Strong suppression of two-dimensional superconductivity in metals by impurities

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The twinning plane in a superconductor is considered in the situation when, in the absence of scattering, the critical temperature of the two-dimensional superconductivity in the plane is higher than the critical temperature in the bulk. It is shown that scattering of electrons from the plane into the volume strongly suppresses the two-dimensional superconductivity and correspondingly lowers the critical temperature of the superconductivity localized in the twinning plane.

1. INTRODUCTION

Two-dimensional superconductivity on surface Tamm levels was considered a quarter of a century ago by Ginzburg and Kirzhnits.[1] Even then, it was noted that, in principle, two-dimensional Cooper pairing of electrons in Tamm levels in a metal can be characterized by a higher critical temperature than for pairing in the bulk, on account of the increase of the density of the electron states of the surface levels or on account of the enhancement of the interaction of the surface electrons with phonons. The question of the proximity effect, i.e., of the suppression of the superconductivity of the electrons localized on the surface by normal electrons from the bulk, was not discussed (this effect, naturally, is absent if we are speaking of Tamm levels on the surface of a dielectric).

Not so long ago, the question of two-dimensional superconductivity was posed by Nabutovskiĭ and Shapiro[2] in connection with the experimental discovery of superconductivity in twinning planes in tin, niobium, and certain other metals at temperatures somewhat higher than the corresponding critical temperatures in the bulk.[3,4] In this case, the existence of electrons localized on a twinning plane is entirely possible, and such electrons, generally speaking, can go over into the superconducting state at a higher temperature than do electrons in the bulk. The proximity effect here is just as important as for superconductivity on the surface levels in a metal. In both cases, hybridization of states localized on the plane with bulk states (because of electron scattering by defects of the sample surface or of the twinning plane, and also by impurities) leads to suppression of the two-dimensional superconductivity.

To investigate the proximity effect in a system of two-dimensional (localized on a plane) and three-dimensional electrons we shall consider a model of such a system that takes the electron scattering into account with the use of a single parameter—the electron mean free time τ. The case of large values of τ (absence of scattering) was studied in the cited paper (Ref. 2). Of greatest interest is the situation when the two-dimensional superconductivity is characterized by a higher critical temperature T_2 than is the superconducting state in the bulk (we denote the critical temperature of the latter by T_{c0}). Then below T_2 Cooper pairing of two-dimensional electrons occurs, with an order parameter $\Delta_2(x)$ that is nonzero in the twinning plane, i.e., in a layer with a thickness a of the order of the atomic scale (the system of coordinates is chosen in such a way that $x = 0$ on the twinning plane, the x axis being perpendicular to the twinning plane).

Near such a layer, because of the BCS interaction, pairing of bulk electrons occurs, and the corresponding order parameter has the form

$$\Delta_{ind}(x)\sim\Delta_2(0)\,\frac{a}{\xi t^{1/2}}\exp(-|x|t^{1/2}/\xi), \tag{1}$$

where $t=\ln(T/T_{c0})$ and ξ is the correlation length (see Ref. 2). As the temperature is lowered the induced superconductivity encompasses an ever greater volume, and near $t\sim(a/\xi)^2$ a weakly localized superconducting state is established, with an order parameter comparable to $\Delta_2(0)$ in a large region of order ξ^2/a (while for $t\sim1$ the superconductivity has a surface character and the complete Meissner effect is proportional to a/l, where l is the spacing between twinning planes).

The limiting case of a strong proximity effect (small τ) was considered in Refs. 5 and 6. This case can be described in the framework of a model with a dimensionless electron-phonon interaction parameter that is locally enhanced near a twinning plane. Now, as the temperature is lowered, the twinning plane immediately goes over from the normal state to a weakly localized superconducting state with order parameter

$$\Delta(x)\sim T_{c0}(t_c-t)^{1/2}\exp(-|x|t^{1/2}/\xi), \tag{2}$$

where $t_c\sim(a/\xi)^2$.

In the present paper we trace how increase of the concentration of scattering centers (decrease of τ) is accompanied by the establishment of a proximity effect that leads to rapid suppression of the two-dimensional superconductivity in the twinning plane. As a result, the critical temperature T_c of the transition of the twinning plane to the superconducting state decreases sharply with decrease of τ, being shifted from the value T_2 to the value $T_{c0}(1+t_c)$.

