Electrodynamic properties of interface between media and surface electromagnetic waves on a plane defect of a crystal

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(Submitted 1 February 1989)
Zh. Eksp. Teor. Fiz. **96**, 353–362 (July 1989)

The electromagnetic properties of a transition layer of microscopic thickness at the interface between two media are investigated within the framework of a macroscopic theory. Independent thermodynamic variables are found and an expansion is obtained for the surface electromagnetic energy of the interface between two dielectric crystals. A system of gauge-invariant boundary conditions is derived for the Maxwell equations (electrostatic and magnetostatic) on a two-dimensional transition layer. The interaction of bulk electromagnetic waves (including glancing ones) with a plane two-dimensional crystal defect such as a stacking fault or a twinning plane, and the propagation of surface waves near such a $2D$ defect, are investigated. A possible existence of two types of deeply penetrating surface electromagnetic waves with TM and TE polarization on a plane crystal defect is predicted. The conditions are found under which the dispersion law of the surface electromagnetic wave on a polarizable defective $2D$ layer takes the form $\omega \propto k^{1/2}$ typical of the spectrum of two-dimensional plasmons.

The physical properties of a transition layer between two media are in general different from the properties of the media themselves. Although the thickness of the transition region is as a rule small and is of the order of the interatomic distance, its distinctive electromagnetic properties (e.g., polarization or conductivity) are manifested in macroscopic electrodynamic phenomena such as reflection and refraction of light,[1,2] propagation of surface electromagnetic waves,[3,4] and others. The description of these phenomena does not require the use of microscopic models of the thin transition layer, for its properties can be taken into account in the effective boundary conditions for the electromagnetic fields on both sides of the layer. Such an approach obviates the need for calculating in explicit form the distribution of the electromagnetic fields over the layer thickness, something impossible for a layer of atomic thickness in the framework of the macroscopic theory. In this question, however, one still encounters contradictions between the description of a thin dielectric layer of macroscopic thickness smaller than the wavelength,[3,4] on the one hand, and the phenomenological description of the dielectric properties of a transition layer of vanishingly small thickness,[5] on the other.

We derive in the present paper a system of gauge-invariant macroscopic boundary conditions for the Maxwell equations on a two-dimensional transition layer of microscopic (atomic) thickness in transparent dielectric media. The proposed system of equations does not contradict the description of a transition layer with macroscopic thickness smaller than the wavelength,[3,4] and supplements the phenomenological description of the dielectric properties of a surface.[5] By way of example of the use of the macroscopic-electrodynamics equations for a $2D$ layer we consider the interaction between a plane defect of a dielectric crystal and propagation of surface waves near such a defect. The possibility is predicted of the existence of two (in contrast to the analysis in Ref. 5) types of deeply penetrating electromagnetic surface waves—with TE and TM polarization— on a plane crystal defect. The considered surface waves on a plane crystal defect are analogous with respect to their polarization and dispersion to two fundamental (zero-gap) modes of a symmetric dielectric planar waveguide of macroscopic thickness smaller than the wavelength (see, e.g., Ref. 6).

1. EQUATIONS OF MACROSCOPIC ELECTRODYNAMICS OF A 2D POLARIZABLE OR CONDUCTING TRANSITION LAYER

We begin the discussion of the electromagnetic properties with the electrostatics and magnetostatics of dielectrics. In this case the boundary conditions for the bulk equations in the contacting dielectric media can be obtained by variation of the total volume and surface free energy of the system. To vary the density of the volume free energy F it is convenient to use the known expression[7]

$$\delta F = -(\mathbf{D}\delta\mathbf{E}+\mathbf{B}\delta\mathbf{H})/4\pi, \tag{1}$$

