Edge magnetoplasmons at a periodically perturbed boundary of a 2D conducting system

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A simple solution is found in the limit $\omega \ll \omega_c$ for the problem of the spectrum of edge magnetoplasmons for a wave which is traveling along an ideal boundary of a semi-infinite 2D system with a stepfunction electron density profile, without screening. A generalization of this theory to the case in which the boundary of a 2D system is perturbed by a periodic small perturbation $\xi(y)$ with $\xi'(y) \ll 1$ is proposed. This perturbation is shown to enter the theory in the combination $\xi'(y)\sigma_{xy}/\sigma_{xx}$, which may be large under the condition $\omega \ll \omega_c$, by virtue of the ratio $\sigma_{xy}/\sigma_{xx} \gg 1$. An integral equation whose solution determines the properties of edge magnetoplasmons in the case $\xi(y) \neq 0$ is constructed. An approximate solution of this equation shows that a deformation $\xi(y)$ of the free boundary of a 2D system can have a significant effect on the spectrum of edge magnetoplasmons.

The theory of edge magnetoplasmons in bounded two-dimensional (2D) charged systems has been derived under the assumption that the corresponding boundaries—of half-planes, disks, etc.—are ideal.[1–8] The boundaries of actual 2D conducting systems in various semiconductor devices are of course not ideal and must be characterized by a certain degree of roughness. The basic purpose of the present study is to learn how a weak (in the sense $\partial\xi/\partial y \ll 1$) periodic perturbation $\xi(y)$ described by

$$\xi(y)=\xi_0 \sin q_0 y, \quad \xi_0 q_0 \ll 1, \quad \int \xi(y)\,dy=0, \tag{1}$$

of a rectilinear boundary of a semi-finite 2D electron system which occupies the half-plane $x \geqslant 0$ with a free boundary along the y axis affects the properties of edge magnetoplasmons which are traveling along the y axis. As we will see below, this effect turns out to be far from trivial.

Since the problem of the properties of edge magnetoplasmons has an abundance of external parameters, we begin with a determination of the range of applicability of the results which have been derived previously. To begin with, we are talking about a 2D system with a step electron density profile $n(x) = n_s\theta(x)$, where n_s is the average electron density far from the boundary, and $\theta(x)$ is a θ-function without screening (the case of a heterostructure). The magnetic field H, which is directed normal to the surface of the 2D system, is classically strong (i.e., the condition $\omega_c\tau > 1$ holds, where ω_c is the cyclotron frequency, and τ is the momentum relaxation time), but the length scale l [defined in (10)], over which most of the charge of the edge magnetoplasmons is concentrated, is still significantly greater than the cyclotron radius r_c. In describing the edge magnetoplasmons we can thus use a local Ohm's law (see the discussion in Ref. 8) with conductivity tensor components σ_{ik} from the Drude formulas

$$\sigma_{xx}=i\omega n_s e^2/m^*(\omega^2-\omega_c^2),$$
$$\sigma_{xy}=\omega_c n_s e^2/m^*(\omega^2-\omega_c^2), \quad \omega_c=eH/m^*c. \tag{2}$$

Here e is the electron charge, ω is the wave frequency, and m^* is the effective mass of the electrons. The wave number q of the waves is much smaller than the value of q_0 from (1): $q \ll q_0$.

This paper is organized in the following way. In the first section we find the spectrum of edge magnetoplasmons for waves which are traveling along an unperturbed boundary of a 2D system. Our reason for discussing this problem, which has already been studied in detail by Volkov and Mikhaĭlov,[4,7] is that we need to put the theory in a form convenient for a generalization to the case of a perturbed boundary. In the second section of the paper we generalize the theory of edge magnetoplasmons for an ideal boundary to the case in which the shape of the boundary undergoes a nonzero perturbation $\xi(y)$ of the form (1).

EDGE MAGNETOPLASMONS AT A PLANE BOUNDARY

A. Our starting point is a system of equations for the electric potential $\varphi(x, y, t)$ and the oscillatory part of the density, $\delta n(x, y, t)$:

$$\varphi(x,y,t)=\varphi(x)\exp\{i(qy-\omega t)\},$$
$$\delta n(x,y,t)=\delta n(x)\exp\{i(qy-\omega t)\}, \tag{3}$$

$$\varphi(x)=2\frac{e}{\varkappa}\int_0^\infty K_0(q|x-s|)\,\delta n(s)\,ds, \tag{4}$$

$$ie\omega\delta n=\sigma_{xx}\Delta\varphi, \quad \varphi_x'' \gg \varphi_y''\equiv -q^2\varphi. \tag{5}$$

Here $\varkappa$ is the dielectric constant of the substrate, which occupies the half-space $z \leqslant 0$, and $K_0(x)$ is the Bessel function of imaginary argument.

