Quantum-chromodynamics sum rules and nuclear matter

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An attempt is made to use the sum rules of quantum chromodynamics to obtain basic properties of nuclear matter. It is shown that nuclear matter has an equilibrium state with a density that depends substantially on the magnitude of the πN σ-term. An expansion of characteristics of nuclear matter in powers of $\rho^{1/3}$, where ρ is the nucleon density, is obtained, and includes terms $\sim\rho^{8/3}$. For $\sigma\sim 60$ MeV the values of the nuclear-matter parameters are close to those which can be obtained from nuclear models. Estimates of nucleon swelling in the nucleus are given.

1. INTRODUCTION

The construction of a consistent theory of nuclear matter is one of the principal tasks of nuclear physics. Numerous investigations[1] have been based on the nucleon-nucleon interaction and, therefore, have required the introduction of certain phenomenological concepts. These models appeared long before the advent of quantum chromodynamics (QCD). Since we believe today that QCD is the true theory of the strong interactions, it is desirable to have a model of nuclear matter based on QCD.

In the present paper we make an attempt to obtain the basic properties of nuclear matter starting from QCD sum rules.[2] Why have we used this particular method, and not one of the others that are also based on QCD, e.g., that in Ref. 3? There are several reasons for this.

1. The sum-rule method takes into account, albeit in a very "averaged" manner, the confinement of the quarks and gluons. The majority of chiral theories[3] do not take confinement into account. The principal events of nuclear physics are played out at distances 0.3 fm $\lesssim r \lesssim$ 2 fm. To construct a theory of nuclear matter one requires knowledge of the contribution of both short distances $r\sim 0.3$ fm, where perturbative QCD is valid, and distances $r\gtrsim 1$ fm, where it is necessary to allow for confinement.

2. The sum rules are based on the QCD Lagrangian and use asymptotic freedom. Therefore, the method accords with our current understanding of the strong interactions.

3. The accuracy of the method can be monitored in the framework of the method itself, without additional phenomenological hypotheses.

4. The sum rules successfully describe a series of properties of nucleons, from their static characteristics to deep inelastic scattering.[4]

5. Owing to the sum rules, a new picture of the structure of the QCD vacuum has arisen.[5] This may be a sign that the study of the sum rules is a necessary stage in the construction of a true model of the nucleus.

In the present paper we have expressed the parameters of nuclear matter in terms of the expectation values of certain QCD operators over the nucleus. This is an explicit expression of Migdal's idea, which is that the properties of nuclear matter are determined principally by the short-wavelength contributions associated with the interaction of the hadrons. The long-wavelength excitations (principally of the pion type) should be studied separately, after the question of the stability of nuclear matter has been solved. The method makes it possible to solve the first part of the problem, while the second part has already been solved by the method of quasiparticles.

We note that the sum-rule method has been used to study the bound states of two nucleons.[7]

In the paper we calculate the equilibrium density of nuclear matter, which depends on the magnitude of the pion-nucleon σ-term. The potential energy U, binding energy ε, and bulk modulus K are calculated as functions of the equilibrium density. For $\sigma = 60$ MeV,

$$\rho_0=0.197\ \text{fm}^{-3},\quad \varepsilon=-13\ \text{MeV},$$

$$U=-67\ \text{MeV},\quad K=154\ \text{MeV}. \tag{1}$$

2. THE SUM-RULE METHOD

The sum rules describing the principal static properties of hadrons[8] take the form of dispersion relations for the polarization operator Π_0 of the hadronic current $j(y)$ that have been subjected to a Borel transformation. Here,

$$\Pi_0(q^2)=i\int d^4y\langle 0|T\{\overline{j(y)}j(0)\}|0\rangle e^{i(qy)}, \tag{2}$$

and the hadronic (henceforth, proton) current is given by

$$j(y)=u^a(y)C\gamma_\mu u^b(y)\gamma_5\gamma_\nu d^c(y)\varepsilon^{abc}g^{\mu\nu}, \tag{3}$$

where C is the charge-conjugation operator and $u(y)$ and $d(y)$ are quark fields. In the dispersion relation the contribution of the nucleon pole has been separated out, while the other states are approximated by a continuum starting at a certain point W^2, i.e.,

$$\Pi_0(q^2)=-\frac{\lambda^2}{q^2-m^2}+\frac{1}{\pi}\int_{W^2}^{\infty}\frac{F(Q^2)dQ^2}{Q^2-q^2}. \tag{4}$$

