Nonequilibrium fluctuations in semiconductors associated with energy scattering of charge carriers by lattice optical phonons

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We investigate theoretically the nonequilibrium fluctuations in a semiconductor electron gas under conditions such that the main energy relaxation mechanism for charge carriers is instantaneous spontaneous emission of optical phonons with energy $\hbar\omega_0$, while the carrier momentum is scattered by ionized impurities or by acoustic phonons. We discuss the low-temperature case ($kT \ll \hbar\omega_0$, where T is the lattice temperature), for which we obtain the fluctuation spectrum and investigate its field dependence. We calculate the linear response of the nonequilibrium electron system to an external probe, and identify certain nonequilibrium fluctuation-dissipation relations. We show that in the low-frequency limit the spectral density of current fluctuations saturates with increasing electric field in pure semiconductors, while in doped semiconductors it passes through a maximum and then decreases. For practical electric fields, the convective contribution to the fluctuations vanishes in pure semiconductors and the fluctuations become isotropic, whereas in doped semiconductors the fluctuations are substantially anisotropic. The small-signal conductivity in the first case also approaches saturation and becomes isotropic, while in the second case the conductivity anisotropy changes sign with increasing electric field. In the high-frequency limit the intensity spectrum of fluctuations and the small-signal conductivity are different for doped semiconductors, and have a non-Lorentzian form, which leads to an explicit dependence of the noise temperature on frequency.

1. INTRODUCTION

A kinetic theory of current fluctuations in semiconductors under strong electric fields in which electron-electron interactions were neglected was constructed in Refs. 1–3.

The authors of Refs. 1, 2 investigated fluctuations in an electron-phonon system in which the nonequilibrium steady-state distribution was quasi-isotropic in momentum space, and the symmetric part of the distribution function was a Davydov-Dryuvestein function. The primary relaxation mechanisms for electronic energy ε and momentum **p** in this case are interactions with accustic phonons. The symmetric part of the distribution function $F_0(\varepsilon)$ is characterized, apart from the heat-bath temperature T, by the single parameter

$$E_0=[{}^3/_2 m k_0 T/[e^2\tau_p(k_0T)\tau_\varepsilon(k_0T)]]^{1/2}, \quad (1.1)$$

which has the sense of a characteristic field for electron-gas heating (i.e., the electrons are strongly heated for $E \gg E_0$). Here $\tau_p(k_0T)$ and $\tau_\varepsilon(k_0T)$ are the equilibrium electron relaxation times for momentum **p** and energy ε. Even though no fluctuation-dissipation relations of Callen-Welton type are satisfied for nonequilibrium systems, for the situation investigated in Refs. 1. 2 they retain an order-of-magnitude validity if we simply replace the lattice temperature T in them by $2\bar{\varepsilon}/3k_0$, where $\bar{\varepsilon}$ is the average electron energy. As remarked in Ref. 3, the reason for this is the fact that the symmetric part of the electron distribution function coincides with the equilibrium distribution function $\exp(-\varepsilon/k_0T)$ if in the latter we carry out the substitution described above. However, in the nonequilibrium state, in addition to a growth of the fluctuation power due to increase of the average electron energy $\bar{\varepsilon}$, there also appears an anisotropy in the fluctuations caused by the mutual influence of the electron current and energy fluctuations (i.e., the convective contribution to the fluctuations[4]). In energy and momentum scattering of electrons by acoustic phonons, the convective contribution is negative and the fluctuation intensity along the current **j** is found to be smaller than the intensity of transverse fluctuations. This implies that the fluctuations in energy partially suppress the current fluctuations. Consequently, the anisotropy of the fluctuations is an effect which is characteristic of the nonequilibrium state.

In Ref. 3, the current fluctuations were studied in a situation where the primary mechanism for scattering both the energy and momentum of the electrons was spontaneous emission of optical phonons of energy $\hbar\omega_0$; in contrast to Refs. 1, 2, these authors found the electron distribution function has a strong anisotropy (i.e., the streaming effect[5]). This regime is realized at low lattice temperatures ($k_0T \ll \hbar\omega_0$) in the electric field interval

$$E^- \ll E \ll E^+, \quad (1.2)$$

where the characteristic fields E^- and E^+ are determined by the relations

$$eE^-\tau_p=P_0, \quad eE^+\tau_0=P_0, \quad P_0=(2m\hbar\omega_0)^{1/2}, \quad (1.3)$$

in which τ_p is the momentum relaxation time for an electron in the passive energy region ($\varepsilon < \hbar\omega_0$), while τ_0 is the time for emission of an optical phonon. In this case, relations of Callen-Welton type are violated in a substantial fashion. To lowest order in the scattering, the spectral density of current fluctuations $(\delta j_i \delta j_k)_\omega$ in the passive energy region is different from zero only in the resonance regions at frequencies which are multiples of the time-of-flight frequency $\omega_E = 2\pi/\tau_E$ ($\tau_E = P_0/eE$ is the time of flight of an electron to the boundary of the passive region), while the small-signal conductivity $\sigma_{ij}(\omega)$ is zero.

Noting that the inelastic scattering is weak in the pas-

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sive region and that few electrons penetrate into the active region of energies, as was done in Ref. 6, leads to a finite value of the longitudinal component of the small-signal conductivity $\sigma_{\parallel}(\omega)$ in the same resonance regions; the resonance line shapes both for $(\delta j \delta j)^{\parallel}_{\omega}$ and for $\sigma_{\parallel}(\omega)$ are Lorentzian in this case. The finiteness of $\sigma_{\parallel}(\omega)$ allows us to maintain the Callen-Welton relation in the neighborhood of each resonance if we correctly choose the coefficient of proportionality between $(\delta j_i \delta j_k)_{\omega}$ and $\sigma_{ik}(\omega)$; this coefficient is usually interpreted as a noise temperature T^n.[4] We emphasize that T^n will not coincide with $2\bar{\varepsilon}/3k_0$ even in order of magnitude for the case of streaming, i.e., in this respect the differences between this situation and the situations discussed in Refs. 1, 2, 3, 6 are substantial.

In this paper we will investigate the nonequilibrium spatially-inhomogeneous current fluctuations in a semiconductor electron gas under conditions which are intermediate between those investigated in Refs. 1–3. We will assume that the electron distribution remains quasi-isotropic, which is ensured by efficient scattering of the electron momentum by acoustic phonons or by ionized impurities. The basic energy loss mechanism for electrons is emission of optical phonons with energy $\hbar\omega_0$. We investigate the low-temperature case $(k_0 T \ll \hbar\omega_0)$. The concentration of electrons is low enough that the electron-electron interactions can be neglected, so that we cannot introduce an electron temperature. The coupling constant with optical phonons is assumed to be large, and the distribution of electrons is different from zero only in the passive energy region $\varepsilon < \hbar\omega_0$; few electrons penetrate into the active energy region $\varepsilon > \hbar\omega_0$, and the distribution function there is close to zero. In other words, an electron in the passive region, as in Refs. 1, 2, moves diffusively in momentum space; however, the energy loss due to quasielastic scattering mechanisms is small in this case and, as in Ref. 3, is determined basically by the emission of optical phonons. The kinetic equation for this case was solved in Refs. 7, 8; the realism of the model used there is confirmed, e.g., by recent experimental data.[9] A new characteristic electric field E_c emerges as a consequence of the theory presented here (Ref. 5), at which energy scattering of electrons by optical phonons becomes significant; the range of relevant electric fields is determined by the conditions

$$E_c \ll E \ll E^{-}. \tag{1.4}$$

Depending on the specific mechanism of electron momentum scattering, the solution to the kinetic problem leads to two qualitative results which are fundamentally new.[7–9] If the dominant momentum scattering is by acoustic phonons, then the symmetric part of the electron distribution function does not depend on electric field, and correspondingly the field dependence of all the kinetic coefficients saturate. In particular, there appears a second "ohmic" region in the current-voltage characteristics. If, however, momentum scattering by ionized impurities dominates, then the current-voltage characteristics saturate, while the mean electron energy decreases with increasing electric field E (i.e., the carrier-cooling effect). The physical reason for this behavior of the field dependence of the average energy lies in the "matched" effects of impurity scattering of the electron momentum and emission of optical phonons: after emission of an optical phonon, the electron falls into the region of low energies $\varepsilon \to 0$, where the probability is large for scattering by the ionized impurities, and this hinders the departure of the electron from this region under the action of an electric field. As a result, electrons accumulate in the low-energy regime, and consequently their average energy decreases as the electric field increases, since there is also an increase in the electron flux into the low-energy region in this case.

