Effects of quantum coherence of electromagnetic radiation in interactions with electrons

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The dynamics of an electron in a quantized monochromatic field is analyzed. The state of the field is described in various representations. The wave functions of the system are derived. They show that as an electron moves in a coherent quantized field with a linear polarization a squeezed state of photons is formed. If the external field is circularly polarized, a coherent state forms. A squeezed state of photons also forms as an electron interacts with individual linearly polarized vacuum modes in a bounded volume. A study is made of the photon statistics for various states of the quantized external field. Expressions are derived for the radiation of a classical field and for the Compton effect.

1. INTRODUCTION

Effects of the intensity of electromagnetic radiation in interactions with electrons have been studied in depth in many places (e.g., Refs. 1–3) on the basis of exact Volkov solutions for the Dirac and Klein-Gordon equations for a charged particle in the field of a classical plane electromagnetic wave. New aspects of many-photon processes and intensity effects have been found for free electrons in analyses of various phenomena in quantum electrodynamics. The classical treatment of the external field in these studies, however, has made it impossible to see effects stemming from quantum coherence.

The advent of highly coherent sources of electromagnetic radiation opens up some interesting possibilities for new experiments to detect subtle effects of optical quantum coherence. The conventional mathematical apparatus of quantum electrodynamics in the occupation-number representation, i.e., with a diagonal photon-number operator, does not provide information about the phase state of photons. Glauber[4] and Klauder and Sudarshan[5] have studied the statistical properties of a photon beam by means of a coherent-state representation.

The new coherent photon states referred to as "squeezed" or "two-photon" states have recently been the subject of active theoretical and experimental research. These states are like ordinary coherent states of photons in that they will, under certain conditions, minimize the uncertainties in the "coordinate" and the "momentum," but they have no classical analog.

Studies of the quantum coherence of electromagnetic radiation as it interacts with atoms have revealed some new and specific effects.[6,7] The interaction of a quantized monochromatic electromagnetic field with electrons was analyzed in Refs. 7–12. Bergou and Varro[10] analyzed Compton scattering, carrying out a complete calculation of the scattering cross section for the case of a circular polarization of the external field. Bergou and Varro showed that the interaction of an electron with quantized radiation introduces a new statistics, and they demonstrated some of its properties in the case of circular polarization.

Becker *et al.*[11] carried out a particularly interesting study of the dynamics of a charged particle in a quantized monochromatic electromagnetic field in a bounded volume (in a cavity). Restricting their analysis to the long-wavelength and nonrelativistic approximations, and ignoring recoil effects, they studied the phenomenon of a "squeezed charged" vacuum and the quantum-coherence effects which result from an interaction with an electron. They showed that sufficiently bright electron beams and sufficiently long waves would make it possible to observe a squeezing effect in the interaction of quantized electromagnetic radiation with electrons in a bounded volume.

In the present paper we use exact solutions of the Dirac and Klein-Gordon equations for a particle in a quantized monochromatic radiation field to study quantum-coherence effects as electromagnetic radiation interacts with an electron. We examine classical radiation and Compton scattering for various states of the external field.

2. QUANTUM EFFECTS IN THE MOTION OF A CHARGED PARTICLE IN A RADIATION FIELD

The motion of a charged particle in a quantized monochromatic radiation field was first studied by Berson.[12] However, the "coordinate" representation for the quantized field was used to derive exact solutions of the Dirac equation there, so further applications of these solutions were difficult. In the present paper we use a solution of the Dirac equation in the occupation-number representation to study quantum-mechanical effects of the interaction of electrons with a quantized field

$$A_\mu = \frac{e_\mu}{(2\omega V)^{1/2}} (ce^{ikx} + c^+ e^{-ikx}),$$

inside a volume V which has the shape of a rectangular parallelepiped. We impose periodic conditions at the boundaries. In the case of a linear polarization of the photons the solution is

$$\psi_{np}(x) = e^{if_n x}\left[1 + \frac{e\hat{k}\hat{A}}{2kp}\right]\frac{u(p)}{V^{1/2}} e^{-ikxc^+c} U(-z) D(\alpha) |n\rangle. \quad (2.1)$$

Here $u(p)$ is a free Dirac bispinor which satisfies the equation

$$(ip+m)u(p) = 0, \quad (2.2)$$

and $D(\alpha)$ and $U(-z)$ are respectively a unitary displacement operator and a unitary squeezing operator, given by[4,13]

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$$D(\alpha)\equiv\exp[\alpha(c^+-c)],\tag{2.3}$$

$$U(-z)\equiv\exp\left[-\frac{z}{2}(c^2-(c^+)^2)\right],\tag{2.4}$$

$$\alpha=-\frac{pb}{pk}\left(1-\frac{2b^2}{kp}\right)^{-3/4},\qquad e^{2z}=\left(1-\frac{2b^2}{kp}\right)^{-1/2}$$

$$b_\mu=\frac{ee_\mu}{(2\omega V)^{1/2}}.\tag{2.5}$$

In (2.1), f_n is the 4-momentum of an electron in the radiation field, where

$$f_{n\mu}=p_\mu+k_\mu\left(1-\frac{2b^2}{kp}\right)^{1/2}\left[n+\frac{1}{2}-\alpha^2-\frac{1}{2}\left(1-\frac{2b^2}{kp}\right)^{-1/2}\right].\tag{2.6}$$

In the case of a circular polarization of quantized external radiation inside a volume V with periodic conditions at the boundaries, the solution of the Dirac equation is

$$\psi_{np}(x)=e^{if_nx}\left[1+\frac{e\hat{k}\hat{A}}{2kp}\right]\frac{u(p)}{V^{1/2}}e^{-ikxc^+c}D(\alpha)\,|n\rangle,\tag{2.7}$$

where

$$\alpha=-\frac{pb^*}{pk}\left(1-\frac{|b|^2}{kp}\right)^{-1}.\tag{2.8}$$

The 4-momentum of an electron in the radiation field is described in this case by

$$f_{n\mu}=p_\mu+k_\mu\left(1-\frac{|b|^2}{kp}\right)\left(n+\frac{1}{2}-|\alpha|^2-\frac{1}{2}\frac{1+|b|^2/kp}{1-|b|^2/kp}\right).\tag{2.9}$$

Solutions of the Klein-Gordon equation can be found from expressions (2.1) and (2.7), for the cases of linear and circular polarization, respectively, by assuming that the expression in square brackets is unity and by omitting the Dirac bispinors.

