Dynamics of a Bloch point (point soliton) in a ferromagnet

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A dynamic theory is derived for the motion of a Bloch point, i.e., a point topological soliton which is homotopically equivalent to a "hedgehog," along a Bloch line in a ferromagnet. The effective Lagrangian for motion of this type is derived from the Landau-Lifshitz equations and the Slonczewski equations. An expression is derived for the mass of a Bloch point for a medium with a large quality factor (i.e., a large ratio of the uniaxial-anisotropy energy to the magnetostatic energy). Oscillations of a Bloch point in a potential well formed by magnetostatic fields in a film or plate of finite thickness are analyzed. The possibility of observing such oscillations experimentally is discussed.

1. INTRODUCTION

A number of topologically stable intrinsic defects, or topological solitons, can exist in ferromagnets: plane defects (domain walls), line defects (Bloch lines), and point defects (Bloch points).

By now the dynamics of domain walls has been the subject of a very long list of theoretical and experimental studies.[1] There have also been theoretical and experimental studies of the dynamics of Bloch lines (e.g., Refs. 2–4 and the bibliographies there). It is now time for a theoretical and experimental study of topological solitons of more complicated structure, Bloch points. In Ref. 5 we derived a theory for the motion of a Bloch point normal to the Bloch line in which it is positioned. The Bloch point was treated as the boundary between sections of a Bloch line with different topological charges. The intrinsic mass of the Bloch point was ignored, so the dynamics of this micromagnetic structure was determined not by the dynamics of the Bloch point but by the dynamics of the Bloch line, which necessarily participates in the transverse motion of a Bloch point.

In the present paper we analyze the longitudinal motion of a Bloch point along a Bloch line, i.e., the motion associated with a displacement which is a continuous parameter of the energy degeneracy in the absence of magnetic fields. The dynamics of the Bloch point itself is thus the governing factor for the motion. We use the procedure of transforming from a description of this micromagnetic configuration with a Bloch point in terms of Landau-Lifshitz field equations to a description in terms of generalized coordinates of the Bloch point. A feature which distinguishes this procedure in a fundamental way from corresponding procedures which have been developed for domain walls and Bloch lines is that in the cases of these walls and lines it is sufficient to know the unperturbed (i.e., immobile) distribution of the magnetic moment in order to derive a dynamic theory in terms of generalized coordinates. In those cases it is not necessary to find corrections which are linear in the velocity of the motion, and the equations in generalized coordinates are constructed from the condition under which the equations for these corrections can be solved. In deriving dynamic equations for a Bloch point, in contrast, we must have explicit solutions for these corrections to the field of the magnetic moment $\mathbf{M}(r)$, which are linear in the velocity of motion. The equation of motion found for a Bloch point constitutes Newton's second law with a mass for which we will derive an expression here for the case of a ferromagnetic medium with a large quality factor $Q = K/2\pi M^2$ (i.e., the uniaxial anisotropy energy K is considerably larger than the magnetostatic energy $2\pi M^2$, where M is the magnetic moment), of the type used in magnetic-bubble technology.

A Bloch point is an analog of a "hedgehog" in field theory.[6] An isotropic soliton of this sort exists in the isotropic Heisenberg model. As follows from the results of the present study, however, as the anisotropy energy tends toward zero the size of the soliton increases without bound, and the mass becomes infinite. The result is the peculiar dynamics of a hedgehog in the isotropic Heisenberg model.

As an experiment in which the dynamics of a Bloch point might be manifested, one might attempt to observe oscillations—excited by an oscillatory external field—of a Bloch point in a potential well formed by magnetostatic fields. We will discuss such an experiment at the end of this paper.

