Bremsstrahlung in a nonuniform light field

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A derivation is given of the cross sections for stimulated multiphoton bremsstrahlung and absorption when nonrelativistic electrons are scattered by atoms placed in a light focus. It is shown that the emission and absorption processes become asymmetric when the electron dynamics in the strong spatially inhomogeneous light field is taken into account.

INTRODUCTION

The scattering of an electron by an atom located in a strong electromagnetic field is accompanied by the stimulated bremsstrahlung effect (SBE) whereby the energy of the electron changes during the scattering process as it absorbs or emits photons.

The cross sections for the multiphoton SBE were first derived in Ref. 1 for fast electrons. Many attempts were subsequently made to abandon the Born approximation[2,3] (see also the reviews given in Refs. 4 and 5 and the references therein) in situations in which the external field was a spatially uniform light field with a time-independent amplitude. However, the SBE cross sections are normally measured in focused laser pulses, and depend significantly on the space-time structure of the light field. Expressions for the average SBE cross sections were obtained in Refs. 6 and 7 for a light field with a time-dependent amplitude.

In order to take into account the spatial inhomogeneity of the light field, the authors of Refs. 8 and 9 assumed that the field amplitude in the SBE cross sections was a function of the position of the scattering atoms, and then averaged their results over the volume of the focal region. However, this way of allowing for the inhomogeneity of the strong light field is inadequate because it does not take into account the dynamic manifestations of the inhomogeneity of the field. The point is that the translational motion of electrons takes place in the effective (ponderomotive) potential which is equal to the mean energy of the oscillations in the light field. Effects associated with this potential must be taken into account in the analysis of all physical processes involving electrons in focused fields. In particular, they play an important role in the interpretation of ionization spectra above the threshold.[10]

When the SBE is analyzed, it is important to allow for the fact, that, both before and after scattering by the atom, the electron is scattered by the ponderomotive potential in such a way that its initial and final momenta are not equal to the electron momentum in the incident beam and the momentum of the detected electron, respectively. Moreover, an electron with energy less than the height U_0 of the ponderomotive barrier will not penetrate the central part of the focus, but will be scattered by atoms on the periphery of the region, in which the field strength is low. This tends to suppress the SBE.

In Sec. 1, we present a classical and a quantum-mechanical description of the motion of an electron in a planar light field. Thomson emission and soft bremsstrahlung produced when an electron is scattered by the light focus are briefly considered in Sec. 2. The results obtained for the SBE are presented in Sec. 3. Possible experimental confirmations of the dynamic manifestations of the light field inhomogeneity are analyzed in the concluding Section.

1. ELECTRON DYNAMICS

Consider the motion of an electron in an inhomogeneous light field. Suppose that an electromagnetic wave propagates in the direction of the z axis. Its amplitude is independent of y, but has a bell-shaped maximum of width $2R$ when plotted as a function of x. The focal radius R is assumed to be large in comparison with translational motion. The planar geometry of the focus ensures that we can avoid mathematical difficulties and carry out an analytic treatment. Since we shall confine our attention to nonrelativistic electrons, we can take the vector potential in the dipole approximation:

$$\mathbf{A}(x,\ t)=-[c\mathbf{E}(x)/\omega]\sin\omega t. \tag{1}$$

The direction of linear polarization in (1) lies along the direction of spatial variation ($\hbar = m_e = e = 1$).

A uniform beam of electrons with momentum $\mathbf{p} = (p_x, \mathbf{p}_\perp)$, where $p_x = p\cos\theta$, is incident from the half-space $x<0$ at an angle θ with respect to the x axis on the plane layer occupied by the field (1). The time of an interaction between the electron with longitudinal velocity v_x, and the field (1) is determined by the time taken to cross the focus. The time-independent approximation used in (1) (field amplitudes independent of time) implies that the laser pulse length (the field source is assumed to be a laser) is much longer than the time to cross the focus

