Phonon-pumped nonequilibrium states in superfluid ^{3}He

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We consider stationary nonequilibrium states induced in the superfluid phases of ^{3}He by phonon pumping at frequencies lower than the average pair-binding energy and far from the resonances of the collective triplet-condensate modes. The kinematic exclusion of the phonon single-particle absorption channel in He-3 interfers with the simulation mechanism known from superconductivity physics. Superfluidity can nonetheless be stimulated by phonon-absorption events in which several Fermi excitations participate. The calculations are performed for both the *B* and *A* phases of He-3. For the latter, account is taken of "competing" production of excess excitations by phonons via direct condensate-pair breaking. The stimulation is found to predominate at low frequencies, so that phonon emission can induce an *A–B* phase transition.

1. INTRODUCTION

The superfluidity of helium-3 is akin to electron superconductivity in metals, but stimulation of the order parameter in ^{3}He has been so far neither explained theoretically nor studied experimentally. Such a study is hindered by certain fundamental differences between superfluid ^{3}He and superconductors. These interfere with direct use of the known results of superconductivity physics for analyzing the situation in helium-3, and have prompted our present study.

The stimulation effect is strongest in a superconductor acted upon by a high-frequency electromagentic field.[1,2] For a neutral Fermi liquid such as ^{3}He, the analog of electromagnetic pumping can be pumping by sound. "Sound" means here high-frequency or "zero" sound of frequency ω_q higher than the Fermi-excitation damping rate; these sound quanta can be regarded as longitudinal phonons subject to acoustic dispersion.[3] Gap stimulation in acoustic pumping of a superconductor was considered in Ref. 4. Why is this theory not applicable to superfluid ^{3}He? The point is that the zero-sound velocity in ^{3}He is significantly higher than the Fermi velocity, and this closes the single-particle relaxation channel for phonon absorption.[1)] Yet it is precisely through this channel that the usual stimulation proceeds in superconductors (for phonon–electron interactions).[2]

Bearing the foregoing in mind, we pay principal attention here to the mechanism of phonon absorption in superfluid ^{3}He when several Fermi excitations take part. It is just this mechanism which predominates in the normal state.[3] There are two more substantial sound-absorption channels-Cooper-pair breaking and buildup of collective triplet-condensate modes in ^{3}He with superfluid ordering. At frequencies ω_q comparable with the pair binding energy both absorption mechanisms can predominate over the multiparticle one.[6] Under certain circumstances, however, the multiparticle channel prevails. Thus, collective-mode excitation is resonant. Far from resonances we can disregard phonon absorption by collective modes (study of this mechanism as a source of the departure from equilibrium would in itself be of considerable interest). As for absorption with pair breaking, it has a threshold in the pseudo-isotropic (B) phase and is absent for $\omega_q < 2|\Delta|$, where $|\Delta|$ is the gap in the single-particle-excitation spectrum. In the axial (A) phase this threshold is indistinct (since the gap has "punctures" in momentum space), and the mechanism of absorption with pair breaking must be analyzed on a par with the multiparticle mechanism.

To study the stimulation effect in superfluid ^{3}He it is necessary to obtain first an expression for the source of the departure from equilibrium in a system of single-particle excitations. It is convenient to do this by using a quantum description of the high-frequency sound field. In the kinetic equation for single-particle excitations, the operator for fermion collisions with external-field quanta is given by [2)]

$$J = \frac{1}{8\pi}\,\mathrm{Sp}\,\{\hat g\hat\Sigma^A - \hat\Sigma^R\hat g + \hat g^R\hat\Sigma - \hat\Sigma\hat g^A\}, \tag{1}$$

where

$$\hat g_{\alpha\beta} = \begin{pmatrix} g_{\alpha\beta} & f_{\alpha\beta} \\ -f_{\alpha\beta}^+ & \bar g_{\alpha\beta} \end{pmatrix}$$

is a propagator describing the superfluid state of the Fermi liquid.[8–11] The trace in (1) is taken over the spin indices α and β. The self-energy matrix

$$\hat\Sigma_{\alpha\beta} = \begin{pmatrix} \Sigma_{1,\alpha\beta} & \Sigma_{2,\alpha\beta} \\ -\Sigma^+_{2,\alpha\beta} & \bar\Sigma_{1,\alpha\beta} \end{pmatrix}, \tag{2}$$

which corresponds to fermion scattering by phonons, contains the phonon Green's function, which we express in the form

$$D_\omega(\mathbf{q}) = (1+2N_{\omega_\mathbf{q}})\,\mathrm{sign}\,\omega\,(D^R-D^A)_\omega,$$
$$D^{R(A)} = \frac{2\omega_\mathbf{q}}{\omega_\mathbf{q}^2-(\omega\pm i\delta)^2}, \tag{3}$$

bearing in mind that the average phonon occupation numbers $N_{\omega_\mathbf{q}}$ are determined in this equation by the electric field. The functions $N_{\omega_\mathbf{q}}$ and $D_\omega(\mathbf{q})$ specify the reason for the equilibrium departure from in fermion–boson collisions. The subsequent transition to the classical limit for Bose fields can be made in (1) and (3), as usual, by putting

$$N_{\omega_\mathbf{q}} \gg 1. \tag{4}$$

We derive in Sec. 2 an expression for the collision integral in a multiparticle channel for phonon-emission absorption. The cause of the departure from equilibrium in ^{3}He-B is discussed in Sec. 3. The stationary states resulting from

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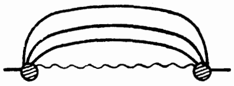
phonon pumping are considered in Sec. 4, where the possibility of stimulating the order parameter in a pseudo-isotropic phase is demonstrated. An analysis of the same process in the axial phase, with allowance for direct production of excess excitations when a Cooper pair is broken by a phonon is carried out in Sec. 5.