The boundary conditions on systems (3)–(5) are

$$j_x|_{x=0}=0, \quad \text{or} \quad \sigma_{xx}\partial\varphi/\partial x|_0+iq\sigma_{xy}\varphi|_0=0, \tag{6}$$

$$\varphi(x)|_{+\infty}\to 0, \quad \delta n(x)|_{+\infty}\to 0. \tag{6a}$$

In writing relations (4)–(6) we assumed that the equilibrium electron density $n(x)$ has a nonzero limit as $x\to 0$, i.e., $n(0)\to n_s$. The alternative possibility, $n(x)|_0\to 0$, ultimately leads to results which are the same as (4)–(6), although the intermediate calculations are slightly different (see the Appendix).

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To study Eqs. (4)–(6), we substitute δn from (5) into (4) and carry out a single integration by parts. We then take the approach of Ref. 9 to regularize the singular part of the integral equation which arises for $\varphi(x)$:

$$\varphi(x)=\frac{2\sigma_{xx}}{i\omega\varkappa}\left\{K_0(qx)[\varphi'(0)-\varphi'(x)]\right.$$

$$\left.+\int_0^\infty[\varphi'(x)-\varphi'(s)]\frac{\partial K_0(q|x-s|)}{\partial s}ds\right\}. \quad (4a)$$

According to (4a), at large distances $x\gg l$ the function $\varphi(x)$ has the basic behavior

$$\varphi(x)\infty\varphi'(0)K_0(qx). \quad (7)$$

All of the other terms on the right side of (4a) decay more rapidly than e^{-qx}, e.g., as e^{-2qx}, at large distances. The asymptotic expression for $\varphi(x)$ in (7) has the form of the potential of a periodically charged rectilinear filament. The length scale l is determined by (10).

In the region $x<l$ the behavior $\varphi(x)$ is determined primarily by the integral term on the right side of (4a). The point of greatest interest here is $x=0$, at which Eq. (4a) becomes

$$\varphi(0)\approx\frac{2\sigma_{xx}}{i\omega\varkappa}\varphi'(0)\int_0^\infty\left[1-\frac{\varphi'(s)}{\varphi'(0)}\right]\frac{\partial K_0(qs)}{\partial s}ds. \quad (8)$$

Assuming

$$\frac{\varphi(0)}{\varphi'(0)}=-l,\quad\left(1-\frac{\varphi'(x)}{\varphi'(0)}\right)=\begin{cases}0, & x<l\\ 1, & x>l,\end{cases} \quad (9)$$

we can easily rewrite (8) and (9) in terms of l:

$$-l=\frac{2\sigma_{xx}}{i\omega\varkappa}\ln\frac{1}{ql},\quad l=\frac{\sigma_{xx}}{iq\sigma_{xy}},\quad ql\ll 1. \quad (10)$$

From (10) we immediately find the dispersion law for edge magnetoplasmons for long-wave excitations which are traveling along a semi-infinite $2D$ system:

$$\omega=-\frac{2q\sigma_{xy}}{\varkappa}\ln\frac{1}{ql}=-\frac{2q\sigma_{xy}}{\varkappa}\ln\frac{i\sigma_{xy}}{\sigma_{xx}}. \quad (10a)$$

Results (10) and (10a) reproduce qualitatively the basic conclusions of the theory of Refs. 4 and 7 for this limiting case.[1)]

B. We now return to our orginal equations, (4)–(6), and make use of the inequality $ql\ll 1$, which holds quite well in the region of classically strong magnetic fields. We put these equations in a form convenient for subsequent generalization to the case $\xi\neq 0$. Here it is natural to assume that the oscillatory density $\delta n(x,y,t)$ has the form of a δ-function in the x direction, i.e.,

$$e\delta n(x,y,t)=Q(y)\delta(x)e^{-i\omega t}, \quad (11)$$

where $Q(y)$ is the linear charge density of the edge magnetoplasmons. Substituting δn from (11) into the general Poisson integral for $\varphi(x,y,t)$, we find

$$\varphi(x,y,t)=\frac{e^{-i\omega t}}{\varkappa}\int_{-\infty}^{\infty}\frac{Q(y_1)dy_1}{[x^2+(y-y_1)^2]^{1/2}}. \quad (12)$$