2. THE FLUCTUATION SPECTRUM

In order to calculate the fluctuation spectrum we will use the method developed in Refs. 1, 2, 4. Let us introduce the auxiliary correlation function

$$\langle \delta F(\mathbf{p},\tau)\delta j_k(0)\rangle = -e\sum_{\mathbf{p}'} v_k(\mathbf{p}')\langle \delta F(\mathbf{p},\tau)\delta F(\mathbf{p}',0)\rangle \tag{2.1}$$

for a stationary random process which, according to the Wiener-Khintchin theorem, is related to its spectral density $\gamma_k(\mathbf{p},\omega)$ by the relation

$$\gamma_k(\mathbf{p},\omega) = \int_0^{\infty} \langle \delta F(\mathbf{p},\tau)\delta j_k(0)\rangle \exp(i\omega\tau)\,d\tau, \tag{2.2}$$

where $\delta F(\mathbf{p},\tau)$ is a fluctuation in the distribution function at the point $\mathbf{p}$ in momentum space and the time τ, and $\delta j_k(0)$ is the fluctuation of the kth component of the current density at the time $\tau = 0$. The angular brackets denote averaging with the total density matrix of the electron subsystem. The spectral density of current fluctuations is expressed through $\gamma_k(\mathbf{p},\omega)$:

$$(\delta j_i \delta j_k)_{\omega} = -\frac{e}{V}\sum_{\mathbf{p}} [v_i(\mathbf{p})\gamma_k(\mathbf{p},\omega) + v_k(\mathbf{p})\gamma_i(\mathbf{p},-\omega)]. \tag{2.3}$$

Let us write the equation for the quantity $\gamma_k(\mathbf{p},\omega)$:

$$-i\omega\gamma_k(\mathbf{p},\omega) - e\mathbf{E}\frac{\partial}{\partial \mathbf{p}}\gamma_k(\mathbf{p},\omega) - \hat{I}\gamma_k(\mathbf{p},\omega) = -\frac{e}{V}F(\mathbf{p})[v_k(\mathbf{p}) - V_k], \tag{2.4}$$

where V_k and $v_k(\mathbf{p})$ are the kth components of the drift velocity of an electron and the velocity of an electron with momentum $\mathbf{p}$, $\hat{I}$ is the collision operator, $F(\mathbf{p})$ is the total single-particle electron distribution function, and V is the volume of the region of the crystal under study. The operator $\hat{I}$ includes the interaction with acoustic and optical phonons and the scattering with ionized impurities.

The method of solving Eq. (2.4) is similar to that discussed in Ref. 8 for solving the kinetic equation. Representing $\gamma_k(\mathbf{p},\omega)$ in the form of a sum of symmetric and antisymmetric parts

$$\gamma_k(\mathbf{p},\omega) = \gamma_k^0(\varepsilon,\omega) + \gamma_k^1(\mathbf{p},\omega), \tag{2.5}$$

we obtain equations for these parts:

$$\gamma_k^1(\mathbf{p},\omega) = -\frac{e\tau_p(\varepsilon)}{1 - i\omega\tau_p(\varepsilon)}\left[\frac{1}{V}v_k(\mathbf{p})F_0(\varepsilon) - Ev_z(\mathbf{p})\frac{d}{d\varepsilon}\gamma_k^0(\varepsilon,\omega)\right], \tag{2.6}$$

$$i\omega\tau_\varepsilon(k_0T)g(\varepsilon)\gamma_k^0(\varepsilon,\omega)-\frac{d}{d\varepsilon}\left\{g(\varepsilon)\left[\varepsilon\left(\frac{\varepsilon}{k_0T}\right)^{1/2}\gamma_k^0(\varepsilon,\omega)\right.\right.$$
$$\left.\left.+\left(\frac{2}{3}\frac{e^2}{m}E^2\varepsilon\tau_p(\varepsilon)\tau_\varepsilon(k_0T)+\varepsilon(\varepsilon k_0T)^{1/2}\right)\frac{d}{d\varepsilon}\gamma_k^0(\varepsilon,\omega)\right]\right\}$$
$$=g(\varepsilon)\frac{\tau_\varepsilon(k_0T)}{\tau_{op}(\varepsilon)}(N_0+1)\gamma_k^0(\varepsilon+\hbar\omega_0,\omega)$$
$$-\frac{\delta_{kz}}{V}\left[\frac{2}{3}\frac{e^2}{m}E\varepsilon\tau_p(\varepsilon)\tau_\varepsilon(k_0T)\right.$$
$$\times g(\varepsilon)\frac{dF_0(\varepsilon)}{d\varepsilon}+\frac{d}{d\varepsilon}\left(\frac{2}{3}\frac{e^2}{m}E\varepsilon\tau_p(\varepsilon)\tau_\varepsilon(k_0T)g(\varepsilon)F_0(\varepsilon)\right)$$
$$\left.-eg(\varepsilon)F_0(\varepsilon)V_k\right]. \quad (2.7)$$

Here $g(\varepsilon)$ is the density of electron states, and the electric field is assumed to lie along the z-axis ($E_z=E$). The first term of the right side of (2.7) describes relaxation to $\gamma_k{}^0(\varepsilon,\omega)$ by way of spontaneous emission of optical phonons; N_0 is the equilibrium Planck distribution for the optical phonons, and $\tau_{op}(\varepsilon)$ is the time for scattering of an electron with energy ε by optical phonons. The terms which include absorption of optical phonons can be neglected in the present problem.[8]

The symmetric part $F_0(\varepsilon)$ of the distribution function which appears in (2.6) was derived in Eq. (8). It has the form

$$F_0(x)=C\exp[-I(x)]\int_x^{x_0}\frac{\exp[I(u)-I(x_0)]}{u^2[\theta(u)+1]}du, \quad (2.8)$$

where C is a normalization constant,

$$\theta(u)=\mathscr{E}_0\frac{u}{u^2+B}, \quad \mathscr{E}_0=\frac{E^2}{E_0^2},$$
$$B=6\frac{\mu_a}{\mu_I}, \quad I(x)=\int_0^x\frac{du}{\theta(u)+1},$$
$$x=\frac{\varepsilon}{k_0T}, \quad x_0=\frac{\hbar\omega_0}{k_0T}. \quad (2.9)$$