Of particular interest is the wave function of a charged particle in vacuum in a bounded volume. It follows from expression (2.1) that electron interaction with individual linearly polarized vacuum modes in a bounded volume produces a squeezed state

$$U(-z)D(\alpha)\,|0\rangle=|-z;\alpha\rangle.\tag{2.10}$$

This state generates some special statistical properties of the quantized field in an interaction with an electron with the electromagnetic vacuum. This effect was found by Becker *et al.*[11] for a nonrelativistic electron in the long-wave approximation without recoil. They called it a "squeezed charged vacuum."

As an electron interacts with individual circularly polarized vacuum modes in a bounded volume, an ordinary "one-photon" coherent state forms:

$$D(\alpha)\,|0\rangle=|\alpha\rangle.\tag{2.11}$$

If we discard the spinor parts of solutions (2.1) and (2.7) (for the Klein-Gordon equation, for example), the wave function of an electron in vacuum with a bounded volume is thus expressed in terms of a squeezed state (2.10) or a coherent state (2.11).

Up to this point we have been discussing (as in Ref. 10) the case in which the external field is specified in the occupation-number representation. A topic of particular interest is the dynamics of a charged particle in quantized external fields in states which minimize the uncertainties in the coordinate and momentum of a photon.

Among such quantized field states the most important are coherent and squeezed field states.

We first consider the case of a coherent external field. Following Ref. 12, we accordingly construct a packet of wave functions (2.1) and (2.7):

$$\psi_{\beta p}(x)=\exp\left(-\frac{|\beta|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\beta^n}{(n!)^{1/2}}\psi_{np}(x),\qquad \beta\equiv|\beta|e^{i\varphi}.\tag{2.12}$$

In the case of linearly polarized electromagnetic radiation we find the following expression for the wave function:

$$\psi_{\beta p}(x)=\exp[i(f_0x-\alpha\,\mathrm{Im}\,\beta(x))]\left[1+\frac{e\hat{k}\hat{A}}{2kp}\right]\frac{u(p)}{V^{1/2}}\times|-z\exp(2ikx);(\alpha+\beta(x))\exp(-ikx)\rangle,\tag{2.13}$$

where f_0 is given by (2.6) with $n=0$,

$$\beta(x)=|\beta|e^{i\varphi}e^{i\tilde{k}x},\quad \tilde{k}=ke^{-2z}=k(1-2b^2/kp)^{1/2},\tag{2.14}$$

and $|\beta(x)|^2=|\beta|^2=\bar{n}$ is the expectation value of the number of photons.

In the case of circularly polarized radiation, the following expression is found for the wave function:

$$\psi_{\beta p}(x)=\exp[i(f_0x-\mathrm{Im}\,\alpha^*\beta(x))]\times\left[1+\frac{e\hat{k}\hat{A}}{2kp}\right]\frac{u(p)}{V^{1/2}}\,|(\alpha+\beta(x))\exp(-ikx)\rangle,\tag{2.15}$$

where f_0 is given by (2.9) with $n=0$,

$$\beta(x)=|\beta|e^{i\varphi}e^{i\tilde{k}x},\quad \tilde{k}=k(1-|b|^2/kp),\tag{2.16}$$

and $|\beta(x)|^2=|\beta|^2=\bar{n}$ is the expectation value of the number of photons in the radiation mode when the interaction is turned off.

It can be seen from (2.13) and (2.15) that the electron wave function is expressed in terms of a squeezed field state in the case of linear polarization in the field of a coherent wave. In the case of circularly polarized radiation the wave function of the electron-plus-field system is expressed in terms of a coherent state. As in the case of the motion of an electron in vacuum, the photon part of the wave function is a squeezed or coherent state if we ignore the spin part of expression (2.13) or (2.15), respectively (for the Klein-Gordon or Schrödinger equation, for example).

We make the transformation from wave functions (2.13) and (2.15) to the case of a classical field—the Volkov wave functions[3]—by projecting these states onto the coherent field state $|\beta\rangle$, i.e., by using $\langle\beta\,|\,\psi_{\beta p}\rangle$, and by taking the limits $\bar{n}\to\infty$, $V\to\infty$, while the photon density $\bar{n}/V$ remains finite.

We turn now to the construction of wave packets for the case in which the quantized external field is in a squeezed state $|\zeta;\beta\rangle$:

$$\psi_{\zeta\beta p}(x)=\sum_{n=0}^{\infty}\langle n|\zeta;\beta\rangle\psi_{np}(x). \quad (2.17)$$

The coefficients $\langle n|\zeta;\beta\rangle$ are expressed in terms of Hermite polynomials. Explicit expressions for them are given in Ref. 13, among other places.

For a linearly polarized external field we find the following result for the wave function:

$$\psi_{\zeta\beta p}(x)=\exp(if_0x)\left[1+\frac{e\hat{k}\hat{A}}{2kp}\right]\frac{u(p)}{V^{1/2}}\exp(-ikxc^+c)$$
$$\times U(-z)D(\alpha)|\zeta(x);\beta(x)\rangle, \quad (2.18)$$

where $\beta(x)$ is given by (2.14), and

$$\zeta(x)=\zeta e^{-2i\tilde{k}x}=re^{i\theta}e^{-2i\tilde{k}x}. \quad (2.19)$$

For a circularly polarized external electromagnet field we find

$$\psi_{\zeta\beta p}(x)=\exp(if_0x)\left[1+\frac{e\hat{k}\hat{A}}{2kp}\right]\frac{u(p)}{V^{1/2}}$$
$$\times\exp(-ikxc^+c)D(\alpha)|\zeta(x);\beta(x)\rangle, \quad (2.20)$$