where **E**, **H**, **D**, and **B** are the electric and magnetic fields and inductions. To obtain an expression for the variation of the surface-energy density α of the boundary of the dielectrics it must be recognized that the contribution of the singular properties of the transition layer to the electromagnetic phenomena in the system is relatively small. Therefore the surface energy α can depend only on those independent electromagnetic variables that are defined on the interface in the absence of its singular properties.[1)] In our case these variables are the electric and magnetic field components E_μ and H_μ tangential to the interface and continuous on it, and the normal components D_n and B_n of the induction vectors (the Greek subscripts, unlike the Latin ones, will take on hereafter the values 1 and 2 and will number the coordinate axes in the tangent plane; $D_n = D_i n_i$, where n_i is a unit vector normal to the boundary and directed from medium 1 to medium 2). The variation of the density of the interface surface electromagnetic energy α takes therefore in the general case the form

$$\delta\alpha = (D_n\delta\tilde{\varphi} - C_\mu^*\delta E_\mu + B_n\delta\tilde{\psi} - A_\mu^*\delta H_\mu)/4\pi, \tag{2}$$

i.e., $\alpha = \alpha(\tilde{\varphi}, E_\mu, \tilde{\psi}, H_\mu)$; the meaning of the variables $\tilde{\varphi}$, $\tilde{\psi}$, C_μ^*, A_μ^* in the identity (2) will be made clear below.

 0038-5646/89/070200-05$04.00

Taking into account (2) and the bulk electrostatics and electromagnetics equations

$$\operatorname{div}\mathbf{D}^{(1,2)}=\operatorname{div}\mathbf{B}^{(1,2)}=0,\ \mathbf{E}^{(1,2)}=-\nabla\varphi^{(1,2)},\ \mathbf{H}^{(1,2)}=-\nabla\psi^{(1,2)}$$

the variation δF_s of the total surface energy takes the form

$$\delta F_s=\int\delta\alpha\,dS=\int\frac{dS}{4\pi}\{D_n\delta\tilde\varphi+C_\mu^*\nabla_\mu\delta\varphi+B_n\delta\tilde\psi+A_\mu^*\nabla_\mu\delta\psi\}$$

$$=\int\frac{dS}{4\pi}\{D_n\delta\tilde\varphi-\delta\varphi\nabla_\mu C_\mu^*+B_n\delta\tilde\psi-\delta\psi\nabla_\mu A_\mu^*\},\qquad(3)$$

where φ and ψ are the electric and magnetic potentials on the interface (they are continuous on it if the surface layer has no singular properties).

Taking (1) and the electro- and magnetostatics equations into account, the variation δF_V of the volume energy of the contacting dielectric media 1 and 2 is equal to

$$\delta F_V=\int\delta F\,dV=\int\frac{dS}{4\pi}\{D_n^{(1)}\delta\varphi^{(1)}-D_n^{(2)}\delta\varphi^{(2)}$$

$$+B_n^{(1)}\delta\psi^{(1)}-B_n^{(2)}\delta\psi^{(2)}\}$$

$$=\int\frac{dS}{4\pi}\{(D_n^{(1)}-D_n^{(2)})\delta\varphi-D_n(\delta\varphi^{(2)}-\delta\varphi^{(1)})$$

$$+(B_n^{(1)}-B_n^{(2)})\delta\psi-B_n(\delta\psi^{(2)}-\delta\psi^{(1)})\}.\qquad(4)$$

Using (3) and (4), from the condition that the total free energy of the system is a minimum, $\delta F_V+\delta F_S=0$, we obtain the following quasi-static boundary conditions on the interface surface $Z=$ const of 2 dielectric media:

$$D_n^{(1)}-D_n^{(2)}=\nabla_\mu C_\mu^*,\qquad\varphi^{(2)}-\varphi^{(1)}=\tilde\varphi,\qquad(5)$$

$$B_n^{(1)}-B_n^{(2)}=\nabla_\mu A_\mu^*,\qquad\psi^{(2)}-\psi^{(1)}=\tilde\psi.\qquad(6)$$