$$\tau \gg R/v_x. \tag{2}$$

The electron executes a translational motion in the light field, and also oscillates with the optical frequency. Since the focal region is macroscopically large ($R \gtrsim 10^{-4}$ cm), the oscillation amplitude is small in a wide range of electron energy, light intensity, and field frequency, and the oscillations themselves are rapid in comparison with the translational motion

$$\xi \ll R, \quad \omega^{-1} \ll R/v_x. \tag{3}$$

The oscillatory motion is determined by the local field strength at the point x that is the center of the oscillation:

$$\xi(t)=[\mathbf{E}(x)/\omega^2]\cos\omega t. \tag{4}$$

The translational motion alters the position of the center of oscillation $[x = x(t)]$ and, in accordance with (4), the oscillations follow adiabatically the translational motion. Conditions (3) then enable us to average over the rapid motion,

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which leads to an effective separation of translational and vibrational motions in both classical[11] and quantum-mechanical[12] cases. The translational motion can be treated classically or by the WKB approximation of quantum mechanics, since the condition

$$\lambda \sim p_x^{-1} \ll R \tag{5}$$

is satisfied for all electron energies of interest to us here.

The dynamics of electrons in the field (1) is determined by the Hamiltonian

$$\hat{H} = \frac{1}{2}\left(\hat{\mathbf{p}} + \frac{1}{c}\mathbf{A}(x,t)\right)^2. \tag{6}$$

The nonoscillating part of this Hamiltonian

$$\hat{H}_{tr} = \hat{\mathbf{p}}^2/2 + U(x), \tag{7}$$

describes the translational motion in the ponderomotive potential[13]

$$U(x) = E^2(x)/4\omega^2. \tag{8}$$

The conserved quantities are then the component of translational momentum along the yz plane

$$\mathbf{p}_\perp = \text{const} \tag{9}$$

and the energy of longitudinal motion (along the x axis)

$$p_x^2/2 = p_x^2(x)/2 + U(x). \tag{10}$$

The constants of motion (9) and (10) determine the translational trajectory of the electron interacting with the field (1). This point is discussed in Ref. 14, which also examines questions related to the finite length of the laser pulse and the nonconservation of translational energy. The momentum of the electron at the point x is then

$$\mathbf{p}(x) = (p_x(x), \mathbf{p}_\perp), \tag{11}$$

and its longitudinal component can be found from (10):

$$p_x(x) = (p_x^2 - 2U(x))^{1/2}. \tag{12}$$

Electrons whose longitudinal energy is less than the barrier height, i.e., $\varepsilon_x = p_x^2/2 < U_0 = U(0)$, are reflected by the focus.

Integration of the equations of fast motion, generated by the Hamiltonian (6), gives rise to the displacement (4).

The solution of the Schrödinger equation with the Hamiltonian (6) will now be sought in the form $\psi = \chi\varphi$. The function χ describes the translational motion and satisfies the Schrödinger equation with the Hamiltonian (7). In the quasiclassical approximation, the progressive-wave solution is

$$\chi_p(\mathbf{x},t) = \left(\frac{p_x}{p_x(x)}\right)^{1/2} \exp\left[i\int^x p_x(x')\,dx' + i\mathbf{p}_\perp\mathbf{x}_\perp - i\varepsilon_p t\right], \tag{13}$$

where $\mathbf{p}$ and $\varepsilon_p = \mathbf{p}^2/2$ are, respectively, the momentum and the energy of the free electron well away from the focus.

The factor φ describes the oscillations and depends on the coordinates only via the field amplitude. The equation for this function can be found by substituting $\psi = \chi\varphi$ into the original Schrödinger equation. If we neglect corrections that are small if (3) and (5) are satisfied, this equation becomes

$$i\partial\varphi/\partial t = \{U(x)\cos 2\omega t + (\dot{\boldsymbol{\xi}}\hat{\mathbf{p}})\ln\chi\}\varphi. \tag{14}$$

The solution of (14) that corresponds to the traveling wave (13) is

$$\varphi_p(x,t) = \exp\left\{i\frac{U(x)}{2\omega}\sin 2\omega t - i\frac{E(x)p_x(x)}{\omega^2}\cos\omega t\right\}. \tag{15}$$