In Eq. (4) λ^2 is the residue at the nucleon pole;

$$F(Q^2)=\frac{1}{2i}\Delta\Pi_0(Q^2)\theta(Q^2-W^2), \tag{5}$$

and the function $\Pi_0(q^2)$ is calculated using QCD perturbation theory.

The Borel transformation

$$\hat{B}_{M^2}f(x)=\lim\frac{x^{n+1}}{n!}\left(-\frac{d}{dx}\right)^n f(x)$$
$$\text{as}\quad x\to\infty,\quad n\to\infty,\quad M^2=x^2/n \tag{6}$$

annihilates the unknown residue polynomials and improves

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the convergence. After the transformation (6) the dispersion relation should be fulfilled for all values of M^2. In Ref. 8 the first few terms of the expansion of Π_0 in powers of q^{-2} were calculated, corresponding to the expansion of j in powers of y^2—the operator expansion of Ref. 9. The sum rules are found to be valid in the stability interval

$$0{,}8\ \text{GeV}^2 \leqslant M^2 \leqslant 1{,}4\ \text{GeV}^2. \qquad (7)$$

In Ref. 8 the following values were found for the quantities m, λ^2, and W^2:

$$m=1\ \text{GeV},\quad \lambda^2=2{,}1\ \text{GeV}^6,\quad W^2=2{,}3\ \text{GeV}^2 \qquad (8)$$

We want to answer the question: How are the quantities (8) changed in nuclear matter? To answer this question we shall solve the equations, analogous to (4), for the change of the polarization operator in a medium (nuclear matter).

The polarization operator Π_m in a medium depends not only on q^2 but also on the energy q_0, i.e.,

$$\Pi_m(q^2, q_0) = i\int e^{i(qy)} \langle \mathcal{M} | T\{\bar{j}(y) j(0)\} | \mathcal{M}\rangle d^4y, \qquad (9)$$

where $|\mathcal{M}\rangle$ is the nuclear-matter state. The value of the variable q_0 is determined by the fact that the nucleon corresponding to the pole in Eq. (4) is a constituent of nuclear matter. In the lowest orders in ρ (see Sec. 6) we can assume the nucleons of nuclear matter to be at rest. Therefore, representing the matter by a "particle" with baryon number $A \gg 1$, momentum p_A, and mass Am, we determine q_0 from the condition

$$s_A = (q+p_A)^2 = (A+1)^2 m^2. \qquad (10)$$

For the interaction with each of the nucleons the pair energy is

$$s = (q+p)^2 = 4m^2, \qquad (11)$$

where p is the momentum of the nucleon in the medium.

We assume that the interval of stability of the sum rules for the change of the polarization operator in the medium

$$\Pi(q^2, q_0) = \Pi_m(q^2, q_0) - \Pi_0(q^2) \qquad (12)$$

coincides with the interval (7) for the vacuum.

The sum rules for the operator (12), after the Borel transformation (6), will give the values of the changes Δm, $\Delta\lambda^2$, and ΔW^2 in nuclear matter. Since in the nonrelativistic approximation, to lowest order in ρ,

$$\Delta m = U, \qquad (13)$$

where U is the one-particle potential energy of the nucleon, we use the above nucleon-density function $\Delta m(\rho) = U(\rho)$ to find the equilibrium nucleon density ρ_0, and then calculate for ρ_0 the nuclear-matter characteristics (1).