Equations (5) and (6) make clear the meaning of the parameters in the thermodynamic identity (2): the quantities $\tilde\varphi$ and $\tilde\psi$ have the meaning of surface jumps of the electric and magnetic scalar potentials (analogs of the potential of an electric double layer[9]); the quantities C_μ^* and A_μ^* have the meaning of surface jumps of the tangential components of the electric and magnetic vector potentials $\mathbf{C}$ and $\mathbf{A}$:

$$C_\mu^*=e_{n\mu\nu}(C_\nu^{(1)}-C_\nu^{(2)}),\qquad\mathbf{D}^{(1,2)}=\operatorname{rot}\mathbf{C}^{(1,2)},\qquad(7)$$

$$A_\mu^*=e_{n\mu\nu}(A_\nu^{(1)}-A_\nu^{(2)}),\qquad\mathbf{B}^{(1,2)}=\operatorname{rot}\mathbf{A}^{(1,2)},\qquad(8)$$

where e_{ikl} is a unit completely antisymmetric tensor. In addition, it follows from the form of the identity (2) that in the case of electro- and magnetostatics the quantities $\tilde\varphi$ and $\tilde\psi$ are proportional to the densities of the normal components of the surface electric and magnetic polarizations, and the quantities C_μ^* and A_μ^* are proportional to the densities of the tangential components of the corresponding surface polarizations. The identity (2) itself can then be regarded as a generalization, to include the case of an interface of two dielectrics, of the Gibbs–Lippmann relation for electrocapillary phenomena on the interface of two conductors: $E_\mu=0$, $\tilde\varphi=$ const.[7]

The set of boundary conditions (5) and (6) for the electro- and magnetostatics equations is closed by the thermodynamic expansions for the surface parameters $\tilde\varphi$, $\tilde\psi$, C_μ^*, A_μ^*. These expansions can be easily obtained by using a new thermodynamic potential $\tilde\alpha=\alpha-(D_n\tilde\varphi+B_n\tilde\psi)/4\pi$ that satisfies the thermodynamic identity

$$\delta\tilde\alpha=-(\tilde\varphi\delta D_n+C_\mu^*\delta E_\mu+\tilde\psi\delta B_n+A_\mu^*\delta H_\mu)/4\pi,\qquad(9)$$

i.e., $\tilde\alpha=\tilde\alpha(D_n,E_\mu,B_n,H_\mu)$.

From this identity we can obtain the following linear expansions:

$$\tilde\varphi=\tilde\varphi^{(0)}+aD_n+d_\mu E_\mu+mB_n+p_\mu H_\mu,\qquad(10)$$

$$C_\mu^*=C_\mu^{*(0)}+b_{\mu\nu}E_\nu+d_\mu D_n+q_\mu B_n+r_{\mu\nu}H_\nu,\qquad(11)$$

$$\tilde\psi=\tilde\psi^{(0)}+fB_n+l_\mu H_\mu+mD_n+q_\mu E_\mu,\qquad(12)$$

$$A_\mu^*=A_\mu^{*(0)}+g_{\mu\nu}H_\nu+l_\mu B_n+p_\mu D_n+r_{\mu\nu}E_\nu.\qquad(13)$$

In these expansions, the parameters $\tilde\varphi^{(0)}$, $C_\mu^{*(0)}$, $\tilde\psi^{(0)}$, $A_\mu^{*(0)}$ have, in accordance with the foregoing, the meaning of components of residual surface electric and magnetic polarizations. The coefficients a, $b_{\mu\nu}$, d_μ, f, $g_{\mu\nu}$, l_μ are $t\to -t$ invariant, and the coefficients m, p_μ, q_μ, $r_{\mu\nu}$ are $t\to -t$ noninvariant, i.e., they differ from zero only on surfaces having a magnetic structure (these coefficients describe the surface magnetoelectric effect). The vectors d_μ, l_μ, p_μ, q_μ differ from zero only on boundaries having no inversion centers (e.g., on asymmetric crystal media). Note that the electromagnetic properties of a plane isotropic centrosymmetric transition layer are described in general by the four independent parameters a, b_1, f, g_1, ($b_{\mu\nu}=b_1\delta_{\mu\nu}$, $g_{\mu\nu}=g_1\delta_{\mu\nu}$).