In (2.9), μ_a and μ_I are electron mobilities in weak electric fields associated with scattering by acoustic phonons and ionized impurities, respectively. The parameter B determines the relative contribution of each of these scattering mechanisms to the function $\theta(u)$, which is a characteristic of the carrier heating. When the two scattering mechanisms (i.e., scattering by acoustic phonons and by ionized impurities) are included, the momentum relaxation time $\tau_p(\varepsilon)$ which enters into (2.6) and (2.7) equals

$$\tau_p(x)=\tau_a(k_0T)\frac{x^{3/2}}{x^2+B}. \quad (2.10)$$

For low-frequency fluctuations, i.e., those which satisfy the condition

$$\omega\tau_\varepsilon^*\ll1, \quad (2.11)$$

we can neglect the first term on the left side by Eq. (2.7). [In what follows, we will discuss criterion (2.11), along with what it means to introduce this characteristic relaxation time τ_ε^* for the charge carrier energy.] After this, Eq. (2.7) can be integrated once, and after a few transformations it takes the form

$$\frac{d}{dx}\gamma_k^0(x,\omega)+\frac{\gamma_k^0(x,\omega)}{\theta(x)+1}=\frac{1}{V}\frac{k_0T}{E}\delta_{kz}[\lambda(x)-G(x)], \quad (2.12)$$

where the notations

$$\lambda(x)=\frac{\theta(x)}{\theta(x)+1}F_0(x)+\frac{\varkappa(x)}{x^2[\theta(x)+1]},$$
$$\varkappa(x)=\int_0^x\left[u^2\theta(u)\frac{dF_0(u)}{du}-\Psi_Eu^{1/2}F_0(u)\right]du, \quad (2.13)$$

$$\Psi_E=\frac{2}{\pi^{1/2}N}\int_0^{x_0}u^2\theta(u)\frac{dF_0(u)}{du}du, \qquad N=\frac{2}{\pi^{1/2}}\int_0^{x_0}u^{1/2}F_0(u)du, \quad (2.14)$$

$$G(x)=\frac{G_0}{x^2[\theta(x)+1]},$$
$$G_0=V\frac{E}{k_0T}\int_0^x\frac{\tau_\varepsilon(k_0T)}{\tau_{op}(u)}(N_0+1)\gamma_k^0(u+x_0,\omega)du. \quad (2.15)$$

have been introduced. Because of the weak penetration of the electrons into the active region of energies, the function $\gamma_k{}^0(u+x_0,\omega)$ rapidly decreases as u increases in this region, and therefore the integral in (2.15), and correspondingly G_0, are practically independent of x. The constant G_0, and also the constant associated with the final integration of Eq. (2.12), are found from the conditions

$$\int_0^{x_0}x^{1/2}\gamma_k^0(x,\omega)dx=0, \quad (2.16)$$
$$\gamma_k^0(x_0,\omega)=0. \quad (2.17)$$

The first of these follows from the requirement that the electron concentration equal the constant n, while the second follows from the requirement that few electrons penetrate into the active energy region (an analogous condition was used in Ref. 8 in solving the kinetic equation). We note that the assumption that G_0 is independent of x, together with condition (2.17), leads to a distortion of $\gamma_k{}^0(u+x_0,\omega)$ in small neighborhoods of the points $x=0$ and $x=x_0$. However, in view of the smallness of the electron penetration into the active region of energies, this distortion also turns out to be small. An exact analytic solution to this problem, where in place of Eq. (2.17) we use the condition that the solutions match at $x=x_0$, would require that we know the function $\gamma_k{}^0(u+x_0,\omega)$ in the active energy region as well, which is possible only in certain special cases. This question was investigated in detail for the kinetic equation in Ref. 10.

The solution to Eq. (2.12) including conditions (2.16) and (2.17) has the form

$$\gamma_k^0(x,\omega)$$
$$=\frac{1}{V}\frac{k_0T}{E}\exp[-I(x)]\left[-\int_x^{x_0}\lambda(u)\exp[I(u)]du\right.$$
$$\left.+G_0\int_x^{x_0}\frac{\exp[I(u)]}{u^2[\theta(u)+1]}du\right], \quad (2.18)$$

while the constant G_0 equals

$$G_0=\int_0^{x_0} x^{1/2}\exp[-I(x)]\left[\int_x^{x_0}\lambda(u)\exp[I(u)]du\right]dx \Big/ \int_0^{x_0} x^{1/2}\exp[-I(x)] \times\left[\int_x^{x_0}\frac{\exp[I(u)]du}{u^2[\theta(u)+1]}\right]dx. \tag{2.19}$$

We now calculate the special density of current fluctuations. From (2.3), (2.6), and (2.12), taking into account the fact that $i\omega\tau_p(\varepsilon)$ can be neglected in (2.6) when condition (2.19) is fulfilled, we find that

$$(\delta j_i\delta j_k)_\omega=\frac{2\sigma_a}{V}k_0T\frac{\delta_{ik}}{N}[\mathscr{P}_0-\delta_{kz}\mathscr{P}_1], \tag{2.20}$$

where

$$\mathscr{P}_0=\int_0^{x_0}\Phi(x)F_0(x)dx,\qquad \Phi(x)=\frac{\tau_p(x)}{\tau_a(k_0T)}x^{3/2}, \tag{2.21}$$

$$\mathscr{P}_1=\int_0^{x_0}\lambda(x)\exp[I(x)][\mathscr{K}(x)+\Phi(x)\exp[-I(x)]]dx$$
$$-G_0\int_0^{x_0}\frac{\exp[I(x)]}{x^2[\theta(x)+1]}[\mathscr{K}(x)+\Phi(x)\exp[-I(x)]]dx, \tag{2.22}$$

$$\mathscr{K}(x)=\int_0^x\exp[-I(u)]\left[\frac{\Phi(u)}{\theta(u)+1}-R_0u^{1/2}\right]du, \tag{2.23}$$

$$R_0=\int_0^{x_0}\frac{\Phi(x)\exp[-I(x)]}{\theta(x)+1}dx\Big/\int_0^{x_0}x^{1/2}\exp[-I(x)]dx. \tag{2.24}$$

Here $\sigma_a=en\mu_a$. The second term in (2.20) determines the convective contribution to the fluctuations.[4] The resulting expressions (2.20)–(2.24) give a solution to the fluctuation problem in the low-frequency region for the nonequilibrium system under discussion here.

In the intermediate-frequency region, when the condition

$$(\tau_\varepsilon^*)^{-1}\ll\omega\ll(\tau_p^*)^{-1}, \tag{2.25}$$

is fulfilled, the first term on the left side of Eq. (2.7) is the important one. Then, according to Ref. 1,

$$\gamma_k^0(x,\omega)\approx 0 \tag{2.26}$$

are correspondingly the second term of (2.6) and (2.20) in the angular brackets can be neglected:

$$(\delta j_i\delta j_k)_\omega=\frac{2\sigma_a}{V}k_0T\delta_{ik}\frac{\mathscr{P}_0}{N}. \tag{2.27}$$

For the high-frequency fluctuations, when the condition

$$\omega\tau_p^*\gg 1, \tag{2.28}$$

if fulfilled, the solution (2.26) remains valid, as previously; now, however, we cannot neglect the term $i\omega\tau_p(x)$ in the denominator of (2.6). Then in this range of frequencies we obtain for the spectral density

$$(\delta j_i\delta j_k)_\omega=\frac{2\sigma_a}{V}k_0T\delta_{ik}\frac{\mathscr{P}_\omega}{N}, \tag{2.29}$$

where

$$\mathscr{P}_\omega=\int_0^{x_0}\frac{\Phi(x)}{1+\omega^2\tau_p^2(x)}F_0(x)dx. \tag{2.30}$$

The high-frequency region is bounded from above by the condition $\omega\ll\bar{\varepsilon}/\hbar$ (Ref. 1), i.e., the region where the classical equation (2.4) is applicable.

Let us now investigate the field and frequency dependence of the spectral density of the current fluctuations, $(\delta j_i\delta j_k)_\omega$, based on the expressions obtained above. As we already noted in the Introduction, the kinetic behavior of the system depends significantly on which momentum-scattering mechanism dominates. Therefore when investigating fluctuations it is reasonable to discuss the cases of acoustic and impurity scattering separately.

