Nonstationary-image scale transformations by photon-echo signals

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Recording and reconstruction of nonstationary optical images by signals produced by photon echo or its modifications have been investigated. Scale transformations were observed on excitation of the resonant medium by light pulses having spherical wave fronts. The properties of the scale transformations as functions of the exciting-pulse wave-front curvature are discussed.

1. INTRODUCTION

Photon echo (PE) is the optical analog of the spin echo well known in the rf band and first discovered and explained by Hahn in 1950.[1] Many features of the two phenomena have clearly pronounced similarities manifested primarily in the temporal properties of the coherent spontaneous-emission signals. In particular, comprehensive studies of the correlation between the temporal form of the excited pulses and the echo signals were made quite recently for the optical band.[2–15] A feature of the photon echo is a large variety of spaciotemporal properties that are of practical interest for optical methods of information processing. These include the reconstruction of the temporal profile of a coding light pulse in the forward and mirror-reversed directions,[2–15] temporal correlation analysis of the excitation-field envelopes,[16–20] compression of light pulses in PE signals,[21,22] and others.

Dynamic-holography methods based on reconstruction of coherent pulsed fields with the aid of PE have been extensively developed by now. In traditional holography it is required that a reference wave interfere with a scattered object wave to form a stationary field distribution. In echo holography, however, this requirement is not obligatory. On the contrary, owing to the phase memory, an echo hologram can be recorded when the object and reference waves are separated in time, and the sensitivity of the resonant medium to the spectral composition of the light makes it possible to record and reconstruct nonstationary object scenes. These are precisely the properties demonstrated in a cycle of studies of spaciotemporal holography in highly selective photometric media, based on photochemical hole burning.[30–35] In particular, recording of a wave-front structure using a photo-chemically accumulated stimulated optical echo is an inherent property of the long-time photon echo observed in crystals with paramagnetic impurities.[18,29,36,37] As a result, reconstruction of nonstationary object scenes is possible in the forward and time-reversed directions.

New possibilities of recording and reconstructing nonstationary images are afforded by the use of multilevel resonant media. It is known that reproduction of the temporal form of exciting pulses is accompanied under these conditions by a characteristic scale transformation that depends on the specific mechanisms of the inhomogeneous broadening of the resonance energy levels.[16] Planar object scenes turn out in this case to be spatially scale-invariant, notwithstanding the frequency difference between the recording and reconstruction procedures.[38]

We show in the present paper that when light pulses with spherical wave fronts are used the character of the nonstationary images reconstructed by using PE signals can be spacelike with different scale factors. We determine for the primary stimulated PE, and also for the modified stimulated PE, the parameters of the scale transformations of nonstationary images of the paraxial approximation, as functions of the curvature radii of the excitation-pulse wave fronts.

2. PRINCIPAL EQUATIONS

We shall describe interaction of coherent light pulses with a resonant medium in a quasi-optical approximation, using the parabolic diffraction-theory equation

$$\frac{\partial E}{\partial z}-\frac{i}{2k}\Delta_\perp E=i\frac{2\pi\omega}{cn}N_0\langle d_e\rangle,\qquad \Delta_\perp=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},\tag{1}$$

where $E(\mathbf{r},z,\tau)$ and $\langle d_e(\mathbf{r},z,\tau)\rangle$ are slowly varying amplitudes of the light-wave electric-field strength and of the induced dipole moment of the impurity atom or ion in the resonance medium ($\mathbf{r}=x\mathbf{i}+y\mathbf{j}$), k and ω are the wave number and carrier frequency of the optical pulses, the z coordinate coincides with the light-beam propagation direction, the angle brackets in the right-hand side of Eq. (1) denote averaging of the atom's dipole resonance moment over the frequency spread, N_0 is the density of the resonance atoms, and n is a refractive index of nonresonant type. The time τ is introduced as $\tau=t-zn/c$.

We define the Green's function $G(\mathbf{r},z)$ as the solution of the equation

$$\frac{\partial G}{\partial z}-\frac{i}{2k}\Delta_\perp G=\delta(\mathbf{r})\delta(z).\tag{2}$$

The function $G(\mathbf{r},z)$ turns out here to equal[39]

$$G(\mathbf{r},z)=\theta(z)\int\frac{d\mathbf{q}}{(2\pi)^2}\exp\left(i\mathbf{qr}-iz\frac{\mathbf{q}^2}{2k}\right)\tag{3}$$

or else

$$G(\mathbf{r},z)=-\frac{ik\theta(z)}{2\pi z}\exp\left(\frac{ik\mathbf{r}^2}{2z}\right),\tag{4}$$

where $\theta(z)$ is the unit step function.