2. THE LANDAU-LIFSHITZ THEORY AND THE SLONCZEWSKI EQUATIONS

Our derivation starts with the Landau-Lifshitz equations in polar coordinates for the magnetic moment M ($m = \cos\vartheta = M_z/M$, $\varphi = \operatorname{arctg} M_y/M_x$):

$$\frac{M}{\gamma}\frac{\partial m}{\partial t} = \frac{\delta H}{\delta\varphi}, \tag{1}$$

$$-\frac{M}{\gamma}\frac{\partial\varphi}{\partial t} = \frac{\delta H}{\delta m} \tag{2}$$

where γ is the gyromagnetic ratio. The Lagrangian density for these equations is

$$\mathscr{L} = -\frac{M}{\gamma} m \frac{\partial\varphi}{\partial t} - \mathscr{H}(m,\varphi), \tag{3}$$

and the energy density $\mathscr{H}$ is given by the expression[1]

$$\mathscr{H} = A\,[(\nabla\vartheta)^2 + \sin^2\vartheta\,(\nabla\varphi)^2] + [K + 2\pi M^2 \sin^2\varphi]\,\sin^2\vartheta + MH_z'ym. \tag{4}$$

Here we are taking into account the inhomogeneous-exchange energy ($\sim A$), the uniaxial-anisotropy energy ($\sim K$), the magnetostatic energy in the Winter approximation[1] ($\sim M^2$), and the energy of the interaction with the weak nonuniform magnetic field $H_z = H_z'y$, which creates a potential well for a domain wall, whose central surface coincides with the xz plane. The micromagnetic structure which

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we are discussing here is shown in Fig. 1. The domain wall (the xz plane) is separated by a Bloch line along the z axis. The Bloch point is at the origin of coordinates, which breaks up the Bloch line into two regions differing in the sign of the topological charge.

Research on both the statics and dynamics of domain walls makes extensive use of a simplified description which leads to the Slonczewski equations.[1] This description is based on the assumption that the thickness of the domain wall, Δ, is much smaller than the other length scales in the theory. The change in $m = \cos\vartheta$ as the domain wall is crossed is then described by the Landau-Lifshitz solution

$$\theta_0 = 2\,\mathrm{arctg}\,\exp\left(\frac{y-q}{\Delta}\right), \tag{5}$$

and the wall thickness Δ, the wall displacement q, and the angle φ are smooth functions in the xz plane. The dynamic behavior of this system is described by the pair of canonical-conjugate variables $q - \varphi$ (coordinate-momentum), which are defined in a plane (instead of the $m - \varphi$ canonical pair, which are defined throughout space in the Landau-Lifshitz theory).

The effective Hamiltonian for the Slonczewski equations

$$\frac{2M}{\gamma}\frac{\partial q}{\partial t} = \frac{\delta\sigma}{\delta\varphi}, \tag{6}$$

$$-\frac{2M}{\gamma}\frac{\partial\varphi}{\partial t} = \frac{\delta\sigma}{\delta q} \tag{7}$$

is the energy of the domain wall, with a density (per unit area)

$$\sigma=\sigma_0+2A\Delta_0\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial z}\right)^2\right]+4\pi M^2\Delta_0\sin^2\varphi \tag{8}$$

$$+MH_z'q^2+\frac{1}{2}\sigma_0(\nabla q)^2,$$

where $\sigma_0 = 4(AK)^{1/2}$ and $\Delta_0 = (A/K)^{1/2}$ are the energy density and thickness of the Bloch domain wall in the ground state ($\varphi = 0$, $q = 0$), and the last term is the energy of the surface tension of the domain wall, which increases as the wall bends.

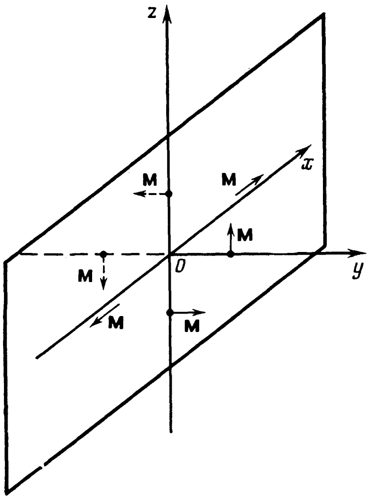

FIG. 1. Domain wall in the xz plane. The Bloch line runs along the z axis; the Bloch point is the origin of coordinates.