3. MOMENTUM SCATTERING BY ACOUSTIC PHONONS

In this case $B\ll 1$ in Eq. (2.9), and

$$\theta(x)=\mathscr{E}_0/x,\quad I(x)=x-\mathscr{E}_0\ln(1+x/\mathscr{E}_0). \tag{A}$$

If the range of electric fields is such that the condition

$$I(x_0)\gg 1, \tag{3.1}$$

is fulfilled, then it follows from (2.8) that the distribution function has the form of the Davydov-Druyvestein function:

$$F_0(x)\approx C'\exp(-x)[x+\mathscr{E}_0]^{\mathscr{E}_0}, \tag{3.2}$$

where C' is a normalization constant which is redefined in comparison to (2.8). This implies that the interaction with optical phonons in these electric fields is not yet able to significantly affect the behavior of the electron subsystem. Correspondingly, we can neglect the contribution of optical-phonon scattering in all the expressions for the spectral density of fluctuations. Then from (2.22)–(2.24), including (3.2), we obtain

$$\mathscr{P}_1=-\mathscr{E}_0\int_0^{x_0}\frac{\mathscr{K}'(x)-xF_0(x)}{F_0(x)[\mathscr{E}_0+x]}[\mathscr{K}'(x)+xF_0(x)]dx, \tag{3.3}$$

$$\mathscr{K}'(x)=\int_0^x\left[u\frac{dF_0(u)}{du}-\frac{\Psi_E}{\mathscr{E}_0}u^{1/2}F_0(u)\right]du, \tag{3.4}$$

where we have used the fact that when $B\ll 1$ we have $\Phi(x)=x$. All the remaining expressions retain their previous form; however, $F_0(x)$ in these expressions is now determined by Eq. (3.2). Hence, in the region of electric fields for which conditions (3.1) is satisfied, all the results obtained here coincide with the results obtained in Refs. 2 and 4. The characteristic energy relaxation time $\tau_\varepsilon^*(\varepsilon)$ in (2.11) in this case approximately equals $\tau_\varepsilon(k_0T\mathscr{E}_0^{1/2})$, i.e., $\bar{\varepsilon}\approx k_0T\mathscr{E}_0^{1/2}$. Because the problem here is to obtain the field and frequency dependences for the spectral density of fluctuations, in order to complete the picture and to make possible a comparison of the results obtained here with the results derived in Refs. 2 and 4, we will briefly summarize the results of these references.

In the "warm electron" region (i.e., for $\mathscr{E}_0 \ll 1$), the transverse (⊥) and longitudinal (||) components of the tensor $(\delta j_i\ \delta j_k)_\omega$ equal

$$(\delta j_i \delta j_i)_\omega^\perp = \frac{2\sigma_a}{V} k_0 T[1+0{,}39\mathscr{E}_0] \quad (i\neq z), \tag{3.5}$$

$$(\delta j_z \delta j_z)_\omega^\parallel = \frac{2\sigma_a}{V} k_0 T[1-0{,}44\mathscr{E}_0]. \tag{3.6}$$

Correspondingly, for "hot electrons" (i.e., for $\mathscr{E}_0 \gg 1$),

$$(\delta j_i \delta j_i)_\omega^\perp = \frac{2\sigma_a}{V} k_0 T\left[\frac{2^{3/4}\Gamma(5/4)}{\pi^{1/2}}\mathscr{E}_0\right] \quad (i\neq z), \tag{3.7}$$

$$(\delta j_z \delta j_z)_\omega^\parallel = 0{,}49\frac{2\sigma_a}{V} k_0 T\left[\frac{2^{3/4}\Gamma(5/4)}{\pi^{1/2}}\mathscr{E}_0\right]. \tag{3.8}$$

In the range of electric fields which satisfy the condition opposite to (3.1), i.e., for

$$I(x_0) \ll 1, \tag{3.9}$$

according to (2.8) the distribution function has the form

$$F_0(x) = C\int_x^{x_0} \frac{du}{u^2[\theta(u)+1]}. \tag{3.10}$$

The criterion (3.9) determines the electric field region in which inelastic interactions with optical phonons play a fundamental role in the energy relaxation. The condition $I(x_0) = 1$ determines a new characteristic field which enters into (1.4). When (3.9) is fulfilled it is necessary to use the following limiting forms in Eqs. (2.22)–(2.24):

$$\exp[I(x)] \to 1, \quad \left[\frac{1}{\theta(x)+1}\right] \to 0, \tag{3.11}$$

which gives

$$R_0 \approx 0, \ \mathscr{K}(x) \approx 0. \tag{3.12}$$

From (2.19), using the first expression in (3.11), we obtain

$$G_0 = \int_0^{x_0} x^{3/2}\lambda(x)\,dx \Big/ \int_0^{x_0} \frac{dx}{x^{1/2}[\theta(x)+1]}. \tag{3.13}$$

From this it follows that in the limit defined by (3.11) the expression for $\mathscr{P}_1$, which determines the convective contribution to the fluctuations, has the form

$$\mathscr{P}_1 = \int_0^{x_0} \lambda(x)\Phi(x)\,dx - G_0\int_0^{x_0} \frac{\Phi(x)}{x^2[\theta(x)+1]}\,dx. \tag{3.14}$$

In the approximation (3.9) for $\lambda(x)$, from (2.13)–(2.18) and (3.10) and $B \ll 1$ we obtain

$$\lambda(x) = -\frac{C}{\mathscr{E}_0}\left[1+\ln\frac{x}{x_0} - \left(\frac{x}{x_0}\right)^{3/2}\left(1-\frac{3}{2}\ln\frac{x}{x_0}\right)\right]. \tag{3.15}$$

Then from (3.13) and (3.15) we find for G_0 the value

$$G_0 = 0{,}39Cx_0. \tag{3.16}$$

If we now calculate $\mathscr{P}_1$ from (3.14)–(3.16), we find that in the range of electric fields which satisfy condition (3.9), for the case of momentum scattering of an electron by acousltic lattice vibrations the convective contribution to the fluctuations disappears:

$$\mathscr{P}_1 \approx 0. \tag{3.17}$$

We emphasize that this result is numerical, i.e., it follows only after we evaluate the integrals entering into (3.14). However, the difference in the integrands in (3.14) or in the values of the indefinite integrals is not zero. Calculations show that the first nonvanishing terms, which determine the relative convective contributions to the current fluctuations $\mathscr{P}_1/\mathscr{P}_0$ in (2.20), are on the order of $10^{-3}x_0/\mathscr{E}_0$ (as a comparison, the criterion (3.9) in explicit form gives $x_0^2/2\mathscr{E}_0 \ll 1$). The fact that the convective term reduces to zero implies, according to (2.20), that the fluctuation properties of the sample under study are isotropic when the conditions described above are fulfilled. Apparently, this is the first time that the existence of such a situation in a nonequilibrium system has been demonstrated.

Let us now calculate the contribution to the spectral density connected with fluctuations of the antisymmetric part of the distribution function, which is determined by Eq. (2.21). Using (3.10), we obtain for $\mathscr{P}_0$ and N

$$\mathscr{P}_0 = \frac{1}{4}C\frac{x_0^2}{\mathscr{E}_0}, \quad N = \frac{8}{9\pi^{1/2}}C\frac{x_0^{3/2}}{\mathscr{E}_0}, \tag{3.18}$$

where we have used the criterion (3.9) and carried out a series expansion of the corresponding expressions in the small parameter $x_0/\mathscr{E}_0 \ll 1$. The spectral density of current fluctuations in this case equals

$$(\delta j_s \delta j_s)_\omega^\perp = (\delta j_z \delta j_z)_\omega^\parallel = (\delta j_i \delta j_k)_\omega = \frac{2\sigma_a}{V} k_0 T \cdot \frac{9}{32}(\pi x_0)^{1/2}\delta_{ik}, \quad (s\neq z). \tag{3.19}$$

Hence, the field dependence of $(\delta j_i\ \delta j_k)_\omega$ in the range of electric fields (3.9), which is manifested by an intense generation of optical phonons, approaches saturation.

