Electromagnetic-wave diffraction by multilayer media with rough interfaces

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The Green's function formalism is used for a novel formulation of the problem of electromagnetic-wave diffraction by multilayer media with rough interfaces. The problem is reduced to a solution of the traditional quantum-mechanics problem of T-matrix scattering with $T = V + VG_0T$. The derived equation is solved in an approximation linear in the amplitudes of the roughnesses.

An all-inclusive theoretical analysis of strong resonant electromagnetic (EM) effects discovered recently on rough interfaces of condensed media, such as surface-enhanced Raman scattering,[1] anomalous suppression of the specular component,[2] wave localization and their backward scattering,[3] and others call for the use of the most powerful formalism of modern theoretical physics—the Green's function method. However, even the first attempt to solve the problem of EM wave diffraction by the Green's function method[4] revealed a number of problems that hinder further development of this method, namely, the singular behavior of the Green's function and the discontinuous behavior of its constituent fields. In the general case this introduces inevitably in the solution (even in the first term of the expansion in powers of the roughness amplitude) mathematically ill-posed expressions such as $\int dx\theta(x)\delta(x)$, where $\theta(x)$ is the Heaviside step function and $\delta(x)$ the Dirac function.[5] It is this circumstance which prompted the authors of Ref. 6 to confine themselves, in the solution of the problem of EM-wave diffraction by rough interfaces of three-layer media by the Green's function method, to the case of only normal incidence, where indeterminacies of this type do not occur, at least in the linear terms of the expansion.

A complete solution of the problem for the case of EM-wave diffraction by one rough interface of homogeneous isotropic media was given in Ref. 7, where the problem was reduced, for arbitrary interface roughnesses, to a solution of a traditional quantum-mechanics equation for the scattering T-matrix. The modified Green's function contained in this equation is free of singularities, the basis functions for this function are continuous on the interface, while the iterative solution is free of mathematically ill-posed expressions in any order in the perturbation.

The aim of the present paper is to generalize the results of Ref. 7 to include multilayer media containing an arbitrary number of rough interfaces. In order not to clutter up the exposition with many subscripts and symbols, we consider first three-layer media with rough interfaces (Sec. 1), followed by a generalization to multilayer media (Sec. 3). In Secs. 2 and 4 we obtain, by way of example, a solution of the equation for the T matrix in an approximation linear in the roughness amplitudes. The results are discussed in Sec. 5. Some of the intermediate equations are relegated to Appendices A and B.

It is assumed below that the media in contact are homogeneous, isotropic, without spatial dispersion, and in the general case with a dissipation $\operatorname{Im}\varepsilon_j(\omega)\geqslant 0$, where ε_j is the dielectric constant of the layer $j = 1,2,..., n$ (Fig. 1) and ω is the frequency of the incident monochromatic EM wave.

1. FORMULATION OF PROBLEM, THREE-LAYER MEDIUM

The propagation of a monochromatic EM wave $\mathbf{E}(\mathbf{r})e^{-i\omega t}$ in a three-layer medium (Fig. 1a) having a dielectric constant

$$\varepsilon(\mathbf{r})=\varepsilon_1+(\varepsilon_2-\varepsilon_1)\theta(h_1(\boldsymbol{\rho})-z)+(\varepsilon_3-\varepsilon_2)\theta(h_2(\boldsymbol{\rho})-z)$$

and containing interfaces $z = h_j(\boldsymbol{\rho})$ ($\boldsymbol{\rho} = (x,y)$ is a two-dimensional vector lying in the plane $z = 0$) is determined by solving the macroscopic-electrodynamics equations

$$(\mathrm{rot\,rot}-k_0^2\varepsilon_z)\mathbf{E}(\mathbf{r})=v(\mathbf{r})\mathbf{E}(\mathbf{r}), \tag{1}$$

where $k_0 = \omega/c$ is the wave number in vacuum, ε_z is the dielectric constant in the absence of perturbations of the interfaces

$$\varepsilon_z=\varepsilon_1+(\varepsilon_2-\varepsilon_1)\theta(\bar{h}_1-z)+(\varepsilon_3-\varepsilon_2)\theta(\bar{h}_2-z),$$

$\bar{h}_j = \langle h_j(\boldsymbol{\rho})\rangle$, $\langle ...\rangle$ is the average over the ensemble of the rough surfaces, and $v(\mathbf{r})$ is the perturbation of the problem, determined by the change $\Delta\varepsilon(\mathbf{r}) = \varepsilon(\mathbf{r}) - \varepsilon_z$, of the dielectric constant of the medium in the region of the rough layer,

$$v(\mathbf{r})=k_0^2\Delta\varepsilon(\mathbf{r})=(k_2^2-k_1^2)\lambda_1(\mathbf{r})+(k_3^2-k_2^2)\lambda_2(\mathbf{r}),$$
$$\lambda_j(\mathbf{r})=\theta(h_j(\boldsymbol{\rho})-z)-\theta(\bar{h}_j-z),$$

and $k_j = k_0\varepsilon_j^{1/2}$ is the wave number in the layer j.

We rewrite Eq. (1) in integral form

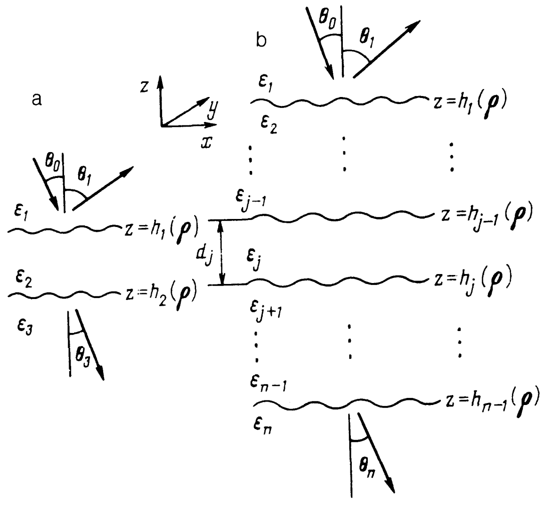

FIG. 1. Diffraction of electromagnetic waves in three-layer (a) and in n-layer (b) media.