The Slonczewski equations (6) and (7) could also be derived from a Lagrangian with a density (per unit area of the domain wall)

$$\mathscr{L}_s = -\frac{2M}{\gamma}q(x,z)\frac{\partial\varphi}{\partial t} - \sigma. \tag{9}$$

The densities σ and $\mathscr{L}_s$, of the energy and the Lagrangian, are found by integrating $\mathscr{L}$ and $\mathscr{H}$, respectively, over y [Eqs. (3) and (4)]. As ϑ here we are using the Landau-Lifshitz solution ϑ_0 from (5).

We can use the Slonczewski equations at distances from the center of the Bloch point which exceed the thickness of the domain wall, Δ_0; at distances less than Δ_0, we must switch to the more general Landau-Lifshitz equations. When we move closer to the center of the Bloch point and reach small distances on the order of the correlation length, we enter a region in which the micromagnetic approach (which starts from the assumption that the modulus of the vector magnetic moment remains constant) must be abandoned, since topological considerations[1] show that the moment **M** must vanish at the center of a Bloch point. However, as will become clear below, the contribution of small distances is by no means dominant for the mass of the Bloch point, so there is no need to analyze the structure at atomic scales, where we cannot use the Landau-Lifshitz equation.

3. STATIC SOLUTION

We now consider the static structure in the ground state. In the range of applicability of the Slonczewski equations, the static structure corresponds to an undisplaced domain wall ($q = 0$) with an azimuthal-angle (φ) field which satisfies the sine-Gordon equation found by minimizing the energy, $\delta\sigma/\delta\varphi = 0$;

$$\Delta\varphi - \frac{\sin 2\varphi}{\Lambda_0^2} = 0, \tag{10}$$

where the length

$$\Lambda_0=\Delta_0 Q^{1/2}=(A/2\pi M^2)^{1/2} \tag{11}$$

is the thickness of the Bloch line.[1]

A Bloch line along the z axis with a given topological charge is described by a one-dimensional solution of Eq.

$$\varphi(x) = \pm 2\,\mathrm{arctg}\,\exp\left(\frac{x}{\Lambda_0}\right). \tag{12}$$

For our structure, on the other hand, with a Bloch point dividing the Bloch line into regions with different topological charges [$z > 0$ corresponds to the upper sign in (12), and $z < 0$ to the lower sign], the field of the azimuthal angle, $\varphi(x,z)$, asymptotically approaches the solution (12) only at distances from the Bloch point greater than Λ_0 (Fig. 2). The solution of the sine-Gordon equation (10), on the other hand, at small distances from the Bloch point is described approximately by the solution of the Laplace equation and constitutes a "vortex solution" which does not depend on the distance from the center of the Bloch point, $r = (x^2 + z^2)^{1/2}$

$$\varphi_0=\Phi, \tag{13}$$

where $\Phi = \mathrm{arctg}\, z/x$ is the azimuthal angle in the xz plane.

We now consider extremely small distances from the center of the Bloch point—small not only in comparison

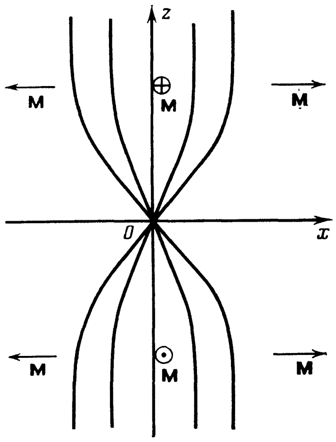

FIG. 2. Contour lines of φ in the xz plane which satisfy the sine-Gordon equation.

with Λ_0 but also in comparison with the wall thickness Δ_0. Here we cannot use the Slonczewski equations, and we should use the more general Landau-Lifshitz equations. The term $\sim A$ (the inhomogeneous-exchange energy) is now the dominant one in expression (4) for the energy density. We know quite well[1] that a static solution of the Landau-Lifshitz equations (1), (2) in this case is

$$\vartheta_0=\theta, \quad \varphi_0=\Phi, \tag{14}$$

where we have introduced the spherical coordinates $R^2 = x^2 + y^2 + z^2$, $\Phi = \operatorname{arctg} z/x$, $\theta = \arccos y/R$ with polar axis y in configuration space (cf. the polar coordinates $\vartheta = \arccos M_y/M$, $\varphi = \operatorname{arctg} M_z/M_x$ in the space of magnetic moments **M**). A solution of (14) is a solution for a hedgehog soliton in the isotropic Heisenberg model. By performing various arbitrary three-dimensional rotations of the structure which we have found, we find all possible structures of this homotopic class.