In the preceding discussion, we have carried out an investigation of the spectral density of current fluctuations in the low-frequency region of the spectrum, as defined by the condition (2.11). However, by virtue of the result (3.17), and as is also clear from (2.27), Eq. (3.19) also remains valid in the intermediate frequency range determined by conditions (2.25). This implies that at frequencies $\omega \sim (\tau_\varepsilon^*)^{-1}$ there is no dispersion in the longitudinal component of the spectral density of the current fluctuations in the electric field range (3.9), whereas such dispersion is present[4] in the field range defined by condition (3.1): for example, according to (3.7), (3.8), the intensity of longitudinal fluctuations increases by approximately a factor of two in the region of strong heating.

It should also be noted that in the electric field interval (3.9) the characteristic relaxation time τ_ε^* is determined by inelastic interactions of the electrons with optical phonons, and therefore it differs from the τ_ε^* for the range of fields (3.1), where, as we have already pointed out, τ_ε^* is determined by quasielastic scattering by acoustic phonons. The τ_ε^* of interest to us can be estimated by using the kinetic equation for $F_0(\varepsilon)$. It follows from this equation that the constant C in (2.8) is connected with the flux of electrons in energy space J_{op} caused by emission of optical phonons through the following relation:

$$C=\frac{\tau_\varepsilon(k_0T)}{g(k_0T)k_0T}J_{op},\qquad(3.20)$$

where $g(k_0T)$ is the density of states at $\varepsilon=k_0T$. The flux J_{op} determines the rate of energy loss by all the electrons because of optical phonon emission, and we can assume that $J_{op}=n/\tau_\varepsilon^*$. Then, using (3.20), we obtain from the normalization condition

$$\tau_\varepsilon^*\approx\frac{\tau_\varepsilon(k_0T)}{C}\int_0^{x_0}x^{1/2}F_0(x)\,dx.\qquad(3.21)$$

For $F_0(x)$, we find from (3.10) and $\theta(u)=\mathscr{E}_0/u$ that the time τ_ε^* decreases as the electric field increases

$$\tau_\varepsilon^*\approx\frac{4}{9}\tau_\varepsilon(k_0T)\frac{x_0^{3/2}}{\mathscr{E}_0}.\qquad(3.22)$$

In the high-frequency region, when condition (2.28) is fulfilled, the current-fluctuation spectral density according to (2.29), (2.30), equals

$$(\delta j_i\delta j_k)_\omega=\frac{2\sigma_a}{V}k_0T\frac{\pi^{1/2}}{8}x_0^{3/2}\frac{\delta_{ik}}{\omega^2\tau_a^2(k_0T)}.\qquad(3.23)$$

For comparison, in the range of electric fields (3.1) we have according to Ref. 4 that

$$(\delta j_i\delta j_k)_\omega=\frac{2\sigma_a}{V}k_0T[2+1,78\mathscr{E}_0]\frac{\delta_{ik}}{\omega^2\tau_a^2(k_0T)}\quad(\mathscr{E}_0\ll1),\qquad(3.24)$$

$$(\delta j_i\delta j_k)_\omega=\frac{2\sigma_a}{V}k_0T[2^{1/4}\Gamma(5/4)\mathscr{E}_0^{3/4}]\frac{\delta_{ik}}{\omega^2\tau_a^2(k_0T)}\quad(\mathscr{E}_0\gg1).\qquad(3.25)$$

The physical reason for the saturation in the field dependence of the current-fluctuation spectral density (3.19), (3.23) over the entire range of frequencies is the same as that which operates when the second "ohmic" region appears. When (3.9) is fulfilled, $F_0(\varepsilon)$ practically ceases to depend on electric field (if $B\ll1$, then in (3.10) we can neglect the one in the denominator for the energy values of interest, and hence the electric field can be included in the normalization constant; in this way, we remove the explicit dependence of the distribution function $F_0(\varepsilon)$ on the electric field E).

4. MOMENTUM SCATTERING BY IONIZED IMPURITIES

The general scheme for investigating fluctuations in this case is analogous to that used in the previous section. For $B\gg x_0^2$, we find from (2.9) that

$$\theta(x)=\mathscr{E}_0'x,\quad I(x)=\frac{1}{\mathscr{E}_0'}\ln(\mathscr{E}_0'x+1),\qquad(D)$$

in which we introduce a new characteristic electric field

$$\mathscr{E}_0'=\mathscr{E}_0/B.\qquad(4.1)$$

In the range of electric fields (3.1), from (2.8) we obtain for the distribution function

$$F_0(x)\approx C'(\mathscr{E}_0'x+1)^{-1/\mathscr{E}_0'}.\qquad(4.2)$$

This function is normalized only for $\mathscr{E}_0'<2/3$, since scattering by ionized impurities is a nonconfining mechanism, and as the electric field increases electrons runaway into the high-energy region.

Let us discuss the low-frequency region (2.11). For $\mathscr{E}_0'\ll1$ we can discard the second term in (2.22), and obtain from (2.20)–(2.24) and (4.2):

$$(\delta j_i\delta j_i)_\omega^\perp=\frac{2\sigma_I}{V}k_0T[1+18\mathscr{E}_0']\quad(i\neq z),\qquad(4.3)$$

$$(\delta j_z\delta j_z)_\omega^\parallel=\frac{2\sigma_I}{V}k_0T[1+(18+D_i)\mathscr{E}_0']\quad D_i=15,25,\qquad(4.4)$$

where $\sigma_i=en\mu_i$. Here, in contrast to the previous case, the convective contribution to the current fluctuations is positive and

$$(\delta j_z\delta j_z)_\omega^\parallel>(\delta j_i\delta j_i)_\omega^\perp.$$

The interaction with optical phonons is a confining mechanism, which limits the electron runaway, and its inclusion makes possible an investigation of the region of strong electric fields satisfying condition (3.9). In this case, from (2.14), (2.12), (3.10), and (3.14), after uncomplicated but tedious calculations we obtain for the quantities entering into (2.20),

$$N=\frac{4}{3}\left(\frac{\pi}{\mathscr{E}_0'}\right)^{1/2}\frac{C}{B},\quad\mathscr{P}_0=\frac{1}{8}\frac{x_0^2}{\mathscr{E}_0'}\frac{C}{B},$$

$$\mathscr{P}_1=\frac{1}{8}\frac{x_0^2}{\mathscr{E}_0'}\frac{C}{B}\left[1-\frac{8}{3\pi^2}(2\pi^2-3)\right].\qquad(4.5)$$

Substituting (4.5) into (2.20), we find the spectral density of current fluctuations

$$(\delta j_i\delta j_i)_\omega^\perp=\frac{2\sigma_I}{V}k_0T\frac{x_0^2}{64(\pi\mathscr{E}_0')^{1/2}}\quad(i\neq z),\qquad(4.6)$$

$$(\delta j_z\delta j_z)_\omega^\parallel=\frac{2\sigma_I}{V}k_0T\frac{x_0^2}{64(\pi\mathscr{E}_0')^{1/2}}K_D,\qquad(4.7)$$

where

$$K_D=\frac{8}{3\pi^2}(2\pi^2-3)\qquad(4.8)$$

is the anisotropy coefficient for the current-fluctuation spectral density (or the diffusion coefficient for hot electrons[4]).