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We neglect hereafter the reaction of the medium as well as the diffraction, and assume satisfaction of the inequality

$$l < ka_r^2,\ \alpha_0^{-1},$$

where a_r is the characteristic transverse dimension of the resonant medium, l is its length, and α_0 is the absorption coefficient. The amplitude of the echo-signal electric field at the exit from the resonant medium can be represented in this case in the form

$$E(\mathbf{r},z,\tau)=i\frac{2\pi\omega}{cn}N_0\langle d_\varepsilon(\mathbf{r},\tau)\rangle l. \quad (5)$$

The solution of the homogeneous equation (1), which describes the propagation of the light beam in free space, can be expressed by using specified boundary conditions given by $E(\mathbf{r},z,\tau) = E_0(\mathbf{r},\tau)\delta(z)$:

$$E(\mathbf{r},z,\tau)=\int dz'\,d\mathbf{r}'\,G(\mathbf{r}-\mathbf{r}',z-z')E_0(\mathbf{r}',\tau)\delta(z')$$
$$=\int d\mathbf{r}'\,G(\mathbf{r}-\mathbf{r}',z)E_0(\mathbf{r}',\tau). \quad (6)$$

We consider now the formation and transformation of optical images by PE signals.

3. SCALE TRANSFORMATIONS PRODUCED BY PRIMARY PHOTON-ECHO SIGNALS

Let a spherical light wave with wave front curvature center at the point $\mathbf{r}=0$, $z=-R_1$ illuminate a flat transparency located in the plane $z=0$ and having an amplitude transmissivity $T(\mathbf{r},\tau)$. Note that the dependence of T on the time τ allows us to consider nonstationary object scenes. A resonant medium of small length l (e.g., a thin layer of resonant atoms) is placed at a distance L from the transparency and serves as a dynamic phase hologram.

We write the solution of the homogeneous Eq. (1) that describes the propagation of the light beam in the region from the transparency to the hologram in the form

$$E_1(\mathbf{r},L,\tau)=-\left(\frac{ik}{2\pi L}\right)E_{01}\int d\mathbf{r}'\,T(\mathbf{r}',\tau)$$
$$\times\exp\left[\frac{ik\mathbf{r}'^2}{2R_1}+\frac{ik(\mathbf{r}-\mathbf{r}')^2}{2L}\right]. \quad (7)$$

The solution obtained for E_1 serves as the first exciting pulse for the resonant medium (hologram). Note that this solution was obtained under the simplifying assumption that the aperture delay is short compared with the pulse duration, i.e., $a^2/cL < \delta$, where a is the characteristic transverse dimension of the information transparency, and δ is the duration of the exciting light pulse.

Assume that after a time interval τ_0 the resonant medium (hologram) is acted upon by a second exciting light pulse, having likewise a spherical front with curvature radius R_2:

$$E_2(\mathbf{r},L,\tau)=E_{02}f_2(\tau)\exp\left[\frac{ik(\mathbf{r}-\mathbf{r}_0)^2}{2R}\right]. \quad (8)$$

The determination of the resonant response of the medium to external pulses of form (7) and (8) is a complicated electrodynamic problem, in which it is necessary to take into account the quantum properties of the resonant medium. A solution of a similar but simplified problem in which the exciting pulses have plane wave fronts, was discussed in the literature long ago and is the basis of echo-spectroscopy of resonant media.[40]

We use an approximation well known in photon-echo theory (see, e.g., Refs. 38 and 39), and write for the electric field of the PE signal at the exit from the resonant medium

$$E(\mathbf{r},L+l,\tau)=\frac{\pi^2\omega|d_{ab}|^4N_0l}{cn\hbar^3}\int\frac{d\varepsilon}{2\pi}g(\varepsilon)E_2^{F2}(\mathbf{r},L,\varepsilon)$$
$$\times E_1^{F*}(\mathbf{r}\ L,\varepsilon)\exp[-i\varepsilon(\tau-\tau_e)], \quad (9)$$

where d_{ab} is a transition dipole-moment matrix element, τ_e is the time of onset of the PE, and the functions $E^F_{1,2}(\mathbf{r},L,\varepsilon)$ are the Fourier amplitudes of the frequency spectrum:

$$E_\alpha^F(\mathbf{r},L,\varepsilon)=\int_0^\delta E_\alpha(\mathbf{r},L,\tau)\exp(i\varepsilon\tau)\,d\tau,\qquad \alpha=1,2. \quad (10)$$