We have thus derived an explicit expression for the field of the moment **M** around a Bloch point in spherical coordinates for three regions:

$$1 - R \gg \Lambda_0, \quad 2 - \Lambda_0 \gg R \gg \Delta_0, \quad 3 - \Delta_0 \gg R.$$

4. DETERMINATION OF DYNAMIC CORRECTIONS; MASS OF THE BLOCH POINT

Our problem is to derive the effective Lagrangian of a Bloch point in terms of the displacement z and velocity $V = \partial z/\partial t$ of the Bloch point along the z axis. In the derivation of an effective Lagrangian or Hamiltonian of this sort in generalized coordinates in the construction of a dynamic theory of a domain wall and a Bloch line, it has proved to be sufficient to substitute into these expressions a static solution for the corresponding structure expressed in terms of generalized coordinates. In the problem of interest here, i.e., the dynamics of a Bloch point, the static solution proves to be inadequate for deriving an effective Lagrangian. The substitution of the static solution into the Lagrangian does not result in terms which depend on the velocity of the Bloch point, since the kinetic term in Lagrangian (1) disappears in this approximation, by virtue of the symmetry. It is thus necessary to seek a solution for the field of the magnetic moment M by expanding it in the velocity of the Brillouin point, V, i.e., by setting $m = m_0 + m_1$ and $\varphi = \varphi_0 + \varphi_1$. We find the corrections which are linear in V, m_1 and φ_1, initially in the region of applicability of Slonczewski equations (6) and (7), i.e., under the condition $r \gg \Delta_0$. In this case we have $m_1 = q\partial m_0/\partial y$, and we must solve Eqs. (6) and (7) by setting $\partial q/\partial t = - V\partial q_0/\partial z = 0$ (since there is no displacement of the wall in the static case) and $\partial\varphi/\partial t = - V\partial\varphi_0/\partial z$. We immediately find $\varphi_1 = 0$, and q is found from an equation which follows from (7):

$$\frac{2M}{\gamma\sigma_0} V \frac{\partial \varphi_0}{\partial z} = -\frac{1}{l^2} q + \Delta q, \tag{15}$$

where the length l is determined by the "rigidity" of the domain wall:

$$l^2 = \frac{\sigma_0}{2MH_z'}.$$

We can write a solution of Eq. (15) by making use of the Green's function of this linear, inhomogeneous equation:

$$q(\mathbf{r}) = \frac{2M}{\gamma\sigma_0} V \int d\mathbf{r}' \, K_0\left(\frac{|\mathbf{r}-\mathbf{r}'|}{l}\right) \frac{\partial\varphi_0(\mathbf{r}')}{\partial z}, \tag{16}$$

where $K_0(z)$ is the modified Bessel function of index zero, and $\mathbf{r}$ and $\mathbf{r}'$ are two-dimensional radius vectors in the xz plane.

Going back to Lagrangian density (9) for the Slonczewski equations, and substituting q from (16) and the static solution $\varphi = \varphi_0$, $\partial\varphi_0/\varphi t \approx - V\partial\varphi_0/\partial z$ into that expression, we find the following Lagrangian for a Bloch point after integrating over the xz plane:

$$L_{\mathrm{Bp}} = m_{\mathrm{Bp}} \frac{V^2}{2} - H_{\mathrm{Bp}}, \tag{17}$$

where

$$m_{\mathrm{Bp}} = \frac{1}{\sigma_0}\left(\frac{M}{\gamma}\right)^2 \int\int d\mathbf{r}\, d\mathbf{r}_1 \frac{\partial\varphi_0(\mathbf{r})}{\partial z} K_0\left(\frac{|\mathbf{r}-\mathbf{r}_1|}{l}\right) \frac{\partial\varphi_0(\mathbf{r}_1)}{\partial z},$$

and where

$$H_{\mathrm{Bp}} = 2\pi A \Delta_0 \ln Q \tag{18}$$

is the static energy of the Bloch point [see (9.14) in Ref. 1]. Expression (17) thus gives us the Lagrangian for a Bloch point in free motion. The "potential energy" of the Bloch point, which depends on its coordinates, appears if the magnetic field has a y component (Sec. 5).