We proceed now to describe the electrodynamic properties of interfaces of media (with allowance for the retardation of the electromagnetic waves). We know that the Maxwell equations in dielectrics can be rewritten in the gauge-invariant form

$$\mathbf{E}=-\nabla\varphi-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t},\qquad\mathbf{B}=\operatorname{rot}\mathbf{A},$$
$$\mathbf{H}=-\nabla\psi+\frac{1}{c}\frac{\partial\mathbf{C}}{\partial t},\qquad\mathbf{D}=\operatorname{rot}\mathbf{C},\qquad(14)$$

where φ, ψ, $\mathbf{C}$, and $\mathbf{A}$ are the scalar and vector electric and magnetic potentials. Recognizing that the transition-layer parameters $\tilde\varphi$, $\tilde\psi$, C_μ^*, A_μ^* introduced above have been defined as surface discontinuities of the corresponding potentials, it follows from (14) that the gauge-invariant boundary conditions for the Maxwell equations on the interface of two media take the form

$$E_\mu^{(1)}-E_\mu^{(2)}=\nabla_\mu\tilde\varphi-\frac{1}{c}\frac{\partial}{\partial t}\tilde A_\mu,\qquad(15)$$

$$H_\mu^{(1)}-H_\mu^{(2)}=\nabla_\mu\tilde\psi+\frac{1}{c}\frac{\partial}{\partial t}\tilde C_\mu,\qquad(16)$$

where

$$\tilde A_\mu=e_{n\nu\mu}A_\nu^*,\ \tilde C_\mu=e_{n\nu\mu}C_\nu^*.$$

The system of boundary conditions (15) and (16) is closed by the linear expansions (10)–(13), in which now all the coefficients must in general be regarded as functions of the frequency ω. It follows from the form of the boundary conditions (15) and (16), with the expansions (10)–(13)

taken into account, that a description of the electrodynamic (in the general case, dielectric) properties of the transition layer actually contains the expansion of the boundary conditions for the Maxwell equations in terms of the small parameters $kd^* \ll 1$ and $\omega d^*/c \ll 1$, which are proportional to the effective thickness d^* of the transition layer (k is the two-dimensional wave number). In such a description one must take into account the surface discontinuities of the tangential components of both the magnetic field H_μ and the electric field E_μ, which are in general of the same order on a dielectric (polarizable) $2D$ transition layer. This property of a $2D$ polarizable layer will be demonstrated below in the description of the surface electromagnetic waves localized near a plane defect of a dielectric crystal.

On the other hand, in the case of a $2D$ conducting layer in a dielectric layer (e.g., an inversion layer in a heterojunction), the tensor $b_{\mu\nu}$ in the expansion (11) is proportional to $\sigma_{\mu\nu}/\omega$, where $\sigma_{\mu\nu} = \sigma_{\mu\nu}(\omega,H_0)$ is the tensor of the $2D$ conductivity (dissipative and Hall-effect, or dynamic) of the layer. Therefore in the considered low-frequency region $\omega d^*/c \ll 1$ the coefficients $b_{\mu\nu}$ are considerably larger than all other coefficients (which depend little on frequency) in the expansions (10)–(13), i.e., in this case, as follows from (15) and (16), the surface discontinuities of the tangential components of the electric field are negligibly small compared with the discontinuities of the magnetic-field tangential components. The macroscopic boundary conditions for the Maxwell equation on a $2D$ conducting layer in a dielectric medium become much simpler compared with a $2D$ polarizable layer, and take the form (see, e.g., Refs. 5 and 10)

$$[\mathbf{H}_1-\mathbf{H}_2, \mathbf{n}]=\frac{4\pi}{c}j, \quad E_\mu^{(1)}=E_\mu^{(2)}\equiv E_\mu, \tag{17}$$

where $j_u = \sigma_{\mu\nu}E_\nu$ is the surface current density in the $2D$ conducting layer.