At the boundary $x=0$, Eq. (12) is rewritten as follows $[\varphi(0,y,t)=\varphi(y)e^{-i\omega t}]$:

$$\varphi(y)=\frac{1}{\varkappa}\int_{-\infty}^{\infty}\frac{Q(y_1)dy_1}{[l^2+(y-y_1)^2]^{1/2}}. \quad (12a)$$

Here we have "manually" introduced a cutoff factor l from (10), which guarantees a correct description of the logarithmic singularity of the integral (12).

Equation (5), integrated over x from 0 to ∞, gives us

$$i\omega Q(y)=\sigma_{xx}\partial\varphi/\partial x|_0. \quad (13)$$

Using (13) and (6), we can then relative $Q(y)$ and $\partial\varphi/\partial y$:

$$i\omega Q(y)=-\sigma_{xy}\partial\varphi/\partial y|_{x=0}. \quad (14)$$

Finally, using (13) and (14), we can put Eq. (12a) in the following form, which is convenient for generalizations:

$$\varphi(y)=-\frac{\sigma_{xy}}{i\omega\varkappa}\int_{-\infty}^{+\infty}\frac{(\partial\varphi/\partial y_1)dy_1}{[l^2+(y-y_1)^2]^{1/2}}. \quad (15)$$

A solution $\varphi=\varphi_0e^{iqy}$ of this integral equation quickly leads to a dispersion law $\omega(q)$ which is the same as the expression for $\omega(q)$ in (10a).

EDGE MAGNETOPLASMONS AT A PERIODICALLY PERTURBED BOUNDARY

A. We assume that the boundary of the $2D$ system is perturbed by the periodic ripple $\xi(y)$ given by (1). We thus wish to find an analog of Eq. (15) which holds in the case $\xi\neq 0$.

The oscillator density δn takes the form

$$e\delta n(x,y,t)=Q(y)\delta(x-\xi(y))e^{-i\omega t}, \quad (16)$$

which is a generalization of expression (11) for δn. Substituting δn from (16) into the general Poisson integral, we find

$$\varphi(xy)=\frac{e}{\varkappa}\int_{\xi(y_1)}^{\infty}dx_1\int_{-\infty}^{+\infty}dy_1\frac{\delta n(x_1y_1)}{[(x-x_1)^2+(y-y_1)^2]^{1/2}}$$

$$=\frac{1}{\varkappa}\int_{-\infty}^{+\infty}\frac{Q(y_1)dy_1}{[(x-\xi(y_1))^2+(y-y_1)^2]^{1/2}}. \quad (17)$$

Equation (5), integrated over x from $\xi(y)$ to ∞, along with the relations $\varphi''_x\gg q_0^2\varphi\gg q^2\varphi$, gives us

$$i\omega Q(y)=\sigma_{xx}\partial\varphi/\partial x|_{x=\xi}. \quad (18)$$

In this case the boundary condition (6) takes the form

$$(j_x+\xi'(y)j_y)|_{x=\xi}=0 \quad (19)$$

or, in expanded form, where we are using (18),

$$i\omega Q(y)=-\left(1+\frac{\partial\xi}{\partial y}\frac{\sigma_{xy}}{\sigma_{xx}}\right)^{-1}\sigma_{xy}\frac{\partial\varphi}{\partial y}. \quad (20)$$

Substituting $Q(y)$ from (20) into expression (17) for $\varphi(x,y)$, taken at $x=\xi(y)$, we find the expression for $\varphi(y)$ which we have been seeking:

$$\varphi(y)=-\frac{\sigma_{xy}}{i\omega\varkappa}\int_{-\infty}^{+\infty}\left(1+\frac{\partial\xi}{\partial y_1}\frac{\sigma_{xy}}{\sigma_{xx}}\right)^{-1}[[\varepsilon(y)-\xi(y_1)]^2+(y-y_1)^2]^{-1/2}\frac{\partial\varphi}{\partial y_1}dy_1. \tag{21}$$

The solution of this equation should provide information about the structure of the spectrum of edge magnetoplasmons in the case $\xi\neq 0$.

