a=W_0\omega\,\mathrm{Re}\int_0^\infty dt\,I_a(t)\exp[-i(\omega_{21}-\Omega)t], \tag{11}$$

$$W_f=W_0\frac{QB_0}{\omega_{32}}\omega\,\mathrm{Re}\int_0^\infty dt\,I_f(t)\exp[-i(\omega_{31}-\Omega)t], \tag{12}$$

$$I_a(t)=\left\langle \exp\left[iQ\int_0^t F^2(t')\,dt'\right]\right\rangle, \tag{13}$$

$$I_f(t)=\left\langle F(0)F(t)\exp\left[-iQ\int_0^t F^2(t')\,dt'\right]\right\rangle B_0^{-1}, \tag{14}$$

where $W_0=|d_{12}|^2\mathscr{E}_0^2/2\hbar^2\omega$.

Expression (14) can be written in the form of the functional derivative

$$I_f(t)=\frac{1}{B_0}\frac{\delta^2}{\delta v(0)\,\delta v(t)}I([v])\,|_{v=0}, \tag{15}$$

$$I([v])=\left\langle \exp\left[-iQ\int_0^t F^2(t')\,dt'+2\int_0^t v(t')F(t')\,dt'\right]\right\rangle, \tag{16}$$

where

$$I_a(t)=I^*([v])\,|_{v=0}. \tag{17}$$

The generating function (16) is conveniently written in the form of the functional integral[29]

$$I([v])=\int DF\exp\left[-F\left(\frac{1}{2}B^{-1}+iQ\right)F+2vF\right]\Big/\int DF \times\exp\left[-\frac{1}{2}FB^{-1}F\right]. \tag{18}$$

The "matrix" notation used in (18) symbolizes the products of functions $(\alpha\beta)$, of operators $\hat{A}\hat{B}$, and of a function and an operator ($\alpha\hat{A}$ or $\hat{A}\alpha$). They are defined by the following formulas:

$$\alpha\beta=\int_0^t\alpha(t')\beta(t')\,dt',\qquad \hat{A}\hat{B}=\int_0^t\hat{A}(t_1,t')\hat{B}(t',t_2)\,dt',$$
$$\alpha\hat{A}=\int_0^t\alpha(t')\hat{A}(t',t_1)\,dt',\qquad \hat{A}\alpha=\int_0^t\hat{A}(t_1,t')\alpha(t')\,dt'. \tag{19}$$

The quantity $B^{-1}=B^{-1}(t_1,t_2|t)$ is the kernel of the integral operator that is the inverse of $B(t_1-t_2)$ on the interval $\langle 0,t)$, i.e.,

$$\int_0^t B(t_1-t')B^{-1}(t',t_2|t)\,dt'=\delta(t_1-t_2). \tag{20}$$

The expression $\exp[-\frac{1}{2}FB^{-1}F]$ is a functional of the distribution of the Gaussian random process F with pair correlation function B.

If we perform the shift operation

$$F=\tilde{F}+(^1/_2B^{-1}+iQ)^{-1}v,$$

under the functional integral sign in the numerator in (18), we can readily show that the generating function is given by

$$I([v])=[\det(1+2iQB)]^{-1/2}\exp(vLv), \tag{21}$$

where $L=(\frac{1}{2}B^{-1}+iQ)^{-1}$ satisfies the integral equation

$$L(t_1,t_2|t)=2B(t_1-t_2)-2iQ\int_0^t B(t_1-t')L(t',t_2|t)\,dt'. \tag{22}$$

Thus, according to (15) and (17), we have

$$I_a(t)=[\det(1-2iQB)]^{-1/2}, \tag{23}$$

$$I_f(t)=\frac{1}{2B_0}\frac{L(0,t|t)}{[\det(1+2iQB)]^{1/2}}. \tag{24}$$

We can now use the determinant $\det(1\pm 2iQB)$, given by

$$\det(1+2iQB)=\exp[\mathrm{Sp}\ln(1+2iQB)],$$

to show that

$$\frac{d}{dt}\ln\det(1+2iQB)=\frac{d}{dt}\mathrm{Sp}\left[2iQB+2Q^2B^2-i\frac{8}{3}Q^3B^3+\ldots\right]. \tag{25}$$