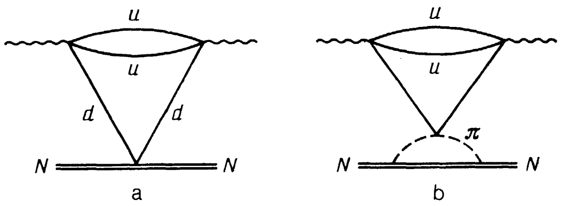

FIG. 2. Contribution to the function Π_2.

3. CALCULATION OF THE POLARIZATION OPERATOR

The polarization operator has the form

$$\Pi(q, p) = \hat{q}\Pi_1(q, p) + 1\cdot\Pi_2(q, p) + \hat{p}\Pi_3(q, p). \qquad (14)$$

We shall be interested in the first two structures (see Figs. 1 and 2).

1. The calculation of Π_1

The leading term of the operator expansion is shown in the diagram of Fig. 1a:

$$\Pi_{1a} = i\int d^4y\, e^{i(qy)} \frac{\gamma_\alpha y \gamma_\beta}{y^4(2\pi)^4} \langle \mathcal{M} | T\{\overline{j'^{ab}_\alpha}(y)\, j'^{a'b'}_\beta(0)\} | \mathcal{M}\rangle c_{aba'b'}, \qquad (15)$$

where

$$j'^{ab}_\alpha(y) = u^a(y) C\gamma_\alpha u^b(y), \qquad c_{aba'b'} = \delta_{ab}\delta_{a'b'} - \delta_{aa'}\delta_{bb'}.$$

Expanding in first order of the operator expansion

$$u(y) = \sum_{n=0} y_{\mu_1} \ldots y_{\mu_n} \frac{(-1)^n}{n!} \partial^{\mu_1} \ldots \partial^{\mu_n} u(0) \qquad (16)$$

and transforming (15) in a manner analogous to that used in the case of deep inelastic scattering (see, e.g., Ref. 9), we obtain

$$\Pi_1 = \int d^4y\, e^{i(qy)} \frac{\hat{y}}{y^2} \times \sum_{n=1} \langle \mathcal{M} | \bar{u}(0) \gamma^{\mu_1} D^{\mu_2\ldots\mu_n} u(0) | \mathcal{M}\rangle \frac{y_{\mu_1\ldots\mu_n}}{y^2} \qquad (17)$$

(D^μ is the covariant derivative with respect to y). In the expansion in the moments of the matter structure function

$$\Pi_1 = -2\pi\cdot 2A(pq) \sum_{n=0} a_{n+1} \left(2A(pq)\frac{\delta}{\delta q^2}\right)^n \ln q^2,$$

$$(pq) = \frac{s-m^2-q^2}{2} \qquad (18)$$

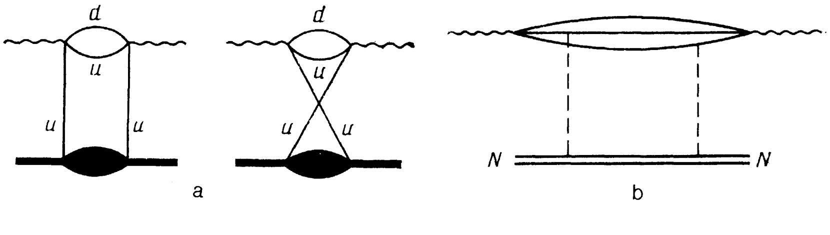

FIG. 1. Contribution to the function Π_1.