The most nontrivial circumstance which arises in a study of Eq. (21) is obviously the appearance of the combination

$$1+\frac{\partial\xi}{\partial y}\frac{\sigma_{xy}}{\sigma_{xx}} \tag{22}$$

in the integrand. The presence of this combination means that despite the small value of the parameter $\xi'(y)\ll 1$ [it must be small if our derivation of Eq. (21) is to be legitimate] the effect of a perturbation ξ on the properties of edge magnetoplasmons can be quite strong, because in the region of classically strong magnetic fields we have a ratio $\sigma_{xy}/\sigma_{xx}\gg 1$, so a situation with $\xi'\sigma_{xy}/\sigma_{xx}\lesssim 1$ or even $\xi'\sigma_{xy}/\sigma_{xx}>1$ is completely possible. We can make use of the latter comments to simplify Eq. (21) slightly, retaining in it only the most important contribution from the perturbation $\xi(y)$:

$$\varphi(y)=-\frac{\sigma_{xy}}{i\omega\varkappa}\int_{-\infty}^{+\infty}\left(1+\frac{d\xi}{dy_1}\frac{\sigma_{xy}}{\sigma_{xx}}\right)^{-1}[l^2+(y-y_1)^2]^{-1/2}\frac{\partial\varphi}{\partial y_1}dy_1. \tag{23}$$

An equation for $\varphi(y)$ written in the form (23) is valid for an arbitrary perturbation $\xi(y)$ with $d\xi/dy\ll 1$ and $\int\xi dy=0$. However, this equation can be solved only for certain specific functions $\xi(y)$.

B. Let us assume that the perturbation of the boundary is specified by expression (1) for $\xi(y)$. To solve Eq. (23) in this case we can use a version of the moment method.

We assume that $\varphi(y)$ can be written in the form

$$\varphi(y)=(\varphi_0+\varphi_1\cos q_0y+\tilde{\varphi}_1\sin q_0y)e^{iqy}. \tag{24}$$

We substitute this expression for φ into Eq. (23) and integrate both sides with respect to y (Ref. 10):

$$i\omega\left(\varphi_0y-\frac{\varphi_1}{q_0}\sin q_0y+\frac{\tilde{\varphi}_1}{q_0}\cos q_0y\right)=\frac{y}{\varepsilon}(2iq\varphi_1+q_0\tilde{\varphi}_1)\sigma_{xy}K_0(ql)+\int_{-\infty}^{+\infty}d\xi\frac{e^{-iq\xi}}{(l^2+\xi^2)^{1/2}}\left\{\frac{A(\xi)\beta(\xi)-B(\xi)\alpha(\xi)}{\varepsilon^2}\times\ln[1+\alpha(\xi)\cos q_0y+\beta(\xi)\sin q_0y]+\left[q\varphi_0-\frac{A(\xi)\alpha(\xi)+B(\xi)\beta(\xi)}{\varepsilon^2}\right]\times\int\frac{dy}{1+\alpha(\xi)\cos q_0y+\beta(\xi)\sin q_0y}\right\}, \tag{25}$$

$$A(\xi)=iq\varphi_1\cos q_0\xi+q_0\varphi_1\sin q_0\xi+q_0\tilde{\varphi}_1\cos q_0\xi,$$
$$B(\xi)=iq\varphi_1\sin q_0\xi-q_0\varphi_1\cos q_0\xi+iq\tilde{\varphi}_1\cos q_0\xi,$$
$$\alpha(\xi)=\varepsilon\cos q_0\xi,\ \beta(\xi)=\varepsilon\sin q_0\xi,\ \varepsilon=q_0\xi\sigma_{xy}/\sigma_{xx},\ \varepsilon^2=\alpha^2+\beta^2, \tag{25a}$$

$$\int\frac{dx}{1+\alpha\cos x+\beta\sin x}=\begin{cases}-\dfrac{2}{\beta+(1-\alpha)\,\mathrm{tg}(x/2)}, & \varepsilon^2=1\\ \dfrac{2}{(1-\varepsilon^2)^{1/2}}\mathrm{arctg}\dfrac{(1-\alpha)\,\mathrm{tg}(x/2)+\beta}{(1-\varepsilon^2)^{1/2}}, & \varepsilon^2<1.\\ \dfrac{1}{(\varepsilon^2-1)^{1/2}}\ln\dfrac{(1-\alpha)\,\mathrm{tg}(x/2)+\beta-(\varepsilon^2-1)^{1/2}}{(1-\alpha)\,\mathrm{tg}(x/2)+\beta+(\varepsilon^2-1)^{1/2}}, & \varepsilon^2>1\end{cases} \tag{25b}$$