Direct differentiation of the series in (25) yields (see Ref. 30)

$$d\ln\det(1+2iQB)/dt=iQL(t,\ t|t), \tag{26}$$

so that

$$\det(1+2iQB)=\exp\left[iQ\int_0^t L(t',t'|t')\,dt'\right]. \tag{27}$$

Formulas (23), (24), and (27), taken together with (11) and (12), determine the probabilities W_a and W_f in their general form. In this sense, they do not require the explicit expression (1) for the correlation function of the random process $F(t)$. Subsequent calculations based on these formulas require the solution of the integral equation (22). Using (1) and (22), we can readily show that, for $t'<t$, $L(t',t|t)$ satisfies the differential equation

$$\begin{gathered} d^4L/dx^4-2(\varkappa^2-1+2i\varkappa b)\,d^2L/dx^2 \\ +[(1+\varkappa^2)^2+4i(1+\varkappa^2)\varkappa b]L=0, \\ \varkappa=\gamma/\omega,\ b=QB_0/\omega,\ x=\omega t', \end{gathered} \tag{28}$$

whose particular solutions have the form $\exp(\pm s_1x)$, $\exp(\pm s_2x)$, where

$$s_{1,2}=\{\varkappa^2-1+2i\varkappa b\pm 2i[\varkappa^2+2i\varkappa b+\varkappa^2b^2]^{1/2}\}^{1/2}. \tag{29}$$

Accordingly, we seek the solutions of the integral equation (22) for $t'<t$ in the form

$$L(t',\ t|t)=k_1\exp(-s_1x)+p_1\exp(s_1x) +k_2\exp(-s_2x)+p_2\exp(s_2x). \tag{30}$$

Substituting (30) in (22), we obtain a set of inhomogeneous linear algebraic equations for the coefficients $k_{1,2}(t)$, $p_{1,2}(t)$. Simple but laborious algebra then yields

$$iQL(t'\to t-0,\ t|t)=d\ln\Phi(t)/dt+(s_1+s_2-2\varkappa)\omega, \tag{31}$$

$$\Phi(\tau)=A_+^2C_-^2(s_1+s_2)^2\exp(-2s_2\tau)$$
$$+A_-^2C_+^2(s_1+s_2)^2\exp(-2s_1\tau)$$
$$-A_-^2C_-^2(s_1-s_2)^2\exp[-2(s_1+s_2)\tau]$$
$$-8A_+A_-C_+C_-s_1s_2\exp[-(s_1+s_2)\tau]-A_+^2C_+^2(s_1-s_2)^2,$$
$$\tau=\omega t,\ A_\pm=(\varkappa\pm s_1)^2+1,\ C_\pm=(\varkappa\pm s_2)^2+1 \qquad (32)$$

In view of (17), (21), and (31), we have

$$I_a(\tau)=\left\{\left[\frac{\Phi(0)}{\Phi(\tau)}\right]^{1/2}\exp\left[-\frac{1}{2}(s_1+s_2)\tau\right]\right\}^{*}\exp(\varkappa\tau). \quad (33)$$