where $\beta(x)$ and $\zeta(x)$ are given by expressions (2.16) and (2.19), respectively, and $\tilde{k}_\mu = k_\mu(1-|b|^2/kp)$. In contrast with the linear-polarization case, state (2.20) is expressed in terms of a squeezed state of a photon:

$$\psi_{\zeta\beta p}(x)=\exp[i(f_0x+\operatorname{Im}\tilde{\alpha}(x)\beta^*(x))]\left[1+\frac{e\hat{k}\hat{A}}{2kp}\right]$$
$$\times\frac{u(p)}{V^{1/2}}\exp(-ikxc^+c)|\zeta(x);\beta(x)+\tilde{\alpha}(x)\rangle, \quad (2.20a)$$

where

$$\tilde{\alpha}(x)=\alpha\operatorname{ch}r-\alpha^*\exp[i(2\tilde{k}x-\theta)]\operatorname{sh}r, \quad \theta=\arg\zeta. \quad (2.21)$$

3. SOME STATISTICAL PROPERTIES OF PHOTONS AS THEY INTERACT WITH ELECTRONS

As we have already mentioned, the interaction of photons with electrons results in substantial changes in the statistical properties of the photons. Bergou and Varro[10] showed that the interaction of an incoherent photon beam (in the occupation-number representation) with electrons leads to some new effects. They examined certain statistical aspects of the electron-photon system for the case in which states are specified in the occupation-number representation.

Let us examine the statistics of the photons produced in an interaction with an electron for various states of the original photon beam. We begin with the case in which the photons of the external field are linearly polarized. Working from wave functions (2.1), (2.13), and (2.18), and taking an average over the states of the electron, we find the following expression for the expectation value of the number of photons:

$$\langle c^+c\rangle=\langle \mathrm{ph}|c^+c|\mathrm{ph}\rangle(\mu^2+\nu^2)+\nu^2+\varkappa^2, \quad (3.1)$$

where

$$\mu=\operatorname{ch}z, \quad \nu=\operatorname{sh}z, \quad \varkappa=\alpha e^z. \quad (3.2)$$

There are three ways to specify the photon state $|\mathrm{ph}\rangle$ here: in the occupation-number representation, $|n\rangle$; in the Glauber representation, $|\beta\rangle$; and in a squeezed state, $|\zeta;\beta\rangle$. We have the following expressions for these representations, respectively:

$$\langle\psi_{np}(x)|c^+c|\psi_{np}(x)\rangle=n(\mu^2+\nu^2)+\nu^2+\varkappa^2, \quad (3.3a)$$
$$\langle\psi_{\beta p}(x)|c^+c|\psi_{\beta p}(x)\rangle=|\beta|^2(\mu^2+\nu^2)+\nu^2+\varkappa^2, \quad (3.3b)$$
$$\langle\psi_{\zeta\beta p}(x)|c^+c|\psi_{\zeta\beta p}(x)\rangle=(|\tilde{\beta}|^2+\nu'^2)(\mu^2+\nu^2)+\nu^2+\varkappa^2, \quad (3.3c)$$

where r and θ are defined by (2.19), and

$$\tilde{\beta}=\mu'\beta-\nu'\beta^*e^{-i\theta}, \quad \mu'=\operatorname{ch}r, \quad \nu'=\operatorname{sh}r. \quad (3.4)$$

It can be seen from expressions (3.3) that as an electron moves in vacuum the expectation value of the number of photons is nonzero.

We turn now to the second-order correlation function

$$G^{(2)}=\langle(c^+)^2c^2\rangle-(\langle c^+c\rangle)^2, \quad (3.5)$$

for which we find the following expression:

$$G^{(2)}=G^{(2)}_{\mathrm{ph}}(\mu^2+\nu^2)^2+\langle \mathrm{ph}|(c^+)^2c^2|\mathrm{ph}\rangle 2\mu^2\nu^2$$
$$+2\langle \mathrm{ph}|c^+c|\mathrm{ph}\rangle[\nu^4$$
$$+3\mu^2\nu^2+\varkappa^2(\mu+\nu)^2]+\nu^2(\mu^2+\nu^2)+2\varkappa^2(\mu+\nu)\nu, \quad (3.6)$$

where $G^{(2)}_{\mathrm{ph}}$ is correlation function (3.5) in which the average is taken over photon states. When the photon state is specified in the occupation-number representation, $|n\rangle$, we have

$$G^{(2)}_{\mathrm{ph}}=-n,$$
$$\langle n|(c^+)^2c^2|n\rangle=n^2-n, \quad \langle n|c^+c|n\rangle=n. \quad (3.7)$$

If the photon state is instead expressed in the coherent-state representation, $|\beta\rangle$, we have

$$G^{(2)}_{\mathrm{ph}}=0,$$
$$\langle\beta|(c^+)^2c^2|\beta\rangle=|\beta|^4, \quad \langle\beta|c^+c|\beta\rangle=|\beta|^2. \quad (3.8)$$

For the motion of an electron in a field in a squeezed state we have

$$G^{(2)}_{\mathrm{ph}}=2|\nu'\tilde{\beta}|^2-2\operatorname{Re}\nu'\mu'\tilde{\beta}^2e^{-i\theta}+\nu'^4+\mu'^2\nu'^2,$$
$$\langle\zeta;\beta|(c^+)^2c^2|\zeta;\beta\rangle=|\tilde{\beta}^2-\mu'\nu'e^{-i\theta}|^2+4|\nu'\tilde{\beta}|^2+2\nu'^4, \quad (3.9)$$
$$\langle\zeta;\beta|c^+c|\zeta;\beta\rangle=|\tilde{\beta}|^2+\nu'^2.$$

Taking the expectation value of the correlation function over the phases θ and φ, we find

$$\overline{G^{(2)}_{\mathrm{ph}}}=\nu'^2(\mu'^2+\nu'^2)(2|\beta|^2+1),$$
$$\overline{\langle\zeta;\beta|(c^+)^2c^2|\zeta;\beta\rangle}=(\mu'^4+\nu'^4)|\beta|^4+4\nu'^2(\mu'^2+\nu'^2)$$
$$\times|\beta|^2+2\nu'^4+\mu'^2\nu'^2, \quad (3.10)$$
$$\overline{\langle\zeta;\beta|c^+c|\zeta;\beta\rangle}=(\mu'^2+\nu'^2)|\beta|^2+\nu'^2.$$