In the case of greatest practical interest, a resonant medium excited by two pulses that are spectrally narrow compared with the inhomogeneous linewidth described by the function $g(\varepsilon)$, it is possible to reconstruct the time structure of the coding pulse in the mirror-reversed time direction. It is assumed that the spectral width of the code pulse is the smallest of all those possible in the integral equation (9), where the two functions $g(\varepsilon)$ and $E_2^F(\varepsilon)$ can be taken outside the integral sign. We get as a result

$$E_e(\mathbf{r},z,\tau)=\gamma\left(\frac{ik}{2\pi L}\right)E_{02}{}^2E_{01}{}^*\exp\left[\frac{ik(\mathbf{r}-\mathbf{r}_0)^2}{2R_2}\right]$$
$$\times\int d\mathbf{r}'\,T^*(\mathbf{r}',\tau_e-\tau)\exp\left[-\frac{ik\mathbf{r}'^2}{2R_1}+\frac{ik(\mathbf{r}-\mathbf{r}')^2}{2L}\right], \quad (11)$$
$$z=L+l,$$

$$\gamma=\frac{\pi^2\omega|d_{ab}|^4N_0lg(0)}{cn\hbar^3}\left(\int_0^{\delta_2}f_2(\tau)\,d\tau\right)^2 \quad (12)$$

For the amplitude of the echo-response electric field in the observation plane $\mathbf{r}_\alpha$ characterized by the arbitrary coordinate z we can easily obtain in similar fashion

$$E_e(\mathbf{r}_\alpha,z,\tau)$$
$$=\gamma\left(\frac{ik}{2\pi L}\right)\left(\frac{-ik}{2\pi(z-L)}\right)E_{02}{}^2E_{01}{}^*\int d\mathbf{r}\,d\mathbf{r}'\,T^*(\mathbf{r}',\tau_e-\tau)$$
$$\times\exp\left[-\frac{ik\mathbf{r}'^2}{2R_1}-\frac{ik(\mathbf{r}-\mathbf{r}')^2}{2L}+\frac{ik(\mathbf{r}-\mathbf{r}_0)^2}{R_2}+\frac{ik(\mathbf{r}_\alpha-\mathbf{r})^2}{2(z-L)}\right]. \quad (13)$$

It should be noted that in the important particular case $T(\mathbf{r}')=1$ Eq. (13) describes a spherical wave of the form

$$\exp[ik\mathbf{r}_\alpha^2/(z-L+R^*)],\qquad (R^*)^{-1}=2(R_2)^{-1}-(R_1+L)^{-1}.$$

For $z=L$, i.e., at the exit from the resonant medium, the curvature radius of the given spherical wave is equal to R^*, in agreement with the results of Ref. 41. When the resonant medium is excited by a sequence of pulses with plane and spherical wave fronts, i.e., for $R_2=\infty$, we obtain for the PE signal a spherical wave that converges at the point

$z = 2L + R_1$, meaning reversal of the wave front.[25]

From (13) follows the condition for the onset of sharp images:

$$-\frac{1}{2L}+\frac{1}{R_2}+\frac{1}{2(z-L)}=0, \tag{14}$$

i.e., the plane of the image formed by the PE signal should be separated from the resonant medium by a distance

$$z-L=L'=\left(\frac{1}{L}-\frac{2}{R_2}\right)^{-1}=\frac{LR_2}{R_2-2L}. \tag{15}$$