2. CALCULATION OF T_c ON THE BASIS OF EILENBERGER'S EQUATIONS

To describe two-dimensional superconductivity in a twinning plane in a superconductor we choose the dimensionless electron-phonon interaction parameter in the form

$$\lambda(x,\theta)=\lambda_0+2a\lambda_2\delta(x)\delta(\cos\theta), \tag{3}$$

where λ_0 is the electron-phonon interaction constant in the bulk, λ_2 is the electron-phonon interaction constant for electrons localized in the plane, and θ is the angle between the

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direction of the electron velocity and the x axis. The second term in (3) takes into account the additional attraction of electrons with trajectories in the twinning plane. Since this term depends on the direction of the electron velocity, it is natural to write the self-consistency equation for the isotropic order parameter as follows.:

$$\Delta(x)=\pi T\sum_{\omega}\int\frac{d\Omega}{4\pi}\lambda(x,\theta)f(\mathbf{r},\mathbf{v}), \tag{4}$$

where the summation is performed over the Matsubara frequencies up to the Debye frequency ω_D. The function f introduced satisfies Eilenberger's equations.[7] Near the critical temperature these equations acquire the form

$$(2|\omega|+1/\tau+\mathbf{v}\nabla)f(\mathbf{r},\mathbf{v})=2\Delta+\bar{f}(\mathbf{r},\mathbf{v})/\tau,$$

$$\bar{f}(\mathbf{r},\mathbf{v})/\tau=n\int d\Omega'\,v'|W(\mathbf{v},\mathbf{v}')|^2 f(\mathbf{r},\mathbf{v}'), \tag{5}$$

where $\mathbf{v}$ is the velocity of the electrons on the Fermi surface, n is the concentration of scattering centers, and W is the scattering amplitude.

In the Fourier representation the linearized self-consistency equation takes the form

$$\Delta_p=\frac{2a\lambda_2}{1-\lambda_0 K_p}\int\frac{dp'}{2\pi}S_{p'}\Delta_{p'}. \tag{6}$$

The functions introduced here are easily determined in the case when in the Eilenberger equations (5) we can neglect the anisotropy of the scattering amplitude and of the Fermi velocity:

$$K_p=\pi T\sum_{\omega}\frac{2A}{1-A/\tau},\qquad A=\frac{1}{pv}\operatorname{arctg}\frac{pv}{2|\omega|+1/\tau},$$
$$S_p=\pi T\sum_{\omega}\frac{1}{(2|\omega|+1/\tau)(1-A/\tau)}. \tag{7}$$

In the integration over the momentum in (6) the cutoff should be made at a value $p_c\sim 1/a$, inasmuch as the δ-function in (3) replaces the actual dependence of λ_2 on x (the parameter λ_2 is nonzero in a layer of thickness $\sim a$ determined by the wave function of the surface electrons, which is localized in the direction of the x axis).

The approach within the framework of the quasiclassical Eilenberger equations, generally speaking, does not permit a quantitative analysis of electrons localized on an atomic scale. However, for $1/\tau = 0$ the Gor'kov equations for surface electrons give the same results for T_c (Ref. 2) as does Eq. (6) with a cutoff at momentum $p_c = \pi/a$. This cutoff will be used below. For small values of $1/\tau$ Eqs. (6) and (7) reproduce the exact results of Refs. 5 and 6 for T_c. Therefore, there is every reason to suppose that the Eilenberger equations give qualitatively correct results in the region of intermediate values of $1/\tau$ as well. Here, for so long as $v\tau\gg a$, the parameters p_c and λ_2 can be assumed to be independent of the quantity τ, in accordance with the theory of dirty alloys of Abrikosov and Gor'kov.[8]

Now, from the integral equation (6), we obtain the relation determining the critical temperature of the transition of the twinning plane to the superconducting state ($t\lesssim 1$):

$$\lambda_2 L(1/\tau)+\frac{a\lambda_2}{2\xi t^{1/2}}L^2(0)=1,$$

$$L(1/\tau)=2\pi T\sum_{\omega}\frac{1}{2|\omega|+1/\tau},$$

$$\xi^2=\frac{v^2}{6}\pi T\sum_{\omega}\frac{1}{\omega^2(2|\omega|+1/\tau)}. \tag{8}$$