It can be seen from the expression for the expectation value of the correlation function over phases that the following relation always holds:

$$\overline{G^{(2)}}\geqslant 0. \quad (3.11)$$

We see from (3.6) that in the linear-polarization case the original correlation function is supplemented by some positive quantities which depend on the parameters of the electron and the field. If the second-order correlation function $G_{\rm ph}^{(2)}$ is negative before the interaction, i.e., if there is a photon debunching, this correlation function may become positive after the interaction with the electron, and a bunching of the field photons may be observed. If the field correlation function is nonnegative, $G_{\rm ph}\geqslant 0$, in the initial state, i.e., if there is a bunching of photons, the interaction with the electron will result in an intensification of the bunching of the external-field photons. The interaction of a quantized electromagnetic field with an electron can thus lead to some substantial effects in the photon statistics.

Calculations can be carried out in a corresponding way for a circular polarization of the external field. In this case we should start from wave functions (2.7), (2.15), and (2.20). After taking an average over the parameters of the electron, we find the following expression for the expectation value of the photon number operator:

$$\langle c^+c\rangle=\langle \text{ph}\,|c^+c|\,\text{ph}\,\rangle+|\alpha|^2. \qquad (3.12)$$

Here α is given by (2.8), and the photon states $|\text{ph}\rangle$ are given in one of three ways: in the occupation-number representation, in the representation of coherent states, or in the representation of squeezed states. In these three cases we have the following respective expressions:

$$\langle\psi_{np}(x)\,|c^+c|\,\psi_{np}(x)\rangle=n+|\alpha|^2, \qquad (3.13a)$$

$$\langle\psi_{\beta p}(x)\,|c^+c|\,\psi_{\beta p}(x)\rangle=|\beta|^2+|\alpha|^2, \qquad (3.13b)$$

$$\langle\psi_{\zeta\beta p}(x)\,|c^+c|\,\psi_{\zeta\beta p}(x)\rangle=|\beta|^2+\nu'^2+|\alpha|^2. \qquad (3.13c)$$

After an average is taken over the phases θ and φ, the latter expression becomes

$$\overline{\langle\psi_{\zeta\beta p}(x)\,|c^+c|\,\psi_{\zeta\beta p}(x)\rangle}=(\mu'^2+\nu'^2)\,|\beta|^2+\nu'^2+|\alpha|^2. \qquad (3.14)$$

The second-order correlation function in the case of circular polarization is given by

$$G^{(2)}=G_{\rm ph}^{(2)}+2|\alpha|^2\langle\,\text{ph}\;|c^+c|\;\text{ph}\,\rangle, \qquad (3.15)$$

where the average over the photon states is carried out in one of three representations: the photon number representation $|n\rangle$, the coherent-state representation $|\beta\rangle$, or the squeezed-state representation $|\zeta;\beta\rangle$.

For these states, the correlation function (3.15) takes the following respective forms:

$$G^{(2)}=n(2|\alpha|^2-1), \qquad (3.16a)$$

$$G^{(2)}=2|\alpha|^2|\beta|^2, \qquad (3.16b)$$

$$G^{(2)}=2|\nu'\tilde{\beta}|^2-2\,\text{Re}\,\mu'\nu'\tilde{\beta}^2e^{-i\theta}+\nu'^4+\mu'^2\nu'^2+2|\alpha|^2(|\tilde{\beta}|^2+\nu'^2). \qquad (3.16c)$$

Taking the average of the latter expression over the phases θ and φ, we find

$$\overline{G^{(2)}}=2\nu'^2(\mu'^2+\nu'^2)\,|\beta|^2+\nu'^4+\mu'^2\nu'^2+2|\alpha|^2[(\mu'^2+\nu'^2)\,|\beta|^2+\nu'^2]. \qquad (3.16d)$$

It can be seen from (3.16b) and (3.16d) that the correlation function is positive, i.e., that the sign of the correlation functions of the free field is retained. In the case of an incoherent field (3.16a) the correlation function is negative, like the correlation function of the free field at values $|\alpha|^2<1/2$.

4. CLASSICAL RADIATION BY AN ELECTRON MOVING IN A QUANTIZED MONOCHROMATIC FIELD

Let us examine the classical radiation by an electron moving in a quantized monochromatic field, specified in various representations. The current density j_μ and the operator representing the kinetic momentum of the electron, π_μ, are related by[3]

$$j_\mu(x)=(e/mV)\langle\,\text{ph}\;|\pi_\mu(x)|\;\text{ph}\,\rangle, \qquad (4.1)$$

where π_μ is an operator averaged over the spin states of the electron which is moving in a quantized electromagnetic field. If the quantized field is linearly polarized, we have

$$\pi_\mu(x)=p_\mu-e^z(eA_\mu+2\alpha b_\mu)+k_\mu\left[\frac{e^z}{kp}\,p_\nu(eA_\nu+2\alpha b_\nu)-\frac{e^{2z}}{2kp}\,(eA_\nu+2\alpha b_\nu)^2\right], \qquad (4.2)$$

where the wave vector k_μ in A_μ is replaced by $\tilde{k}_\mu$ [see (2.14)].