From (12) we find $\partial\varphi_0/\partial z \to 0$ at large distances $r \gg \Lambda_0$ from the center of the Bloch point, and we find that the integral (18) is dominated by the region $r \ll \Lambda_0$ of the vortex solution (13), in which the relation $\partial\varphi_0/\partial z = (\cos\Phi)/r$ holds. The kinetic term $m_{\mathrm{Bp}} V^2/2$ in the Lagrangian of the Bloch point is formed by both the kinetic term $\sim q\partial\varphi/\partial t$ in the Lagrangian of the Slonczewski theory, (9), and the contribution to the energy density σ which is quadratic in q [see (8)].

For $\Lambda_0 \gg l$, the term $\sim \Delta q$ in (15), which results from the surface tension, can be ignored, and Eq. (15) transforms from a differential equation into an algebraic equation for q. The solution of this algebraic equation is

$$q = -\frac{2M}{\gamma\sigma_0} l^2 \frac{\partial\varphi_0}{\partial z} V = -\frac{1}{\gamma H_z'} \frac{\partial\varphi_0}{\partial z} V. \tag{19}$$

The Green's function for Eq. (15) in this approximation is a δ-function in the xz plane, and expression (18) reduces to a logarithmically divergent integral

$$m_{\mathrm{Bp}}=\frac{1}{\sigma_0}\left(\frac{M}{\gamma}\right)^2 l^2\int dr\left(\frac{\partial\varphi_0}{\partial z}\right)^2=\frac{\pi}{2}\frac{M}{\gamma^2 H_z'}\ln\frac{\Lambda_0}{r_m},\qquad(20)$$

where the lower limit of the cutoff within the logarithm is $r_m = l$ if $l > \Delta_0$ or $r_m = \Delta_0$ if $\Delta_0 > 1$.

In the other limit $\Lambda_0 \ll l$, in which the displacement q is determined exclusively by the surface tension, the value of m_{Bp} can be estimated in order of magnitude from

$$m_{\mathrm{Bp}}\sim\left(\frac{M}{\gamma}\right)^2\frac{\Delta_0\Lambda_0^2}{A}=\left(\frac{M}{\gamma}\right)^2\frac{\Delta_0^3}{A}Q\sim\frac{\Delta_0}{\gamma^2}.\qquad(21)$$

According to (21) the mass of a Bloch point is the product of the Döring mass $1/\gamma^2\Delta_0$ and the area Δ_0^2. It should be kept in mind, however, that the area of the domain wall which lies within the Bloch point (i.e., in the region in which the static solution is distorted by the Bloch point) is Λ_0^2, not Δ_0^2. Accordingly, inside a Bloch point the mass density (per unit area) is smaller than the Döring mass by a factor of $Q = \Lambda_0^2/\Delta_0^2$.

Let us estimate the contribution to the mass of a Bloch point from the region $r < \Delta_0$, in which we cannot use the Slonczewski equations, and in which we should make use of the more general Landau-Lifshitz equations (1) and (2), replacing $\partial m/\partial t$ by $-V\partial m_0/\partial z$ and $\partial\varphi/\partial t$ by $-V\partial\varphi_0/\partial z$ in them. The corrections m_1 and φ_1 (linear in the velocity V) found from these equations should then be substituted into the kinetic term of the Lagrangian, and an integration should be carried out over the region $R < \Delta_0$. In this region, a leading role is played by the energy of the inhomogeneous exchange, $\sim A$. A dimensional estimate of the corresponding contribution to the mass of Bloch point yields

$$\delta m_{\mathrm{Bp}}\approx\left(\frac{M}{\gamma}\right)^2\frac{\Delta_0^3}{A}.\qquad(22)$$