The solution of the Schrödinger equation with the Hamiltonian (6) is then obtained by multiplying (13) and (15) together:

$$\psi_p(\mathbf{x},t) = \chi_p(\mathbf{x},t)\varphi_p(x,t), \tag{16}$$

which describes both oscillations in the light field and the acceleration of the electron by the ponderomotive potential.[12,15]

The scattered states $\psi_p^{(+)}(\mathbf{x},t)$ that describe the scattering of the electron wave by the light focus are obtained from (16) as follows. First, for the translational motion, we construct states $\chi_p^{(+)}(\mathbf{x},t)$ that are linear combinations of solutions such as (13) that satisfy the boundary conditions at infinity and the conditions of quasiclassical continuity near the turning points. The corresponding factor (15) is then introduced into these solutions for each traveling wave.

For a quasiclassical barrier, and with exponential accuracy, we can neglect the transmission and over-barrier reflection. The scattered state with $\varepsilon_x > U_0$ is then given by (16) in all space, and the state with $\varepsilon_x < U_0$ is a standing wave on one side of the barrier and is exponentially attenuated inside it.

In a small neighborhood of an arbitrary point x_0, the over-barrier state reduces to a wave solution with momentum $\mathbf{p}(x_0)$ (apart from a phase factor that depends on x_0).[16]

2. EMISSION BY AN ELECTRON IN AN INHOMOGENEOUS LIGHT FIELD

The motion of an electron in an electromagnetic wave is accompanied by the emission of radiation. In the quantum-mechanical approach to this emission, the states of the electron in the plane-wave field are described by the Volkov solutions.[16] A similar evaluation of the probability of emission of a photon by an electron in an inhomogeneous light field was carried in Ref. 17 where the electron wave functions were taken to be the states discussed in Sec. 1. The results obtained in Ref. 17 show that the radiation emitted by a nonrelativistic electron in an inhomogeneous field with a smooth envelope can be treated classically.

In the classical approach, the Thomson emission by an electron in the field (1) is determined by the acceleration $\ddot{\xi} = \mathbf{E}(x)\cos\omega t$. Passage through the focus, described by the function $x(t)$, gives rise to broadening of the emission spectrum by the amount $\Delta\omega \sim v_x/R$ around the field frequency ω. We note that the broadening of the spectrum due to the finite time taken to cross the focus will occur even when the ponderomotive acceleration is not taken into account, i.e., when the focal region is traversed with constant velocity. The total energy of Thomson radiation emitted as the focal region is transversed is

$$\Delta\varepsilon = \frac{2}{3c^3}\int_{x_0}^{\infty} E^2(x)\frac{dx}{v_x(x)}, \tag{17}$$

where the velocity is found from energy conservation (10), and the lower limit x_0 is zero for $\varepsilon_x > U_0$, or is given by the turning point ($U(x_0) = \varepsilon_x$) for $\varepsilon_x < U_0$. For fast electrons ($\varepsilon_x \gg U_0$), we can put $v_x(x) \approx v_x$, in which case (17) becomes identical with the expression for the radiated energy in the constant-velocity approximation. The radiated energy decreases with increasing velocity: $\Delta\varepsilon \propto v_x^{-1}$.

The slowing down of the incident electron by the ponderomotive potential barrier increases the time spent by the electron in the field and, consequently, gives rise to an increase in the radiated energy. This effect is at its maximum when the electron energy is comparable with the barrier height.

Electrons with sub-barrier energy ($\varepsilon_x < U_0$) do not reach the center of the focal region where the field strength is high, and this gives rise to a reduction in the radiated energy. For such electrons, the maximum intensity $E^2(x_0) = 4\omega^2\varepsilon_x$ is reached at the turning point. The radiated energy is then practically independent of the maximum field strength at the center of the focus, and is proportional to the velocity of the incident electron: $\Delta\varepsilon \propto v_x$. The radiated energy is therefore a maximum for an electron with energy $\varepsilon_x \sim U_0$.