In the region $q_0y\gg 1$ the left side of Eq. (25) increases linearly as a function of y:

$$\int\varphi(y)dy\propto\varphi_0y.$$

On the right side, the first term is again linear in y, while the second depends on y in a complicated way, but with dependence weaker than linear. Finally, the third term is linear in y in the region $\varepsilon\ll 1$ but not for $\varepsilon\gtrsim 1$. Consequently, to first order in y, Eq. (25) is satisfied if we equate the coefficients of the terms on the two sides of Eq. (25) which are linear in y. As a result we find a relation among the amplitudes φ_0, φ_1, and $\tilde{\varphi}_1$:

$$\omega\varphi_0=-\frac{2\sigma_{xy}}{\varkappa}qK_0(ql)\times\begin{cases}\varphi_0(1+\varepsilon^2/2)-\varepsilon(\varphi_1+q_0\tilde{\varphi}_1/2q), & \varepsilon\ll 1\\ \varphi_1/\varepsilon, & \varepsilon>1\end{cases}. \tag{26}$$

To find a second and a third independent relationship among φ_0, φ_1, and $\tilde{\varphi}_1$ we have to multiply both sides of Eq. (23) by $\cos q_0y$ or $\sin q_0y$, integrate the resulting relations over y, and equate to zero the resultant coefficient of the term which is linear in y:

$$-\frac{i}{2}\omega\varphi_1\varkappa=\sigma_{xy}K_0(q_0l)\begin{cases}iq\varphi_1+q_0\tilde{\varphi}_1-iq\varepsilon\varphi_0, & \varepsilon\ll 1\\ iq\varphi_0/\varepsilon, & \varepsilon>1\end{cases}, \tag{27}$$

$$-\frac{i}{2}\omega\tilde{\varphi}_1\varkappa=\sigma_{xy}K_0(q_0l)\begin{cases}iq\tilde{\varphi}_1-q_0\varphi_1, & \varepsilon<1\\ 0, & \varepsilon>1\end{cases}. \tag{27a}$$

On the basis of (26) and (27) we find a dispersion law for edge magnetoplasmons at a periodically deformed boundary:

$$\omega=\frac{-2q\sigma_{xy}}{\varkappa}K_0(ql)\left(1+\frac{\varepsilon^2}{2}\right),\quad \varepsilon\ll 1, \tag{28}$$

$$\omega^2=\frac{4q^2\sigma_{xy}^2}{\varkappa\varepsilon^2}K_0(ql)K_0(q_0l),\quad \varepsilon>1. \tag{28a}$$

In the limit $\varepsilon>1$ the roughness of the boundary obviously changes the spectrum of edge magnetoplasmons dramatically. We should stress that a transition from the case $\varepsilon<1$ to the limiting case $\varepsilon > 1$ can be observed on a given sample with a fixed ripple $\xi(y)$ as the magnetic field is increased monotonically.

In discussing results (28) and (28a), we recall an important assertion from Refs. 4 and 7: For a bounded $2D$ system with dimensions $L_1, L_2 \gg l$ one can determine the discrete spectrum of edge magnetoplasmons by using expressions (10a) for $\omega(q)$ with wave numbers q_n represented in the form

$$q_n = 2\pi n/P, \quad n = \pm 1, \ \pm 2, \ldots, \tag{29}$$

where P is the parameter of the sample. According to this hypothesis, the effect of a periodic perturbation $\xi(y)$ with $\xi' \ll 1$ on the dispersion law for an edge magnetoplasmon which is traveling along the edge of a semi-infinite $2D$ system reduces to the substitution $q \to q^*$:

$$\omega|_{\xi(y)\neq 0} = \omega(q^*), \quad q^* \approx q(1 - {}^1/_2(\xi')^2). \tag{29a}$$

Comparing this expression for $\omega(q)$ with the results (28) and (28a), we easily see that (29a) is untenable. We are thus led to doubt the universal applicability of hypothesis (29). For example, the spectra of edge magnetoplasmons on a square and on a circle with identical perimeters can, generally speaking, be different, while according to (29) they are completely identical.