2. ZERO SOUND AS A SOURCE OF DISEQUILIBRIUM

To calculate the self-energy expressions in (1), which correspond to fermion pair collisions with participation of a phonon, we use the techniques of analytic continuation of diagrams drawn in the discrete-imaginary frequency representation. The graphs we need are of the form


(5)

By running through the possible intersections in (5) with four lines and taking into account in them the dependence on the imaginary frequencies, on the superfluid ordering parameters, and on the phonon field, we obtain the diagrams of Fig. 1. The presence of a superfluid condensate in the system leads to the matrix structure of $\hat{\Sigma}$ [see (2)]. The contribution made to Σ_1 by the first diagrams of Fig. 1 is shown in Fig. 2. In the discrete-imaginary-frequency representation

$$\varepsilon_n = i\pi T(2n+1)$$

we can write for the elements of the matrix $\hat{\Sigma}$ (we omit the spin indices, which are still immaterial)

$$\begin{aligned}\hat{\Sigma}_1(P,P-K) &= T^3\sum_{\varepsilon_1\varepsilon_2\varepsilon_3}\iiint\frac{d^3\mathbf{p}_1\,d^3\mathbf{p}_2\,d^3\mathbf{p}_3}{(2\pi)^9}\\&\quad\times\{XG_1G_2G_3D_4-YG_1F_2F_3^+D_4\},\\ \hat{\Sigma}_2(P,P-K) &= T^3\sum_{\varepsilon_1\varepsilon_2\varepsilon_3}\iiint\frac{d^3\mathbf{p}_1\,d^3\mathbf{p}_2\,d^3\mathbf{p}_3}{(2\pi)^9}\\&\quad\times\{XF_1F_2^+F_3D_4-YF_1G_2\bar{G}_3D_4\}.\end{aligned} \tag{6}$$

The arguments in (6) were chosen in symmetric form, and then the 4-momenta of the phonon propagator are defined as $P_4 = P - P_1 - P_2 - P_3$ and $K_4 = K - K_1 - K_2 - K_3$. The quadratic forms for the amplitudes X and Y are

$$\begin{aligned}X &= (W_{\mathbf{p}-\mathbf{p}_1}+W_{\mathbf{p}_2+\mathbf{p}_3})(W_{\mathbf{p}-\mathbf{p}_3}+W_{\mathbf{p}_1+\mathbf{p}_2})-|W_{\mathbf{p}-\mathbf{p}_1}|^2-|W_{\mathbf{p}_2+\mathbf{p}_3}|^2\\&\quad-W_{\mathbf{p}-\mathbf{p}_1}W_{-\mathbf{p}_2-\mathbf{p}_3}-W_{\mathbf{p}_1-\mathbf{p}}W_{\mathbf{p}_2+\mathbf{p}_3},\\ Y &= |W_{\mathbf{p}-\mathbf{p}_1}|^2+|W_{\mathbf{p}_2+\mathbf{p}_3}|^2+W_{\mathbf{p}-\mathbf{p}_1}W_{-\mathbf{p}_2-\mathbf{p}_3}\\&\quad+(W_{\mathbf{p}-\mathbf{p}_2}+W_{\mathbf{p}_1+\mathbf{p}_3})(W_{\mathbf{p}-\mathbf{p}_3}+W_{\mathbf{p}_1+\mathbf{p}_2})\\&\quad+W_{\mathbf{p}_1-\mathbf{p}}W_{\mathbf{p}_2+\mathbf{p}_3}-(W_{\mathbf{p}-\mathbf{p}_1}+W_{\mathbf{p}_2+\mathbf{p}_3})\\&\quad\times(W_{\mathbf{p}-\mathbf{p}_2}+W_{\mathbf{p}-\mathbf{p}_3}+W_{\mathbf{p}_1+\mathbf{p}_2}+W_{\mathbf{p}_1+\mathbf{p}_3}).\end{aligned} \tag{7}$$

In Eqs. (7) $W_{\mathbf{q}}$ is the effective fermion–fermion collision potential defined with account taken of the single-phonon processes.

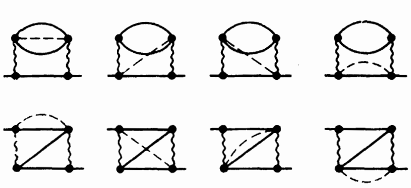


FIG. 1. Diagrams corresponding to the self-energy part (1). A wavy line corresponds to a fermion–fermion interaction potential, and a dashed line to a phonon propagator.

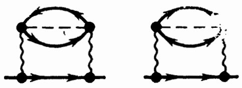


FIG. 2. Diagrams, corresponding to the first skeleton diagram of Fig. 1, for the Σ_1 component of the self-energy matrix $\hat{\Sigma}$.

We transform now from expressions (6), written in the discrete-imaginary-frequency representation, to expressions on the real axis using the Gor'kov–Eliashberg technique.[12] To this end we consider the Nth order perturbation-theory diagram as a function of the complex variable ε for fixed imaginary frequencies of the field vertices. For each frequency the analytic continuation should be from the upper half-plane to the real axis. It can therefore be assumed that to this diagram there correspond cuts located between the lines $\mathrm{Im}\,\varepsilon = 0$ and $\mathrm{Im}(\varepsilon-\omega) = 0$ in the upper half-plane. Setting the Green's functions in (6) in correspondence with the sets of cuts

$$\mathrm{Im}(\varepsilon_1-\omega_{1i})=0,\ \mathrm{Im}(\varepsilon_2-\omega_{2k})=0,$$
$$\mathrm{Im}(\varepsilon_3-\omega_{3l})=0,\ \mathrm{Im}(\omega_4-\omega_{4m})=0,$$

we transform the sum over the frequencies into a triple integral. Since the direction of the arrows does not influence the analytic continuation, we rewrite (6), leaving out the irrelevant symbols, in the form

$$\begin{aligned}\Sigma &= T^3\sum_{\varepsilon_1\varepsilon_2\varepsilon_3}G_{\varepsilon_1}G_{\varepsilon_2}G_{\varepsilon_3}D_{\varepsilon-\varepsilon_1-\varepsilon_2-\varepsilon_3}\\&=\oint\oint\oint\frac{d\varepsilon_1\,d\varepsilon_2\,d\varepsilon_3}{(4\pi i)^3}\,\mathrm{th}\frac{\varepsilon_1}{2T}\,\mathrm{th}\frac{\varepsilon_2}{2T}\,\mathrm{th}\frac{\varepsilon_3}{2T}\,G_{\varepsilon_1}G_{\varepsilon_2}G_{\varepsilon_3}D_{\varepsilon-\varepsilon_1-\varepsilon_2-\varepsilon_3},\end{aligned} \tag{8}$$

where the integration contours enclose the poles of the hyperbolic tangents. Further stage-by-stage transformation of (8) into integration along the real axis leads to