Similarly, using (24), (30), and (31), we obtain

$$I_f(\tau)=\frac{1}{2b}\frac{[\Phi(0)]^{1/2}}{[\Phi(\tau)]^{3/2}}[A_+A_-\Delta_1(\tau)+C_+C_-\Delta_2(\tau)]$$
$$\times\exp\left[\frac{1}{2}(2\varkappa-s_1-s_2)\tau\right], \qquad (34)$$

$$\Delta_1(\tau)=2A_+C_+C_-s_2\exp(-s_2\tau)$$
$$-[A_-C_+^2(s_1+s_2)+A_+C_+^2(s_1-s_2)]\exp(-s_1\tau)$$
$$+[A_-C_-^2(s_1-s_2)+A_+C_-^2(s_1+s_2)]\exp[-(s_1+2s_2)\tau]$$
$$-2A_-C_+C_-s_2\exp[-(2s_1+s_2)\tau], \qquad (35)$$
$$\Delta_2(\tau)=2A_+A_-C_+s_1\exp(-s_1\tau)$$
$$-[A_+^2C_-(s_1+s_2)+A_+^2C_+(s_2-s_1)]\exp(-s_2\tau)$$
$$+[A_-^2C_+(s_1+s_2)+A_-^2C_-(s_2-s_1)]\exp[-(2s_1+s_2)\tau]$$
$$-2A_+A_-C_-s_1\exp[-(s_1+2s_2)\tau].$$

2. ASYMPTOTIC ESTIMATES OF THE SHAPES OF SATELLITES OF ALLOWED AND FORBIDDEN LINES; DISCUSSION OF RESULTS

Let us now examine limiting cases of the above expressions. Let $\varkappa\to 0$, which corresponds to very slow fluctuations in the parameters of the random field $F(t)$. If we pass to the limit as $\varkappa\to 0$ in (33) and (34), we find that

$$I_a(\tau)=[(1-ib\tau)^2+b^2\sin^2\tau]^{-1/2}, \qquad (36)$$

$$I_f(\tau)=[\cos\tau-ib\sin\tau+ib\tau\cos\tau][(1+ib\tau)^2+b^2\sin^2\tau]^{-3/2}. \qquad (37)$$

It is readily verified that (36) and (37) can be obtained directly in the quasistatic approximation, i.e., by averaging the corresponding expressions for the monochromatic field:

$$F=F_0\cos(\omega t-\varphi)=X\cos\tau+Y\sin\tau,$$
$$X=F_0\cos\varphi,\ Y=F_0\sin\varphi. \qquad (38)$$

Using the expression within the angle brackets in (13) and (14) for this field, we obtain

$$I_{am}(X,Y,\tau)=\exp(P_+X^2+P_-Y^2-RXY), \qquad (39)$$

$$I_{fm}(X,Y,\tau)=(1/B_0)(X^2\cos\tau+XY\sin\tau)$$
$$\times\exp(-P_+X^2-P_-Y^2+RXY), \qquad (40)$$

$$P_\pm=i\frac{Q\tau}{2\omega}\pm i\frac{Q}{4\omega}\sin 2\tau,\qquad R=i\frac{Q}{2\omega}(\cos 2\tau-1)$$

where, assuming that F_0 is a random quantity with the Rayleigh distribution function

$$\mathscr{P}(F_0)=\frac{F_0}{B_0}\exp\left(-\frac{F_0^2}{2B_0}\right), \qquad (41)$$

and that the phase of the field φ is distributed uniformly on the interval $\langle 0,2\pi\rangle$, we can show that

$$I_{a,f}(\tau)=\frac{1}{2\pi B_0}\int_{-\infty}^{+\infty}dX\int_{-\infty}^{+\infty}dY\exp\left(-\frac{X^2+Y^2}{2B_0}\right)I_{am,fm}(X,Y,\tau). \qquad (42)$$

Direct evaluation of the Gaussian integrals in (42) leads to (36) and (37).