C. Equation (23) is also convenient for describing the interaction of edge magnetoplasmons with a local perturbation of a boundary. We assume that this perturbation is a single step:

$$\frac{\sigma_{xy}}{\sigma_{xx}} \xi'(y) = \begin{cases} 0, & -\infty < y < -d \\ \varepsilon, & -d \leq y \leq 0, \\ 0, & 0 < y < +\infty \end{cases} \qquad \xi'(y) \ll 1. \tag{30}$$

The distance d, over which the relation $\xi' \neq 0$ holds, is assumed to be quite large in comparison with l but small in comparison with the length scale of the variation in $\varphi(y)$ near the perturbation.

Using expression (30) for $\xi'(y)$, and assuming that φ varies slowly in the vicinity of the perturbed region, we can rewrite the general equation (23) as

$$i\omega\varphi(y) = -\frac{\sigma_{xy}}{\varkappa} \int_{-\infty}^{+\infty} \frac{(\partial\varphi/\partial y_1)\, dy_1}{[l^2 + (y - y_1)^2]^{1/2}} - \frac{\sigma_{xy}}{\varkappa} F(y) \frac{\partial\varphi}{\partial y}\bigg|_0, \tag{31}$$

$$F(y) = \frac{\varepsilon}{1+\varepsilon} \int_{-d}^{0} \frac{dy_1}{[l^2 + (y - y_1)^2]^{1/2}}. \tag{31a}$$

In writing (31) we assumed that the derivative $\partial\varphi/\partial y$ has no discontinuities along the y axis.

We solve Eq. (31) by means of Fourier transforms. Assuming in this connection

$$\varphi(q) = \int_{-\infty}^{+\infty} \varphi(y) e^{iqy}\, dy, \quad F(q) = \int_{-\infty}^{+\infty} F(y) e^{iqy}\, dy, \tag{32}$$

$$F(q) = \frac{2\varepsilon d}{1+\varepsilon} K_0(ql), \tag{32a}$$

we easily find from (31)

$$\varphi(q) = -\frac{(\sigma_{xy}/\varkappa)\, qF(q)\varphi(0)}{\omega + 2q(\sigma_{xy}/\varkappa) K_0(ql)}. \tag{33}$$

Now using for $\varphi(0)$ the expression

$$\varphi(0) = 2\pi \int_{-\infty}^{+\infty} \varphi(q)\, dq,$$

and also using the explicit expression for $\varphi(q)$ in (33), we find a dispersion relation which determines the natural frequency of a localized edge magnetoplasmon:

$$1 = -2\pi \int_{-\infty}^{+\infty} \frac{\sigma_{xy}\varkappa^{-1} qF(q)\, dq}{\omega + 2\sigma_{xy}\varkappa^{-1} qK_0(ql)}. \tag{34}$$

Equation (34) is reminiscent of equations which arise in the theory of quasilocal vibrations of a lattice with defects.[11] Accordingly, a general analysis of this equation can be carried out by analogy with the well-studied problem of quasilocal modes. In this paper we will instead restrict the discussion to the appearance of a solution of Eq. (34) at small values of ε.

Determining whether a solution of (34) exists in the limit $\varepsilon \ll 1$ reduces to testing the convergence of the integral

$$I = \int_{-\infty}^{+\infty} \frac{F(q)}{K_0(ql)}\, dq.$$

If this integral does converge, Eq. (34) has no solution as $\varepsilon \to 0$. If, on the other hand, it diverges, there must be such a solution. In the particular case in which $F(q)$ is given by (32a), integral I divergences linearly at large q. For this reason, a solution of Eq. (34) exists for arbitrarily small ε; the natural frequency ω of the localized edge magnetoplasmon is linear in ε.

Interestingly, the problem of the reflection of edge magnetoplasmons from a single step, or from a barrier, which is similar in physical content, cannot be solved by a formalism which uses Eq. (31) by itself. The reason is that this equation has no standing-wave solutions in the unperturbed region [the same is true of Eq. (23)], so the standard mechanism for the passage of a wave through an obstacle, involving incident, reflected, and transmitted waves, does not operate in its pure form in this case.

D. To complete this discussion of the properties of edge magnetoplasmons at a deformed boundary of a $2D$ system, we examine the effect of a deformation of the boundary on the properties of edge magnetoplasmons in a $3D$ problem.