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$$\mathbf{E}(\mathbf{r})=\mathbf{E}_0(\mathbf{r})+\int \hat{G}(\mathbf{r},\mathbf{r}')v(\mathbf{r}')\mathbf{E}(\mathbf{r}')d^3\mathbf{r}', \tag{2}$$

using for this purpose the Green's function $\hat{G}(\mathbf{r},\mathbf{r}')$ of a three-layer medium satisfying the equation

$$(\mathrm{rot\,rot}-k_0^2\varepsilon_z)\hat{G}(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'). \tag{3}$$

A solution of Eq. (3) is known.[6,8] It contains, however, a very cumbersome expanded expression of the tensor $G_{\alpha\beta}(\mathbf{b},z,z')$ determined by the Fourier transform of the function $G_{\alpha\beta}(\mathbf{r},\mathbf{r}')$ with respect to the difference argument, for different values of the subscripts $\alpha,\beta=x,y,z$ and for regions of space $z,z'\in V_1$, V_2, V_3, (V_j is the region belonging to the layer j). This hinders the algebraic transformations and makes the subsequent generalizations difficult. We present a more compact form of the function $G_{\alpha\beta}(\mathbf{b},z,z')$, which will be generalized below to include multilayer media. In dyadic notation we obtain

$$\hat{G}(\mathbf{b},z,z')=-\frac{\widehat{\mathbf{z}\mathbf{z}}}{k_0^2\varepsilon_z}\delta(z-z')+\hat{G}'(\mathbf{b},z,z'), \tag{4}$$

$$\hat{G}'(\mathbf{b},z,z')=\frac{i}{2\eta_1}\sum_{\gamma=s,p}t_\gamma[\mathbf{E}_{n\gamma}^+(\mathbf{b},z)\mathbf{E}_{1\gamma}^-(\mathbf{b},z')\theta(z-z')$$
$$+\mathbf{E}_{1\gamma}^+(\mathbf{b},z)\mathbf{E}_{n\gamma}^-(\mathbf{b},z')\theta(z'-z)], \tag{5}$$

where $n=3$ for the considered case of three-layer media. The system of basic functions $\mathbf{E}_{m\gamma}^{\pm}(\mathbf{b},z)$, where $m=1$, n and $\gamma=s,p$, consists of independent solutions of the homogeneous equation (1) with zero right-hand part, and is given together with other symbols in Appendix A. The amplitude transmission coefficients $t_\gamma=t_\gamma(\mathbf{b})$ are specified by Eqs. (A4). Normalization of the functions (A1) and the form of (5) are mutually consistent, and the poles of the Green's function (5), which correspond to the eigenmodes of the medium, enter only in the transmission coefficient t_γ. The functions $\mathbf{E}_{m\gamma}^{\pm}(\mathbf{b},z)$ made no pole singularities and specify the polarization structure of (5).

Direct utilization of expressions (4) and (5) for an iteration solution of Eq. (2) by expanding the field $\mathbf{E}(\mathbf{r})$ in powers of the roughness-amplitude fluctuations $\tilde{h}_j(\boldsymbol{\rho})=h_j(\boldsymbol{\rho})-\bar{h}_j$ leads in general, just as for two-layer media,[7] to mathematicially ill-posed expressions. Equation (2) is thus hardly suitable for analysis.

Separating in (2) the singular term (4), and carrying out transformations similar to those made in Ref. 7, we obtain an equivalent equation for the scattering T-matrix

$$\hat{T}(\mathbf{b},\mathbf{b}_0,z_1,z_2)=\hat{V}(\mathbf{b}-\mathbf{b}_0,z_1)\delta(z_1-z_2)$$
$$+\int d^2\mathbf{b}'\,dz'\,\hat{V}(\mathbf{b}-\mathbf{b}',z_1)\hat{G}_0(\mathbf{b}',z_1,z')\hat{T}(\mathbf{b}',\mathbf{b}_0,z',z_2), \tag{6}$$

where $\hat{G}_0(\mathbf{b},z,z')$ is a modified Green's function

$$\hat{G}_0(\mathbf{b},z,z')=\frac{i}{2\eta_1}\sum_{\gamma=s,p}t_\gamma[\mathbf{X}_{n\gamma}^+(\mathbf{b},z)\mathbf{X}_{1\gamma}^-(\mathbf{b},z')\theta(z-z')$$
$$+\mathbf{X}_{1\gamma}^+(\mathbf{b},z)\mathbf{X}_{n\gamma}^-(\mathbf{b},z')\theta(z'-z)], \tag{7}$$

that contains no singular terms and consists only of continuous components of the field $\mathbf{E}$

$$\mathbf{X}_{ms}^{\pm}=\mathbf{E}_{ms},\quad \mathbf{X}_{mp}^{\pm}=\varepsilon_z\mathbf{E}_{mz}\pm\mathbf{E}_{mb}\equiv\mathbf{D}_{mz}\pm\mathbf{E}_{mb}, \tag{8}$$

where $\mathbf{E}_{ms,b,z}$ are the components of the initial fields $\mathbf{E}_{m\gamma}^{\pm}$ defined in (A5). The perturbation $\hat{V}(\mathbf{q},z)$ in (6) is the Fourier transform of $\hat{V}(\mathbf{r})=\hat{V}(\boldsymbol{\rho},z)$ with respect to the argument $\boldsymbol{\rho}$, where

$$\hat{V}(\mathbf{r})=(k_2^2-k_1^2)\left(\hat{P}_\perp+\frac{1}{\varepsilon_1\varepsilon_2}\hat{P}_\parallel\right)\lambda_1(\mathbf{r})$$
$$+(k_3^2-k_2^2)\left(\hat{P}_\perp+\frac{1}{\varepsilon_2\varepsilon_3}\hat{P}_\parallel\right)\lambda_2(\mathbf{r}), \tag{9}$$

$\hat{P}_\parallel=\hat{z}\hat{z}$ and $\hat{P}_\perp=1-\hat{z}\hat{z}$ are the operators for projection on the normal direction z and on the plane $z=\mathrm{const}$, respectively.

The matrix elements $T_{\alpha\beta}^{lm}(\mathbf{b},\mathbf{b}_0)$ of the solution of Eq. (6) for the scattering operator $\hat{T}(\mathbf{b},\mathbf{b}_0,z,z')$

$$T_{\alpha\beta}{}^{lm}(\mathbf{b},\mathbf{b}_0)=\int_{-\infty}^{\infty}dz\,dz'\,\mathbf{X}_{l\alpha}^-(\mathbf{b},z)\hat{T}(\mathbf{b},\mathbf{b}_0,z,z')\mathbf{X}_{m\beta}^+(\mathbf{b}_0,z'), \tag{10}$$

taken over the same system of basis functions (8) on which the modified Green's function is constructed, determine uniquely the amplitude of the diffracted waves $\mathbf{E}(\mathbf{r})$ in the media l, $m=1$, n that border the layers ($n=3$ for a three-layer medium).

$$\mathbf{E}(\mathbf{r})=\sum_{\beta=s,p}C_\beta\mathbf{E}_{1\beta}^+(\mathbf{b}_0,z)e^{i\mathbf{b}_0\boldsymbol{\rho}}+i\sum_{\alpha,\beta=s,p}C_\beta\int\frac{d^2\mathbf{b}}{2\eta_1}t_\alpha e^{i\mathbf{b}\boldsymbol{\rho}}$$
$$\times\begin{bmatrix}\exp[i\eta_1(z-\bar{h}_1)]\,\hat{\mathbf{e}}_{1-}^{\alpha}T_{\alpha\beta}^{11}(\mathbf{b},\mathbf{b}_0), & z\geqslant h_{1,\,max},\\ \exp[-i\eta_n(z-\bar{h}_{n-1})]\,\hat{\mathbf{e}}_{n+}^{\alpha}T_{\alpha\beta}^{n1}(\mathbf{b},\mathbf{b}_0), & z\leqslant h_{n-1,\,min}\end{bmatrix}, \tag{11}$$

where C_β are arbitrary constants that determine the intensity and the polarization state of the incident EM waves, $\mathbf{e}_{j\pm}^{\alpha}$ are the diffracted-wave polarization vectors (A3), and η_j are the projections of the wave vectors (A2) on the direction of the normal in the layer j. It is assumed in (11), without loss of generality, that the incident plane wave is specified in layer 1 (Fig. 1a).