The evaluation of the Fourier integrals (11) and (12) in the quasistatic case (36) and (37) can be carried out explicitly (see also the special cases discussed in Ref. 21). The final results are

$$\frac{W_a}{W_0}=\frac{1}{b}\sum_{n=-\infty}^{+\infty}\exp\left(-\frac{\omega_a^{(n)}-\Omega}{\omega b}\right)J_n^2\left(\frac{\omega_a^{(n)}-\Omega}{2\omega}\right)\theta(\omega_a^{(n)}-\Omega), \qquad (43)$$

$$\frac{W_f}{W_0}=\frac{\omega}{b\omega_{32}}\sum_{n=-\infty}^{+\infty}\exp\left(-\frac{\Omega-\omega_f^{(n)}}{\omega b}\right)\left\{J_n^2\left(\frac{\Omega-\omega_f^{(n)}}{2\omega}\right)\right.$$
$$\left.+J_{n-1}^2\left(\frac{\Omega-\omega_f^{(n)}}{2\omega}\right)+4J_{n-1}\left(\frac{\Omega-\omega_f^{(n)}}{2\omega}\right)J_{n-2}\left(\frac{\Omega-\omega_f^{(n)}}{2\omega}\right)\right\}$$
$$\times\theta(\Omega-\omega_f^{(n)})\frac{\Omega-\omega_f^{(n)}}{\omega}, \qquad (44)$$

$$\omega_a^{(n)}=\omega_{21}+2n\omega,\ \omega_f^{(n)}=\omega_{31}+(2n+1)\omega, \qquad (45)$$

where $J_n(x)$ is the Bessel function, $\theta(x)=1$ for $x>0$ and $\theta(x)=0$ for $x<0$. It then follows from (43) and (44) that, in relatively weak fields $\langle b\ll 1\rangle$, the frequency function $W_{a,f}(\Omega)$ has a multiphoton satellites at frequencies $\Omega=\omega_{21}\pm 2\omega,\ \omega_{21}\pm 4\omega,...$ for the allowed transitions and $\Omega=\omega_{31}\pm\omega,\ \omega_{31}\pm 3\omega,...$ for the forbidden transition. In general, all these satellites have asymmetric shapes, but the asymmetry becomes less well defined as the satellite number ($|n|$) increases. Near a satellite of an allowed transition with large $|n|$, the shape of the absorption line is shown by (43) to have the Gaussian form

$$W_a^{(n)}(\Omega)\propto\exp\left\{-\frac{[\Omega-\omega_a^{(n)}+2\omega|n|b]^2}{4|n|\omega^2b^2}\right\}, \qquad (46)$$
$$|n|\gg 1,\ b|n|\ll 1.$$

This Gaussian is shifted relative to the combination frequency $\omega_a^{(n)}$ by the amount $-2\omega|n|b$, where ωb is the average Stark shift, and has a characteristic width of order $2\omega b|n|^{1/2}$. It is readily verified with the help of (44) that satellites of a forbidden transition with large $|n|$ have a similar Gaussian shape. Figure 2 shows the forbidden-line satellites for the quasistatic case with a small natural width $\Gamma(\Gamma\ll b)$ introduced phenomenologically and obtained by direct numerical integration, using (12) and (37). As the field strength increases (b increases), the satellite structure of the absorption band is found to broaden and become smoother (see Fig. 2).

Let us now consider the other special case of fast fluctuations in the parameters of the random field $F(t)$, and suppose that $b\ll\varkappa\ll 1$. Expanding in powers of the small parameter $b/\varkappa$ in (33) and (34), we find in the leading order that

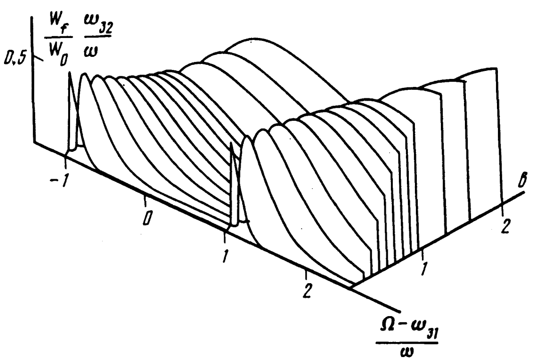

FIG. 2. Evolution of the first two satellites of a forbidden transition with increasing noise field strength in the quasistatic case ($\varkappa = 0$, $\Gamma/\omega = 0.002$).