The kinetic momentum (4.2) takes different forms in different representations of the quantized field. In the case of an incoherent field, in which the state is specified in the occupation-number representation, we can use the relation

$$\langle n|\pi_\mu(x)\,|n\rangle=p_\mu+2\left(\tilde{k}_\mu\frac{bp}{\tilde{k}p}-b_\mu\right)\alpha e^z-\tilde{k}_\mu\frac{b^2}{\tilde{k}p}\,e^{2z}\left(n+\frac{1}{2}+2\alpha^2\right). \qquad (4.3a)$$

In the case of a coherent field, in the Glauber-state representation, we find

$$\langle\beta\,|\,\pi_\mu(x)\,|\,\beta\rangle=p_\mu+2\left(\tilde{k}_\mu\frac{bp}{\tilde{k}p}-b_\mu\right)e^z\,(\text{Re}\,\beta e^{i\tilde{k}x}+\alpha)-\tilde{k}_\mu\frac{2b^2}{\tilde{k}p}\,e^{2z}\left[(\text{Re}\beta e^{i\tilde{k}x}+\alpha)^2+\frac{1}{4}\right]. \qquad (4.3b)$$

Finally, in the case of a squeezed state of the external field we have

$$\langle\zeta;\beta\,|\,\pi_\mu(x)\,|\,\zeta;\beta\rangle=p_\mu+2\left(\tilde{k}_\mu\frac{bp}{\tilde{k}p}-b_\mu\right)e^z\,(\text{Re}\,\tilde{\beta}e^{i\tilde{k}x}+\alpha)-\tilde{k}_\mu\frac{2b^2}{\tilde{k}p}\,e^{2z}\left[(\text{Re}\,\tilde{\beta}e^{i\tilde{k}x}+\alpha)^2+\frac{1}{4}\,(\text{ch}\,2r-\text{sh}\,2r\cos(\theta+2\tilde{k}x))\right]. \qquad (4.3c)$$

Averaging over $\tilde{k}x$ in (4.3), we find the respective values of the average kinetic momenta or quasimomenta of the electron:

$$q_\mu(n)=p_\mu+2\left(\tilde{k}_\mu\frac{bp}{\tilde{k}p}-b_\mu\right)\alpha e^z-\tilde{k}_\mu\frac{b^2}{\tilde{k}p}\,e^{2z}\left(n+\frac{1}{2}+2\alpha^2\right),$$

$$q_\mu(\beta)=p_\mu+2\left(\tilde{k}_\mu\frac{bp}{\tilde{k}p}-b_\mu\right)\alpha e^z-\tilde{k}_\mu\frac{b^2}{\tilde{k}p}\,e^{2z}\left(|\beta|^2+\frac{1}{2}+2\alpha^2\right) \qquad (4.4a)$$

$$q_\mu(\zeta;\beta)=p_\mu+2\left(\tilde{k}_\mu\frac{bp}{\tilde{k}p}-b_\mu\right)\alpha e^z-\tilde{k}_\mu\frac{b^2}{\tilde{k}p}\,e^{2z}\left(|\beta|^2+\frac{1}{2}\,\text{ch}\,2r+2\alpha^2\right).$$

The last expression in (4.4a) for the quasimomentum of an electron in a squeezed field depends on the phases of the external field, θ and φ. Taking an average over θ and φ, we find that this exnression becomes

$$\bar{q}_\mu(\bar{n}; r) = p_\mu + 2\left(\tilde{k}_\mu \frac{bp}{\tilde{k}p} - b_\mu\right)\alpha e^{r} - \tilde{k}_\mu \frac{b^2}{\tilde{k}p} e^{2z}\left[\left(\bar{n}+\frac{1}{2}\right)\operatorname{ch} 2r + 2\alpha^2\right]. \tag{4.4b}$$

At high intensities of the external field, as $\bar{n}\to\infty$, we have $V\to\infty$, while $\bar{n}/V$ (the photon density) remains constant, the first two relations in (4.4a) become the expressions for the quasimomentum of an electron moving in an intense classical monochromatic field. These expressions were derived by means of a Volkov wave function.[3] In the case of a squeezed external field ($r\neq 0$), on the other hand, there is no such transition.

Corresponding calculations for the case of a circularly polarized external field starting from wave functions (2.7), (2.15), and (2.20) lead to the following expression for the kinetic-momentum operator of the electron, π_μ:

$$\pi_\mu(x) = p_\mu - (eA_\mu + 2\operatorname{Re}\alpha b_\mu) + \frac{\tilde{k}_\mu}{\tilde{k}p}\Big\{ e(pA) - \frac{|b|^2}{2} + 2\operatorname{Re}\alpha(bp) - (eA_\nu^- + \alpha^* b_\nu^*)(eA_\nu^+ + \alpha b_\nu)\Big\}, \tag{4.5}$$

where $A_\mu^\pm$ are the positive- and negative-frequency parts of the quantized monochromatic field, in which k_μ has been replaced by $\tilde{k}_\mu$ [see (2.16)].

In the particle-number, coherent-state, and squeezed-state representations, the average kinetic momentum is given by the following respective expressions, where we are using (4.5):

$$q_\mu(n) = p_\mu + 2\operatorname{Re}\alpha\left(\tilde{k}_\mu\frac{bp}{\tilde{k}p} - b_\mu\right) - \tilde{k}_\mu\frac{|b|^2}{\tilde{k}p}\left(n+\frac{1}{2}+|\alpha|^2\right),$$

$$q_\mu(\beta) = p_\mu + 2\operatorname{Re}\alpha\left(\tilde{k}_\mu\frac{bp}{\tilde{k}p} - b_\mu\right) - \tilde{k}_\mu\frac{|b|^2}{\tilde{k}p}\left(|\beta|^2+\frac{1}{2}+|\alpha|^2\right),$$

$$q_\mu(\zeta;\beta) = p_\mu + 2\operatorname{Re}\alpha\left(\tilde{k}_\mu\frac{bp}{\tilde{k}p} - b_\mu\right) - \tilde{k}_\mu\frac{|b|^2}{\tilde{k}p}\left(|\beta|^2+\frac{1}{2}\operatorname{ch} 2r+|\alpha|^2\right). \tag{4.6}$$

As in the case of a linearly polarized external field, the last expression in (4.6) contains the phases θ and φ. After an average is taken over these phases, that expression becomes

$$\bar{q}_\mu(\bar{n}; r) = p_\mu + 2\operatorname{Re}\alpha\left(\tilde{k}_\mu\frac{bp}{\tilde{k}n} - b_\mu\right) - \tilde{k}_\mu\frac{|b|^2}{\tilde{k}p}\left[\left(\bar{n}+\frac{1}{2}\right)\operatorname{ch} 2r+|\alpha|^2\right]. \tag{4.7}$$