Comparing (22) with (20) and (21), we see that if the quality factor Q is large, and if the condition $l \gg \Delta_0$ holds, the mass is dominated by the region $r > \Delta_0$. These estimates of the mass of a Bloch point show that when we switch to the isotropic Heisenberg model, i.e., when we let $\Delta_0, \Lambda_0 \to \infty$, the mass of the point tends toward infinity. In the isotropic Heisenberg model it is thus not possible to derive particle-like equations of motion for a point soliton.

5. OSCILLATIONS OF A BLOCH POINT; POSSIBILITIES FOR EXPERIMENTAL OBSERVATION

The mass calculated above for a Bloch point should determine the frequency of the oscillations of this point in a nonuniform field $H_y = H_y'z$. Such a field, interacting with the magnetic moment at the core of the Bloch line (this moment has opposite directions on the parts of the line above and below the Bloch point), distinguishes the position of the Bloch point at the origin or coordinates. The Lagrangian of the Bloch point in such a field is

$$L_{\mathrm{Bp}}=\frac{1}{2}m_{\mathrm{Bp}}\left(\frac{\partial z}{\partial t}\right)^2-MH_y'z^2\pi^2\Lambda_0\Delta_0,\qquad(23)$$

where z is the coordinate of the Bloch point. The energy H_{Bp}, which does not depend on z, has been omitted here. The frequency of the resonant oscillations for a Lagrangian of this sort is

$$\omega_{\mathrm{Bp}}^2=\frac{2MH_y'\pi^2\Lambda_0\Delta_0}{m_{\mathrm{Bp}}}.\qquad(24)$$

As an example we consider a Bloch point with a twisted domain wall in a film of thickness h. According to (8.33) of Ref. 1, the field H_y at the center of the film has a gradient $H_y' = 16M/h$. Using expression (21) for the mass of the Bloch point, we then find

$$\omega_{\mathrm{Bp}}=4\pi M\left(\frac{2\Lambda_0\Delta_0}{hm_{\mathrm{Bp}}}\right)^{1/2}=4\pi\gamma M\left(\frac{2\Lambda_0}{h}\right)^{1/2}.\qquad(25)$$

Whether such oscillations can be observed depends on the magnitude of the viscous loss during the motion of the Bloch point, i.e., on the resonance quality factor $\overline{Q} = \omega_{\mathrm{Bp}}/2\,\mathrm{Im}\,\omega$. The quality factor $\overline{Q}$ is related to the mobility of the Bloch point in the field H_y, which is given by the following expression according to (9.20) from Ref. 1:

$$\mu_{\mathrm{Bp}}\approx\pi\gamma(\Delta_0\Lambda_0)^{1/2}/\alpha M$$

We then have

$$\overline{Q}=\frac{m_{\mathrm{Bp}}\omega_{\mathrm{Bp}}\mu_{\mathrm{Bp}}}{\pi^2\Lambda_0\Delta_0}=\frac{4}{\alpha}\left(\frac{\Delta_0}{h}\right)^{1/2}.\qquad(26)$$

Taking $M \approx 100$ G, $\Delta_0 \approx 10^{-5}$ cm, $h \approx 10^{-3}$ cm, and $\alpha \approx 0.01$ (Ref. 1) for an estimate, we find the frequency of the resonant oscillations to be on the order of 4 MHz, and we find a quality factor $\overline{Q} \sim 40$. These results indicate that it would be possible to experimentally observe resonant oscillations of a Bloch point. These oscillations might be excited by an oscillatory field H_y.

6. CONCLUSION

We have derived an effective Lagrangian for the motion of a Bloch point along a Bloch line. The mass calculated for a Bloch point here can be determined experimentally through the excitation of oscillations of a Bloch point in a potential well formed by magnetostatic fields. The mass of the Bloch point should also be manifested in dynamic transformations of the Bloch line, in which it would determine the rate of these processes.

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