If condition (14) is met, the integral with respect to **r** can be easily taken

$$\int d\mathbf{r}\exp\left(\frac{ik\mathbf{r}\mathbf{r}'}{L}-\frac{2ik\mathbf{r}\mathbf{r}_0}{R_2}-\frac{ik\mathbf{r}_\alpha\mathbf{r}}{L'}\right)=(2\pi)^2\left(\frac{L}{k}\right)^2 \times\delta\left(\mathbf{r}'-\frac{L}{L'}\mathbf{r}_\alpha-\frac{2L}{R_2}\mathbf{r}_0\right),$$

and we obtain ultimately

$$E(\mathbf{r}_\alpha, L+L', \tau) =\gamma\left(\frac{L}{L'}\right)E_{02}^2E_{01}^*T^*\left(\frac{L}{L'}\mathbf{r}_\alpha+\frac{2L}{R_2}\mathbf{r}_0,\tau_e-\tau\right)\exp(i\Phi), \tag{16}$$

where the phase Φ turns out to be a complicated function of the radius vectors $\mathbf{r}_\alpha$ and $\mathbf{r}_0$, and also of the curvature radii R_1 and R_2 and of the distances L and L':

$$\Phi=-\frac{k}{2L}\left(\frac{L}{L'}\mathbf{r}_\alpha+\frac{2L}{R_2}\mathbf{r}_0\right)^2 -\frac{k}{2R_1}\left(\frac{L}{L'}\mathbf{r}_\alpha+\frac{2L}{R_2}\mathbf{r}_0\right)^2+\frac{k}{R_2}\mathbf{r}_0^2+\frac{k}{2L'}\mathbf{r}_\alpha^2. \tag{17}$$

It follows then that the PE signal intensity in the image-formation plane is

$$I(r_\alpha,\tau)=\Gamma I_{02}^2I_{01}\left|T\left(\frac{L}{L'}\mathbf{r}_\alpha+\frac{2L}{R_2}\mathbf{r}_0,\tau_e-\tau\right)\right|^2, \tag{18}$$

where Γ is a coefficient that depends on γ, L, and L'.

The primary-PE signal from thus nonstationary images, the time sequence in which is mirror-inverted, and the images themselves are spatially similar with a scale factor M:

$$M=\frac{L'}{L}=\frac{R_2}{R_2-2L}=\left(1-\frac{2L}{R_2}\right)^{-1}. \tag{19}$$

The character of the transformation of the initial image (contraction or elongation) depends on $|M|$: elongation if $|M|>1$ and contraction if $|M|<1$. If the scale factor $M<0$, the image will be inverted.

We emphasize that when a resonant medium is excited by plane waves the character of the images formed by the PE signals remains spatially scale-invariant, i.e., $M\equiv 1$, in agreement with the results of Fig. 38.

4. SCALE TRANSFORMATIONS PRODUCED BY STIMULATED PE SIGNALS

Stimulated photon echo (SPE) is extensively used at present both in spectroscopy and in applications (optical information processing), and determines the physical mechanism of formation of long-time PE. Since three optical pulses are used to excite SPE signals, the possibilities of scale transformations in the images are more extensive.

We shall assume that the coding light pulse, which can be any one of a sequence of three exciting pulses, is similar to the one given by Eq. (7), and the two other pulses are described by expressions of the type (8). Using then the results of Ref. 16, we obtain for the amplitude of the stimulated PE at the exit from the resonant medium

$$E(\mathbf{r},z,\tau)=\frac{2\pi^2\omega|d_{ab}|^4N_0l}{cn\hbar^3}\int\frac{d\varepsilon}{2\pi}g(\varepsilon)E_3^F(\mathbf{r},z,\varepsilon) \times E_2^F(\mathbf{r},z,\varepsilon)E_1^{F*}(\mathbf{r},z,\varepsilon)\exp[-i\varepsilon(\tau-\tau_e)]. \tag{20}$$

Again, when the spectral width of the coding pulse turns out to be the smallest among all possible widths, including the line width, we can obtain from (20)

$$E_e(\mathbf{r},z,\tau)=\gamma'\left(\frac{ik}{2\pi L}\right)E_{03}E_{02}E_{01}^*\exp\left[\frac{ik(\mathbf{r}-\mathbf{r}_{02})^2}{2R_2}+\frac{ik(\mathbf{r}-\mathbf{r}_{03})^2}{2R_3}\right]\int d\mathbf{r}'\,T^*(\mathbf{r}',\tau_e-\tau)\exp\left[-\frac{ik\mathbf{r}'^2}{2R_1}-\frac{ik(\mathbf{r}-\mathbf{r}')^2}{2L}\right] \tag{21a}$$