$$\begin{aligned}\Sigma = \iiint\frac{dz_1\,dz_2\,dz_3}{(4\pi i)^3}&\Bigg\{\sum_{i,k,l}\delta_i(G_{z_1+\omega_{1i}})\,\delta_k(G_{z_2+\omega_{2k}})\,\delta_l(G_{z_3+\omega_{3l}})\\&\times D_{\varepsilon-z_1-z_2-z_3-\omega_{1i}-\omega_{2k}-\omega_{3l}}\,\mathrm{th}\frac{z_1}{2T}\,\mathrm{th}\frac{z_2}{2T}\\&\times\mathrm{th}\frac{z_3}{2T}-\sum_{i,l,m}\delta_i(G_{z_1+\omega_{1i}})\\&\times G_{\varepsilon+z_2-z_1-\omega_{1i}-\omega_{3l}-\omega_{4m}}\Big[\delta_l(G_{z_3+\omega_{3l}})\,\delta_m(D_{-z_2-z_3+\omega_{4m}})\,\mathrm{th}\frac{z_4}{2T}\\&-\delta_l(G_{z_3-z_1+\omega_{3l}})\,\delta_m(D_{-z_3+\omega_{4m}})\,\mathrm{cth}\frac{z_3}{2T}\Big]\,\mathrm{th}\frac{z_1}{2T}\,\mathrm{th}\frac{z_2}{2T}\\&-\sum_{k,l,m}G_{\varepsilon+z_1-\omega_{2k}-\omega_{3l}-\omega_{4m}}\delta_k(G_{z_2+\omega_{2k}})\\&\times\Big[\delta_l(G_{z_3+\omega_{3l}})\,\delta_m(D_{-z_1-z_2-z_3+\omega_{4m}})\\&\times\mathrm{th}\frac{z_3}{2T}-\delta_l(G_{z_3-z_1-z_2+\omega_{3l}})\\&\times\delta_m(D_{-z_3+\omega_{4m}})\,\mathrm{cth}\frac{z_2}{2T}\Big]\,\mathrm{cth}\frac{z_1}{2T}\,\mathrm{th}\frac{z_2}{2T}\\&+\sum_{k,l,m}G_{\varepsilon+z_1-\omega_{2k}-\omega_{3l}-\omega_{4m}}\delta_k(G_{z_2-z_1+\omega_{2k}})\\&\times\Big[\delta_l(G_{z_3+\omega_{3l}})\,\delta_m(D_{-z_3-z_1+\omega_{4m}})\\&\times\mathrm{th}\frac{z_3}{2T}-\delta_l(G_{z_3-z_2+\omega_{3l}})\\&\times\delta_m(D_{-z_3+\omega_{4m}})\,\mathrm{cth}\frac{z_3}{2T}\Big]\,\mathrm{cth}\frac{z_1}{2T}\,\mathrm{th}\frac{z_2}{2T}\Bigg\}.\end{aligned} \tag{9}$$

The external variable ε and the field frequencies have remained imaginary here. Continuing (9) with respect to ε from the regions $\mathrm{Im}(\varepsilon-\omega)>0(\mathrm{Im}\varepsilon<0)$ and then with respect to all the frequencies from the upper half-plane, we obtain for $\Sigma^{R(A)}$ the expression

$$\Sigma^{R(A)}=\iiint_{-\infty}^{\infty}\frac{d\varepsilon_1\,d\varepsilon_2\,d\varepsilon_3}{(4\pi i)^3}$$
$$\times\{G_1^{R(A)}(G^R-G^A)_2(G^R-G^A)_3D_4-G_1G_2G_3D^{R(A)}$$
$$+G_1^{R(A)}(G^R-G^A)_2G_3(D^R-D^A)_4$$
$$+G_1^{R(A)}G_2(G^R-G^A)_3(D^R-D^A)_4$$
$$+G_1G_2^{R(A)}(G^R-G^A)_3(D^R-D^A)_4$$
$$+G_1^{R(A)}G_2G_3D_4+G_1G_2^{R(A)}G_3D_4+G_1G_2G_3^{R(A)}D_4\}. \quad (10)$$

Using for Σ the definition

$$\Sigma_{\varepsilon,\varepsilon-\omega}=\sum_{N=0}^{\infty}\sum_k\delta_k(\Sigma^{(N)}_{\varepsilon,\varepsilon-\omega})\,\mathrm{th}\frac{\varepsilon-\Omega_k}{2T}, \quad (11)$$

where Ω_k is a certain combination of field-vertex frequencies, we get from (9)

$$\Sigma=\iiint_{-\infty}^{\infty}\frac{d\varepsilon_1\,d\varepsilon_2\,d\varepsilon_3}{(4\pi i)^3}$$
$$\times\{(G^R-G^A)_1(G^R-G^A)_2(G^R-G^A)_3(D^R-D^A)_4$$
$$+G_1G_2G_3D_4+G_1G_2(G^R-G^A)_3(D^R-D^A)_4$$
$$+G_1(G^R-G^A)_2G_3(D^R-D^A)_4$$
$$+G_1(G^R-G^A)_2(G^R-G^A)_3D_4+(G^R-G^A)_1G_2(G^R-G^A)_3D_4$$
$$+(G^R-G^A)_1(G^R-G^A)_2G_3D_4+(G^R-G^A)_1G_2G_3(D^R-D^A)_4\}. \quad (12)$$

The expressions for the quantities $\Sigma^{R(A)}$ and Σ that determine the collision integral follow from (6) when account is taken of (10) and (12). Before presenting the results, we integrate in (6) over the variables $\xi=v_F(p-p_F)$; this procedure is allowed because of the short-range character of the effective interaction, which permits the amplitudes X and Y to be regarded as dependent only on the angles. We can therefore write[3)]

$$\iiint\frac{d^3\mathbf{p}_1\,d^3\mathbf{p}_2\,d^3\mathbf{p}_3}{(2\pi)^9}XG_1G_2G_3D_4=\left(\frac{mp_F}{2\pi^2}\right)^3\iiint\frac{dO_{\mathbf{p}_1}dO_{\mathbf{p}_2}dO_{\mathbf{p}_3}}{(4\pi)^3}$$
$$\times XD_4\iiint d\xi_1\,d\xi_2\,d\xi_3G_1G_2G_3$$
$$=\left(\frac{mp_F}{2\pi^2}\right)^3\iiint\frac{dO_{\mathbf{p}_1}dO_{\mathbf{p}_2}dO_{\mathbf{p}_3}}{(4\pi)^3}Xg_1g_2g_3D_4, \quad (13)$$

where g is the quasiclassical Green's function.[8] Defining the operator $\hat{L}$ as

$$\hat{L}=\left(\frac{mp_F}{2\pi^2}\right)^3\iiint\frac{d\varepsilon_1\,d\varepsilon_2\,d\varepsilon_3}{(4\pi i)^3}\iiint\frac{dO_{\mathbf{p}_1}dO_{\mathbf{p}_2}dO_{\mathbf{p}_3}}{(4\pi)^3} \quad (14)$$

and restoring the previously omitted symbols, we represent the results in the form

$$\Sigma_1^{R(A)}=\hat{L}[X\{g_1\bar{g}_2g_3D_4\}_1-Y\{g_1f_2f_3^+D_4\}_1],$$
$$\Sigma_1=\hat{L}[X\{g_1\bar{g}_2g_3D_4\}_2-Y(g_1f_2f_3^+D_4\}_2],$$
$$\Sigma_2^{R(A)}=\hat{L}[X\{f_1f_2^+f_3D_4\}_1-Y\{f_1g_2\bar{g}_3D_4\}_1],$$
$$\Sigma_2=\hat{L}[X\{f_1f_2^+f_3D_4\}_2-Y\{f_1g_2\bar{g}_3D_4\}_2]. \quad (15)$$