the leading contributions $\sim q^2\ln q^2$ and $\sim \ln q^2$ are given by the term with $n = 0$, while the terms with $n \geqslant 1$ give contributions $\sim q^{-2}$ (polynomials in q^2 are annihilated by the Borel transformation). Confining ourselves to the term with $n = 1$ in Eq. (17), we obtain

$$\Pi_{1a}=-2\pi\cdot 2(pq)n_u\ln q^2, \qquad (19)$$

where n_u is the difference of the numbers of quarks and antiquarks in the medium. Thus, the relation (19) does not assume that the matter consists of individual nucleons. In the following, however, it will be more convenient for us to work with relations explicitly containing the nucleon densities:

$$\Pi_{1a}=-2\pi[s-m^2-q^2](2\rho_p+\rho_n)\ln q^2, \qquad (20)$$

where $\rho_{p(n)}$ is the density of protons (neutrons) in the matter. The Borel transformation gives

$$\hat{B}\Pi_{1a}=2\pi[(s-m^2)M^2-M^4](2\rho_p+\rho_n). \qquad (21)$$

The leading corrections to (21) arise, clearly, when contributions of the form $((s-m^2)/q^2)^n a^N_{n+1}$ are taken into account, where a^N_{n+1} are the moments of the nucleon structure function. These corrections correspond to the calculation of the graphs for Π_{1a} for a fixed

$$x=-q^2/2(pq)=-q^2/(s-m^2-q^2). \qquad (22)$$

In the momentum representation it is easy to obtain[10]

$$\Pi_{1a}(q^2,x)=2\pi\frac{q^2}{x}\rho\int_0^1 d\alpha\, f(\alpha)\ln q^2\left(1+\frac{\alpha}{x}\right), \qquad (23)$$

where f is the nucleon structure function. After the Borel transformation we have

$$\hat{B}\Pi_{1a}=2\pi[(s-m^2)M^2-M^4]\rho\int_0^1 f_q(\alpha)\exp\{-(s-m^2)\alpha/M^2(1+\alpha)\}d\alpha, \qquad (24)$$

where f_q describes the valence quarks. Using the parametrization[11]

$$f_q(\alpha)=3.9\alpha^{-0.45}(1-\alpha)^{3.2}, \qquad \int_0^1 f_q(\alpha)\,d\alpha=3, \qquad (25)$$

we find that the magnitude of the discarded terms in (20) is at most 20% in the interval (7). We note that the relation (24) makes it possible to calculate contributions that depend explicitly on s, as we shall need to do when corrections $\sim\rho^{5/3}$ are taken into account (see Sec. 6).

A space-time picture of the process and its relationship to deep inelastic scattering are given in the Appendix. Terms $\sim\ln q^2$ also arise when the diagrams of Fig. 1b are taken into account:

$$\Pi_{1b}=\pi\langle\mathcal{M}|\frac{\alpha_s}{\pi}G^2|\mathcal{M}\rangle\ln q^2. \qquad (26)$$

The quantity $\langle N|\pi^{-1}\alpha_s G^2|N\rangle$ can be determined from the following relationship for the trace T^μ_μ of the energy-momentum tensor[12]:

$$T_\mu{}^\mu=\sum_i m_i\bar{q}_i q_i-\frac{9\alpha_s}{8\pi}G^2,$$

where q_i (m_i) are the fields (masses) of the quarks u, d, and s. Since $\langle N|T^\mu_\mu|N\rangle = m\langle N/N\rangle$, we obtain[12]

$$\langle N|\frac{\alpha_s}{\pi}G^2|N\rangle=\frac{8}{9}\left(m-\sum_i m_i\langle N|\bar{q}_i q_i|N\rangle\right). \qquad (27)$$

Analogously, for matter we obtain

$$\langle\mathcal{M}|\frac{\alpha_s}{\pi}G^2|\mathcal{M}\rangle=\left(Am-\sum_i m_i\langle\mathcal{M}|\bar{q}_i q_i|\mathcal{M}\rangle\right)\frac{8}{9}. \qquad (28)$$

Since the second term in (27) gives no more than 10% of the total contribution to Π_1, the entire structure of Π_1 is independent of the circumstance that nuclear matter consists of nucleons. In terms of nucleon densities,

$$\Pi_1=4\pi[(s-m^2)-q^2][(2\rho_p+\rho_n)\ln q^2]-\pi^2\left\langle N\left|\frac{\alpha_s}{\pi}G^2\right|N\right\rangle(\rho_p+\rho_n)\ln q^2. \qquad (29)$$