Equation (6) and expressions (10) and (11) solve completely the problem of the diffraction of EM waves in a three-layer medium by the rough interfaces, but in contrast to the initial equation (2) they determine the field only in the media that border the layer. On going from (2) to (6), (10), and (11) no restrictions whatever were imposed on the character of the roughnesses and on the layer thickness.

Equation (6) for the T matrix agrees is form with the analogous equation of Ref. 7 for a contact between two semi-infinite media with one rough interface. In contrast to Ref. 7, however, the perturbation of $\hat{V}(\mathbf{r})$ in (6) in the presence of several rough interfaces is not reduced to a scalar form, but is given by the tensor (9).

2. LINEAR APPROXIMATION FOR A THREE-LAYER MEDIUM. ANGLE SPECTRUM

In the limiting case of not too rough interfaces, Eq. (6) for the T matrix can be iterated by expanding (9) and (6) in powers of the roughness-fluctuation amplitudes $\tilde{h}_j(\boldsymbol{\rho})=h_j(\boldsymbol{\rho})-\bar{h}_j$ (Ref. 7). In the approximation linear in $\tilde{h}_j$ we obtain

$$\hat{T}(\mathbf{b}, \mathbf{b}_0, z_1, z_2)$$

$$=(k_2^2-k_1^2)\left(\hat{P}_\perp+\frac{1}{\varepsilon_1\varepsilon_2}\hat{P}_\parallel\right)\delta(z_1-\tilde{h}_1)\delta(z_2-\tilde{h}_1)\tilde{h}_{1,\mathbf{b}-\mathbf{b}_0}$$

$$+(k_3^2-k_2^2)\left(\hat{P}_\perp+\frac{1}{\varepsilon_2\varepsilon_3}\hat{P}_\parallel\right)\delta(z_1-\tilde{h}_2)\delta(z_2-\tilde{h}_2)\tilde{h}_{2,\mathbf{b}-\mathbf{b}_0},$$

where $\tilde{h}_{j,\mathbf{q}}$ is the Fourier transform of the profile $\tilde{h}_j(\boldsymbol{\rho})$. The bilinear combination of $\delta(z)$ functions eliminates all the integration with respect to z in the matrix elements (10). As a result, the $T^{lm}_{\alpha\beta}(\mathbf{b}, \mathbf{b}_0)$ contain only terms made up of scalar products of the form

$$\mathbf{X}_{l\alpha}^-(\mathbf{b}, \tilde{h}_j)\left(\hat{P}_\perp+\frac{1}{\varepsilon_j\varepsilon_{j+1}}\hat{P}_\parallel\right)\mathbf{X}_{m\beta}^+(\mathbf{b}_0, \tilde{h}_j).$$

The projection operators $\hat{P}_\perp$ and $\hat{P}_\parallel$ renormalize the $\hat{\mathbf{z}}$ components of the fields $\mathbf{X}^\pm_{m\alpha}$ and leave the $\hat{\mathbf{s}}$ and $\hat{\mathbf{b}}$ components unchanged. With allowance for (8) it is convenient to introduce a local basis on the jth interface

$$\mathbf{Z}_{ms}^\pm(\mathbf{b}, \tilde{h}_j)=\mathbf{E}_{ms}(\mathbf{b}, \tilde{h}_j),\ \mathbf{Z}_{mp}^{\pm}(\mathbf{b}, \tilde{h}_j)$$
$$=(\varepsilon_j\varepsilon_{j+1})^{-1/2}\mathbf{D}_{mz}(\mathbf{b}, \tilde{h}_j)\pm\mathbf{E}_{mb}(\mathbf{b}, \tilde{h}_j). \qquad (12)$$

The matrix elements (10) reduce then finally, in the approximation linear in $\tilde{h}_j$, to the form

$$T_{\alpha\beta}^{lm}(\mathbf{b}, \mathbf{b}_0)=(k_2^2-k_1^2)(\mathbf{Z}_{l\alpha}^-(\mathbf{b}, \tilde{h}_1)\mathbf{Z}_{m\beta}^+(\mathbf{b}_0, \tilde{h}_1))\tilde{h}_{1,\mathbf{b}-\mathbf{b}_0}$$
$$+(k_3^2-k_2^2)(\mathbf{Z}_{l\alpha}^-(\mathbf{b}, \tilde{h}_2)\mathbf{Z}_{m\beta}^+(\mathbf{b}_0, \tilde{h}_2))\tilde{h}_{2,\mathbf{b}-\mathbf{b}_0}. \quad (13)$$

Expressions for the fields of the local basis $\mathbf{Z}^\pm_{m\alpha}(\mathbf{b}\ \tilde{h}_j)$ follow directly from Eqs. (12), (8), (A5), and (A1):

$$\mathbf{Z}_{1s}^\pm(\mathbf{b}, \tilde{h}_1)=E_3^+\hat{\mathbf{s}},\quad \mathbf{Z}_{1s}^\pm(\mathbf{b}, \tilde{h}_2)=\hat{\mathbf{s}},$$

$$\mathbf{Z}_{3s}^\pm(\mathbf{b}, \tilde{h}_1)=\hat{\mathbf{s}},\quad \mathbf{Z}_{3s}^\pm(\mathbf{b}, \tilde{h}_2)=E_1^+\hat{\mathbf{s}},$$

$$\mathbf{Z}_{1p}^\pm(\mathbf{b}, \tilde{h}_1)=\frac{k_3 b}{k_1k_2}H_3^+\hat{\mathbf{z}}\pm\frac{\eta_3}{k_3}H_3^-\hat{\mathbf{b}},$$

$$\mathbf{Z}_{1p}^\pm(\mathbf{b}, \tilde{h}_2)=\frac{b}{k_2}\hat{\mathbf{z}}\pm\frac{\eta_3}{k_3}\hat{\mathbf{b}},$$

$$\mathbf{Z}_{3p}^\pm(\mathbf{b}, \tilde{h}_1)=\frac{b}{k_2}\hat{\mathbf{z}}\mp\frac{\eta_1}{k_1}\hat{\mathbf{b}},$$

$$\mathbf{Z}_{3p}^\pm(\mathbf{b}, \tilde{h}_2)=\frac{k_1 b}{k_2k_3}H_1^+\hat{\mathbf{z}}\mp\frac{\eta_1}{k_1}H_1^-\hat{\mathbf{b}}.$$