$$E_3(\mathbf{r},z,\tau)=\gamma''\left(\frac{-ik}{2\pi L}\right)E_{03}E_{02}E_{01}^*\exp\left[\frac{-ik(\mathbf{r}-\mathbf{r}_{01})^2}{2R_1}+\frac{ik(\mathbf{r}-\mathbf{r}_{03})^2}{2R_3}\right] \times\int d\mathbf{r}'\,T(\mathbf{r}',\tau-\tau_e)\exp\left[\frac{ik\mathbf{r}'^2}{2R_2}+\frac{ik(\mathbf{r}-\mathbf{r}')^2}{2L}\right]\quad z=L+l, \tag{21b}$$

where

$$\gamma'=\frac{2\pi^2\omega|d_{ab}|^4N_0lg(0)}{cn\hbar^3}\left(\int f_2(t)\,dt\int f_3(t)\,dt\right),$$

and the expression for γ'' is obtained from γ' by the substitution $f_2\to f_1$.

Note that the information pulse is the first exciting light pulse in Eq. (21a) and the second in (21b).

The expression for the amplitude of the electric field intensity with arbitrary coordinate z is similarly modified, and the result is

$$(L')^{-1}=(L)^{-1}-(R_3)^{-1}-(R_2)^{-1}, \tag{22a}$$

$$(L'')^{-1}=-(L)^{-1}+(R_1)^{-1}-(R_3)^{-1}. \tag{22b}$$

The PE field distribution in the $\mathbf{r}_\alpha$ observation plane takes then the form

$$E_e(\mathbf{r}_\alpha,L+L',\tau)=\gamma'\left(\frac{L}{L'}\right)E_{03}E_{02}E_{01}^*T^*\left(\frac{L}{L'}\mathbf{r}_\alpha+\frac{L}{R_2}\mathbf{r}_{02}+\frac{L}{R_3}\mathbf{r}_{03},\tau_e-\tau\right)\exp(i\Phi'), \tag{23a}$$

$$\Phi' = -\frac{k}{2}\left(\frac{1}{R_1}+\frac{1}{L}\right)\left(\frac{L}{L'}\mathbf{r}_\alpha+\frac{L}{R_2}\mathbf{r}_{02}+\frac{L}{R_3}\mathbf{r}_{03}\right)^2 + \frac{k\mathbf{r}_\alpha^2}{2L'}+\frac{k\mathbf{r}_{02}^2}{2R_2}+\frac{k\mathbf{r}_{03}^2}{2R_3} \quad (23b)$$

or else

$$E(\mathbf{r}_\alpha, L+L'', \tau) = \gamma''\left(\frac{L}{L''}\right)E_{03}E_{02}E_{01}^{*}T\left(-\frac{L}{L''}\mathbf{r}_\alpha - \frac{L}{R_3}\mathbf{r}_{03}+\frac{L}{R_1}\mathbf{r}_{01}, \tau-\tau_c\right)\exp(i\Phi''), \quad (24a)$$

$$\Phi'' = \frac{k}{2}\left(\frac{1}{R_2}+\frac{1}{L''}\right)\left(-\frac{L}{L''}\mathbf{r}_\alpha-\frac{L}{R_3}\mathbf{r}_{03}+\frac{L}{R_1}\mathbf{r}_{01}\right)^2 - \frac{k\mathbf{r}_{01}^2}{2R_1}+\frac{k\mathbf{r}_{03}^2}{2R_3}+\frac{k\mathbf{r}_\alpha^2}{2L''}. \quad (24b)$$

It follows then that the intensity in the $\mathbf{r}_\alpha$ observation plane is

$$I'(\mathbf{r}_\alpha, \tau) = \Gamma' I_{01}I_{02}I_{03}\left|T\left(\frac{L}{L'}\mathbf{r}_\alpha+\frac{L}{R_2}\mathbf{r}_{02}+\frac{L}{R_3}\mathbf{r}_{03}, \tau_c-\tau\right)\right|^2, \quad (25a)$$

$$I''(\mathbf{r}_\alpha, \tau) = \Gamma'' I_{01}I_{02}I_{03}\left|T\left(-\frac{L}{L''}\mathbf{r}_\alpha+\frac{L}{R_3}\mathbf{r}_{03}+\frac{L}{R_1}\mathbf{r}_{01}, \tau-\tau_c\right)\right|^2. \quad (25b)$$