In the most interesting situation when $\lambda_2>\lambda_0$, for sufficiently weak scattering ($\tau>\tau_c$), two-dimensional superconductivity is realized in the twinning plane, and the critical temperature is determined from the equation [see (8)]

$$\ln\frac{T_2}{T_c}+\psi\left(\frac{1}{2}\right)-\psi\left(\frac{1}{2}+\frac{1}{4\pi\tau T_c}\right)=0, \tag{9}$$

where ψ is the logarithmic derivative of the Γ-function, $T_2 = (2\gamma/\pi)\omega_D\exp(-1/\lambda_2)$, and $\ln\gamma = C$ is the Euler constant. The critical value τ_c is determined from Eq. (9) as that value of τ at which $T_c = T_{c0}$. In the limiting case $T_2 - T_{c0}\ll T_{c0}$ the critical temperature $T_c = T_2 - \pi/8\tau$, and the critical value $\tau_c - \pi/8(T_2 - T_{c0})$. If, however, $T_{c0}\ll T_2$, we obtain $\tau_c = \gamma/\pi(T_2 - 2\gamma T_{c0})$. For $\tau<\tau_c$ the proximity effect suppresses the two-dimensional superconductivity, and near the twinning plane we obtain a weakly localized superconducting state. The critical temperature in this case is close to T_{c0} [see (8)]:

$$t_c=\frac{T_c-T_{c0}}{T_{c0}}=\left(\frac{a\lambda_2}{2\lambda_0^2(1-\lambda_2 L)\xi}\right)^2, \tag{10}$$

and for $\tau T_{c0}\ll 1$ we obtain

$$L=\ln[(1+2\tau\omega_D)/(1+2\pi\tau T_{c0})].$$

The neighborhood of the critical value τ_c in which the change from the relation (9) to the relation (10) occurs is equal in order of magnitude to $\tau_c(a/\lambda_0^2\xi)^{2/3}$. Thus, the transition between the two regimes (9) and (10) turns out to be rather sharp. It is this which makes it possible to consider only two superconducting states—a surface state and a weakly localized state (see the figure).

To avoid misunderstandings, we draw attention to the fact that in the limit $T_{c0} = 0$ we arrive at the case of a twinning plane in a normal metal. Therefore, for $\tau\gtrsim 1/T_2$ the proximity effect suppresses the two-dimensional superconductivity. Inasmuch as for electrons in the bulk we assumed the condition $v\tau\gg a$, the results obtained do not describe a

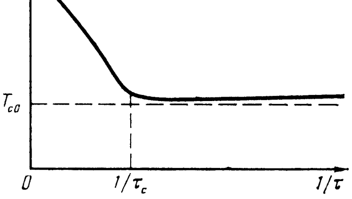

FIG. 1. Schematic dependence of T_c on $1/\tau$. The linear increase of T_c at large values of $1/\tau$ is related to the decrease of ξ in this limiting case.

twinning plane in a dielectric. In the latter case we must go over to the two-dimensional Eilenberger equations, and this will lead immediately to Anderson's theorem, i.e., to the τ-independence of $T_c = T_2$.

3. CONCLUSION

In this paper we have considered a twinning plane in a superconductor, with the assumption that in this plane localization of electrons occurs and that below a critical temperature T_2 the localized electrons undergo Cooper pairing. We have investigated the most interesting situation, when the value of T_2 is higher than the critical temperature T_{c0} in the bulk. For large values of the electron mean free time τ, below T_2 surface superconductivity is established in a layer of thickness a and induced bulk superconductivity is established in a region of the order of the correlation length ξ, and T_2 can be considerably higher than T_{c0}. In the "dirty" limit (small τ), below a critical temperature T_c very close to T_{c0} ($t_c \equiv T_c/T_{c0} - 1 \sim (a/\xi)^2$), a weakly localized superconducting state appears in a region $\sim \xi^2/a$. It is clear that with decrease of τ the point T_c is shifted from T_2 to $T_{c0}(1 + t_c)$ because of the proximity effect. In this case, the suppression of the two-dimensional superconductivity is analogous to the suppression of singlet Cooper pairing by magnetic impurities, and in the neighborhood of the critical value τ_c the transition from the surface solution to the weakly localized solution occurs very sharply. Therefore, in principle, two-dimensional superconductivity in a metal can be realized only in systems with highly perfect surfaces or twinning planes.

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