Substituting (15) in (1) and using the factorization of the Green's functions of the superfluid state

$$g_{\alpha\beta}=2\pi i\frac{\varepsilon\theta(\varepsilon^2-|\Delta_\mathbf{p}|^2)}{(\varepsilon^2-|\Delta_\mathbf{p}|^2)^{1/2}}(1-2n_\varepsilon)\delta_{\alpha\beta},$$
$$f_{\alpha\beta}=2\pi i\frac{\theta(\varepsilon^2-|\Delta_\mathbf{p}|^2)}{(\varepsilon^2-|\Delta_\mathbf{p}|^2)^{1/2}}(1-2n_\varepsilon)(\Delta_\mathbf{p})_{\alpha\beta}, \quad (16)$$
$$(\Delta_\mathbf{p})_{\alpha\beta}=\Delta_\mathbf{p}\{i(\boldsymbol{\sigma}\sigma^{(y)})_{\alpha\beta}\mathbf{d}\},$$

$$g_{\alpha\beta}^{R(A)}=i\pi\frac{\varepsilon}{\xi_\varepsilon^{R(A)}}\delta_{\alpha\beta},\qquad f_{\alpha\beta}^{R(A)}=i\pi\frac{(\Delta_\mathbf{p})_{\alpha\beta}}{\xi_\varepsilon^{R(A)}};$$

$$\xi_\varepsilon^R=-(\xi_\varepsilon^A)^*=\begin{cases}(\varepsilon^2-|\Delta_\mathbf{p}|^2)^{1/2}\,\mathrm{sign}\,\varepsilon+i\delta, & \varepsilon^2>|\Delta_\mathbf{p}|^2\\ i(|\Delta_\mathbf{p}|^2-\varepsilon^2)^{1/2}, & \varepsilon^2<|\Delta_\mathbf{p}|^2\end{cases} \quad (17)$$

(here n_ε is the nonequilibrium distribution function of the single-particle excitations, $|\Delta|=|\Delta_\mathbf{p}|$ is a (generally speaking) anisotropic gap in their spectrum, and $\mathbf{d}$ is the anisotropy vector) we get the collision integral that will serve. as stipulated in Sec. 1, as the source of single-particle excitations in the fermion system.

3. PROPERTIES OF THE SOURCE OF THE DEPARTURE FROM EQUILIBRIUM

In the pseudo-isotropic B phase the spectrum of the single-particle excitations is isotropic with a gap $|\Delta_\mathbf{p}|$, which is independent of the direction of the momentum $\mathbf{p}$. The anisotropy vector $\mathbf{d}$ in the B phase can be chosen in the form $\mathbf{d}=\mathbf{p}/p_F$. The collision integral is then

$$J^{(f\text{-}f\text{-}ph)}(n_\varepsilon)=-8\pi\left(\frac{mp_F}{2\pi^2}\right)^3\int\frac{d^3\mathbf{q}}{(2\pi)^3}N_{\omega_\mathbf{q}}\iiint\frac{dO_{\mathbf{p}_1}dO_{\mathbf{p}_2}dO_{\mathbf{p}_3}}{(4\pi)^3}$$
$$\times\iiint_{|\Delta|}^{\infty}d\varepsilon_1\,d\varepsilon_2\,d\varepsilon_3\,[M_1\delta(\mathbf{p}-\mathbf{p}_1-\mathbf{p}_2-\mathbf{p}_3-\mathbf{q})$$
$$\times\{[(1-n)\,n_1n_2n_3N_{\omega_\mathbf{q}}$$
$$-n(1-n_1)(1-n_2)(1-n_3)(N_{\omega_\mathbf{q}}+1)]$$
$$\delta(\varepsilon-\varepsilon_1-\varepsilon_2-\varepsilon_3-\omega_\mathbf{q})$$
$$+[(1-n)\,n_1n_2n_3(N_{\omega_\mathbf{q}}+1)$$
$$-n(1-n_1)(1-n_2)(1-n_3)N_{\omega_\mathbf{q}}]$$
$$\times\delta(\varepsilon-\varepsilon_1-\varepsilon_2-\varepsilon_3+\omega_\mathbf{q})\}$$
$$+M_2\delta(\mathbf{p}+\mathbf{p}_1-\mathbf{p}_2-\mathbf{p}_3-\mathbf{q})\{[(1-n)(1-n_1)$$
$$\times n_2n_3N_{\omega_\mathbf{q}}-nn_1(1-n_2)(1-n_3)$$
$$\times(1+N_{\omega_\mathbf{q}})]\delta(\varepsilon+\varepsilon_1-\varepsilon_2-\varepsilon_3-\omega_\mathbf{q})$$
$$+[(1-n)(1-n_1)\,n_2n_3(N_{\omega_\mathbf{q}}+1)$$
$$-nn_1(1-n_2)(1-n_3)N_{\omega_\mathbf{q}}]$$
$$\times\delta(\varepsilon+\varepsilon_1-\varepsilon_2-\varepsilon_3+\omega_\mathbf{q})\}$$
$$+M_3\delta(\mathbf{p}-\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3-\mathbf{q})$$
$$\times\{[(1-n)(1-n_2)(1-n_3)\,n_1N_{\omega_\mathbf{q}}$$
$$-nn_2n_3(1-n_1)(N_{\omega_\mathbf{q}}+1)]$$
$$\times\delta(\varepsilon-\varepsilon_1+\varepsilon_2+\varepsilon_3-\omega_\mathbf{q})$$
$$+[(1-n)(1-n_2)(1-n_3)\,n_1(N_{\omega_\mathbf{q}}+1)$$
$$-nn_2n_3(1-n_1)N_{\omega_\mathbf{q}}]\delta(\varepsilon-\varepsilon_1+\varepsilon_2+\varepsilon_3+\omega_\mathbf{q})\}$$
$$+M_4\delta(\mathbf{p}+\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3-\mathbf{q})[(1-n)$$
$$\times(1-n_1)(1-n_2)(1-n_3)N_{\omega_\mathbf{q}}$$
$$-nn_1n_2n_3(N_{\omega_\mathbf{q}}+1)]\delta(\varepsilon+\varepsilon_1+\varepsilon_2+\varepsilon_3-\omega_\mathbf{q})]. \quad (18)$$