If we take the model envelope $E(x) = E_0 \times \exp(-|x|/R)$, we find that the radiated energy can be evaluated exactly, and the result is

$$\Delta\varepsilon = \frac{2}{3c^3} E^2(x_0) \frac{R}{v_{eff}}, \qquad v_{eff} = \frac{v_x + v(x_0)}{2}$$

which, for an electron with $\varepsilon_x \to U_0$, is greater by a factor of two than the energy found in the uniform-velocity approximation.

It is well-known that allowance for relativistic effects leads to the appearance of harmonics of the fundamental frequency.[16] In a plane wave, the intensity ratio of two successive harmonics for $\lambda \gg \xi$ is of the order of $(\xi/\lambda)^2$. Apart from relativistic effects, harmonics can also appear in the field (1) when the amplitude is inhomogeneous.[17] The intensities of the harmonics are then determined by the parameter $(\xi/R)^2$. Since the focal radius is greater than the wavelength $R \gg \lambda$, the intensities of the harmonics in an inhomogeneous light field are largely determined by relativistic effects. The influence of spatial inhomogeneity on the broadening of the harmonics and on the amount of radiated energy is similar to that described above in the case of the fundamental frequency.

The acceleration of the electron in the inhomogeneous light field consists of the acceleration associated with the fast oscillations $\ddot{\xi}$ and the acceleration associated with the translational motion in the ponderomotive potential, $\ddot{\mathbf{x}}$. This means that, as the electron crosses the focal region, the emission of Thomson radiation and its harmonics is accompanied by bremsstrahlung that has a continuous spectrum and is due to the scattering of the electron by the ponderomotive potential.[17,18] Since $\ddot{x} \sim U_0/R$, the radiation emitted by an electron with energy above the barrier

$$\Delta\varepsilon \sim \frac{1}{c^3} \left(\frac{U_0}{R}\right)^2 \frac{R}{v_x}$$

is proportional to the square of the field strength at the focus. For a sub-barrier electron ($\varepsilon_x < U_0$), the radiated energy is independent of the maximum intensity at the center of the focus and is proportional to v_x^3.

3. SBE CROSS SECTIONS

We must now examine the scattering of electrons by atoms at the focus of a high-intensity light beam. In contrast to Ref. 1, the Volkov solutions will now be replaced with wave functions that take into account the acceleration of the electron by the ponderomotive potential. This approach corresponds to the use of the distorted-wave method for calculating the cross sections for scattering by two potentials.[19]

The electron-atom interaction will be described by the potential $V(r)$. This implies that the field has no effect on the atom and that the state of the atom remains the same during the scattering process. Moreover, the atoms are stationary and uniformly distributed throughout the focal region. The geometry of this region and of the scattered electron beam are the same as in Sec. 1.

The particular feature of the present problem is that the electron is scattered by two potentials, namely, the atomic and the ponderomotive. The effect of the latter is accurately taken into account by the wave functions (see Sec. 1), so that, to first order in the electron-atom interaction, the transition amplitude for scattering by an atom at the point $\mathbf{x}_0$ is given by

$$C_{\mathbf{p}_f \mathbf{p}_i} = -i \int dt \int d\mathbf{x}\, \psi_{\mathbf{p}_f}^{(-)*}(\mathbf{x}, t)\, V(|\mathbf{x}-\mathbf{x}_0|)\, \psi_{\mathbf{p}_i}^{(+)}(\mathbf{x}, t), \quad (18)$$

where $\mathbf{p}_i$ is the momentum of the incident electron and $\mathbf{p}_f$ is the final momentum of the free electron detected at a large distance from the focal region.