2. INTERACTION OF ELECTROMAGNETIC WAVES WITH A PLANE DEFECT OF A CRYSTAL. SURFACE WAVES

The obtained electrodynamic boundary conditions on a $2D$ transition layer can be used to calculate the coefficients of reflection and transformation of electromagnetic waves incident on a plane $2D$ lattice defect such as a stacking fault or a twinning plane, and also to investigate surface waves on such a plane defect. The unique property of a plane crystal defect is that the dielectric constants and magnetic permeabilities of the media on the two sides of its surface are exactly equal so that if the defect layer has no unique (excess) electrodynamic properties the electromagnetic waves pass through its plane without any change and no surface waves propagate near such a defect. A similar situation obtains also in the case of an elastocapillary interaction of bulk acoustic waves with a plane crystal defect, and also when surface elastic waves propagate near such a defect.[8,11,12]

When the boundary conditions (15) and (16) are used to describe the electrodynamic properties of a plane crystal defect it must be recognized that in this case the electromagnetic fields E_μ, D_n, H_μ, and B_n on the singular surface must be regarded as the arithmetic means of the corresponding fields near the singular plane in the adjoining media 1 and 2:

$$E_\mu={}^1/_2(E_\mu^{(1)}+E_\mu^{(2)}), \tag{18}$$

etc.

This property of a plane defect reflects its symmetry with respect to interchange of the media that make contact through its surface (cf. the analogous description of the elastocapillary properties of a plane crystal defect[11,12]).

Let the plane of a $2D$ defect be perpendicular to the optic axis Z of a uniaxial nonmagnetic dielectric crystal. Using the boundary conditions (15) and (16) and taking the expansions (10)–(13) and the definitions (18) into account, it can be shown that the reflection coefficient R of the amplitude of the electric field of a linearly polarized electromagnetic wave normally incident on the plane defect is equal to

$$R=\frac{i\omega}{2c}\left(\frac{b_1}{\varepsilon_{xx}^{1/2}}-g_1\varepsilon_{xx}^{1/2}\right)\ll 1, \tag{19}$$

and the transmission coefficient D is given by

$$D=1+\frac{i\omega}{2c}\left(\frac{b_1}{\varepsilon_{xx}^{1/2}}+g_1\varepsilon_{xx}^{1/2}\right), \tag{20}$$

where $\varepsilon_{xx} = \varepsilon_{yy} \neq \varepsilon_{zz}$ is the dielectric constant of the uniaxial crystal.

On the other hand, for an incident glancing wave (incidence angle $\theta\to\pi/2$) the amplitude reflection coefficient R can be verified to tend to -1, and the transmission coefficient to zero. The parameter $kd^* \ll 1$ defines a glancing-angle interval $\varphi = \pi/2 - \theta$ in which the reflection coefficient is close to unity: $\varphi \lesssim kd^*$. The parameter d^* is equal to $d_p^* = g_1 - \varepsilon_{zz}a$ in the case of reflection of a polarized wave from a plane defect of a uniaxial crystal, and to $d_s^* = b_1/\varepsilon_{xx} - f$ for reflection from an s-polarized wave. If one of the parameters $d_{p,s}^*$ of a plane defect is zero, the reflection coefficient R of glancing waves of the respective polarization tends to zero, and the transmission coefficient D tends to unity (the defect layer is effectively "bleached"). If, however, the relation $g_1 = \varepsilon_{zz}a = b_1/\varepsilon_{xx}$ or $b_1/\varepsilon_{xx} = f = g$ is satisfied for the parameters of a plane defect, we have for interaction of p- or s-polarized bulk electromagnetic waves with such a $2D$ defect $R = 0$ and $|D| = 1$ for all incidence angles.

The presence of even so weak a perturbation as the plane of a $2D$ defect in the volume of a crystal can lead thus to total reflection from it of glancing electromagnetic waves with $R = -1$, i.e., to impossibility of propagation of a homogeneous bulk wave of corresponding polarization along the plane of the defect. This property of the reflection of glancing waves is closely connected with the appearance in a crystal of inhomogeneous waves localized near a plane defect—surface waves.