The factor M_1 in (18), which is a combination of interaction potentials, state densities, and coherence factors, is equal to

$$M_1(P, P_1, P_2, P_3) = X_1 uu_1u_2u_3 + X_2 vv_1v_2v_3 + Y_1 vv_1u_2u_3 + Y_2 uu_1v_2v_3, \quad (19)$$

where

$$X_1 = X,\ X_2 = [(\mathbf{p}_1\mathbf{p}_2)(\mathbf{p}_2\mathbf{p}_3) - (\mathbf{p}\mathbf{p}_2)(\mathbf{p}_1\mathbf{p}_3) + (\mathbf{p}\mathbf{p}_3)(\mathbf{p}_1\mathbf{p}_2)]X/p_F^4,$$
$$Y_1 = (\mathbf{p}\mathbf{p}_1)Y/p_F^2,\ Y_2 = (\mathbf{p}_2\mathbf{p}_3)Y/p_F^2, \quad (20)$$
$$u_\varepsilon = \frac{|\varepsilon|\theta(\varepsilon^2 - |\Delta_\mathbf{p}|^2)}{(\varepsilon^2 - |\Delta_\mathbf{p}|^2)^{1/2}}, \quad v_\varepsilon = \frac{|\Delta_\mathbf{p}|\theta(\varepsilon^2 - |\Delta_\mathbf{p}|^2)}{(\varepsilon^2 - |\Delta_\mathbf{p}|^2)^{1/2}} \operatorname{sign} \varepsilon.$$

The expressions for the other factors M_{2-4} are similar to (19), where the momentum and frequency variables are arranged in accordance with the following rules:

$$M_2 = M_1(P, -P_1, -P_2, P_3) + M_1(P, P_2, -P_1, P_3) + M_1(P, P_3, P_2, -P_1),$$
$$M_3 = M_1(P, P_1, -P_2, -P_3) + M_1(P, -P_2, P_1, -P_3) + M_1(-P, -P_3, -P_2, P_1), \quad (21)$$
$$M_4 = M_1(P, -P_1, -P_2, -P_3),\ P = (\mathbf{p}, \varepsilon).$$

We assume next that $\omega_\mathbf{q} \ll |\Delta|$. It can be concluded in this case that the $J^{(f\text{-}f\text{-}ph)}$ terms proportional to the factor M_1 make up the distribution function n_ε for $\varepsilon \geqslant 3|\Delta|$. The terms proportional to M_2 and M_3 cause the distribution function to depart from equilibrium for $\varepsilon \geqslant |\Delta|$. Participating in the elementary events described by the terms with M_3 are three excitations with energies $\varepsilon \geqslant |\Delta|$ and one excitation with $\varepsilon \geqslant 3|\Delta|$. The processes described by the terms proportional to M_2 correspond to (four) particles with energies $\varepsilon \geqslant |\Delta|$. No terms with M_4 appear for $\omega_\mathbf{q} < 4|\Delta|$.

Assuming monochromatic phonon emission, we define the wave vector of the external phonon field

$$N_{\omega_\mathbf{q}} = N_{\omega_q^0}\delta(\mathbf{q} - \mathbf{q}_0). \quad (22)$$

Since we are interested in the action on ^{3}He by radiation of frequency $\omega_q^0 \ll |\Delta|$, we can assume that the dominant contribution to the distribution function n_ε is made by that part of the collision integral (18) which is proportional to the factor M_2. Retaining in (18) the group of terms that make the main contribution, we express the source of the departure from equilibrium)

$$Q(n_\varepsilon) \equiv J^{(f-f-ph)}(n_\varepsilon)$$

under condition (3) in the form

$$Q(n_\varepsilon) \approx -\frac{8\pi}{\varepsilon_F}\frac{N_{\omega_{\mathbf{q}}^0}}{p_F^3} \times \iiint_{|\Delta|}^{\infty} d\varepsilon_1\, d\varepsilon_2\, d\varepsilon_3 m_{\mathbf{q}_0} [\delta(\varepsilon + \varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \omega_q^0) + \delta(\varepsilon + \varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \omega_q^0)]\{(1-n) \times (1-n_1)n_2n_3 - nn_1(1-n_2)(1-n_3)\}, \quad (23)$$

where

$$m_{\mathbf{q}_0} = \{3x_1uu_1u_2u_3 - 3x_2vv_1v_2v_3 - y_1(vv_1u_2u_3 - 2vu_1u_2v_3) + y_2(uu_1v_2v_3 - 2uv_1u_2v_3)\}, \quad (24)$$

with the quantities x_i and y_i in (24) defined by

$$x_i(y_i) = \left(\frac{mp_F}{2\pi^2}\right)^2 \iiint \frac{dO_{\mathbf{p}_1}\, dO_{\mathbf{p}_2}\, dO_{\mathbf{p}_3}}{(4\pi)^3} \times X_i(Y_i)\,\delta\left(1 - \frac{\mathbf{p}(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \mathbf{q}_0)}{p|\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \mathbf{q}_0|}\right).$$

In the last expression the δ function restricts the integration over the angles. Noting that the momentum of an (external) phonon of frequency $\omega_q^0 \ll |\Delta| \ll \varepsilon_F$ is much smaller than $\mathbf{p}$, and leaving $\mathbf{q}_0$ out of the argument of the δ function, then the quantities x_i (y_i) for the B phase of ^{3}He become constants. If the applied fields are not too strong, we can assume the source $Q(n_\varepsilon)$ of the departure from equilibrium to contain Fermi excitations with equilibrium distribution functions:

$$n_\varepsilon = n_\varepsilon^0 = \left(\exp\frac{|\varepsilon|}{T} + 1\right)^{-1}.$$

Let us examine the properties of such a source, assuming the parameters $|\Delta|/T$ and $\omega_q^0/|\Delta|$ to be small.

Note that the expressions in the curly brackets of (23) vanish (by virtue of familiar trigonometric identity) if the frequency ω_q^0 is neglected in the arguments of the δ functions. We use this circumstance for the subsequent transformations. We integrate in (23) with respect to some variable, say ε_3. Expanding next in terms of ω_q^0 and using the identities

$$\frac{\partial n_\varepsilon^0}{\partial \varepsilon} = -\frac{1}{T} n_\varepsilon^0(1 - n_\varepsilon^0),$$
$$n_\varepsilon^0 n_{\varepsilon_1}^0 (1 - n_{\varepsilon+\varepsilon_1}^0) = (1 - n_\varepsilon^0)(1 - n_{\varepsilon_1}^0) n_{\varepsilon+\varepsilon_1}^0,$$