Let us assume that a metal fills the half-space $x \geq 0$, that a magnetic field is directed along the x axis, and that a plasmon "runs" along the y axis. The problem reduces to one of solving the equations

$$\Delta\varphi = 4\pi e\delta n, \tag{35}$$

$$i\omega e\delta n + \operatorname{div} \mathbf{j} = 0, \quad j_i = -\sigma_{ik} \frac{\partial\varphi}{\partial x_k} \tag{36}$$

with the components σ_{ik} in the Drude approximation [see expression (2) for σ_{ik} with $n_s \to n_0$, where n_0 is the bulk electron density].

While the continuity equation (36) takes the form div

$\mathbf{j} = 0$ in a surface wave in the interior of the metal, it is not difficult to see that this assumption agrees with the Poisson equation (35), which is also satisfied in the interior under the condition $\Delta\varphi = 0$. As a result, an edge magnetoplasmon in the 3D problem has charges which are completely localized at the free boundary of the metal. If the surface charge density is σ, the continuity equation at the free boundary leads to a relationship between σ and j_x:

$$i\omega\sigma = -j_x|_{x=0}. \tag{37}$$

In addition, there are familiar electrodynamic boundary conditions

$$\partial\varphi/\partial x|_{+0} - \partial\varphi/\partial x|_{-0} = 4\pi\sigma, \quad \varphi|_{+0} = \varphi|_{-0}, \tag{38}$$

which relate the derivatives of the potential on the two sides of a boundary at which there is surface charge σ, and also the potentials themselves. The problem of the properties of edge magnetoplasmons in the 3D case thus reduces to one of solving the Laplace equation

$$\Delta\varphi = 0 \tag{39}$$

on the two sides of the boundary of the metal and joining the solutions with the help of the boundary conditions (37) and (38).

At an ideal boundary the dispersion law for edge magnetoplasmons is determined in the 3D case by the equation

$$1 = 2\pi\left[\frac{n_0 e^2}{m^*(\omega^2-\omega_c^2)} - \frac{\omega_c n_0 e^2}{m^*(\omega^2-\omega_c^2)\,\omega}\right]. \tag{40}$$

In the limit $\omega \gg \omega_c$ we find from (40) the familiar expression for the frequency of an edge plasmon in the 3D case:

$$\omega = 2^{-1/2}\omega_0, \quad \omega_0^2 = 4\pi e^2 n_0/m^*. \tag{40a}$$

If, on the other hand, the condition $\omega \ll \omega_c$ holds, we find from (40) that, as in the 2D case, there exists an edge magnetoplasmon with a frequency

$$\omega = 2\pi n_0 e^2/m^*\omega_c, \tag{40b}$$

which falls off with increasing magnetic field. In this sense the presence of a mode ω with $\partial\omega/\partial H < 0$ in a strong magnetic field is not a unique property of a 2D charged system. A similar mode exists in the 3D case.

There is a qualitative difference between edge magnetoplasmons in the 3D and 2D cases: The existence of a surface charge σ means that there will be nonvanishing electric fields in the x direction [the derivatives $\partial\varphi/\partial x$ in boundary conditions (38) are nonzero]. These fields increase slowly with increasing magnetic field. For this reason, the normal derivatives $\partial\varphi/\partial x$ cease to play a significant role in determining the dispersion law for edge magnetoplasmons in the region of strong magnetic fields. Formally, this circumstance corresponds to the legitimacy of ignoring the first term on the right side of general equation (40). In 2D systems, on the other hand, the localization of charge in an edge magnetoplasmon at a free boundary is accompanied by anomalously large values of the derivative $\partial\varphi/\partial x$, which increase with increasing magnetic field. For this reason the derivatives $\partial\varphi/\partial x$ "participate" in shaping the spectrum of edge magnetoplasmons for 2D systems over the entire magnetic field range. For the same reason, the boundary perturbation $\xi(y)$ strongly influences the properties of edge magnetoplasmons in 2D systems and has no anomalous properties in the 3D case. The latter assertion can be tested quite easily by using the expression $\tilde{j}_x = j_x + \xi'(y)j_y$, which is similar to (19), for the component $\tilde{j}_x$ and by going through the obvious calculations to generalize Eqs. (40) to the case $\xi(y) \neq 0$. In this equation, as in the 2D case, a combination $(\sigma_{xy}/\sigma_{xx})\xi'(y)$ arises, but it appears in only the first term on the right side of (40), which, as we have already mentioned, ceases to play any significant role in the case $\omega \ll \omega_c$.