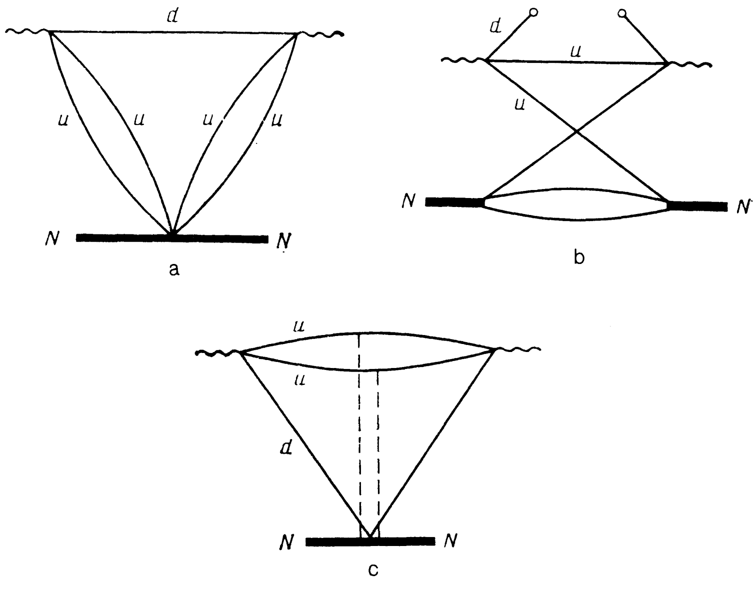

FIG. 3. Contributions not taken into account in the calculations of the functions Π_1 (a) and Π_2 (b, c).

The second term in Eq. (29) gives a contribution equal to 1/8 of the whole quantity Π_1, and one may hope that the series in powers of q^{-2} converges. The graphs of the next order that were not taken into account (Fig. 3a) contain unknown condensates of the form $\langle N|\bar{q}\bar{q}qq|N\rangle$.

2. The calculation of Π_2

The leading term of the operator expansion of the function Π_2 is determined by the diagram of Fig. 2a:

$$\Pi_{2a}=i\int d^4y\, e^{i(qy)}\langle\mathscr{M}|\bar{d}(y)d(0)|\mathscr{M}\rangle\frac{1}{y^6}, \tag{30}$$

where $d(y)$ is given by the formula (16). As in the calculation of Π_1, in the sum (16) over n the leading contribution is given by the term with $n=1$:

$$\Pi_{2a}=2\pi q^2\varkappa\ln(-q^2), \tag{31}$$

where $\varkappa=2\langle\mathscr{M}|\bar{d}(0)d(0)|\mathscr{M}\rangle$. Since the equilibrium density $\rho_{\rm ph}$ of nuclear matter is small in comparison with the close-packing density $\rho_{\rm cp}$,

$$\rho_{ph}/\rho_{cp}\sim 0.3,\quad \rho_{ph}=0.17\ \text{fm}^{-3}\ (\text{Ref. }1), \tag{32}$$

it is natural to attempt to find the solution in the form of an expansion in powers of ρ. In lowest order in ρ,

$$\varkappa=\varkappa_0\rho,\quad \varkappa_0=\langle N|\bar{u}u+\bar{d}d|N\rangle, \tag{33}$$

where $|N\rangle$ is the free-nucleon state.