Following Ritus,[3] who calculated the spectrum of the classical radiation by an electron in a classical external field on the basis of Volkov functions, we carry out corresponding calculations for various representations of the quantized external field. In the present section of the paper we assume that the energy and momentum of the radiated photon are much smaller than the energy and momentum of the electron, and we ignore the inverse effect of the radiation on the electron. We assume that the state of the quantized external field is given. Under these assumptions we can use the equations of classical electrodynamics for the radiation, where the current is determined by quantum-mechanical expectation value (4.1). The intensity of the radiation by the electron is given by

$$\frac{dI_{k'}}{V} = |j_\mu(k')|^2\frac{Vd^3k'}{4\pi^2 T}. \tag{4.8}$$

Here $j_\mu(k')$ is a Fourier component of the 4-current vector of the electron and is given by

$$j_\mu(k') = \int_{-\infty}^{+\infty} j_\mu(s)\exp[-ik'x(s)]\,ds, \tag{4.9}$$

where the integration is over the proper time of the electron. Expression (4.9) for the Fourier component of the current can be written in the following form,[3] where we are making use of (4.1):

$$j_\mu(k') = \frac{e}{(\tilde{k}p)V}\int_{-\infty}^{+\infty}\langle \text{ph}\,|\pi_\mu(y)|\,\text{ph}\rangle e^{f(y)}dy, \tag{4.10}$$

where

$$y = \tilde{k}x = \frac{\tilde{k}p}{m}(s-s_0), \qquad f(y) = -\frac{i}{\tilde{k}p}\int_0^y k_\nu'\langle \text{ph}\,|\pi_\nu(y)|\,\text{ph}\rangle dy. \tag{4.11}$$

Substituting in (4.11) the expectation values of the kinetic momentum for the various representations (4.3) of the linearly polarized quantized field, we find

$$f_n(y) = -i\frac{k'q(n)}{\tilde{k}p}y,$$

$$f_\beta(y) = -\frac{i}{\tilde{k}p}\Big\{k'q(\beta)y - 2(k'b+\alpha e^z kk')e^z \times \operatorname{Im}\beta e^{iy} - \frac{kk'}{2\tilde{k}p}b^2\operatorname{Im}\beta^2 e^{2iy}\Big\},$$

$$f_{\zeta\beta}(y) = -\frac{i}{\tilde{k}p}\Big\{k'q(\zeta;\beta)y - 2(k'b+\alpha e^z kk')e^z\operatorname{Im}\beta e^{iy} - \frac{kk'}{2\tilde{k}p}b^2\operatorname{Im}(\beta^2 - \operatorname{sh} r\operatorname{ch} r e^{-i\theta})e^{2iy}\Big\}. \tag{4.12}$$

In the circular-polarization case we find

$$f_n(y) = -i\frac{k'q(n)}{\tilde{k}p}y,$$

$$f_\beta(y) = -\frac{i}{\tilde{k}p}\{k'q(\beta)y - 2\operatorname{Im}\beta e^{iy}(k'b+\alpha^* kk')\}, \tag{4.13}$$

$$f_{\zeta\beta}(y) = -\frac{i}{\tilde{k}p}\{k'q(\zeta;\beta)y - 2\operatorname{Im}\beta e^{iy}(k'b+\alpha^* kk')\}.$$

We find the intensity of the radiation of the classical field, (4.8), by substituting (4.12) (for a linearly polarized external field) and (4.13) (circularly polarized) into expression (4.10). We then find the following respective expressions for the intensity:

$$dI_{k'}=-\frac{e^2m^2}{4\pi q_0(\tilde{k}p)}\sum_{s=1}^{\infty}\Big\{-|B_0(s)|^2+\frac{b^2}{m^2}(|B_1(s)|^2-\operatorname{Re}B_0(s)B_2^*(s)\Big\}\times\delta\Big(s-\frac{k'q}{\tilde{k}p}\Big)d^3k',\qquad(4.14)$$

$$dI_{k'}=-\frac{e^2m^2}{4\pi q_0(\tilde{k}p)}\sum_{s=1}^{\infty}\Big\{-|B_0(s)|^2+\frac{|b|^2}{m^2}(|B_-(s)|^2+|B_+(s)|^2-\operatorname{Re}B_0^*(s)C(s)\Big\}\delta\Big(s-\frac{k'q}{\tilde{k}p}\Big)d^3k'.\qquad(4.15)$$

The expressions derived here for the intensity of the radiation of a classical field are outwardly the same as the corresponding expressions of Ref. 3, where the calculations were carried out with the help of Volkov solutions. Their coefficients $B_i(s)$, however, are different, depending on the particular representation of the quantized external field. Furthermore, in the argument of the δ-function which imposes a condition on the values of k'_μ, the quantity q_μ is to be understood as expression (4.4a) or (4.6) for the case of a linear or circular polarization, respectively. As we have already mentioned, q_μ differs from its classical analog, so that allowance for the quantum-mechanical nature of the external field results in changes in the form of the discrete spectrum k'_μ.

In the case of linear polarization, the coefficients $B_i(s)$ are related by

$$[s(\tilde{k}p)-k'q+k'p]B_0(s)+\Big(k'k\frac{bp}{kp}-k'b\Big)B_1(s)-\frac{kk'}{2kp}b^2B_2(s)=0,\qquad(4.16)$$

and in the case of circular polarization they are related by

$$(k'b+\alpha^*kk')B_-(s)+(k'b^*+\alpha kk')B_+(s)=[s(\tilde{k}p)+2\operatorname{Re}\alpha(k'b+\alpha^*kk')]B_0(s).\qquad(4.17)$$