the validity of which can be verified directly, we get

$$Q(\varepsilon) \approx -\frac{8\pi N_{\omega_q^0}(1 - n_\varepsilon^0)}{\varepsilon_F p_F^3} \times \int_{|\Delta|}^{\infty} d\varepsilon_1 (1 - n_{\varepsilon_1}^0)\left[2\frac{\omega_q^0}{T}(U_- - U_+) + \frac{(\omega_q^0)^2}{2T^2}(Z_- + Z_+)\right], \quad (25)$$

where

$$U_\mp = \theta(\varepsilon + \varepsilon_1 \mp \omega_q^0 - 2|\Delta|) \times \int_{|\Delta|}^{\varepsilon+\varepsilon_1\mp\omega_q^0-|\Delta|} d\varepsilon_2 n_{\varepsilon_2}^0 n_{\varepsilon+\varepsilon_1-\varepsilon_2}^0 m_{\mathbf{q}_0}(\varepsilon, \varepsilon_1, \varepsilon_2, \varepsilon \mp \omega_q^0 + \varepsilon_1 - \varepsilon_2), \quad (26)$$

$$Z_\mp = \theta(\varepsilon + \varepsilon_1 \mp \omega_q^0 - 2|\Delta|) \times \int_{|\Delta|}^{\varepsilon+\varepsilon_1\mp\omega_q^0-|\Delta|} d\varepsilon_2 n_{\varepsilon_2}^0 n_{\varepsilon+\varepsilon_1-\varepsilon_2}^0 (1 - 2n_{\varepsilon+\varepsilon_1-\varepsilon_2}^0) \times m_{\mathbf{q}_0}(\varepsilon, \varepsilon_1, \varepsilon_2, \varepsilon \mp \omega_q^0 + \varepsilon_1 - \varepsilon_2). \quad (27)$$

The term proportional to $(U_- - U_+)$ in (25) differs from zero if $\varepsilon \sim |\Delta|$. At the same time, the part proportional to $Z_- + Z_+$ is defined in the wider region $\varepsilon \sim T$. In addition, the second of the foregoing terms has an additional small factor in terms of $|\Delta|/T$. Recognizing that the function $m_{\mathbf{q}_0}(\varepsilon, \varepsilon_1, \varepsilon_2, \varepsilon_3)$ is negative-definite[4] for the argument values $\varepsilon_i \sim |\Delta|$, we see that the function $Q(\varepsilon)$ is negative for values directly above the gap, where $\varepsilon \sim \omega_q^0$. For larger ε the function $Q(\varepsilon)$ is positive. It becomes exponentially small for $\varepsilon \gg T$.

4. STIMULATION OF THE SUPERCONDUCTIVITY OF ^{3}He-B

We can now consider stationary and spatially homogeneous solutions of the kinetic equation for the single-particle excitations. In this case the kinetic equation is written in the form

$$0=Q(n_\varepsilon)+J^{(f-f)}(n_\varepsilon). \tag{28}$$

Here $J^{(f\text{-}f)}(n_\varepsilon)$ is the inelastic collision integral of the excitations in ^{3}He-B, which is responsible for relaxation to equilibrium and, finally, a stationary state. The expression for this collision integral is well known (see, e.g., Refs. 10 and 13), and for brevity $J^{(f\text{-}f)}(n_\varepsilon)$ is not given here explicitly. For low-intensity fields, a linearized solution of the kinetic equation suffices. We can use here the $Q(n_\varepsilon)$ properties determined in the preceding section. The fact that the source of the departure from equilibrium breaks up into two groups of terms, one localized with respect to ε near above-gap values and the other (smaller in size) in the temperature-spread region, allows us to assume that a similar breakup in the linearized approximation for the change of the Fermi-excitation distribution function:

$$\delta n_\varepsilon=n_\varepsilon-n_\varepsilon^0\equiv\delta n_\varepsilon^1+\delta n_\varepsilon^2. \tag{29}$$

To calculate the local part δn_ε^1 it suffices to use in (28) the relaxation-time approximation for $J^{(f\text{-}f)}(n_\varepsilon)$:

$$J^{(f-f)}(n_\varepsilon)\approx-\gamma u_\varepsilon\delta n_\varepsilon^1, \tag{30}$$

where $\gamma\propto T^2/\varepsilon_F$ is the excitation-energy damping and depends little on ε when $\varepsilon\sim|\Delta|$. From (30) and (28) an expression for the "local" increment δn_ε^1 follows directly:

$$\delta n_\varepsilon^1\approx\frac{1}{\gamma u_\varepsilon}Q(n_\varepsilon). \tag{31}$$

This expression must be substituted in the self-consistency equation, which for the B phase has the same form as the BCS equation; as a result we have for the change of the gap

$$\delta\Delta=-\frac{T^2}{|\Delta|}\left(\frac{8\pi^2}{7\zeta(3)}\right)\int_{|\Delta|}^{\infty}\frac{d\varepsilon}{(\varepsilon^2-|\Delta|^2)^{1/2}}\delta n_\varepsilon. \tag{32}$$

As to the function δn_ε^2, the relaxation-time approximation for $J^{(f\text{-}f)}(n_\varepsilon)$ in (28) is insufficient for its calculation. Higher accuracy, however, is unnecessary, since the function δn_ε^2 is smaller than δn_ε^1 by a factor $|\Delta|/T$, and this smallness is not offset by the integration with respect to ε in (32). We shall therefore neglect in the source (18) of the departure from equilibrium the contribution of the "tail" of the excitation distribution function, i.e., the contribution connected with δn_ε^2, and also the contribution due to other terms (proportional to the factors $M_{1,3}$ which are also smaller by a factor $|\Delta|/T$).

Let us verify that when (31), (25), and (26) are substituted in (32) the correction to the gap is positive. Recognizing that in the expression resulting from this substitution the integration is sensitive to the values $\varepsilon_i\sim|\Delta|$, and retaining in this expression the functional dependence on ε_i only in the factors that have singularities in the principal order in $\omega_q^0/|\Delta|$, we get

$$\delta\Delta=\frac{32\pi^2}{7\zeta(3)}\frac{\omega_q{}^0T}{|\Delta|^2}\left(\frac{N_{\omega_q{}^0}}{p_F{}^3}\right)\frac{1}{\varepsilon_F\gamma}$$
$$\times\left\{\int_{|\Delta|}^{\infty}d\varepsilon\int_{\max\{|\Delta|,\,2|\Delta|+\omega_q{}^0-\varepsilon\}}^{\infty}d\varepsilon_1\int_{|\Delta|}^{\varepsilon+\varepsilon_1-\omega_q{}^0-|\Delta|}d\varepsilon_2\right.$$
$$\times m_{q_0}(\varepsilon,\varepsilon_1,\varepsilon_2,\varepsilon-\omega_q{}^0+\varepsilon_1-\varepsilon_2)$$
$$-\int_{|\Delta|}^{\infty}d\varepsilon_1\int_{|\Delta|}^{\infty}d\varepsilon\int_{|\Delta|}^{\varepsilon+\varepsilon_1+\omega_q{}^0-|\Delta|}$$
$$\left.\times\, d\varepsilon_2 m_{q_0}(\varepsilon,\varepsilon_1,\varepsilon_2,\varepsilon+\omega_q{}^0+\varepsilon_1-\varepsilon_2)\right\}, \tag{33}$$