$$I_a(\tau)=\exp\left[ib\tau-b^2\tau/2\varkappa+{}^1/_4b^2\exp(-2\varkappa\tau)\cos 2\tau\right], \quad (47)$$

$$I_f(\tau)=\exp(-\varkappa\tau)\cos\tau$$
$$\times\exp\left[-ib\tau-b^2\tau/2\varkappa+{}^1/_4b^2\exp(-2\varkappa\tau)\cos 2\tau\right]. \quad (48)$$

Expanding $\exp\left[\frac{1}{4}b^2\exp(-2\varkappa\tau)\cos 2\tau\right]$ a power series, and retaining the smallest power of the small parameter b^2 in the expansion for each of the factors $\exp(2in\tau)$ ($n = 0, \pm 1, \pm 2, \ldots$), we find that, for $\tau \neq 0$,

$$\exp\left[\frac{1}{4}b^2\exp(-2\varkappa\tau)\cos 2\tau\right]$$
$$\approx \sum_{n=-\infty}^{+\infty}\left(\frac{b^2}{8}\right)^{|n|}\frac{1}{|n|!}\exp(-2\varkappa|n|\tau+2in\tau). \quad (49)$$

The validity of this approximate Fourier expansion is assured by the condition $b \ll \varkappa \ll 1$. Substituting (47), (48), and (49) in (11) and (12), we obtain

$$\frac{W_a(\Omega)}{W_0}=\frac{\omega^2b^2/2\varkappa}{(\Omega-\omega_{21}+\omega b)^2+(\omega b^2/2\varkappa)^2}$$
$$+\sum_{\substack{n=-\infty\\(n\neq 0)}}^{+\infty}\frac{b^{2|n|}}{2^{3|n|}|n|!}\,\frac{4\omega^2|n|\varkappa}{(\Omega-\omega_{21}-2n\omega)^2+(2n\omega\varkappa)^2}, \quad (50)$$

$$\frac{W_f(\Omega)}{W_0}=\frac{\omega}{2\omega_{32}}\sum_{n=0}^{\infty}\frac{b^{2n+1}}{2^{3n}n!}$$
$$\times\left\{\frac{\omega^2(2n+1)\varkappa}{[\Omega-\omega_{31}-(2n+1)\omega]^2+\omega^2(2n+1)^2\varkappa^2}\right.$$
$$\left.+\frac{\omega^2(2n+1)\varkappa}{[\Omega-\omega_{31}+(2n+1)\omega]^2+\omega^2(2n+1)^2\varkappa^2}\right\}. \quad (51)$$