As expected, the occupation-number representation describes a state in which the energy of the electron is completely determined, and there is no radiation. For a coherent state of the external field with a linear polarization, we find the following expressions for the coefficients:

$$B_0(s)=\sum_{l=-\infty}^{\infty}J_{s+2l}(a)J_l(d)e^{is\varphi},$$

$$B_1(s)=e^z\{2\alpha B_0(s)+\beta B_0(s-1)+\beta^*B_0(s+1)\},$$

$$B_2(s)=e^{2z}\{(2n+1-4\alpha^2)B_0(s)+4\alpha e^{-z}B_1(s)+\beta^2B_0(s-2)+\beta^{*2}B_0(s+2)\}.\qquad(4.18)$$

where

$$a=\frac{2|\beta|}{\tilde{k}p}e^z(k'b+\alpha e^zkk'),\quad d=-\frac{kk'}{2(\tilde{k}p)^2}|b|^2|\beta|^2.\qquad(4.19)$$

In the case of circular polarization we find

$$B_0(s)=J_s(|a|)e^{is\tilde{\varphi}},\quad B_-(s)=\alpha B_0(s)+\beta B_0(s-1),\qquad(4.20)$$
$$B_+(s)=\alpha^*B_0(s)+\beta^*B_0(s+1),$$
$$C(s)=(2n+2|\alpha|^2+1)B_0(s)+2\alpha^*\beta B_0(s-1)+2\alpha\beta^*B_0(s+1),$$

where

$$a=\frac{2\beta}{\tilde{k}p}(k'b+\alpha^*kk'),\qquad(4.21)$$

$$\tilde{\varphi}=\arg a.$$

The expressions for the intensity of the classical radiation of an electron in a coherent external field become the corresponding expressions of Ref. 3 when we take the limits $\bar{n}\to\infty$ and $V\to\infty$, holding $\bar{n}/V$ finite.

If we choose the external field in a squeezed state, the expressions for the coefficients in the case of a linear polarization become slightly more complicated:

$$B_0(s)=\sum_{l=-\infty}^{\infty}J_{s+2l}(|a|)J_l(|d|)\exp[i(s+2l)\tilde{\varphi}]\exp(-il\Delta),$$

$$B_1(s)=e^z\{2\alpha B_0(s)+\tilde{\beta}B_0(s-1)+\tilde{\beta}^*B_0(s+1)\},\qquad(4.22)$$

$$B_2(s)=e^{2z}\{(2|\beta|^2+\operatorname{ch}2r-4\alpha^2)B_0(s)+4\alpha e^{-z}B_1(s)+(\tilde{\beta}^2-\operatorname{sh}r\operatorname{ch}re^{-i\theta})B_0(s-2)+(\tilde{\beta}^{*2}-\operatorname{sh}r\operatorname{ch}re^{i\theta})B_0(s+2)\},$$

where

$$a=\frac{2\tilde{\beta}}{\tilde{k}p}e^z(k'b+\alpha e^zkk')=|a|e^{i\tilde{\varphi}},$$
$$d=-\frac{kk'}{2(\tilde{k}p)^2}b^2(\tilde{\beta}^2-e^{-i\theta}\operatorname{sh}r\operatorname{ch}r)^2=|d|e^{i\Delta}.\qquad(4.23)$$

Relations (4.20) and (4.21) remain valid for a circular polarization of a squeezed external field after we make the following substitution everywhere in them:

$$\beta\to\tilde{\beta}\equiv\beta\operatorname{ch}r-\beta^*\operatorname{sh}re^{-i\theta}.\qquad(4.24)$$

With a squeezing parameter $r=0$, expressions (4.22) become (4.18) for a coherent external field.

It can be seen from expressions (4.14) and (4.15) that the wave vector k', of the radiated photon, satisfies the condition

$$k'q=s\tilde{k}q,\qquad(4.25)$$

which has the same form as in the case of a classical field,[3] except that k_μ is replaced by $\tilde{k}_\mu$ and the average kinetic momentum q is given by (4.4) and (4.6).

Denoting by ω_0 the frequency of the wave in the frame of reference in which the electron is at rest on the average ($\mathbf{q}=0, q_0=m_*$), we have $\tilde{k}q=-\tilde{\omega}_0m_*$, where $\tilde{\omega}_0=\omega_0\exp(-2z)$, and m_* is the effective mass of an electron in the field. This mass takes different forms, depending on the polarization and representation of the quantized external field. From (4.25) we find a relation analogous to the expression for the Doppler effect:

$$\omega'=s\tilde{\omega}_0(1-v^2)^{1/2}/(1-v\cos\gamma).\qquad(4.26)$$

Expression (4.26) relates the frequency of the sth harmonic in the frame of reference in which the electron is, on the average, at rest to its frequency (ω') in a frame in which it has an average velocity $\mathbf{v} = \mathbf{q}/q_0$. It can be seen from (4.26) that the radiation spectrum is discrete and that the frequency depends on the angle (γ) between the direction of the radiation and the average momentum $\mathbf{q}$.

5. COMPTON EFFECT IN AN INTENSE QUANTIZED FIELD

The radiation of a photon by an electron in an intense classical field was studied in Refs. 1–3, where the Volkov solution for the Dirac equation was used to calculate the probabilities for a many-photon process in a plane external electromagnetic wave. Since there may be several photons in the interaction region if the external field is intense, the process by which a photon is emitted involves the absorption of several photons of the external field.

The Compton effect for a quantized field was analyzed in Refs. 9 and 10 in the particle-number representation. The probability for the process was calculated in Ref. 10 for the relativistic case, but the calculations were carried out completely there only for a circularly polarized external field.