From Eqs. (4.3), (4.4), (4.6), and (4.7), it follows that the low-frequency current-fluctuation spectral density has a nonmonotonic field dependence with a maximum, and in the region of electric fields satisfying condition (3.9) it decreases with increasing electric field ($(\delta j_i\ \delta j_k)_\omega\sim1/E$). Apparently this is a result obtained analytically for the first time also. The current fluctuations in this case are characterized by significant anisotropy ($K_D\approx5$). The physical reason for the decrease in $(\delta j_i\ \delta j_k)_\omega$ with increasing field is the accumulation of electrons in the low-energy region, whose mechanism was discussed in detail in the Introduction.

In the intermediate-frequency range (2.25), we obtain from (2.27) and (4.5)

$$(\delta j_i\delta j_k)_\omega=\frac{2\sigma_I}{V}k_0T\frac{x_0^2}{64(\pi\mathscr{E}_0')^{1/2}}\delta_{ik}.\qquad(4.9)$$

From a comparison of (4.5) and (4.9) it follows that for $\omega\sim(\tau_\varepsilon^*)^{-1}$ significant dispersion is found in the longitudinal component $(\delta j_z\ \delta j_z)_\omega$. As the frequency increases, $(\delta j_z$

$\delta j_z)_\omega$ decreases by approximately a factor of 5. In this frequency region, however, for $\mathscr{E}_0' \ll 1$ the longitudinal and transverse components of $(\delta j_i\ \delta j_k)_\omega$ are determined by the expressions (4.3).

Let us estiamte the characteristic time τ_ε^* for $I(x_0) \ll 1$ for impurity momentum scattering. From (3.21) and (3.10) with $\theta(u) = \mathscr{E}_0' u$ we find that

$$\tau_\varepsilon^* \approx \frac{2\pi}{3}\tau_\varepsilon(k_0T)\frac{1}{\mathscr{E}_0'^{1/2}}. \quad (4.10)$$

Let us now investigate the high-frequency region (2.28). In weak electric fields, we find for $\mathscr{E}_0' \ll 1$ from (2.29), (2.30), and (4.2) that

$$(\delta j_i \delta j_k)_\omega = \frac{2\sigma_I}{V} k_0T \cdot \frac{1}{6}\left[1 - \frac{7}{8}\mathscr{E}_0'\right]\frac{\delta_{ik}}{\omega^2\tau_I^2(k_0T)}. \quad (4.11)$$

In the region of strong electric fields, when the condition (3.9) is fulfilled the spectrum of fluctuations has a non-Lorentzian form. This is caused by the fact that the main contribution to the integral (2.30) is given by the small-energy region, and in this region we cannot neglect the 1 in the denominator compared to $\omega^2\tau_p{}^2(x)$ even for arbitrarily high frequencies (since $\Phi(x) \sim x^3$, $\tau_p(x) \sim x^{3/2}$; as $x \to 0$, $F_0(x) \sim 1/x$, so that in this case the integral (2.30) diverges logarithmically at its lower limit). For frequencies satisfying the condition

$$\omega^2\tau_I^2(k_0T) \gg \mathscr{E}_0'^3, \quad (4.12)$$

the expression for φ_ω is considerably simplified, and for the current-fluctuation spectral density (2.29) we obtain

$$(\delta j_i \delta j_k)_\omega = \frac{2\sigma_I}{V} k_0T \cdot \frac{1}{4}\left(\frac{\mathscr{E}_0}{\pi}\right)^{1/2} \ln\left[\frac{\omega^2\tau_I^2(k_0T)}{\mathscr{E}_0'}\right]\frac{\delta_{ik}}{\omega^2\tau_I^2(k_0T)}. \quad (4.13)$$

We note that when (4.12) holds the fluctuation spectral density does not depend on the optical phonon energy.

The integral (2.30) does not depend on frequency if the following condition holds;

$$\omega^2\tau_I^2(k_0T)x_0^3 \ll 1. \quad (4.14)$$

Consequently, the order of magnitude of the characteristic time entering into the right side of condition (2.25) is given by $\tau_p^* \approx \tau_I(k_0T)x_0^{3/2}$.

5. SMALL-SIGNAL CONDUCTIVITY AND FLUCTUATION-DISSIPATION RELATIONS

In the previous section we obtained the field and frequency dependences of the current-fluctuation spectral density for two different electron momentum scattering mechanisms. It is interesting to compare the fluctuations in the nonequilibrium system under discussion here with its response to excitation by a weak AC electric field $\mathscr{E}_k = \mathscr{E}_k^0 \exp(-i\omega t)$. The kinetic equaiton which determines the correction $f_k(\mathbf{p},\omega)\exp(-i\omega t)$ to the stationary nonequilibrium distribution function differs from Eq. (2.4) only in the right-hand side, which now equals

$$e\mathscr{E}_k{}^0 \frac{\partial}{\partial p_k} F(\mathbf{p}). \quad (5.1)$$

The procedure used to solve it is analogous to that used to solve Eq. (2.4); therefore we obtain for the small-signal conductivity tensor, including (5.1),

$$\sigma_{ik}(\omega) = -\sigma_a \frac{\delta_{ik}}{N}\left[\int_0^{x_0} \frac{\Phi(x)}{1-i\omega\tau_p(x)}\frac{dF_0(x)}{dx}dx + \delta_{kz}\int_0^{x_0}\frac{\Phi(x)}{1-i\omega\tau_p(x)}\frac{df^0(x,\omega)}{dx}dx\right], \quad (5.2)$$

where $f^0(x,\omega)$ is determined from the equation

$$\frac{i\omega\tau_\varepsilon(k_0T)}{x^2[\theta(x)+1]}\int_0^x u^{1/2}f^0(u,\omega)\,du + \frac{df^0(x,\omega)}{dx} + \frac{f^0(x,\omega)}{\theta(x)+1} = -\left[\frac{2\theta(x)}{\theta(x)+1}\frac{dF_0(x)}{dx} + \frac{g_0}{x^2[\theta(x)+1]}\right]. \quad (5.3)$$

The constant g_0 and the constant of integration in Eq. (5.3) are found from conditions analogous to (2.16) and (2.17).

For nonequilibrium systems it is well-known that no universal relation exists between $(\delta j_i\,\delta j_k)_\omega$ and $\sigma_{ik}(\omega)$ similar to the Callen-Welton fluctuation-dissipation relation for a system in thermodynamic equilibrium. However, a generalization of this relation to nonequilibrium systems serves as a definition of the noise temperature T^n:

$$(\delta j_i \delta j_k)_\omega = \frac{k_0T^n}{V}[\sigma_{ik}(\omega) + \sigma_{ki}^*(\omega)]. \quad (5.4)$$

In equilibrium it is obvious that $T^n = T$. In order to determine T^n under nonequilibrium conditions it is necessary to calculate the spectral and field dependences of the small-signal conductivity. Here, too, we will investigate the two scattering mechanisms for electron momentum separately. All calculations for this case are analogous to those we carried out to determine the spectrum of fluctuations; therefore here we present immediately the final results obtained from (5.2) and (5.3).