Thus, the stimulated PE signals form nonstationary images, in which the time can be either direct (the coding pulse is either the second or the third) or in mirror-inverted time sequence (the coding pulse is the first). The scale factor for the SPE signal is also determined by the ratio of the lengths L and L':

$$M' = \left(1-\frac{L}{R_2}-\frac{L}{R_3}\right)^{-1} \quad (26a)$$

$$M'' = \left(1-\frac{L}{R_1}+\frac{L}{R_3}\right)^{-1}. \quad (26b)$$

Note that the character of the images formed by the PE signals depends on the sign of L': the images are real for $L'>0$ and virtual for $L'<0$.

5. SCALE TRANSFORMATION FOR IMAGES RECONSTRUCTED BY MODIFIED SPE SIGNALS

We know that modified stimulated PE (MSPE) signals are produced in a three-level resonant medium by three exciting light pulses with frequencies $\omega_1=\omega_2\neq\omega_3$ (see, e.g., Ref. 16). We consider first the case when the information signal is the first exciting light pulse. We have them in the hologram plane

$$E_1(\mathbf{r}, L, \tau) = \left(-\frac{ik_a}{2\pi L}\right)E_{01}\int d\mathbf{r}'\, T(\mathbf{r}', \tau)\exp\left[\frac{ik_a(\mathbf{r}-\mathbf{r}')^2}{2L}\right],$$

$$E_2(\mathbf{r}, L, \tau) = E_{02}f_2(\tau)\exp\left[\frac{ik_a(\mathbf{r}-\mathbf{r}_{02}')^2}{2R_2}\right], \quad (27)$$

$$E_3(\mathbf{r}, L, \tau) = E_{03}f_3(\tau)\exp[ik_b(\mathbf{r}-\mathbf{r}_{03})^2/2R_3],$$

where the wave number k_b for the third light pulse, and with it also for the MSPE signal, differs from k_a; the functions $f_{2,3}(\tau)$ describe the temporal forms of the second and third exciting pulses.

Under the same assumptions as for the analysis, in Secs. 3 and 4, of scale transformations of PE signals in two-level resonant systems, we obtain for the MSPE signals at the exit from the resonant medium

$$E_e(\mathbf{r}, z, \tau) = \gamma_M\left(\frac{ik_a}{2\pi L}\right)E_{03}E_{02}E_{01}^{*}\int d\mathbf{r}' T^{*}(\mathbf{r}', (\varkappa-1)(\tau_c-\tau)) \times\exp\left[\frac{-ik_a(\mathbf{r}-\mathbf{r}')^2}{2L}+\frac{ik_a(\mathbf{r}-\mathbf{r}_{02})^2}{2R_2}+\frac{ik_b(\mathbf{r}-\mathbf{r}_{03})^2}{2R_3}\right], \quad z=L, \quad (28)$$

where $\mathbf{r}$ is the position vector in the hologram plane, $\varkappa$ is a timelike scale factor determined by the correlated mechanism of the inhomogeneous broadening of the resonant energy levels, and the coefficient γ_M depends on the properties of the resonant medium and on the parameters of the exciting light pulses:

$$\gamma_M = i\frac{\pi^2\omega N_0|d_{ab}|^2|d_{bc}|^2 g(0) l}{cn\hbar^3}\left(\int_0^{\delta_2} f_2(\tau)d\tau\int_0^{\delta_3} f_3(\tau)d\tau\right). \quad (29)$$

The following equality will hold then in the MSPE-signal observation plane at a distance L' from the hologram:

$$E_e(\mathbf{r}_\alpha, L+L', \tau) = \gamma_M\frac{k_ak_b}{(2\pi)^2LL'}E_{03}E_{02}E_{01}^{*}\int d\mathbf{r}\,d\mathbf{r}' T^{*}(\mathbf{r}', (\varkappa-1)(\tau_c-\tau))\cdot \times\exp\left[-\frac{ik_a(\mathbf{r}-\mathbf{r}')^2}{2L}+\frac{ik_a(\mathbf{r}-\mathbf{r}_{02})^2}{2R_2} +\frac{ik_b(\mathbf{r}-\mathbf{r}_{03})^2}{2R_3}+\frac{ik_b(\mathbf{r}_\alpha-\mathbf{r})^2}{2L'}\right] \quad (30)$$

from which we easily obtain the condition for the formation of sharp images:

$$(L')^{-1} = \frac{k_a}{k_b}(L)^{-1}-\frac{k_a}{k_b}(R_2)^{-1}-(R_3)^{-1}. \quad (31)$$