The transition matrix element for the Compton effect with a linearly polarized external field has the same form as in the case of a classical field[3]:

$$M_{i\to f}=\frac{e}{(2\omega'V)^{1/2}}\bar{u}_{p_f}\left[A_{i\to f}\hat{e}''+B_{i\to f}\left(\frac{\hat{e}''\hat{k}\hat{b}}{2kp_i}+\frac{\hat{b}\hat{k}\hat{e}''}{2kp_f}\right)\right]u_{p_i}, \tag{5}$$

where

$$e''=e'-\frac{ke'}{kk'}k' \quad (ke'')=0, \tag{5.2}$$

and the coefficients are given by

$$A_{i\to f}=\int d^4x\exp[i(f_{0f}+k'-f_{0i})x]\langle \text{ph}\,|i;\,\text{ph}\rangle,$$
$$B_{i\to f}=\int d^4x\exp[i(f_{0f}+k'-f_{0i})x]\langle f;\,\text{ph}\,|c+c^+|\,i;\,\text{ph}\rangle. \tag{5.3}$$

Evaluating the matrix elements in (5.3) in the representation of the number of particles of the quantized external field, we can then go over to the cases of coherent and squeezed fields by means of the transformation functions

$$A_{\beta_i\to\beta_f}=\sum_{n_i,\,n_f}\langle\beta_f|n_f\rangle\langle n_i|\beta_i\rangle A_{n_i\to n_f}\delta(f_{n_f}+k'-f_{n_i}),$$

$$A_{\zeta_i\beta_i\to\zeta_f\beta_f} \tag{5.4}$$

$$=\sum_{n_i,\,n_f}{}'\langle\zeta_f;\beta_f|n_f\rangle\langle n_i|\zeta_i;\beta_i\rangle A_{n_i\to n_f}\delta(f_{n_f}+k'-f_{n_i}).$$

Corresponding relations can be derived for the coefficients $B_{i\to f}$. In the case of the external-field particle-number representation the coefficients A and B are expressed in terms of Hermite polynomials in the following way:

$$A_{n_i\to n_f}=(2\pi)^4\langle\alpha_f|\Delta z;\alpha_i\rangle(n_i!\,n_f!)^{1/2}\left(\frac{\text{th}\,\Delta z}{2}\right)^{(n_i+n_f)/2}$$
$$\times\sum_{k=0}^{(\min n_i,\,n_f)}\frac{H_{n_i-k}(y)H_{n_f-k}(x)[(i/2)\,\text{sh}\,\Delta z]^{-k}}{k!\,(n_i-k)!\,(n_f-k)!}, \tag{5.5}$$

where

$$\Delta z=z_f-z_i,$$
$$x=-\frac{i}{2}\left(-\alpha_i+\alpha_i\,\text{th}\,\Delta z+\frac{\alpha_f}{\text{ch}\,\Delta z}\right)\left(\frac{\text{th}\,\Delta z}{2}\right)^{-1/2}, \tag{5.6}$$
$$y=-\frac{i}{2}\left(\alpha_f-\alpha_f\,\text{th}\,\Delta z+\frac{\alpha_i}{\text{ch}\,\Delta z}\right)\left(-\frac{\text{th}\,\Delta z}{2}\right)^{-1/2}$$

and

$$B_{n_i\to n_f}=\frac{1}{\text{ch}\,\Delta z}\{(\alpha_f e^{-z_i}+\alpha_i e^{-z_f})A_{n_i\to n_f}$$
$$+e^{-z_i}A_{n_i\to n_f-1}+e^{-z_f}A_{n_i-1\to n_f}\}. \tag{5.7}$$

The expression for the probability for the emission of a photon, averaged over the initial polarization states of the electron and summed over the final states, is as follows, for a linearly polarized quantized external field, in the particle-number representation:

$$w_{i\to f}=\frac{e^2(2\pi)^4}{2\omega' m^2}\delta(f_{n_f}+k'-f_{n_i})\Big\{[2(p_ie'')^2+f_if_f-m^2]|A|^2$$
$$+\left[kk'\left(\frac{bp_f}{kp_f}-\frac{bp_i}{kp_i}\right)-4(p_ie'')(be'')\right]\text{Re}\,A^*B$$
$$+\left[\frac{b^2(kk')^2}{2(kp_i)(kp_f)}+2(be'')^2\right]|B|^2\Big\}. \tag{5.8}$$

For a circularly polarized field the matrix element is found in the same way as for a classical field[3]:

$$M_{i\to f}=\frac{e}{(2\omega'V)^{1/2}}\bar{u}_{p_f}\left[B_0\hat{e}''+B_-\left(\frac{\hat{e}''\hat{k}\hat{b}}{2kp_i}+\frac{\hat{b}\hat{k}\hat{e}''}{2kp_f}\right)\right.$$
$$\left.+B_+\left(\frac{\hat{e}''\hat{k}\hat{b}}{2kp_i}+\frac{\hat{b}\hat{k}\hat{e}''}{2kp_f}\right)\right]u_{p_i}. \tag{5.9}$$

Explicit expressions for the coefficients B_0 and $B_\pm$ and also for the probability for the emission of a photon in the case of a circularly polarized quantized external field are given in Ref. 10. It can be shown that these coefficients are related by

$$(n_i-n_f+|\alpha_f|^2+|\alpha_i|^2)B_0=\Delta\alpha B_++\Delta\alpha^*B_-, \tag{5.10}$$

where $\Delta\alpha=\alpha_f-\alpha_i$.

The expressions for the matrix element, (5.9), and for the photon emission probability remain the same in form in the cases of coherent and squeezed external fields; the only change is that the coefficients in these expressions are transformed by means of relations analogous to (5.4).

CONCLUSION

We have analyzed the dynamics of a charged particle in a quantized monochromatic electromagnetic field whose state has been specified in various representations (the particle-number, coherent-state, and squeezed-state representations). It has been shown that a squeezed state is formed in a bounded volume both during the interaction of an electron with individual linearly polarized vacuum modes and during the motion of an electron in a quantized coherent radiation field with linear polarization. In the case of a circular polarization of the quantized external field, on the other hand, a coherent state of photons forms. We have calculated the expectation values of the photon-number operator and the second-order correlation functions. We have analyzed the statistics for the interaction of an electron with quantized electromagnetic radiation for various states.

The results show that the original correlation function is supplemented with some positive quantities which depend on the parameters of the electron and the field. If a debunching of photons occurs before the interaction with the electron, a bunching of the photons of the external field can be observed after the interaction. The interaction of a quantized electromagnetic field with an electron thus results in an intensification of the bunching of the photons and in important changes in the statistics of the external field. Expressions have been derived for the intensity of the radiation of a classical field and for the probability for the Compton effect for an electron which is moving in a quantized field of either linear or circular polarization. It has been shown that the results depend strongly on the state of the quantized external field.

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