Momentum scattering by acoustic phonon

In the low-frequency range (2.11) we obtain the following field dependences for the small-signal conductivity. If $I(x_0) \gg 1$, then

$$\sigma_\perp(\omega) = \sigma_a[1 - 0{,}61\mathscr{E}_0], \quad (5.5)$$

$$\sigma_\parallel(\omega) = \sigma_a[1 - 1{,}83\mathscr{E}_0] \quad (5.6)$$

for $\mathscr{E}_0 \ll 1$, and

$$\sigma_\perp(\omega) = \sigma_a \cdot 2^{1/4}\Gamma(5/4)\mathscr{E}_0^{-1/4}, \quad (5.7)$$

$$\sigma_\parallel(\omega) = {}^1/_2\sigma_a \cdot 2^{1/4}\Gamma(5/4)\mathscr{E}_0^{-1/4} \quad (5.8)$$

for $\mathscr{E}_0 \gg 1$. The expressions (5.5)–(5.8) coincide with those obtained in Refs. 2 and 4, and are presented here for the same reason as expressions (3.5)–(3.8) were presented. Using (5.4), we calculate the longitudinal and transverse noise temperatures:

$$T_\perp{}^n = T[1+\mathscr{E}_0], \quad T_\parallel{}^n = T[1+1{,}4\mathscr{E}_0] \quad (\text{for } \mathscr{E}_0 \ll 1), \quad (5.9)$$

$$T_\perp{}^n = T\left(\frac{2\mathscr{E}_0}{\pi}\right)^{1/2}, \quad T_\parallel{}^n = 0{,}98T\left(\frac{2\mathscr{E}_0}{\pi}\right)^{1/2} \quad (\text{for } \mathscr{E}_0 \gg 1). \quad (5.10)$$

If $I(x_0) \ll 1$, then the convective contribution to (5.2) reduces to zero, and

$$\sigma_\perp(\omega)=\sigma_\parallel(\omega)=\frac{9}{8}\left(\frac{\pi}{x_0}\right)^{1/2}\sigma_a. \qquad (5.11)$$

Hence, the small-signal conductivity, as with the spectral density of current fluctuations, is isotropic. In addition, since here a second "ohmic" region occurs, the small-signal conductivity in the region of electric fields under discussion coincides with its static value. In this case the noise temperature is also isotropic and equals

$$T_\perp{}^n=T_\parallel{}^n=\frac{x_0}{4}T. \qquad (5.12)$$

In the intermediate-frequency region (2.25), the longitudinal and transverse components of the small-signal conductivity are equal and are determined as a function of the range of electric fields by Eqs. (5.5), (5.7) and (5.11), respectively, while the noise temperature here coincides with the transverse components in (5.9), (5.10) and (5.12).

In the high-frequency region (2.28) we obtain the following for $\sigma_{ik}(\omega)$ and T^n: if $I(x_0)\gg1$, then

$$\operatorname{Re}\sigma_\perp(\omega)=\operatorname{Re}\sigma_\parallel(\omega)=\frac{2\sigma_a}{\omega^2\tau_a{}^2(k_0T)}[1+0{,}4\mathscr{E}_0],$$

$$T_\perp{}^n=T_\parallel{}^n=T[1+0{,}5\mathscr{E}_0], \qquad (5.13)$$

for $\mathscr{E}_0\ll1$, and

$$\operatorname{Re}\sigma_\perp(\omega)=\operatorname{Re}\sigma_\parallel(\omega)=\frac{2\sigma_a}{\omega^2\tau_a{}^2(k_0T)}\,\frac{2^{3/4}\Gamma(5/4)}{\pi^{1/2}}\mathscr{E}_0{}^{1/4},$$

$$T_\perp{}^n=T_\parallel{}^n=\frac{T}{2}\left(\frac{\pi\mathscr{E}_0}{2}\right)^{1/2}, \qquad (5.14)$$

for $\mathscr{E}_0\gg1$; if $I(x_0)\ll1$, then

$$\operatorname{Re}\sigma_\perp(\omega)=\operatorname{Re}\sigma_\parallel(\omega)=\frac{2\sigma_a}{\omega^2\tau_a{}^2(k_0T)}\cdot\frac{9}{32}(\pi x_0)^{1/2},$$

$$T_\perp{}^n=T_\parallel{}^n=\frac{2}{9}x_0T. \qquad (5.15)$$

Momentum scattering by ionzied impurities

In the low-frequency region and for weak electric field ($\mathscr{E}_0'\ll1$, $I(x_0)\gg1$) we have for $\sigma_{ik}(\omega)$ and T^n

$$\sigma_\perp(\omega)=\sigma_I\left[1+\frac{33}{8}\mathscr{E}_0'\right], \qquad (5.16)$$

$$T_\perp{}^n=T[1+14\mathscr{E}_0'], \qquad (5.17)$$

$$\sigma_\parallel(\omega)=\sigma_I\left[1+\left(D_2+\frac{33}{8}\right)\mathscr{E}_0'\right],\quad D_2=\frac{33}{4}, \qquad (5.18)$$

$$T_\parallel{}^n=T_\perp{}^n=T[1+(D_1-D_2+14)\mathscr{E}_0']. \qquad (5.19)$$

In strong electric fields, when the condition $I(x_0)\ll1$ is satisfied, the transverse components of the tensor $\sigma_{ik}(\omega)$ and the noise temperature T^n equal

$$\sigma_\perp(\omega)=\sigma_I\frac{x_0}{8(\pi\mathscr{E}_0')^{1/2}}, \qquad (5.20)$$

$$T_\perp{}^n=\frac{x_0}{8}T. \qquad (5.21)$$

It is necessary to note the following with regard to the longitudinal components $\sigma_\parallel(\omega)$: as pointed out in the Appendix, in the electron-cooling regime [i.e., when condition (3.9) is fulfilled] the current-voltage characteristics have a saturation portion; because in the high-frequency region the small-signal conductivity coincides with the differential (DC) conductivity, the longitudinal component $\sigma_\parallel(\omega)$ vanishes in this case. In order to obtain a finite value of $\sigma_\parallel(\omega)$, it is necessary to include the small deviation of the current-voltage relation from saturation. The calculations show that before the onset of saturation the current-voltage characteristics pass through a maximum, and consequently a region with negative differential conductivity (NDC) ($\sigma_\parallel(\omega)<0$) can occur:

$$\sigma_\parallel(\omega)=-\sigma_I\frac{x_0}{8(\pi\mathscr{E}_0')^{1/2}}\left(\frac{2}{\pi(\mathscr{E}_0'x_0)^{1/2}}\right). \qquad (5.22)$$

The presence of a region with NDC can lead to unstable regimes; therefore to observe the effects discussed here it is necessary to take measures to suppress the development of this instability. In experiments this kind of NDC is not observed.[9] Obviously, this is related first of all to the smallness of the differential conductivty compared to the static conductivity ($|\sigma_\parallel(\omega)|\ll\sigma_\perp(\omega)$); and secondly to the fact that inclusion of, e.g., the penetration of electrons into the active region of energies $\varepsilon>\hbar\omega_0$ causes the NDC to disappear. If this penetration is small ($\Delta\varepsilon\ll\hbar\omega_0$), the $\sigma_\parallel(\omega)$ can be calculated within the same approximations as were used to solve the kinetic equation in Ref. 8. Using the distribution function (38) from Ref. 8, we obtain

$$\sigma_\parallel(\omega)=\sigma_I\frac{x_0}{8(\pi\mathscr{E}_0')^{1/2}}\left[B(\mathscr{E}_0')-\frac{2}{(\pi\mathscr{E}_0')^{1/2}}\right]\times\left[1-\frac{2}{(\pi\mathscr{E}_0')^{1/2}}-A(\mathscr{E}_0')\right]^{-2}, \qquad (5.23)$$

where

$$A(\mathscr{E}_0')=\frac{\alpha}{\pi}r_0^{2/5}\int_0^\infty\frac{K_{3/5}(u)}{r_0{}^{4/5}+u^{4/5}}du,$$

$$r_0=4/5(\mathscr{E}_0'x_0)^{5/4}z,\qquad \alpha=2^{7/5}/5\Gamma(3/5), \qquad (5.24)$$

$$B(\mathscr{E}_0')=\frac{\alpha}{\pi}\cdot\frac{7}{5}r_0{}^{2/5}\int_0^\infty\frac{r_0{}^{4/5}-u^{4/5}}{r_0{}^{4/5}+u^{4/5}}K_{3/5}(u)\,du,$$

$$z=\left[\mathscr{E}_0'\frac{\tau_{op}(\hbar\omega_0)}{\tau_\varepsilon(\hbar\omega_0)}\right]^{1/2}. \qquad (5.25)$$

Here $K_\nu(u)$ is the modified Bessel function of the second kind. The function $B(\mathscr{E}_0')$ is positive both for $r_0\gg1$ and $r_0\ll1$. Therefore, inclusion of the penetration of electrons into the active region of energies decreases the NDC, and starting with electric fields which satisfy the condition $r_0<1/z_0{}^{1/2}$, the differential conductivity becomes positive; to within a numerical factor of order unity it is determined by the expression

$$\sigma_\parallel(\omega)\approx\sigma_I\frac{x_0}{8(\pi\mathscr{E}_0')^{1/2}}r_0{}^{-2/5}. \qquad (5.26)$$

Consequently, in this case $\sigma_\parallel(\omega)\sim E^{2/5}$. The condition that few electrons penetrate into the active region of energies is determined by the inequality $z \ll 1$, while the departure from the electron-cooling regime occurs in electric fields at which $r_0 \ll 1$.