Integrating next with respect to the spatial variables $\mathbf{r}$ and $\mathbf{r}'$, we obtain the MSPE signal field distribution in the observation plane:

$$E_e(\mathbf{r}_\alpha, L+L', \tau) = \gamma_M\left(\frac{k_b}{k_a}\right)\left(\frac{L}{L'}\right)^2 E_{03}E_{02}E_{01}^{*} \times\exp(i\Phi_M)T^{*}\left(\frac{k_bL}{k_aL'}\mathbf{r}_\alpha+\frac{k_bL}{k_aR_3}\mathbf{r}_{03}+\frac{L}{R_2}\mathbf{r}_{02}, (\tau_c-\tau)\right), \quad (32)$$

$$\Phi_M=-\frac{k_a}{2L}\left(\frac{L}{R_2}\mathbf{r}_{02}+\frac{k_b}{k_a}\frac{L}{R_3}\mathbf{r}_{03}+\frac{k_b}{k_a}\frac{L}{L'}\mathbf{r}_a\right)^2+\frac{k_a\mathbf{r}_{02}^2}{2R_2}+\frac{k_b\mathbf{r}_{03}^2}{2R_3}+\frac{k_b\mathbf{r}_a^2}{2L'}, \qquad (33)$$

and also the intensity of the MSPE signal:

$$I(\mathbf{r}_a, L+L', \tau)=\Gamma_M' I_{01}I_{02}I_{03}\left|T\left(\frac{k_b}{k_a}\frac{L}{L'}\mathbf{r}_a+\frac{k_b}{k_a}\frac{L}{R_3}\mathbf{r}_{03}+\frac{L}{R_2}\mathbf{r}_{02}, (\tau_c-\tau)\right)\right|^2, \qquad (34)$$

where Γ'_M is some coefficient proportional to γ_M.

The scale coefficient that determines the formation of the initial image in the MSPE signal observation plane is thus equal to

$$M'=\frac{k_a}{k_b}\frac{L'}{L}=\left(1-\frac{k_b}{k_a}\frac{L}{R_3}-\frac{L}{R_2}\right)^{-1}. \qquad (35)$$

This equation for the scale coefficient is a generalization of expression (26a) and takes into account the difference between the MSPE signal wavelength and the wavelength at which the initial image was formed.

If the information is contained in the second light pulse, we have by analogy with (30)

$$E(\mathbf{r}_a, L+L'', \tau)=\gamma_M''\frac{k_ak_b}{(2\pi)^2LL''}E_{03}E_{02}E_{01}^*\times\int d\mathbf{r}\,d\mathbf{r}'\,T(\mathbf{r}', (\varkappa-1)(\tau-\tau_c))\cdot\times\exp\left[-\frac{ik_a}{2R_1}(\mathbf{r}-\mathbf{r}_{01})^2+\frac{ik_a}{2L}(\mathbf{r}-\mathbf{r}')^2+\frac{ik_b}{2R_3}(\mathbf{r}-\mathbf{r}_{03})^2+\frac{ik_b}{2L''}(\mathbf{r}_a-\mathbf{r})^2\right]. \qquad (36)$$

The condition for obtaining sharp images and the expressions for the MSPE signal intensity and the scale-transformation coefficients then take the form

$$(L'')^{-1}=\frac{k_a}{k_b}(R_1)^{-1}-\frac{k_a}{k_b}(L)^{-1}-(R_3)^{-1}, \qquad (37)$$

$$I(\mathbf{r}_a, L+L'', \tau)=\Gamma_M'' I_{01}I_{02}I_{03}\left|T\left(-\frac{k_bL}{k_aL''}\mathbf{r}_a-\frac{k_b}{k_a}\frac{L}{R_3}\mathbf{r}_{03}+\frac{L}{R_1}\mathbf{r}_{01}, \tau-\tau_c\right)\right|^2, \qquad (38)$$

$$M''=-\frac{k_a}{k_b}\frac{L''}{L}=\left(1-\frac{L}{R_1}+\frac{k_b}{k_a}\frac{L}{R_3}\right)^{-1}. \qquad (39)$$

Note that expressions (37)–(39) are generalizations of Eqs. (22b), (25b), and (26b) to include the case of three-level resonance systems.