A significant simplification arises in (18) from the fact that the range of the atomic potential is small in comparison with the dimensions of the focal region, and that the main contribution to the integral with respect to the spatial coordinates is due to the immediate neighborhood of the point x_0. If the atom lies in the region that is classically inaccessible to the initial and final electron, the matrix element is zero with exponential accuracy because of the attenuation of the electron states under the barrier [see (28) below]. If, on the other hand, both wave functions oscillate in the neighborhood of the atom, the phase can be written in the following form in the region that is significant for the integral:

$$\int^{x} p(x')\,dx' = p(x_0)(x - x_0) + \int^{x_0} p(x')\,dx' \quad (19)$$

and we may suppose that the light field is uniform and equal to $\mathbf{E}(x_0)$. The matrix element in (18) is then expressed in terms of the Fourier components of the atomic potential.

The Fourier series for the periodic factor in (18) enables us to carry out the integral with respect to time, so that, after some standard algebra, we obtain the cross section for scattering with emission ($n > 0$) or absorption ($n < 0$) of photons, subject to the energy conservation law

$$\varepsilon_i = \varepsilon_f + n\omega. \quad (20)$$

The cross sections are described by different expressions, depending on whether the wave functions are traveling or standing waves. Let us consider the case where the energy of the incident electron is greater than the height of the barrier, and the motion is longitudinal, i.e.,

$$\varepsilon_{ix} = \varepsilon_i \cos^2\theta_i > U_0, \quad (21)$$

whereas the wave function is given by (16).

Let us first consider the situation where the energy of the scattered electron is greater than the barrier height, i.e.,

$$\varepsilon_{fx}=\varepsilon_f \cos^2\theta_f>U_0 \tag{22}$$

When $\varepsilon_f \gg U_0$, the angular range in (22) covers practically all the directions of emission (except for glancing motion along the plane of the focus).

The cross section for the n-photon SBE is given by

$$\frac{d\sigma^{(n)}(x_0)}{do_{\mathbf{p}_f}}=\frac{1}{4\pi^2}\frac{p_{ix}p_{fx}}{p_{ix}(x_0)p_{fx}(x_0)}\frac{p_f}{p_i}|V(\mathbf{q}(x_0))|^2J_n^2(\lambda(x_0)), \tag{23}$$

where the Fourier components of the atomic potential $V(\mathbf{q}(x_0))$ and the argument of the Bessel function $\lambda(x_0) = \mathbf{E}(x_0)\mathbf{q}(x_0)/\omega^2$ depend on the transferred momentum

$$\mathbf{q}(x_0)=\{p_{fx}(x_0)-p_{ix}(x_0),\ \mathbf{p}_{f\perp}-\mathbf{p}_{i\perp}\}.$$

When the electron in the initial and the final states is fast ($\varepsilon_i \cos^2\theta_i \gg U_0$ and $\varepsilon_f \cos^2\theta_f \gg U_0$), we can neglect $U(x_0)$ in the quasiclassical expressions for the momentum, so that the cross section (23) becomes identical with the SBE cross section for a uniform field.[1] The spatial gradient then manifests itself only through the field amplitude in the argument of the Bessel function.[8] This limiting case is reached if the quantity $\delta q \sim (U_0/v_{ix}v_{fx})q_x$ satisfies the condition $\delta qE/\omega^2 \ll 1$, which allows us to neglect the effect of the ponderomotive acceleration in the Fourier component of the atomic potential and in the argument of the Bessel function.

The cross section (23) can also be obtained in a different way by considering the process in three stages. First, the electron travels along the classical trajectory in the ponderomotive potential and its momentum varies from the initial $\mathbf{p}_i$ to $\mathbf{p}_i(x_0)$, which is the initial momentum in the problem of scattering by an atom. Second, the atom is scattered by the local uniform field with amplitude $\mathbf{E}(x_0)$. The momentum $\mathbf{p}_f(x_0)$ after scattering by the atom is the initial condition for the third stage, i.e., the escape from the focal region. The electron leaving this region is detected with momentum $\mathbf{p}_f$. The factor in parentheses in (23) appears because the cross section is determined for the particle flux density incident on the focus and, in addition, the direction of emission after scattering by the atom is different from the direction of motion of the detected electron.