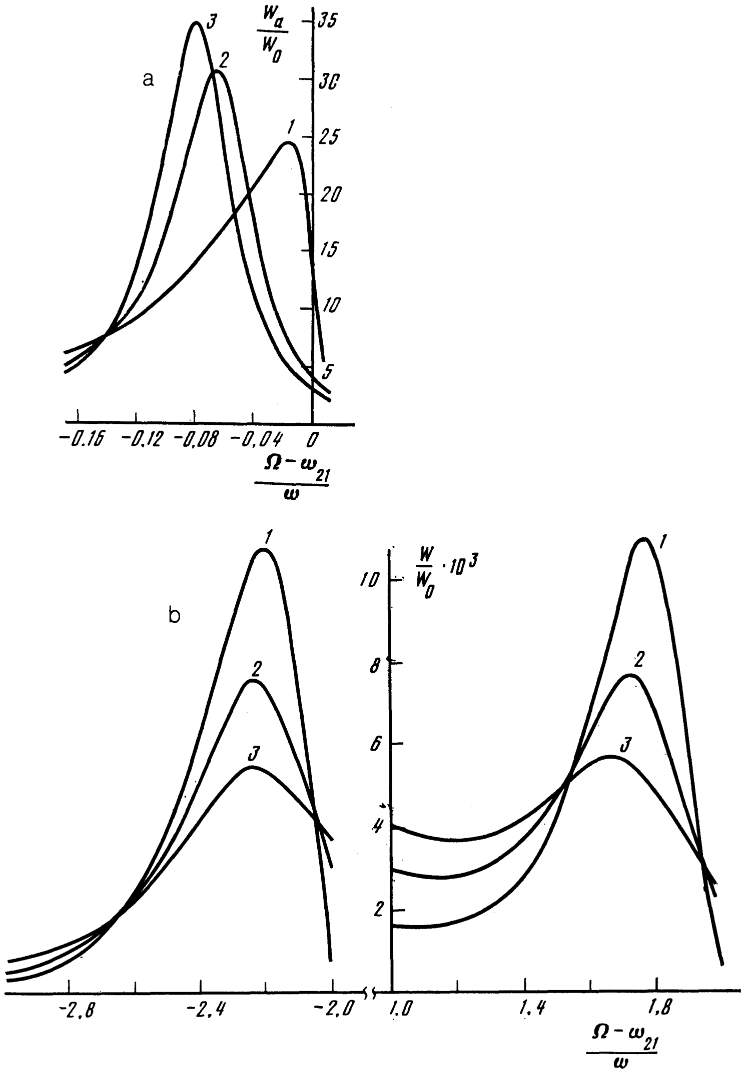

FIG. 3. Evolution of the allowed transition W_a/W_0 ($a - \omega_{21}$, $\Gamma/\omega = 0.005$) and its first two satellites ($b - \omega_{21} \mp 2\omega$, $\Gamma/\omega = 0.002$) with increasing width of the noise field spectrum ($b = 0.1$): $1 - \varkappa = 0.2 - 0.05$; $3 - 0.1$ (in Fig. 3b, W/W_0 should be replaced with W_a/W_0).