Finally, the intitial image can be written by the third light pulse. We obtain then for the reconstructed MSPE signal

$$E(\mathbf{r}_a, L+L''', \tau)=\gamma_M'''\frac{k_ak_b}{(2\pi)^2LL'''}E_{03}E_{02}E_{01}^*\times\int d\mathbf{r}\,d\mathbf{r}'\,T(\mathbf{r}', \tau-\tau_\mathrm{e})\times\exp\left[-\frac{ik_a}{2R_1}(\mathbf{r}-\mathbf{r}_{01})^2+\frac{ik_a}{2R_2}(\mathbf{r}-\mathbf{r}_{02})^2+\frac{ik_b}{2L}(\mathbf{r}-\mathbf{r}')^2+\frac{ik_b}{2L'''}(\mathbf{r}_a-\mathbf{r})^2\right], \qquad (40)$$

from which follows the sharp-image condition

$$(L''')^{-1}=-(L)^{-1}+\frac{k_a}{k_b}(R_1)^{-1}-\frac{k_a}{k_h}(R_2)^{-1}. \qquad (41)$$

For the MSPE signal intensity and for the scale-transformations coefficient we obtain respectively

$$I(\mathbf{r}_a, \tau)=\Gamma_M''' I_{01}I_{02}I_{03}\times\left|T\left(-\frac{L}{L'''}\mathbf{r}_a-\frac{k_a}{k_b}\frac{L}{R_2}\mathbf{r}_{02}+\frac{k_a}{k_b}\frac{L}{R_1}\mathbf{r}_{01}, \tau-\tau_c\right)\right|^2. \qquad (42)$$

$$M'''=-\frac{L'''}{L}=\left(1-\frac{k_a}{k_b}\frac{L}{R_1}+\frac{k_a}{k_b}\frac{L}{R_2}\right)^{-1}. \qquad (43)$$

It follows from expressions (35), (39), and (43) for the scale-transformations coefficients that the reconstructed images are spatially similar, and in the case of excitation by plane waves ($R_i\equiv\infty$) the image formed by the MSPE signal is identical with the initial one ($M\equiv1$).

6. CONCLUSION

The possibilities of image transformation by PE signals are thus substantially expanded when the exciting pulses have spherical wave fronts. Whereas excitation of a resonant medium by plane-front waves does not change the spatial scale of the images, excitation by spherical-front pulses makes the images spatially similar. As a result, the resonant medium plays a dual role: first, that of a spectrally selective hologram, which makes reconstruction of nonstationary images possible, and second, the role of some optical system whose parameters are determined by the wave-front curvature radius. Depending on the relations between the curvature radii of the wave fronts and on the distance between the object scene and the dynamic echo-hologram, it is possible to obtain enlarged (reduced) real (virtual) images.

We present the needed estimates. Using expressions (21) obtained for the spatial properties of the stimulated-PE signals, it is easy to relate the intensity of the received signal with that of the coding pulse or signal that forms the initial image:

$$\frac{I_c}{I_0}\approx\left(\frac{\alpha_0 l}{2}\right)^2\theta_2^2\theta_3^2,$$

where α_0 is the resonant absorption coefficient in the medium and is equal to $4\pi^2\omega|d_{ab}|^2N_0g(0)/cn\hbar$, l is the thickness of the resonance hologram, while θ_2 and θ_3 are the "areas" of the excitation pulses that differ from the coding one ($\theta_i \sim d_{ab}E_i\delta_i/\hbar$).

Strictly speaking, all the quantities in the last relation are small parameters of the considered theory. For estimates, however, we can use the condition $\theta_2 \sim \theta_3 \sim 1$, since linearity is essential for writing the coding pulse, while the "reading" by the second and third pulses can also be nonlinear. Thus, for $\theta_2 \sim \theta_3 \sim 1$ and $\alpha_0 l = 2\cdot 10^{-1}$ we obtain $I_e/I_0 \sim 10^{-2}$.

Note that in a real situation it is necessary also to take into account the relaxation processes that decrease the useful signal. Nonetheless, Mossberg and co-workers[5,20] obtained for I_e/I_0 a value 4%.

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