We now turn to the situation in which the energy of longitudinal motion of the scattered electron is less than the height of the ponderomotive barrier:

$$U(x_0)<\varepsilon_f \cos^2\theta_f<U_0. \tag{24}$$

The final-state wave function is a standing wave, and the matrix element reduces to the superposition of Fourier components of the potential with transferred momenta $\mathbf{q}(x_0)$ and $\tilde{\mathbf{q}}(x_0)$. The quantity $\tilde{\mathbf{q}}(x_0)$ is obtained from $\mathbf{q}(x_0)$ by changing the sign in front of $p_{fx}(x_0)$. The coefficients in this superposition contain quasiclassically large phases [the second term in (19)], so that the interference term is effectively absent from the cross section. All this leads to the following result:

$$\frac{d\sigma^{(n)}(x_0)}{do_{\mathbf{p}_f}}=\frac{1}{4\pi^2}\frac{p_{ix}p_{fx}}{p_{ix}(x_0)p_{fx}(x_0)}\frac{p_f}{p_i}$$

$$\times\{|V(\mathbf{q}(x_0))|^2J_n^2(\lambda(x_0))+|V(\tilde{\mathbf{q}}(x_0))|^2J_n^2(\tilde{\lambda}(x_0))\}. \tag{25}$$

The quantity $\tilde{\lambda}(x_0)$ is expressed in terms of $\tilde{\mathbf{q}}(x_0)$ by analogy with the procedure used for (23). Electrons for which (24) is satisfied are reflected by the ponderomotive barrier, i.e., they remain in the half-space in which scattering by the atom has taken place. Hence, electrons moving in the forward direction, i.e., in the direction of the incident beam ($\theta_f < \pi/2$), are those that have been scattered by atoms at the points $x_0 > 0$. Backward-moving electrons ($\theta_f > \pi/2$) are those that were scattered by points with $x_0 < 0$. We recall that the angle θ_f is measured from the x axis.

If the final-state energy is slightly greater than $U(x_0)$, i.e., the atom lies near the turning point x_f, given by the condition $U(x_f) = \varepsilon_f$, the electrons are scattered into a narrow range of angles θ_f, such that

$$\sin^2\theta_f<\sin^2\theta_c=[\varepsilon_f-U(x_0)]/\varepsilon_f\ll 1. \tag{26}$$

A root singularity appears in (25) because the longitudinal momentum $p_{fx}(x_0)$ vanishes for $\theta_f \to \theta_c$, but the total cross section remains finite and contains the small factor $\sigma^{(n)}(x_0) \sim \sin\theta_c(x_0)$ that vanishes for $x_0 \to x_f$.

The process whereby the electron is reflected by the focus and its direction of motion undergoes a change cannot in principle be described by the Volkov states that are characterized by a conserved quantum number, namely, the momentum.

The dynamic effect of the spatial inhomogeneity of the light field on the SBE is even more radical when

$$\varepsilon_f \cos^2\theta_f<U(x_0), \tag{27}$$

which means that the neighborhood of the atom is classically inaccessible to the final-state electron. As already noted at the beginning of this Section, both the amplitude and the cross section for the n-photon SBE vanish in this situation:

$$\frac{d\sigma^{(n)}(x_0)}{do_{\mathbf{p}_f}}=0. \tag{28}$$

The last expression, taken together with (27), describes the suppression of SBE channels in the inhomogeneous light field. It follows from (27) and (28) that, for the atom at the point x_0, the emission channels ($n>0$) with $\varepsilon_f = \varepsilon_i - n\omega < U(x_0)$ are suppressed absolutely. In other words, atoms located between the turning points determined by the energy ε_f do not contribute to the emission channel with final energy $\varepsilon_f < U_0$. The remaining emission channels ($\varepsilon_f > U(x_0)$), and also the absorption channels for an atom at x_0, are only partially suppressed: scattering through angles satisfying (27) is forbidden, i.e., there is no scattering to states with low energy of motion in the direction of the inhomogeneity.

Expressions (23), (25), and (28) together determine the differential cross section for the SBE by an individual atom in a highly focused light field.