In the range of intermediate frequencies both the longitudinal and transverse conductivities $\sigma_{ik}(\omega)$ and T^n are determined by Eqs. (5.16), (5.17), (5.20), and (5.21). In the high-frequency region (4.12) both components of the tensor $\sigma_{ik}(\omega)$ and T^n are respectively equal to

$$\operatorname{Re}\sigma_{\perp}(\omega)=\operatorname{Re}\sigma_{\parallel}(\omega)=\sigma_I\cdot\frac{1}{6}\left[1+\frac{15}{8}\mathcal{E}_0'\right]\frac{1}{\omega^2\tau_I^2(k_0T)},$$
$$T_\perp^n=T_\parallel^n=T[1+\mathcal{E}_0'] \qquad (5.27)$$

for $\mathcal{E}_0' \ll 1$, $I(x_0) \gg 1$, and

$$\operatorname{Re}\sigma_{\perp}(\omega)=\operatorname{Re}\sigma_{\parallel}(\omega)=\sigma_I\frac{(\pi\mathcal{E}_0')^{1/2}}{12\sqrt{3}}\frac{1}{[\omega^2\tau_I^2(k_0T)]^{3/4}} \qquad (5.28)$$

for $I(x_0) \ll 1$.

It follows from (4.13) and f(5.28) that the current-fluctuations spectral density and the small-signal conductivity in the case under discussion have different frequency dependences. In light of (5.4), this leads to a situation in which the high-frequency noise temperature depends explicitly on frequency;

$$T_\perp^n=T_\parallel^n=T\frac{4\sqrt{3}}{\pi}\frac{1}{[\omega^2\tau_I^2(k_0T)]^{1/4}}\ln\left[\frac{\omega^2\tau_I^2(k_0T)}{\mathcal{E}_0'}\right]. \qquad (5.29)$$

In addition, T^n decreases with increasing electric field in this case.

6. CONCLUSION

The analysis carried out in this paper of the effect of interactions between electrons and optical phonons on nonequilibrium current fluctuations in a semiconductor in strong electric fields shows that inclusion of this interaction substantially influences the spectral and field characteristics of these fluctuations. In Figs. 1 and 2, we present the dependences of (A) the current fluctuation spectral density [here $\hat{S}(\omega,E) = (\delta j_i\delta j_k)_\omega(E)/(\delta j_i \delta j_k)_\omega(0)$], (B) the small-signal conductivity [here $\hat{\Sigma}(\omega,E) = \sigma_{ik}(\omega,E)/\sigma_{ik}(\omega,0)$] and (C) the noise temperature T^n as a function of electric field in the low-frequency limit (2.11).

In pure semiconductors (Fig. 1) the field dependences of all the above-mentioned quantities approach saturation, while their longitudinal ($\parallel$) and transverse ($\perp$) components become equal to each other, which is explained by the vanishign of the convective portion in these quantities. This also leads to the disappearance of the dispersion of the longitudinal components $S_\parallel(\omega,E)$, $\Sigma_\parallel(\omega,E)$ and $T_\parallel^n$ at frequencies $\omega \sim (\tau_\varepsilon^*)^{-1}$. Another interesting circumstance is the fact that the Wannier-Robson relation, which asserts that

$$(\delta j_i \delta j_k)_\omega \approx {}^2/_3\bar{\varepsilon}\sigma_{ik}(\omega)$$

is fulfilled to good accuracy in this case[4,11] (despite the fact that it was proposed for quasielastic scattering); (in practice, this is an assertion that the noise temperature is isotropic). In the present case $\varepsilon = 9/25\,\hbar\omega_0$ (see Ref. 8) and

$$k_0T^n={}^{25}/_{24}({}^2/_3\bar{\varepsilon})\approx{}^2/_3\bar{\varepsilon}.$$

In doped semiconductors (Fig. 2), $S(\omega,E)$ and $\Sigma(\omega,E)$ are nonmonotonic functions of electric field, and in the range of interest to us they decrease with increasing E. In addition, a change of sign occurs in the anisotropic small-signal conductivity. Here the Wannier-Robson relation is

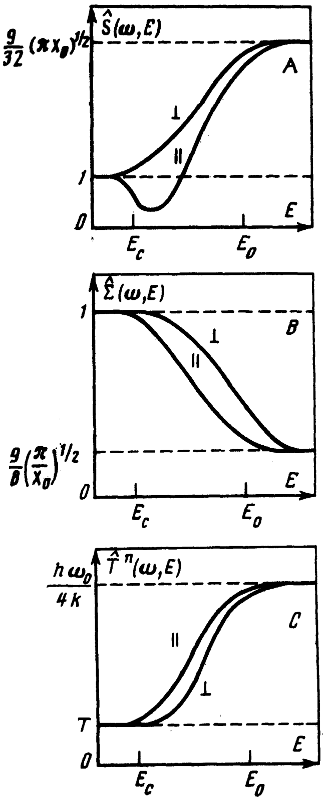

FIG. 1.

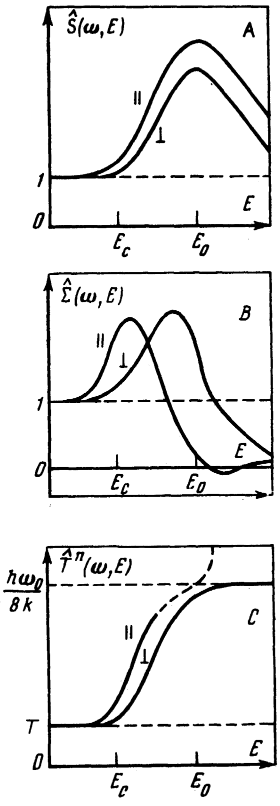

FIG. 2.

violated in a cardinal way; its right and left sides have different field dependences. In the high-frequency limit the spectra of $(\delta j_i \delta j_k)_\omega$ and $\sigma_{ik}(\omega)$ are different and have a non-Lorentzian form, while the noise temperature depends explicitly on frequency and decreases with increasing E.

In conclusion, we note that the distinctive features of the fluctuation phenomena discussed here should be experimentally observable under the same conditions as those in which the kinetic effects are investigated, e.g., in Ref. 9 for various semiconductors (p-Ge, n-InSb, n-GaAs, and n-InP), whose parameters make possible the attainment of the required values of field. In addition, semiconductors of the type AgBr, AgCl, which possess strong coupling to optical phonons, may turn out to be appropriate. In the paper by Bareikis *et al.*, reprinted in Ref. 12, both experimental and numerical (Monte Carlo) results are presented for noise studies in p-Ge and n-InSb under streaming conditions. The good agreement of these results with models[3] allow us at least to postulate that even in electric fields weaker that those investigated in Ref. 12 and determined by the condition (1.4), the required regime will be realized for which the basic role in the energy relaxation of electrons is played by the interaction with optical phonons.

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