where, in the approximation assumed,

$$m_{q_0}(\varepsilon,\varepsilon_1,\varepsilon_2,\varepsilon\mp\omega_q{}^0+\varepsilon_1-\varepsilon_2)=\frac{|\Delta|}{4}\frac{3x_1-3x_2+y_1-y_2}{(\varepsilon-|\Delta|)^{1/2}(\varepsilon_1-|\Delta|)^{1/2}(\varepsilon_2-|\Delta|)^{1/2}(\varepsilon\mp\omega_q{}^0-\varepsilon_1-\varepsilon_2-|\Delta|)^{1/2}}. \tag{34}$$

Noting that the integral with respect to ε_2 in (33) and (34) reduces to quadratures, we express the correction to the gap in the form

$$\delta\Delta\approx-\frac{8\pi^4}{7\zeta(3)}\frac{\omega_q{}^0T}{\varepsilon_F\gamma}\left(\frac{N_{\omega_q{}^0}}{p_F{}^3}\right)(3x_1-3x_2+y_1-y_2)$$
$$\times\int_{|\Delta|}^{|\Delta|+\omega_q{}^0}d\varepsilon\int_{|\Delta|}^{2|\Delta|+\omega_q{}^0-\varepsilon}\frac{\delta\varepsilon_1}{(\varepsilon-|\Delta|)^{1/2}(\varepsilon_1-|\Delta|)^{1/2}}. \tag{35}$$

The remaining integrals are elementary and their contribution is equal to $\pi\omega_q^0$. The final result, after changing to the phonon-radiation energy-flux density, which is connected with $N_{\omega_q^0}$ [Eq. (22)], is

$$w=\int\frac{d^3\mathbf{q}}{(2\pi)^3}\omega_{\mathbf{q}}N_{\omega_{\mathbf{q}}}=\omega_q{}^0N_{\omega_q{}^0}. \tag{36}$$

The stimulation correction to the gap in the B phase is thus

$$\delta\Delta=\frac{8\pi^5|3x_1-3x_2+y_1-y_2|}{7\zeta(3)}\frac{T\omega_q{}^0}{\gamma\varepsilon_F}\frac{w}{p_F{}^3}. \tag{37}$$

Substituting in (37) the typical values of the quantities, we find that measurable changes of $\delta\Delta$ (e.g., $\delta\Delta/|\Delta|\sim1\%$) can be expected at energy flux densities $j=uw$ on the order of a watt per square centimeter.

5. COMPETITION BETWEEN THE PROCESSES IN THE *A* PHASE

As stipulated in Sec. 1, the presence of "punctures" in the gap $|\Delta_{\mathbf{p}}|=\Delta\sin\theta$ smears out the phonon-absorption threshold in the axial phase. Consequently, at arbitrary frequencies ω_q there is a finite probability of phonon absorption with pair breaking. As a result a surplus of fermions is excited and suppresses the superfluid ordering parameter. In addition, a stimulation mechanism similar to that considered in the preceding section acts in the A phase. To ascertain which of the nonequilibrium-action mechanisms predominates we shall analyze the influence of the excitation source corresponding to direct pair-breaking in the A phase.

We write down immediately the self-energy parts corresponding to pair breaking by phonons in the approximation (4) that takes into account only induced processes (cf. Ref. 11),

$$\hat{\Sigma}_1 = g^2 \int_{-\infty}^{\infty} \frac{d\varepsilon'}{4\pi i} \int \frac{d^3\mathbf{p}'}{(2\pi)^3} \hat{G}_\varepsilon(\mathbf{p}')\,\omega_{p-p'} D_{\varepsilon'-\varepsilon}(\mathbf{p}'-\mathbf{p}),$$

$$\hat{\Sigma}_1^{R(A)} = \hat{\Sigma}_1(G \to G^{R(A)}), \quad (38)$$

$$\hat{\Sigma}_2^{(R,A)} = \hat{\Sigma}_1^{(R,A)}(G^{(R,A)} \to F^{(R,A)}),$$

where $g^2\omega_{\mathbf{q}}$ is the square of the fermion–phonon interaction matrix element. Simple transformations reduce (38) to the form

$$\hat{\Sigma}_1 = \lambda N_{\omega_q^0} \omega_q^0 \int \frac{dO_{p'}}{4\pi} \delta(\mathbf{p}' - \mathbf{p} - \mathbf{q}_0) \times \int_{|\Delta_{p'}|}^{\infty} q_{-\varepsilon'}(\mathbf{p}')\,\delta(\varepsilon + \varepsilon' - \omega_q^0)\,d\varepsilon', \quad (39)$$

where $\lambda = g^2(mp_F/2\pi^2)$ is the dimensionless constant of the phonon–fermion interaction in ^{3}He. Substituting the self-energy parts of (38) or (39) in (1) and recognizing that the ^{3}He-A propagators integrated over energy are given by Eqs. (17) in which now $\mathbf{d} = $ const, we obtain for the disequilibrium source

$$Q(\varepsilon) = 2\pi\lambda N_{\omega_q^0} \frac{\omega_q^0}{p_F^3} \int \frac{dO_{p'}}{4\pi} \delta\left(1 - \frac{\mathbf{p}'(\mathbf{p}+\mathbf{q}_0)}{p'|\mathbf{p}+\mathbf{q}_0|}\right) \times \int_{|\Delta_{p'}|}^{\infty} d\varepsilon' \delta(\varepsilon + \varepsilon' - \omega_q^0)(u_\varepsilon u_{\varepsilon'} + v_\varepsilon v_{\varepsilon'})(1 - n_\varepsilon - n_{\varepsilon'}). \quad (40)$$

This source, together with an expression of type (25), is contained in the kinetic equation (28) for ^{3}He at all frequencies $\omega_{\mathbf{q}}^0$. In the linear approximation, the correction due to it is added to δn_ε and is given by a relation of type (31) (here $|\Delta_{\mathbf{P}}| = \Delta \sin\theta$):

$$\delta n_\varepsilon(\mathbf{p}) = \frac{\pi\lambda}{2} \frac{N_{\omega_q^0}}{p_F^3} \frac{(\omega_q^0)^2}{\gamma T} u_{\omega_q^0-\varepsilon}(\mathbf{p}) \left[1 + \frac{\Delta^2 \sin^2\theta}{\varepsilon(\omega_q^0 - \varepsilon)}\right]. \quad (41)$$