The character of the change of the roughness profile (whether it is deterministic or stochastic) was not established in the derivation of (13). In the case of statistically rough interfaces, by substituting (13) and (11), calculating the EM energy flux density in the media bordering the layers, and averaging the result over the ensemble of rough surfaces with allowance for

$$\langle\tilde{h}_{i\mathbf{q}_1}\tilde{h}_{j\mathbf{q}_2}\rangle=S_{ij}(\mathbf{q}_1)\delta(\mathbf{q}_1+\mathbf{q}_2), \qquad (14)$$

where $S_{ij}(\mathbf{q})$ are the spectral densities, we get for the angle spectrum $dP_{\alpha\beta}/d\Omega_m$ of the EM wave scattered into the medium $m = 1, 3$ and having a polarization $\beta = s, p$ upon incidence of a wave with polarization $\beta = s, p$, the following expression

$$\frac{dP_{\alpha\beta}}{P_{0z}\,d\Omega_m}=4\eta_{10}\eta_m k_m^2\{S_{11}(\mathbf{b}-\mathbf{b}_0)\,|Q^m_{1\alpha\beta}|^2$$
$$+S_{22}(\mathbf{b}-\mathbf{b}_0)\,|Q^m_{2\alpha\beta}|^2+2\,\mathrm{Re}[S_{12}(\mathbf{b}-\mathbf{b}_0)Q^m_{1\alpha\beta}\bar{Q}^m_{2\alpha\beta}]\}, \qquad (15)$$

where the overbar denotes complex conjugation, P_{0z} is the normal component of the EM energy flux incident on layer 2 at an angle θ_0; $d\Omega_m = \sin\theta_m d\theta_m d\varphi$ is the solid angle of scattering in the medium $m = 1, 3$; θ_m is the scattering angle in the medium m; $b = k_m \sin\theta_m$; $b_0 = k_1 \sin\theta_0$; φ is the angle between the unit vectors $\hat{\mathbf{b}}$ and $\hat{\mathbf{b}}_0$. Expression (15) is meaningful for $m = 3$ only if the medium 3 is nondissipative and transparent, i.e., if $k_3^2 > 0$. The functions $Q^m_{j\alpha\beta}$ are given in Appendix B.

Comparison of (15) with earlier results[6,9,10] shows that the angle spectrum of an electromagnetic wave scattered into medium 1, in particular cases of normal incidence and for arbitrary S_{ij}, agrees with the result of Ref. 6; for oblique incidence with fully correlated interface roughnesses, when $S_{ij}(\mathbf{q}) = S(\mathbf{q})$, it agrees with Ref. 9. If the interface roughness is assumed uncorrelated, i.e., $S_{12} = S_{21} = 0$, or if the layer is assumed thin, $\eta_2 d \ll 1$, expression (15) for the reflected wave agrees with the result of Ref. 10 provided that the signs of the terms containing the factors pp' (in the notation of Ref. 10) are reversed in Eqs. (A1) and (A3) of that reference for the pp polarization. To confirm the validity of the equations obtained above we note that in the particular case of two-layer media, when for example $\varepsilon_1 = \varepsilon_2$ or $\varepsilon_2 = \varepsilon_3$, or else $\tilde{h}_1(\boldsymbol{\rho}) = \tilde{h}_2(\boldsymbol{\rho})$ and $d\to 0$, Eqs. (15) are transformed into the known results for two-layer media.[7,11] The expressions given in Ref. 1 do not contain such a limiting transition. In addition, Eqs. (15) are a particular case of more general expressions for n-layer medium, which we consider below.

3. FORMULATION OF PROBLEM, MANY-LAYER MEDIUM

The foregoing formulation (6)–(11) of the problem of diffraction of EM waves in a three-layer medium can be directly generalized to include many-layer media with arbitrary number of interfaces $z = h_j(\boldsymbol{\rho})$ (Fig. 1b). The crux here is the statement that the Green's function (4), (5) will be a solution of Eq. (3) also for an n-layer medium having a dielectric constant

$$\varepsilon_z=\varepsilon_1+\sum_{j=1}^{n-1}(\varepsilon_{j+1}-\varepsilon_j)\theta(\tilde{h}_j-z). \qquad (16)$$

This can be verified directly by substituting (4), (5), and (16) in (3). The set of basis functions $\mathbf{E}^\pm_{m\gamma}(\mathbf{b}, z)$, $m = 1,n$ for an n-layer medium, which enters in (5), is given by

$$\mathbf{E}_{m\gamma}^\pm(\mathbf{b}, z)$$
$$=\hat{\mathbf{e}}_{j\mp}{}^{\gamma}a_{j\gamma}^+\exp[i\eta_j(z-\tilde{h}_j)]+\hat{\mathbf{e}}_{j\pm}{}^{\gamma}a_{j\gamma}^-\exp[-i\eta_j(z-\tilde{h}_j)], \qquad (17)$$
$$\tilde{h}_j\leqslant z\leqslant\tilde{h}_{j-1},\quad j=1, 2, \ldots, n,$$

where the coefficients $a^\pm_{j\gamma}$ are connected by the recurrence relations

$$\begin{pmatrix}a^+_{j-1,\gamma}\\ a^-_{j-1,\gamma}\end{pmatrix}=\theta^\gamma_{j-1,j}H_j\begin{pmatrix}a^+_{j\gamma}\\ a^-_{j\gamma}\end{pmatrix},$$

$$(j=n, n-1, \ldots, 2), \quad \begin{pmatrix} a_{n\gamma}^{+} \\ a_{n\gamma}^{-} \end{pmatrix} = H_n^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{18}$$

in the functions $\mathbf{E}_{i\gamma}^{\pm}(\mathbf{b}, z)$ and

$$\begin{pmatrix} a_{j+1,\gamma}^{+} \\ a_{j+1,\gamma}^{-} \end{pmatrix} = \theta_{j+1,j}^{\gamma} H_j^{-1} \begin{pmatrix} a_{j\gamma}^{+} \\ a_{j\gamma}^{-} \end{pmatrix},$$

$$(j=1, 2, \ldots, n-1), \quad \begin{pmatrix} a_{1\gamma}^{+} \\ a_{1\gamma}^{-} \end{pmatrix} = H_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{19}$$