To describe surface electromagnetic waves on a $2D$ defect orthogonal to the optical Z axis of a uniaxial dielectric crystal, we seek the solution for the fields in the form

$$(\mathbf{E}, \mathbf{H})^{(1,2)} \propto \exp\{\pm\varkappa_{o,e}z+i(kx-\omega t)\},$$

where

$$\varkappa_o=(k^2-\varepsilon_{xx}\omega^2/c^2)^{1/2}, \quad \varkappa_e=(\varepsilon_{xx}k^2/\varepsilon_{zz}-\varepsilon_{xx}\omega^2/c^2)^{1/2} \tag{21}$$

are the eigenvalues of the Maxwell equations in the crystal for inhomogeneous ordinary and extraordinary electromagnetic waves, respectively. Using Maxwell's equations and the considered system of boundary conditions for them on the defect plane $Z = 0$ we obtain the following system of equations for the polarization and the spectrum of the surface waves:

$$(H_y^{(1)}+H_y^{(2)})[\varkappa_e-{}^1/_2\varepsilon_{xx}(g_1\omega^2/c^2-ak^2)]=0, \tag{22}$$

$$(H_y^{(1)}-H_y^{(2)})[\varkappa_e b_1/2\varepsilon_{xx}+1]=0, \tag{23}$$

$$(E_y^{(1)}+E_y^{(2)})[\varkappa_o-{}^1/_2(b_1\omega^2/c^2-fk^2)]=0, \tag{24}$$

$$(E_y^{(1)}-E_y^{(2)})[\varkappa_o g_1/2+1]=0. \tag{25}$$

It will be shown below that in the case of a $2D$ polarizable defect layer all four parameters a, b, f, and g_1 are of the order of its effective thickness d^*, it being assumed that $kd^*\sim\omega d^*/c\ll 1$. Therefore, from among the four equations (22)–(25) only Eqs. (22) and (24) describe long-wave surface waves.

Equation (22) describes a symmetric $(H_y^{(1)}=H_y^{(2)})$ TM-polarized surface wave $(E_y^{(1)}=E_y^{(2)}=0)$, the dispersion equation for which is

$$\varkappa_e/\varepsilon_{xx}={}^1/_2(g_1\omega^2/c^2-ak^2). \tag{22'}$$

As seen from (22′), a TM-polarized surface electromagnetic wave exists on a plane defect of a crystal if

$$d_p^*\equiv g_1-a\varepsilon_{zz}>0.$$

This surface wave is similar to the fundamental (zero-gap) TM mode of a symmetric dielectric plane waveguide in the long-wave limit. It can be shown in fact that for a plate of macroscopic thickness d of a uniaxial crystal with dielectric constant $\tilde\varepsilon_{xx}=\tilde\varepsilon_{yy}\neq\tilde\varepsilon_{zz}$ (Z axis normal to the plate), located in a uniaxial medium with dielectric constant $\varepsilon_{xx}=\varepsilon_{yy}\neq\varepsilon_{zz}$, the dispersion equation for the symmetric $(H_y^{(1)}=H_y^{(2)})$ TM mode is

$$\mathrm{ctg}\, p_e\frac{d}{2}=p_e\varepsilon_{xx}/\varkappa_e\tilde\varepsilon_{xx}, \tag{26}$$

where

$$p_e=\left(\tilde\varepsilon_{xx}\frac{\omega^2}{c^2}-\tilde\varepsilon_{xx}k^2/\tilde\varepsilon_{zz}\right)^{1/2}$$

is the eigenvalue of the Maxwell equations for an inhomogeneous extraordinary electromagnetic wave in the crystal from which this plate is taken, and the parameter $\varkappa_e$ is described by (21).

In the long-wave limit $p_e d\ll 1$ we get from (26) the dispersion equation

$$\varkappa_e/\varepsilon_{xx}={}^1/_2 d(\omega^2/c^2-k^2/\tilde\varepsilon_{zz}), \tag{26'}$$

which is similar to (22′). It follows simultaneously from (26) and (26′) that the condition for the existence of a surface TM mode in the considered system is $\tilde\varepsilon_{zz}>\varepsilon_{zz}$. A comparison of (26′) and (22′) yields an estimate of the order of magnitude of the parameters g_1 and a of a plane $2D$ defect:

$$g_1\sim d^*,\ a\sim d^*/\varepsilon_{zz}.$$

Note that it follows from (26), (26′), and (22′) that in the case $\tilde\varepsilon_{zz}\gg\varepsilon_{zz}$, $\tilde\varepsilon_{xx}\ll\varepsilon_{xx}$ and in the wave-number interval

$$(\varepsilon_{zz}/\varepsilon_{xx})^{1/2}\ll kd/2\ll\min\{(\varepsilon_{xx}\varepsilon_{zz})^{1/2}/\tilde\varepsilon_{xx},\ \tilde\varepsilon_{zz}/(\varepsilon_{xx}\varepsilon_{zz})^{1/2}\}$$

the dispersion law of the surface TM mode in a dielectric waveguide system takes the form

$$\omega^2=2c^2k/d(\varepsilon_{xx}\varepsilon_{zz})^{1/2} \tag{27}$$

(or $\omega^2=2c^2k/g_1(\varepsilon_{xx}\varepsilon_{zz})^{1/2}$ for a $2D$ defect) typical of the spectrum of two-dimensional plasma oscillations.

Equation (24) describes a symmetric TE-polarized surface wave $(H_y^{(1)}+H_y^{(2)}=0)$ for which the dispersion equation is

$$\varkappa_o={}^1/_2(b_1\omega^2/c^2-fk^2). \tag{24'}$$

This type of surface electromagnetic wave on a plane $2D$ defect of a dielectrically isotropic crystal is described in Ref. 5 for the case $a=f=g_{\mu\nu}=0$.

It is clear from (24′) that a surface TE-polarized electromagnetic wave exists on a plane defect of a crystal if

$$d_s^*\equiv b_1/\varepsilon_{xx}-f>0.$$

This wave is analogous to the fundamental TE mode of a symmetric dielectric planar waveguide in the long-wave limit. Indeed, the dispersion equation for the symmetric $(E_y^{(1)}=E_y^{(2)})$ TE mode of the macroscopic dielectrically anisotropic plate described above, in an external uniaxial medium, is of the form

$$\varkappa_o=p_o\,\mathrm{tg}(p_o d/2), \tag{28}$$

where

$$p_o=(\tilde\varepsilon_{xx}\omega^2/c^2-k^2)^{1/2}$$

is an eigenvalue of the Maxwell equations for an inhomogeneous ordinary electromagnetic wave in the considered plate, and the parameter $\varkappa_0$ is described by expression (21). In the long-wave limit $p_o d\ll 1$ we obtain from (28) the dispersion equation

$$\varkappa_o={}^1/_2 d(\tilde\varepsilon_{xx}\omega^2/c^2-k^2), \tag{28'}$$

which is similar to (24′) (from (28) and (28′) follows the condition for the existence of a surface TE mode in the considered system: $\tilde\varepsilon_{xx}>\varepsilon_{xx}$). By comparing (28′) with (24′) we can estimate the order of magnitude of the parameters b_1 and f which characterize the plane defect:

$$b_1\sim d^*\varepsilon_{xx},\quad f\sim d^*.$$

The estimates of the parameters a and b_1 of a plane $2D$ defect of a nonmagnetic crystal, made in the framework of the proposed description, correspond to the values of the analogous parameters μ and γ introduced in Refs. 3 and 4 to describe a dielectric transition layer of macroscopic thickness d^* in the limit $kd^*\ll 1$.

Note that, as follows from (28), (28′), and (24′), in the case $\tilde\varepsilon_{xx}\gg\varepsilon_{xx}$ and in the wave-number interval

$$\varepsilon_{xx}/\tilde\varepsilon_{xx}\ll kd/2\ll 1$$

the dispersion law for surface TE modes in a dielectric waveguide system is of the form[10]

$$\omega^2=2c^2k/d\tilde\varepsilon_{xx} \tag{29}$$

(or $\omega^2 = 2c^2k/b_1$ for a 2D defect), which is similar to (27) and is also typical of the spectrum of 2D plasma oscillations.