CONCLUSION

Let us summarize. We have presented an approximate method for solving the problem of the spectrum of edge magnetoplasmons at a rectilinear plane boundary in the limit $\omega \ll \omega_c$. This new method makes it possible to avoid solving the corresponding integral equation, and it reduces to analyzing two transcendental equations, (10), for l and ω. The results of a solution of these equations for a plane boundary agree qualitatively with the results found previously by Volkov and Mikhaĭlov.[4,7] the arguments which led to Eqs. (10) can easily be generalized to the case of cylindrical geometry. As a result, relations analogous to (10) are found for (for example) the minimum dipole frequency of edge magnetoplasmons on a disk:

$$i = \frac{R\sigma_{rr}}{i\sigma_{r\theta}}, \quad l = \frac{2\sigma_{rr}}{i\omega\varkappa}G\left(\frac{R^*}{R}\right), \quad G(x) = \frac{1}{x}[K(x) - E(x)].$$

Here R is the radius of the disk, $K(x)$ and $E(x)$ are elliptic integrals of the first and second kinds, and $R^* = R - l$.

A deformation $\xi(y)$ of the free boundary of a 2D system has an important effect on the properties of edge magnetoplasmons, as we have seen. There is the anomaly that the parameter $\varepsilon = \xi_0 q_0 \sigma_{xy}/\sigma_{xx}$ [see expression (25a) for ε] arises at a perturbed boundary. In principle, this parameter is not small even under the condition $\xi_0 q_0 \ll 1$, since in strong magnetic fields, and at low frequencies $\omega \ll \omega_c$, the inequality $\sigma_{xy}/\sigma_{xx} \gg 1$ holds. We have derived Eq. (23), which determines the properties of edge magnetoplasmons at a deformed boundary. From the approximate solution of this equation we can draw conclusions about the behavior of the spectrum of edge magnetoplasmons for arbitrary values of the parameter ε. The final results for the spectrum of edge magnetoplasmons [see (28) and (28a)] provide evidence that a deformation of the boundary has a significant effect on the properties of edge magnetoplasmons. In addition, this equation can be used to study the localization of edge magnetoplasmons at an isolated local perturbation of the boundary of a 2D system.

APPENDIX

We assume that the equilibrium electron density $n(x)$ vanishes at the boundary $x = 0$. We are then faced with the question of the role played by the boundary condition $j_x|_0 = 0$, which occupies a prominent position in the arguments above but which becomes unimportant in the case $n(x)|_0 \to 0$ (since the requirement $j_x|_0 \to 0$ hold automatically in this case). To answer our question, we write a general linearized continuity equation with $n'(x) \neq 0$:

$$i\omega e\delta n = \sigma_{xx}\Delta\varphi + \left(\sigma_{xx}\frac{\partial\varphi}{\partial x} + iq\sigma_{xy}\varphi\right)\frac{n'(x)}{n_s}. \tag{A1}$$

In all cases of practical interest the derivative $n'(x)$ has a power-law singularity as $x \to 0$. Accordingly, it is not always legitimate to linearize the general continuity equation near the boundary of a $2D$ system. One can, however, require

$$\left(\sigma_{xx} \frac{\partial \varphi}{\partial x} + iq\varphi(x) \right) \Big|_{x\to 0} = 0, \qquad \text{(A2)}$$

thereby arranging conditions for linearization of the continuity equation even at the "most dangerous" point, $x = 0$. In the limit $n(x)|_0 \to 0$, condition (A2), which is formally the same as requirement (6) of the main text, is thus the condition for the existence of a linearized theory of edge magnetoplasmons. With regard to Eq. (A1), on the other hand, we note that in the limit in which $n(x)$ varies rapidly over a distance $l_0 < l$ this equation can be approximated by Eq. (5) of the main text.

[1)]From Refs. 4 and 7 we have

$$\omega = -\frac{2q\sigma_{xy}}{\varkappa} \ln \frac{1}{ql}, \qquad l = \frac{2i\sigma_{xx}}{\omega\varkappa}, \qquad ql \ll 1. \qquad \text{(10b)}$$

Expressions (10a) and (10b) for $\omega(q)$ are essentially identical. Expression (10) for l, however, contains an extra logarithm, $\ln(1/ql)$, not found in expression (10b) for l. For this reason, in discussing Eqs. (10) and (10a) we claim no more than a qualitative agreement with the theory of Refs. 4 and 7. In particular, expression (10a) for $\omega(q)$ is an exactly linear function of q, while expression (10b) for $\omega(q)$ also has a q inside a logarithm.

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