in the functions $\mathbf{E}_{n\gamma}^{\pm}(\mathbf{b}, z)$. The 2×2 matrices θ_{ij}^{γ}, ($\gamma = s, p$) and H_j are defined as

$$\theta_{ij}^{s} = \frac{1}{2\eta_i} \begin{pmatrix} \eta_i+\eta_j & \eta_i-\eta_j \\ \eta_i-\eta_j & \eta_i+\eta_j \end{pmatrix},$$

$$H_j = \begin{pmatrix} \exp(i\eta_j d_j) & 0 \\ 0 & \exp(-i\eta_j d_j) \end{pmatrix},$$

$$\theta_{ij}^{p} = \frac{(\varepsilon_i\varepsilon_j)^{-1/2}}{2\eta_i} \begin{pmatrix} \eta_i\varepsilon_j+\eta_j\varepsilon_i & \eta_i\varepsilon_j-\eta_j\varepsilon_i \\ \eta_i\varepsilon_j-\eta_j\varepsilon_i & \eta_i\varepsilon_j+\eta_j\varepsilon_i \end{pmatrix},$$

where $d_j = \bar{h}_{j-1} - \bar{h}_j$ is the thickness of layer j having a dielectric constant ε_j. The unit vectors $\hat{\mathbf{e}}_{j\pm}^{\gamma}$ are defined by expressions (A3). The initial values of the components $a_{n\gamma}^{\pm}$ in (18) and $a_{1\gamma}^{\pm}$ in (19) normalize the functions (17) to correspond to the form of the Green's function (4), (5), where now $t_\gamma = 1/a_{1\gamma}^{-}$ is the transmission coefficient of the n-layer medium for the passage of a $\gamma = s$-polarized or p-polarized EM wave from layer 1 to layer n.

The boundaries $\bar{h}_0$ and $\bar{h}_n$ which enter formally in (17) and are absent from the initial layered system (Fig. 1b) were introduced to conform with (17). For the chosen initial values of the coefficients $a_{n\gamma}^{\pm}$ and $a_{1\gamma}^{\pm}$ in (18) and (19), the fields (17) are independent of the locations of the boundaries $\bar{h}_0$ and $\bar{h}_n$ so that we can put $\bar{h}_0 = +\infty$, $\bar{h}_n = -\infty$.

Since the solution (4), (5) of Eq. (3) for the Green's function of an n-layer medium is now known, the integral equation (2) for the field is valid also for a many-layer medium, but the perturbation potential $v(\mathbf{r})$ in (2) takes now the form of a sum of pertubations over all the rough interfaces:

$$v(\mathbf{r}) = \sum_{j=1}^{n-1} (k_{j+1}^2 - k_j^2) \lambda_j(\mathbf{r}).$$

The algebraic transformations that effect the transition from the singular integral equation (2) for the field into the equation (6) for the scattering T-matrix remain in force for any number of interfaces, since they are connected only with elimination of the singular component of the Green's function (4) from the integral term of Eq. (2) and with subsequent algebraic transformations of Eq. (8). Expressions (6)–(11) are thus valid also for n-layer media with rough interfaces, but the perturbation (9) must be replaced by a sum over all the rough interfaces:

$$\hat{V}(\mathbf{r}) = \sum_{j=1}^{n-1} (k_{j+1}^2 - k_j^2) \left(\hat{P}_\perp + \frac{1}{\varepsilon_j\varepsilon_{j+1}} \hat{P}_\parallel \right) \lambda_j(\mathbf{r}). \tag{20}$$

Equation (6) for the scattering T-matrix of many-layer media is fully equivalent to the initial equation (1) [or (2)] and does not require that the roughnesses of the interface be small or smooth. In accord with (11), however, its solution determines the field only in the media bordering the layers, $j = 1$ or n.

We consider below by way of example the solution of Eq. (6) for n-layer media in an approximation linear in $\tilde{h}_j$, and obtain expressions for the angle spectrum of the diffracted reflected and transmitted EM waves.

4. LINEAR APPROXIMATION FOR AN n-LAYER MEDIUM. ANGLE SPECTRUM

Substituting (20) in (6) and solving the latter by iteration up to terms linear in $\tilde{h}_j$ (expanding also λ_j in powers of $\tilde{h}_j$ up to terms of the same order of smallness), we obtain for the T-matrix a solution similar to that given in Sec. 2 for a three-layer medium, but containing now a sum over all the rough interfaces:

$$\hat{T}(\mathbf{b}, \mathbf{b}_0, z_1, z_2)$$

$$= \sum_{j=1}^{n-1} (k_{j+1}^2 - k_j^2) \left(\hat{P}_\perp + \frac{1}{\varepsilon_j\varepsilon_{j+1}} \hat{P}_\parallel \right) \tilde{h}_{j,\mathbf{b}-\mathbf{b}_0} \delta(z_1 - \bar{h}_j) \delta(z_2 - \bar{h}_j).$$

When the matrix elements of (10) are calculated, each term of this sum is transformed, just as in the case of three-layer media, by replacing the **X** basis (7) by the local **Z** basis (12). As a result we obtain for the matrix elements $T_{\alpha\beta}^{lm}(\mathbf{b}, \mathbf{b}_0)$, which determine uniquely in accordance with (11) the components of the diffracted EM waves in the media bordering the layers, in an approximation linear in $\tilde{h}_j$, the expression

$$T_{\alpha\beta}^{lm}(\mathbf{b}, \mathbf{b}_0) = \sum_{j=1}^{n-1} (k_{j+1}^2 - k_j^2) (\mathbf{Z}_{l\alpha}^{-}(\mathbf{b}, \bar{h}_j) \mathbf{Z}_{m\beta}^{+}(\mathbf{b}_0, \bar{h}_j)) \tilde{h}_{j,\mathbf{b}-\mathbf{b}_0}.$$