For bound nucleons a large contribution $\sim\rho^{4/3}$ also arises in the first nonvanishing approximation. In fact, all possible emissions of mesons by nucleons have already been taken into account in the states $|N\rangle$ appearing in (33). For bound nucleons we should, however, exclude occupied final states with momentum p smaller than the Fermi momentum p_F:

$$p_F=(^3/_2\rho\pi^2)^{1/3}. \tag{34}$$

In the chiral limit, as is well known,[13] only the one-pion exchange graph (Fig. 2b) survives. Thus, from the contribution (31) we must subtract the contribution of the graph of Fig. 2b, in which the summation over the initial and final states $d^3p_{1(2)}/(2\pi)^3$ is limited by the condition $p_{1(2)}<p_F$. Direct calculation gives

$$\Pi_{2b}=+q^2\ln q^2\cdot\frac{9}{2\pi}\frac{\langle\pi|\bar{q}q|\pi\rangle}{m_\pi^2}p_F\rho, \tag{35}$$

and, to within terms $\sim\rho^2$, we find

$$\Pi_2=q^2\ln q^2\left[2\pi\varkappa_0-\frac{9}{2\pi}\frac{\langle\pi|\bar{q}q|\pi\rangle}{m_\pi^2}p_F\right]\rho. \tag{36}$$

In the graphs of Figs. 2a and 2b we have neglected the interaction of the nucleons with other nucleons of the medium, as manifested in corrections $\sim\rho$ to Eq. (36). Since the nucleons of the medium are moving, the corresponding corrections can be expanded in a series in powers of $p_F^2\sim\rho^{2/3}$. Thus, in general form,

$$\Pi_2=q^2\ln q^2\Big[2\pi\varkappa_0(1+\rho f_1(\rho,p_F^2)) -\frac{9}{2\pi}\frac{\langle\pi|\bar{q}q|\pi\rangle}{m_\pi^2}p_F(1+\rho f_2(\rho,p_F^2))\Big]\rho, \tag{37}$$

where the functions $f_{1,2}$ are not known to us.

The next term of the operator expansion of the quantity $\langle N|\bar{q}(y)q(0)|N\rangle$ contains the operator

$$\langle N|\bar{q}(0)(\lambda/2)^nG_{\alpha\beta}{}^n\sigma_{\alpha\beta}q(0)|N\rangle.$$

This, as in the case of the vacuum, makes no contribution to the structure of Π_2 (Refs. 8, 14), owing to the symmetry of the current (3) under interchange of the indices of the γ-matrices. The neglected graphs of Figs. 3b and 3c contain contributions $\langle N|\bar{q}\gamma_\mu q|N\rangle$ and $\langle N|\bar{q}G^2(\lambda/2)q|N\rangle$. We shall compare each of them with the contribution Π_{2a}. With the natural factorization hypothesis

$$\left\langle N\left|\bar{q}G^2\frac{\lambda}{2}q\right|N\right\rangle=\langle N|\bar{q}q|N\rangle\left\langle N\left|\frac{\alpha_s}{\pi}G^2\right|N\right\rangle, \tag{38}$$

strong cancellation occurs between these graphs.

Thus, QCD perturbation theory gives for $\Pi_{1,2}$ the expressions (20) and (36).

4. CONSTRUCTION OF THE SUM RULES

We now turn our attention to the right-hand sides of the sum rules. In contrast to the case of the vacuum, the polarization operator contains branchings associated with the variable u_A, and therefore the dispersion relation has the form

$$\Pi(q^2,s_A)=-\frac{\lambda_m^2}{q^2-m_m^2}+\frac{1}{\pi}\int_{W_m^2}\frac{F(Q^2,s_A)dQ^2}{Q^2-q^2} +\frac{1}{2\pi i}\int_{Q_0^2}\frac{\Delta_{u_A}\Pi(Q^2,s_A)dQ^2}{Q^2-q^2}. \tag{39}$$