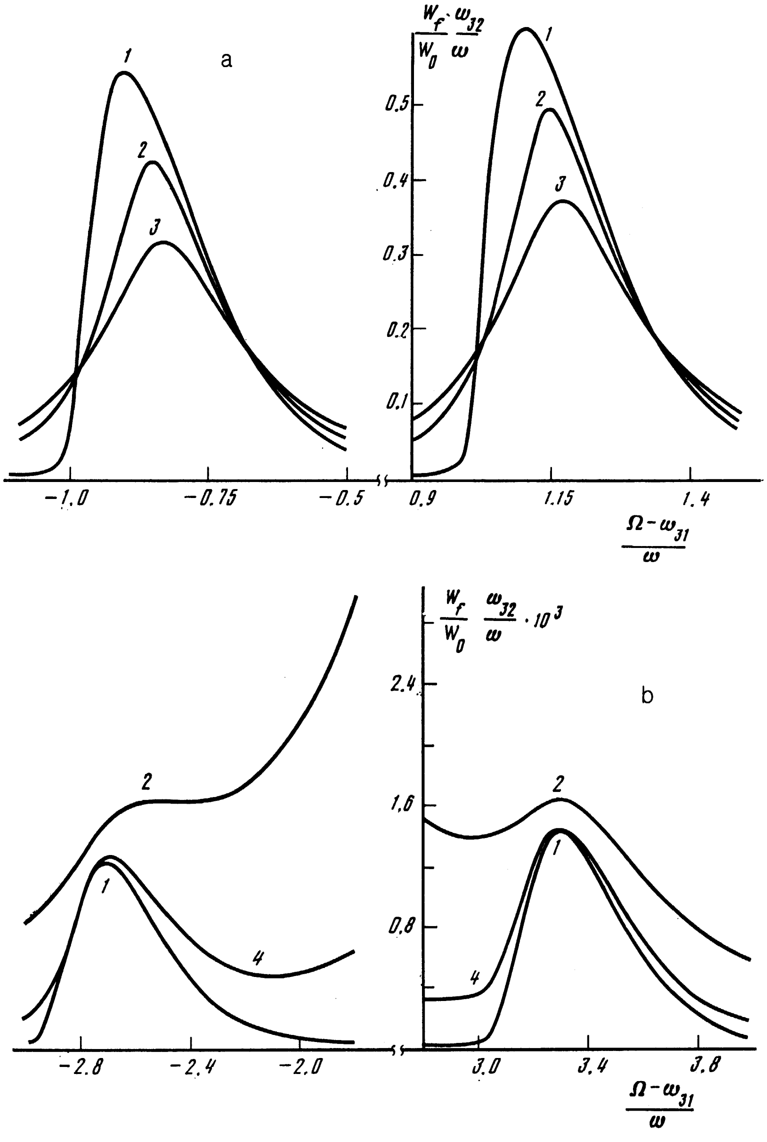

FIG. 4. Evolution of the satellites of a forbidden transition with increasing width of the noise field spectrum ($b = 0.1$):a—first pair of satellites ($\omega_{31} \mp \omega$, $\Gamma/\omega = 0.002$), b—second pair of satellites ($\omega_{31} \mp 3\omega$, $\Gamma/\omega = 0.001$). 1 — $\varkappa = 0$; 2 — 0.05, 3 — 0.1; 4 — 0.01.

It follows from (50) and (51) that the atomic absorption band takes the form of a set of equidistant satellites with a frequency spacing of 2ω. Each of these satellites lies near the combination frequency $\omega_{\alpha f}^{(n)}$ and has the Lorentz shape with half-width that increases with increasing satellite number $|n|$. The central peak that corresponds to the allowed transitions [the first term in (50)] is shifted relative to the undisturbed line $\Omega = \omega_{21}$ by an amount equal to the mean Stark shift ωb. It also has the Lorentz shape, but its half-width $\omega^2 b^2/2\gamma$ is much smaller than both the shift ωb and the satellite half-widths. Moreover, in contrast to the latter, it decreases with increasing γ.

In our case ($b \ll \varkappa \ll 1$), the satellite half-widths are significantly greater than their Stark shifts, so that the latter are omitted from the corresponding expressions under the summation signs in (50) and (51). These expressions give the probabilities of multiphoton transitions with the participation (absorption or stimulated emission) of $m = 2|n|$ photons (for the allowed transitions) and $m = |2n + 1|$ photons (for forbidden transitions). The probabilities of these m-photon transition are then proportional to the mth power of the noise field intensity ($\propto B_0{}^m$). Similar expressions for the m-photon transition probabilities ($m \neq 0$) in a stochastic electromagnetic field with the correlation function (1) were obtained in Refs. 31 and 32 in mth order perturbation theory in the interaction between the atom and the field.

The expressions that we have obtained for the multiphoton transition probabilities contain the results of Refs. 31 and 32 as special cases corresponding to $b \ll \varkappa \ll 1$.

Figure 3 shows the evolution of the line shape in the allowed transition (ω_{21}) and the first satellite of this line ($\omega_{21} \pm 2\omega$), calculated directly from the general expressions (11) and (33) for an increasing width γ of the noise field spectrum, including the case $b \sim \varkappa$ that lies outside the range of validity of the above asymptotic estimates. Figure 4 shows the corresponding evolution of the satellites ($\omega_{31} \pm \omega$ and $\omega_{31} \pm 3\omega$) of forbidden transitions [Eqs. (12) and (34)]. For small values of γ, the line shapes have the typical asymmetric form established in the quasistatic case. However, this asymmetry becomes less well defined as the satellite number increases. The allowed-line peak is found to shift with increasing γ and becomes centered relative to the position of the mean Stark shift. In addition the line undergoes dynamic narrowing with increasing γ. The γ dependence of the satellites is found to be different for allowed and forbidden transitions. The lines are found to broaden and become more symmetric as γ increases, especially for the higher satellite numbers. This behavior of the satellites is in agreement with the above asymptotic estimates.

We have thus developed a theory that can be used to examine the evolution of the shape of multiphoton satellites of atomic transitions in a wide range of parameter ratios corresponding to a strong stochastic electromagnetic field and its interaction with atoms.

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