The functions $\mathbf{Z}_{l\alpha}^{\pm}(\mathbf{b}, \bar{h}_j)$, are expressed, according to (8), (12), and (17), in terms of the initial coefficients $a_{j\gamma}^{\pm}$ of the expansion (17)

$$\mathbf{Z}_{1s}^{\pm}(\mathbf{b}, \bar{h}_j) = \hat{\mathbf{s}}(a_{js}^{-} + a_{js}^{+}), \tag{21}$$

$$\mathbf{Z}_{1p}^{\pm}(\mathbf{b}, \bar{h}_j) = \hat{\mathbf{z}}(a_{jp}^{-} + a_{jp}^{+}) b/k_{j+1} \pm \hat{\mathbf{b}}(a_{jp}^{-} - a_{jp}^{+}) \eta_j/k_j,$$

$$\mathbf{Z}_{ns}^{\pm}(\mathbf{b}, \bar{h}_j) = \hat{\mathbf{s}}(a_{j+1,s}^{-} + a_{j+1,s}^{+}), \tag{22}$$

$$\mathbf{Z}_{np}^{\pm}(\mathbf{b}, \bar{h}_j) = \hat{\mathbf{z}}(a_{j+1,p}^{-} + a_{j+1,p}^{+}) b/k_j \pm \hat{\mathbf{b}}(a_{j+1,p}^{-} - a_{j+1,p}^{+}) \eta_{j+1}/k_{j+1},$$

where $a_{j\gamma}^{\pm}$ are specified in (21) by the recurrence equations (18), and in (22) by Eqs. (19). The basis fields $\mathbf{Z}_{m\gamma}^{\pm}(\mathbf{b}, \bar{h}_j)$ contain only linear combinations of the coefficients $a_{j\gamma}^{\pm}$, in the form $a_{j\gamma}^{-} \pm a_{j\gamma}^{+}$, which have the simple physical meaning of the tangential components of the intensities of the electric

$$E_j^{ms} = \hat{\mathbf{s}}\mathbf{E}_{ms}^{+}(\mathbf{b}, \bar{h}_j), \quad E_j^{mp} = \hat{\mathbf{b}}\mathbf{E}_{mp}^{+}(\mathbf{b}, \bar{h}_j)$$

and magnetic

$$H_j^{ms} = \hat{\mathbf{b}}\mathbf{H}_{ms}^{+}(\mathbf{b}, \bar{h}_j), \quad H_j^{mp} = \hat{\mathbf{s}}\mathbf{H}_{mp}^{+}(\mathbf{b}, \bar{h}_j)$$

fields on the jth interface. Using the introduced definitions of the components $E_j^{m\gamma}$ and $H_j^{m\gamma}$, we obtain for the functions $\mathbf{Z}_{m\gamma}^{\pm}(\mathbf{b}, \bar{h}_j)$

$$\mathbf{Z}_{ms}^{\pm}(\mathbf{b}, \bar{h}_j) = \hat{\mathbf{s}} E_j^{ms}, \tag{23}$$

$$\mathbf{Z}_{mp}^{\pm}(\mathbf{b}, \bar{h}_j) = \hat{\mathbf{z}} H_j^{mp} b/k_0(\varepsilon_j\varepsilon_{j+1})^{1/2} \pm \hat{\mathbf{b}} E_j^{mp}.$$

The fields $E_j^{m\gamma}$ and $H_j^{m\gamma}$ are connected by the recurrence relations

$$E_j^{m\gamma}=(\cos\beta_j+i(Y_{j-1}^{m\gamma}/N_j^\gamma)\sin\beta_j)E_{j-1}^{m\gamma},$$

$$E_1^{ns}=1,\quad E_1^{np}=-\eta_1/k_1, \tag{24}$$

$$E_{j-1}^{m\gamma}=(\cos\beta_j-i(Y_j^{m\gamma}/N_j^\gamma)\sin\beta_j)E_j^{m\gamma},$$

$$E_{n-1}^{1s}=1,\quad E_{n-1}^{1p}=\eta_n/k_n,$$

which follow from (18) and (19), where

$$\beta_j=\eta_j d_j,\quad N_j^s=-\eta_j/k_0,\quad N_j^p=k_0\varepsilon_j/\eta_j,$$
$$Y_j^{m\gamma}=H_j^{m\gamma}/E_j^{m\gamma}$$

is the admittance of the jth boundary and satisfies the relations

$$Y_{j-1}^{m\gamma}=(Y_j^{m\gamma}-iN_j^\gamma\operatorname{tg}\beta_j)/[1-i(Y_j^{m\gamma}/N_j^\gamma)\operatorname{tg}\beta_j],\qquad Y_{n-1}^{1\gamma}=N_n^\gamma,$$

$$Y_j^{m\gamma}=(Y_j^{m\gamma}+iN_j^\gamma\operatorname{tg}\beta_j)/[1+i(Y_{j-1}^{m\gamma}/N_j^\gamma)\operatorname{tg}\beta_j],\qquad Y_1^{n\gamma}=-N_1^\gamma.$$

Substituting the expression for $T_{\alpha\beta}^{lm}$ in (11), normalizing the field $\mathbf{E}(\mathbf{r})$ to unity amplitude of the incident wave, and denoting

$$Q_{j\alpha\beta}^{lm}=\frac{t_\alpha t_{\beta 0}}{4\eta_1\eta_{10}}(k_{j+1}^2-k_j^2)\mathbf{Z}_{l\alpha}^-(\mathbf{b},\bar{h}_j)\mathbf{Z}_{m\beta}^+(\mathbf{b}_0,\bar{h}_j), \tag{25}$$

we obtain for the partial amplitude $E_{\alpha\beta}^{(m)}(\mathbf{b},\mathbf{b}_0)$ of the diffracted wave in the medium $m=1$, in the approximation linear in $\tilde{h}_j$,

$$E_{\alpha\beta}^{(m)}(\mathbf{b},\mathbf{b}_0)=i2\eta_{10}\sum_{j=1}^{n-1}Q_{j\alpha\beta}^{m1}\tilde{h}_{j,\mathbf{b}-\mathbf{b}_0}. \tag{26}$$

Averaging the EM-energy flux density over the ensemble of the rough surfaces and taking (14) and (26) into account, we obtain for the angle spectrum $dP_{\alpha\beta}/d\Omega_m$ of the waves diffracted in layers $m=1$, n

$$\frac{dP_{\alpha\beta}}{P_{0z}\,d\Omega_m}=4\eta_{10}\eta_m k_m^2\sum_{i,j=1}^{n-1}Q_{i\alpha\beta}^{m1}\bar{Q}_{j\alpha\beta}^{m1}S_{ij}(\mathbf{b}-\mathbf{b}_0). \tag{27}$$

The connection between the wave vectors and the scattering angles in the media bordering the layers is similar to that described above [in the text following Eq. (15)] for the case of three-layer media.

Comparison of (25)–(27) with the results of the preceding studies shows that in the particular cases of two- and three-layer media Eqs. (26) and (27) coincide with the results of Refs. 7 and 11 and with Eq. (15). In the general case of n-layer media, Eqs. (25)–(27) agree with the result of Ref. 12 upon correction of a number of misprints [in the notation of Ref. 12, reverse the sign of $N_m^{(0)}$ for a polarization and make the substitution $(\delta\tilde{H}_i - Y_i\delta\tilde{E}_i) \to (\delta\tilde{H}_i - Y_i'\delta\tilde{E}_i)$ in the final Eq. (18)]. To reduce the result (25)–(27) to the form given in Ref. 12 it is necessary to use the recurrence relations (24) and the condition that the Wronskian of the linearly independent soltutions of the homogeneous equation (1) be independent at $v=0$ of the spatial coordinate z.