Scattering of electrons with energies greater than the barrier height ($\varepsilon_i \cos^2\theta_i < U_0$) can be considered in a similar way. Such electrons do not penetrate the center of the focal region (beyond the turning point), and the SBE processes occur in a weaker field. Electrons with final energy $\varepsilon_f < U_0$ are reflected by the ponderomotive barrier in the backward direction, whereas the flux that has passed through the focal region contains only electrons that have absorbed $n \geqslant n^* = [(U_0 - \varepsilon_i \cos^2\theta_i)/\omega] + 1$ photons (the bracket in this expression denotes the integer part).

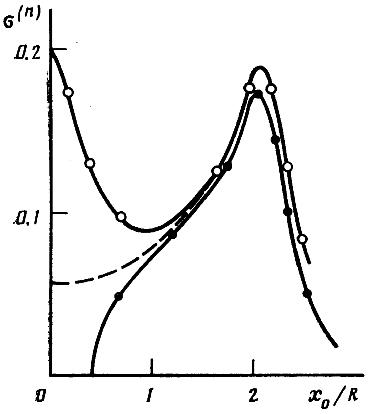

FIG. 1. Spatial dependence of the SBE cross sections: ○, absorption of a photon ($n = -1$), ●—emission of a photon ($n = +1$), dashed line—SB cross section ($n = +1$) in the model without the ponderomotive potential, the absorption and emission cross sections for $n = +1$ are close to one another in magnitude. The value of $\sigma^{(n)}$ is given in units of the Born cross section σ_0 for scattering by the potential $V(r)$ in the absence of the field. $\varepsilon_i = 4.5$ eV, $U_0 = 4$ eV, $\omega = 1$ eV, $p_i a = 0.3$.

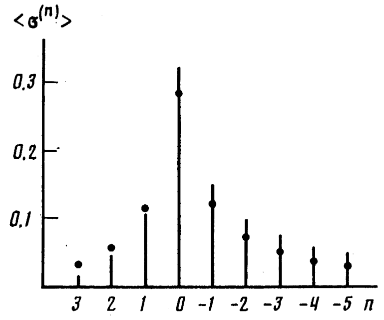

FIG. 2. Total SBE cross sections averaged over the focal region. The values of the parameters are the same as in Fig. 1. The asymmetries are as follows: $\eta^{(1)} = 0.18$, $\eta^{(2)} = 0.39$, $\eta^{(3)} = 0.62$. Points indicate the cross sections in the model without the ponderomotive potential, in which case: $\eta^{(1)} = 0.05$, $\eta^{(2)} = 0.11$, $\eta^{(3)} = 0.22$.

The above effect of the spatial inhomogeneity of a strong light field on electron dynamics gives rise to a greater asymmetry of photon emission and absorption processes. This asymmetry is also exhibited by the angular distributions. The effect is particularly clear for electrons with energies of order U_0.

Figure 1 shows the spatial distribution of total cross sections for the model potential $V(r) = -V_0 \exp(-r^2/a^2)$. It was calculated in the Born approximation for which $a^2 V_0 \ll 1$, $qa \lesssim 1$. The intensity distribution was assumed to be Gaussian. The cross sections calculated without taking into account the ponderomotive acceleration effects are small at the center of the focus. This behavior of the SBE cross section with increasing intensity is typical for electrons scattered by a potential extending to a distance of the order of the Bohr radius.[20] The effect of the field inhomogeneity on the SBE cross section becomes the dominant factor in the central part of the focus, i.e., for $x_0 \lesssim R$. The cross section for scattering with the absorption of a photon has a maximum in this region, whereas the cross section for stimulated emission is zero. The physical reason for the increase in the absorption cross section at the center of the focal region is the slowing down of the electron in the ponderomotive barrier which, first, increases the probability of finding the electron at the center of the focus, $|\psi_{p_i}(x_0)|^2 \sim p_{ix}{}^{-1}(x_0)$, and second, reduces the argument of the Bessel function. For photon emission, the suppression effect is more significant than the slowing down. Experimentally, the scattered electrons are recorded at a large distance from the focus, and the scattered flux consists of particles that have been scattered by different atoms. Since the cross section depends on the position of the atoms, the electron counting rate is expressed in terms of the cross section averaged over the positions of the atoms[8]:

$$\left\langle \frac{d\sigma^{(n)}}{do_{\mathbf{p}_f}} \right\rangle = \frac{1}{2R} \int dx_0 \frac{d\sigma^{(n)}(x_0)}{do_{\mathbf{p}_f}}. \tag{29}$$