The physical reason why just two types of long-wave surface electromagnetic waves can exist on a plane polarizable 2D defect layer in a crystal is quite simple. The point is that in a dielectric crystal, in the absence of a plane defect, there can propagate in a given symmetric direction of two homogeneous bulk electromagnetic waves with mutually orthogonal linear polarizations. Introduction into a crystal, however, of a slow decelerating (with locally increased polarizability) defect layer of proper orientation can transform each of the considered homogeneous electromagnetic waves into a weakly inhomogeneous surface wave. As seen from (22') and (24'), the depth of penetration $\delta = 1/\varkappa_{o,e}$ of each of these surface waves into the crystal exceeds significantly the wavelength λ: $\delta \sim \lambda^2/d^* \gg \lambda$. If, however, $d_p^* = 0$ or $d_s^* = 0$ then, as seen from (22') and (24'), homogeneous ($\varkappa_e = 0$ or $\varkappa_o = 0$) TM or TE bulk electromagnetic waves can propagate along the plane of the 2D defect, and the reflection of glancing p and s waves from the 2D defect have no anomalies in this case. Similar physical factors lead to a possible existence, on a plane crystal defect, of three types of deeply penetrating surface elastocapillary waves (with quasi-longitudinal, quasi-transverse vertical, and pure shear horizontal polarization.[11-13]

The remaining waves, described by Eqs. (23) and (25), of the system considered correspond to antisymmetric gap (activational) modes of a symmetric planar dielectric waveguide in the long-wave limit. In the case of the described macroscopic dielectric plate in an external uniaxial medium the dispersion equations for such modes take the following form for a TM-polarized mode ($H_y^{(1)} = -H_y^{(2)}$)

$$\varkappa_e/\varepsilon_{xx} = -(p_e/\tilde{\varepsilon}_{xx})\,\mathrm{ctg}\, p_e d/2 \tag{30}$$

or in the limit $p_e d \ll 1$

$$\varkappa_e/\varepsilon_{xx} + 2/d\tilde{\varepsilon}_{xx} = 0,$$

which corresponds to Eq. (23); for a TE mode ($E_y^{(1)} = -E_y^{(2)}$) we have

$$\varkappa_o = -p_o\,\mathrm{ctg}\, p_o \frac{d}{2} \tag{31}$$

or in the $p_o d \ll 1$ limit,

$$\varkappa_o + 2/d = 0,$$

which corresponds to Eq. (25).

Thus, the number, polarization, and dispersion of long-wave surface electromagnetic wave on a plane defect of a crystal, described by Eqs. (22)–(25), agree fully with the description [Eqs. (26), (28), (30), and (31)] of inhomogeneous waves in a medium near a dielectrically anisotropic layer of finite macroscopic thickness d in the limit $kd \sim \omega d/c \ll 1$. This circumstance attests, in our opinion, to the validity of the description of the electrodynamic properties of a 2D transition layer in the framework of the proposed system [(10)–(13), (15), (16)] of the macroscopic boundary conditions for the Maxwell equations.

We note in conclusion that if a 2D conducting layer (e.g., of electrons[14–16]) is produced on a plane defect of a crystal, the only parameter that describes its properties in the dynamic regime $\omega\tau \gg 1$, $\omega d^*/c \ll 1$ (τ is the free-path time of the carriers in the 2D layer) will be in the isotropic case, as noted above, the negative parameter $b_{xx}(\omega) = -|b_{xx}(\omega)|$ in the expansion (11). Therefore only one type of TM surface electromagnetic wave can propagate near such a 2D defect in a dielectric (or semiconducting) crystal, and is described by Eq. (23):

$$\varkappa_e = 2\varepsilon_{xx}/|b_1(\omega)|.$$

These properties of surface electromagnetic waves near a 2D electron layer, which follow from the proposed phenomenological description, are also confirmed by more detailed calculations (see, e.g., Refs. 17 and 18).

The author is grateful to V. M. Agranovich, V. I. Al'shits, A. F. Andreev, M. I. Kaganov, A. M. Kosevich, and M. I. Ryazanov for helpful discussions.

[1)]A similar principle of choosing the independent thermodynamic variables of elastic surface energy was used earlier by Andreev and the author to describe elastocapillary properties of crystal interfaces.[8]

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Translated by J. G. Adashko