5. DISCUSSION OF RESULTS. CONCLUSION

Using the Green's-function mathematical formalism, we reformulated the problem of EM-wave diffraction by an arbitrary assembly of rough interface of homogeneous isotropic many-layer media. The problem was reduced to a solution of the traditional quantum-mechanics equation for the scattering T-matrix (6). In symbolic notation we have $T=V+VG_0T$. The modified Green's function G_0 (7), which enters in this equation, is written in covariant form, contains no singular terms, and is not constructed on the system (8) of basis functions containing only field components that are continuous on the interfaces.

A solution of (6) can be obtained by some known method.[13] The matrix elements (10) of the T-matrix, taken in the same system of basis functions on which the Green's function G_0 is constructed, determine uniquely, according to (11), the amplitude of the diffracted waves in the media $z \geqslant \max\{h_1(\boldsymbol{\rho})\}$ or $z\leqslant \min\{h_{n-1}(\boldsymbol{\rho})\}$, bordering the layers. In the derivation of (6), (10), and (11), no restrictions whatever were imposed on the character of the roughnesses (deterministic or stochastic profile, steep or gently sloping, etc.) and on the thicknesses of the layers.

As an example of the use of the developed approach, a solution is obtained in Secs. 2 and 4 of the equation for the T matrix in an approximation linear in the roughness fluctuations, for the cases of three- and n-layer media. Expressions were obtained for the angle spectrum of the diffracted reflected and transmitted EM waves (15) and (27). Comparison with the results of earlier work has revealed in some of them certain misprints whose elimination leads to agreement.

Equation (6) and expressions (10) and (11) serve as the basis for an analysis, outside the scope of perturbation theory, of optical phenomena due to roughness of the interfaces in resonant many-layer systems.

APPENDIX A

The solution of the homogeneous equation (2) in the absence of perturbation of the interfaces ($v=0$) is represented in the form of a linear superposition of independent functions $e^{i\mathbf{bQ}}\mathbf{E}_{m\gamma}^{+}(\mathbf{b},z)$ where the subscripts $m=1$, 3 and $\gamma=s$, p identify the medium in which the incident wave is specified, and the state of its polarization. The functions $\mathbf{E}_{m\gamma}^{\pm}(\mathbf{b},z)$, which form the basis for the construction of the Green's function (8), (9), satisfy the equations

$$\left[\left(i\mathbf{b}\pm\hat{\mathbf{z}}\frac{d}{dz}\right)\times\left(i\mathbf{b}\pm\hat{\mathbf{z}}\frac{d}{dz}\right)\times-k_0^2\varepsilon_z\right]\mathbf{E}_{m\gamma}^{\pm}(\mathbf{b},z)=0,$$

a direct solution of which yields

$$\mathbf{E}_{1\gamma}^{\pm}(\mathbf{b},z)=\begin{cases}\hat{\mathbf{e}}_{1\mp}^{\gamma}a_{1\gamma}^{+}\exp[i\eta_1(z-\bar{h}_1)]+\hat{\mathbf{e}}_{1\pm}^{\gamma}a_{1\gamma}^{-}\exp[-i\eta_1(z-\bar{h}_1)], & z\geqslant\bar{h}_1,\\ \hat{\mathbf{e}}_{2\mp}^{\gamma}a_{2\gamma}^{+}\exp[i\eta_2(z-\bar{h}_2)]+\hat{\mathbf{e}}_{2\pm}^{\gamma}a_{2\gamma}^{-}\exp[-i\eta_2(z-\bar{h}_2)], & \bar{h}_2\leqslant z\leqslant\bar{h}_1,\\ \hat{\mathbf{e}}_{3\pm}^{\gamma}\exp[-i\eta_3(z-\bar{h}_2)], & z\leqslant\bar{h}_2,\end{cases}$$

$$\mathbf{E}^{\pm}_{3\gamma}(\mathbf{b},z)=\begin{cases}\hat{\mathbf{e}}^{\gamma}_{1\mp}\exp[i\eta_1(z-\bar{h}_1)], & z\geqslant\bar{h}_1,\\ \hat{\mathbf{e}}^{\gamma}_{2\mp}b^{+}_{2\gamma}\exp[i\eta_2(z-\bar{h}_1)]+\hat{\mathbf{e}}^{\gamma}_{2\pm}b^{-}_{2\gamma}\exp[-i\eta_2(z-\bar{h}_1)], & \bar{h}_2\leqslant z\leqslant\bar{h}_1,\\ \hat{\mathbf{e}}^{\gamma}_{3\mp}b^{+}_{3\gamma}\exp[i\eta_3(z-\bar{h}_2)]+\hat{\mathbf{e}}^{\gamma}_{3\pm}b^{-}_{3\gamma}\exp[-i\eta_3(z-\bar{h}_2)], & z\leqslant\bar{h}_2,\end{cases} \qquad \text{(A1)}$$

where

$$\eta_j=(k_j^2-b^2)^{1/2},\quad \text{Re, Im}\,\eta_j\geqslant 0 \qquad \text{(A2)}$$

is the normal component of the wave vectors in the medium j, and

$$\hat{\mathbf{e}}^{s}_{j\pm}\equiv\hat{\mathbf{s}}=[\hat{\mathbf{b}},\hat{\mathbf{z}}],\quad \hat{\mathbf{e}}^{p}_{j\pm}\equiv\hat{\mathbf{p}}_{j\pm}=(b\hat{\mathbf{z}}\pm\eta_j\hat{\mathbf{b}})/k_j \qquad \text{(A3)}$$

are the polarization unit vectors of the s- and p-polarized EM waves in the layer. The functions $\mathbf{E}^{\pm}_{m\gamma}(\mathbf{b},z)$ are normalized to unity amplitude of the transmitted wave. The expansion coefficients $a^{\pm}_{j\gamma}$, and $b^{\pm}_{j\gamma}$ follow from the conditions that the fields be continuous on the interfaces