The count rate is obtained by multiplying (29) by the flux density of incident electrons, the density of atoms, and the volume of the focal region $2RS$, where S is the area of the focus in the yz plane. The total cross section $\langle \sigma^{(n)} \rangle$ averaged over the focal region can be obtained in the similar way.

Figure 2 shows the asymmetric SBE spectrum for electrons with energies just greater than U_0. We note that for the field parameters and electron energies employed, the emission channels ($n > 0$) should be absent altogether from Fig. 2 in the case of a uniform field with $U(x_0) \equiv U_0$. Allowance for the spatial inhomogeneity is found to smooth out the picture, but the resulting spectrum is much more asymmetric,

$$\eta^{(n)} = (\langle \sigma^{(-n)} \rangle - \langle \sigma^{(n)} \rangle)/(\langle \sigma^{(-n)} \rangle + \langle \sigma^{(n)} \rangle) \quad (n>0),$$

for processes that result in the emission and absorption of the same number of photons as compared with the situation in the absence of the ponderomotive potential.

CONCLUSION

The dynamic manifestations of the spatial inhomogeneity of a strong light field produce a significant modification of the SBE cross section for electrons with energies of the order of the height U_0 of the ponderomotive barrier, i.e., the mean energy of oscillations in the light field.

It is important to remember that, even if the energy of the free electron incident on the focal region is so high that scattering by an atom in the absence of the field can be calculated in the Born approximation, the slowing down of the electron by a sufficiently high barrier U_0 is so great that the condition for the validity of the Born approximation is no longer satisfied. In the case of Coulomb scattering, the SBE cross section for slow electrons can be calculated using the three-stage scheme (see Sec. 3), with the quasiclassical cross sections for a uniform field, obtained in Ref. 3.

The situation with $\varepsilon_i \approx U_0$ is particularly convenient for experimental investigation of the asymmetry of absorption and emission processes. The number of emitted or absorbed photons per scatter is determined by the argument of the Bessel function, λ, which can be written in the form

$$\lambda^2 = 8(U_0/\omega)(\varepsilon_i/\omega) g^2, \tag{30}$$

where g is the angular factor.

For $U_0 \sim \varepsilon_i \lesssim \omega (\lambda \lesssim 1)$, the SBE spectrum consists of a small number of peaks of the same height. In the optical range ($\omega \sim 1$ eV), this requires an intensity $I \sim 10^{13}$ W/cm^2. Ionization by electrons with energies in excess of the barrier height[10] can serve as a source of monoenergetic electrons with $\varepsilon_i \sim 1$ eV. In the experiment described in Ref. 9, $\lambda \sim 1$ was achieved using low intensity CO_2 laser radiation ($U_0/\omega \sim 10^{-3}$) and fast electrons ($\varepsilon/\omega \sim 10^2$). Under these conditions $\varepsilon_i \gg U_0$, and the effect of the gradient force on the motion of the electron is insignificant.

For energies $\varepsilon_i \sim U_0 \gg \omega$ ($\lambda \gg 1$), the spectrum contains a large number of peaks of comparable height, and the effect can be seen as an asymmetry of the envelope of a large number of peaks even when the electron energy resolution is less than the photon energy.

The well known difficulties encountered in performing such experiments are due to the fact that appreciable ionization of the atomic target can occur in the intensities that are necessary in these experiments. Special measures must therefore be taken to separate scattered electrons from photoelectrons. Alternatively, the target can be a beam of ions with a high double ionization potential.

The above suppression of stimulated bremsstrahlung emission channels may become significant in connection with the heating of electrons by a strong light field.

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