The self-consistency equation for ^{3}He–A (see, e.g., Ref. 10) can be written in the form

$$\frac{\Delta\mathbf{\Psi}}{3} = \left(V \frac{mp_F}{2\pi^2}\right) \int_{-\infty}^{\infty} \frac{d\varepsilon}{4\pi i} \frac{dO_p}{4\pi} \mathbf{p} f_\varepsilon(\mathbf{p}). \quad (42)$$

Here V is the pairing potential, $\hat{\mathbf{p}} = \mathbf{p}/p_F$, and $f_\varepsilon(\mathbf{p})$ is the orbital part of the function $(f_\varepsilon)_{\alpha\beta}$ [Eq. (16)]:

$$f_\varepsilon(\mathbf{p}) = 2\pi i \frac{\Delta_{\mathbf{p}}(1 - 2n_\varepsilon)}{(\varepsilon^2 - |\Delta_{\mathbf{p}}|^2)^{1/2}} \theta(\varepsilon^2 - |\Delta_{\mathbf{p}}|^2), \quad (43)$$

where

$$\Delta_{\mathbf{p}} = \Delta \mathbf{\Psi} \hat{\mathbf{p}}. \quad (44)$$

The vector $\mathbf{\Psi}$ is equal to

$$\mathbf{\Psi} = \mathbf{\Delta}' + i\mathbf{\Delta}'', \quad (45)$$

where $\mathbf{\Delta}'$ and $\mathbf{\Delta}''$ are orthogonal unit vectors that define the symmetry axis $\mathbf{l} = \mathbf{\Delta}' \times \mathbf{\Delta}''$. Substituting in (42) Eq. (43) with (44) taken into account, multiplying both sides of (42) by $\mathbf{\Psi}^*$, and using the relation

$$|\mathbf{\Psi}\hat{\mathbf{p}}|^2 = \sin^2\theta, \quad (46)$$

where θ is the angle between the anisotropy axis and the vector $\mathbf{p}$, we obtain an equation that determines the excitation-spectrum gap for the nonequilibrium function n_ε:

$$1 = \frac{3}{4}\left(V \frac{mp_F}{2\pi^2}\right) \int_0^\pi \sin^3\theta\,d\theta \int_{|\Delta_p|}^{\infty} d\varepsilon \frac{1 - 2n_\varepsilon}{(\varepsilon^2 - |\Delta_p|^2)^{1/2}}. \quad (47)$$

From (47) we get after standard transformations a gap correction linear in the change δn_ε:

$$\delta\Delta = -\frac{15\pi^2}{14\zeta(3)} \frac{T^2}{\Delta} \int_0^\pi \sin^3\theta\,d\theta \int_{|\Delta_p|}^{\infty} d\varepsilon \frac{\delta n_\varepsilon}{(\varepsilon^2 - |\Delta_p|^2)^{1/2}}. \quad (48)$$

Substituting (41) in (48) and using the smallness of the parameter $(\omega_{\mathbf{q}}^0/\Delta)$ we get

$$\delta\Delta = 2a \int_0^{\omega_q^0/2\Delta} \sin^3\theta\,d\theta \times \int_{\Delta\sin\theta}^{\omega_q^0 - \Delta\sin\theta} d\varepsilon \frac{\omega_q^0 - \varepsilon}{(\varepsilon^2 - \Delta^2\sin^2\theta)^{1/2}[(\omega_q^0 - \varepsilon)^2 - \Delta^2\sin^2\theta]^{1/2}}$$
$$\times \left(1 + \frac{\Delta^2\sin^2\theta}{\varepsilon(\omega_q^0 - \varepsilon)}\right) \approx 2a \int_0^{(\omega_q^0/2\Delta)} \sin^3\theta\,d\theta \times \int_{\Delta\sin\theta}^{\omega_q^0 - \Delta\sin\theta} \frac{d\varepsilon}{(\varepsilon - \Delta\sin\theta)^{1/2}[(\omega_q^0 - \varepsilon) - \Delta\sin\theta]^{1/2}} = a\frac{\pi}{2}\left(\frac{\omega_q^0}{2\Delta}\right)^4, \quad (49)$$

$$a = -\frac{15}{28} \frac{\pi^3\lambda}{\zeta(3)} \frac{(\omega_q^0)^2 N_{\omega_q^0}}{\Delta\gamma p_F^3} T.$$

Converting to the phonon-radiation energy density, in accordance with (36), we get

$$\delta\Delta \approx -\frac{15}{28} \frac{\pi^4\lambda}{\zeta(3)} \frac{w}{p_F^3} \frac{T}{\gamma} \left(\frac{\omega_q^0}{2\Delta}\right)^5. \quad (50)$$

We have thus obtained the negative gap correction necessitated by direct phonon depairing events in ^{3}He-A. Let us compare it with the positive stimulation correction. The latter can be found without special calculations, since it suffices to obtain its order of magnitude by using the equation derived in Sec. 4 for ^{3}He-B. Comparing (37) with (50) we conclude that stimulation in the A phase can be observed (accurate to numerical factors) at frequencies

$$\gamma < \omega_q^0 < 2\Delta(2\Delta/\varepsilon_F)^{1/4}, \quad (51)$$

the left-hand inequality of (51) being the condition for the validity of our analysis.

Note that stimulation of superfluid ordering in ^{3}He-A can lead to an interesting phenomenon. It is caused by stabilization of the order parameter in the A phase by a paramagnon field whose amplitude decreases when the superfluid-ordering parameter increases (this is in fact the cause of the transition into the B phase[14]). The stimulated increase of the order parameter of the A phase can destabilize the latter and force a transition into the B phase, provided, of course, that the stimulation is initiated close enough to the thermodynamic limit of the transition.

[1)]The same is true of photons in superconductors. In the latter, however, no kinematic constraints occur usually because many scattering centers on present.[5]

[2)]We consider here only states with a symmetric population of the particle–hole excitation modes, since the unbalance by phonon pumping is small by the factor $|\Delta|/\varepsilon_F$ (cf. Ref. 7).

[3)]The D-function can be taken outside the integrals with respect to ξ because the external and internal momenta in Σ are close to p_F.

[4)]To verify this circumstance we must return to Eq. (18) and interpret it to mean the collision operator of fermions with the self-field of the ^{3}He phonons (i.e., put in (18) $N_{\omega q} = [\exp(\omega_q/T) - 1]^{-1}$, where T is the Fermi-liquid temperature). The channel proportional to the factor M_2 should describe relaxation to equilibrium when the function n_ε departs by a small amount from equilibrium. The positive damping of the excitation energies leads then to a negative sign of the factor m_{q_0}.

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