$$a_{1s}^{\pm}=(\eta_1E_3^{+}\mp\eta_3E_3^{-})/2\eta_1,\quad a_{2s}^{\pm}=(\eta_2\mp\eta_3)/2\eta_2,$$
$$a_{1p}^{\pm}=(\varepsilon_3\eta_1H_3^{+}\mp\varepsilon_1\eta_3H_3^{-})/2\eta_1(\varepsilon_1\varepsilon_3)^{1/2},$$
$$a_{2p}^{\pm}=(\varepsilon_3\eta_2\mp\varepsilon_2\eta_3)/2\eta_2(\varepsilon_2\varepsilon_3)^{1/2},$$
$$b_{3s}^{\pm}=(\eta_3E_1^{+}\pm\eta_1E_1^{-})/2\eta_3,\quad b_{2s}^{\pm}=(\eta_2\pm\eta_1)/2\eta_2,$$
$$b_{3p}^{\pm}=(\varepsilon_1\eta_3H_1^{+}\pm\varepsilon_3\eta_1H_1^{-})/2\eta_3(\varepsilon_1\varepsilon_3)^{1/2},$$
$$b_{2p}^{\pm}=(\varepsilon_1\eta_2\pm\varepsilon_2\eta_1)/2\eta_2(\varepsilon_1\varepsilon_2)^{1/2},$$
$$E_j^{+}=[(\eta_2+\eta_j)\exp(-i\eta_2d)+(\eta_2-\eta_j)\exp(i\eta_2d)]/2\eta_2,$$
$$E_j^{-}=[(\eta_2+\eta_j)\exp(-i\eta_2d)-(\eta_2-\eta_j)\exp(i\eta_2d)]/2\eta_j,$$
$$H_j^{+}=[(\varepsilon_j\eta_2+\varepsilon_2\eta_j)\exp(-i\eta_2d)+(\varepsilon_j\eta_2-\varepsilon_2\eta_j)\exp(i\eta_2d)]/2\varepsilon_j\eta_2,$$
$$H_j^{-}=[(\varepsilon_j\eta_2+\varepsilon_2\eta_j)\exp(-i\eta_2d)-(\varepsilon_j\eta_2-\varepsilon_2\eta_j)\exp(i\eta_2d)]/2\varepsilon_2\eta_j,$$

where $d=\bar{h}_1-\bar{h}_2$ is the layer thickness. The functions $E_j^{\pm}$ and $H_j^{\pm}$ are normalized to unity at $d=0$.

If the incident wave is specified in the medium 1, the amplitude reflection and transmission coefficients r_γ and t_γ are given by

$$r_\gamma=a_{1\gamma}^{+}/a_{1\gamma}^{-},\quad t_\gamma=1/a_{1\gamma}^{-}. \qquad \text{(A4)}$$

Another form of the functions (A1), more convenient for succeeding transformations, is

$$\mathbf{E}_{ms}^{\pm}=\mathbf{E}_{ms},\quad \mathbf{E}_{mp}^{\pm}=\mathbf{E}_{mz}\pm\mathbf{E}_{mb}, \qquad \text{(A5)}$$

where $\mathbf{E}_{ms}$ and $\mathbf{E}_{mb}$ are tangential $\hat{\mathbf{s}}$- and $\hat{\mathbf{b}}$-components that are continuous on the interfaces, and $\mathbf{E}_{mz}$ is the normal $\hat{\mathbf{z}}$-component of the field $\mathbf{E}^{+}_{m\gamma}$ and is discontinuous on the interfaces.

APPENDIX B

The functions $Q^{m}_{j\alpha\beta}=Q^{m}_{j\alpha\beta}(\mathbf{b},\mathbf{b}_0)$ determine the contribution of the jth rough interface ($j=1,2$) and the angle spectrum (15) of diffracted s- or p-polarized EM waves in a medium 1,3 (Fig. a) following incidence of a plane s- or p-polarized EM wave at an angle $\theta_0=\arcsin(b_0/k_1)$ in medium 1. The subscript "0" indicates below a dependence of the quantities on $\mathbf{b}_0$, while the absence of the subscript a dependence on $\mathbf{b}$. For example, $\eta_j=\eta_j(\mathbf{b})$ and $\eta_{j0}=\eta_j(\mathbf{b}_0)$, where η_j is specified (A2), etc.

Defining

$$Q^{m}_{j\alpha\beta}=(k^2_{j+1}-k_j^2)\,t_\alpha t_{\beta0}B^{m}_{j\alpha\beta}/4\eta_1\eta_{10},$$

we obtain

$$B^{1}_{1ss}=E_3^{+}E_{30}^{+}\hat{\mathbf{s}}\hat{\mathbf{s}}_0,\quad B^{3}_{1ss}=E_{30}^{+}\hat{\mathbf{s}}\hat{\mathbf{s}}_0,$$
$$B^{1}_{2ss}=\hat{\mathbf{s}}\hat{\mathbf{s}}_0,\quad B^{3}_{2ss}=E_1^{+}\hat{\mathbf{s}}\hat{\mathbf{s}}_0,$$
$$B^{1}_{1pp}=\frac{k_3^2bb_0}{k_1^2k_2^2}H_3^{+}H_{30}^{+}-\frac{\eta_3\eta_{30}}{k_3^2}H_3^{-}H_{30}^{-}\hat{\mathbf{b}}\hat{\mathbf{b}}_0,$$
$$B^{3}_{1pp}=\frac{k_3bb_0}{k_1k_2^2}H_{30}^{+}+\frac{\eta_1\eta_{30}}{k_1k_3}H_{30}^{-}\hat{\mathbf{b}}\hat{\mathbf{b}}_0,$$
$$B^{1}_{2pp}=\frac{bb_0}{k_2^2}-\frac{\eta_3\eta_{30}}{k_3^2}\hat{\mathbf{b}}\hat{\mathbf{b}}_0,\quad B^{3}_{2pp}=\frac{k_1bb_0}{k_3k_2^2}H_1^{+}+\frac{\eta_1\eta_{30}}{k_1k_3}H_1^{-}\hat{\mathbf{b}}\hat{\mathbf{b}}_0,$$
$$B^{1}_{1sp}=E_3^{+}H_{30}^{-}\frac{\eta_{30}}{k_3}\hat{\mathbf{s}}\hat{\mathbf{b}}_0,\quad B^{3}_{1sp}=H_{30}^{-}\frac{\eta_{30}}{k_3}\hat{\mathbf{s}}\hat{\mathbf{b}}_0,$$
$$B^{1}_{2sp}=\frac{\eta_{30}}{k_3}\hat{\mathbf{s}}\hat{\mathbf{b}}_0,\quad B^{3}_{2sp}=E_1^{+}\frac{\eta_{30}}{k_3}\hat{\mathbf{s}}\hat{\mathbf{b}}_0,$$
$$B^{1}_{1ps}=-H_3^{-}E_{30}^{+}\frac{\eta_3}{k_3}\hat{\mathbf{b}}\hat{\mathbf{s}}_0,\quad B^{3}_{1ps}=E_{30}^{+}\frac{\eta_1}{k_1}\hat{\mathbf{b}}\hat{\mathbf{s}}_0,$$
$$B^{1}_{2ps}=-\frac{\eta_3}{k_3}\hat{\mathbf{b}}\hat{\mathbf{s}}_0,\quad B^{3}_{2ps}=H_1^{-}\frac{\eta_1}{k_1}\hat{\mathbf{b}}\hat{\mathbf{s}}_0.$$

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