We shall show, however, that for a bound system with finite binding energy ε and finite detachment energy ν ($\varepsilon-\nu>0$), such as nuclear matter, the singularities in u_A lie sufficiently far away. In fact, assuming the nucleons to be at rest, we shall set $s_A=(Am+m+\varepsilon)^2$ ($\varepsilon<0$). The singularities in u_A correspond to states with baryon number $A-1$, i.e., lie at $u_A\geqslant(Am-m-\nu)^2$ ($\nu<0$). Therefore, the singularities in u_A lie to the right of the point

$$q^2=Am^2-(s_A+u_A)/2=m^2+Am(\varepsilon-\nu). \tag{40}$$

For large $A\gg1$ the singularity (40) lies far to the right, and, after the Borel transformation, becomes exponentially small. Therefore, we neglect the last term in (39), writing the dispersion relation in the form

$$\Pi(q^2,s)=-\frac{\lambda_m^2}{q^2-m_m^2}+\frac{1}{\pi}\int_{W_m^2}^{\infty}\frac{F(Q^2,s)dQ^2}{Q^2-q^2}, \tag{41}$$

where the function $F(Q^2)$ is determined by the condition (5).

We note that we were able to neglect the singularities in the u-channel because of the condition $A\gg1$. Thus, this method is not applicable to the calculation of the nucleon-nucleon scattering amplitude.

The left-hand sides of the sum rules also contain singularities in u. They appear explicitly when the graph of Fig. 1a is calculated in the momentum representation—see Eqs. (22) and (23).

Substituting

$$\ln q^2\left(1+\frac{\alpha}{x}\right)=\ln q^2+\ln\left(1+\frac{\alpha}{x}\right),$$

we find that the term $\ln q^2$ corresponds to the same singularities as in the case of the vacuum, while the term $\ln(1+\alpha/x)$ leads to the result that the function Π_{1a} calculated using perturbation theory has a discontinuity of the form

$$\Delta\Pi_{1a}\sim\int\limits_{\frac{q^2}{s-m^2-q^2}}^{1} f(\alpha)\,d\alpha \qquad (42)$$

across the cut

$$0\leqslant q^2\leqslant\frac{s-m^2}{2}. \qquad (43)$$

For $q^2\gtrsim m^2$ the discontinuity (42) is small, this being explained by the smallness of the function $f(\alpha)$ for $1-\alpha\ll 1$ [see (24)]. Therefore, the presence of the discontinuity (42) does not affect the structure of the right-hand side of the sum rules.

All the nuclear excitations that could have a strong influence on the description of the nucleon in the medium are related to singularities in the variable s and do not change the dispersion relation (39). Therefore, we expect that the changes Δm, $\Delta\lambda^2$, and ΔW^2 are small, and shall take them into account by expanding in a series.

5. SOLUTION OF THE SUM RULES IN THE LOWEST ORDERS IN ρ

In this section we shall find the change Δm in the nucleon mass with allowance for terms $\sim\rho$ and $\sim\rho^{4/3}$:

$$\Delta m=a\rho+b\rho^{4/3} \qquad (44)$$

and do the same for $\Delta\lambda^2$ and ΔW^2. Substituting the functions (29) and (36) into Eq. (39), performing the Borel transformation, and subtracting the sum rules for the vacuum, we obtain, taking only the terms linear in the unknowns into account,

$$6\pi[(s-m^2)E_0(M^2)-M^2E_1(M^2)]\frac{M^2}{m}\frac{\rho}{L} \qquad (45)$$

$$-\pi^2\left\langle N\left|-\frac{\alpha_s}{\pi}G^2\right|N\right\rangle E_0(M^2)\frac{M^2}{m}\frac{\rho}{L}$$

$$=-\frac{2m}{M^2}\lambda^2e^{-m^2/M^2}\Delta m-e^{-m^2/M^2}\Delta\lambda^2$$

$$-\frac{W^4}{2}\frac{e^{-W^2/M^2}}{L}\Delta W^2,$$

$$-2\pi\varkappa_0M^4E_1(M^2)\rho+\frac{9}{2\pi}\frac{\langle\pi|\bar{q}q|\pi\rangle}{m_\pi}M^4E_1(M^2)p_F\rho$$

$$=\left(1-\frac{2m^2}{M^2}\right)\lambda^2e^{-m^2/M^2}\Delta m+me^{-m^2/M^2}\Delta\lambda^2$$

$$+2aW^2e^{-W^2/M^2}\Delta W^2(-1), \qquad (46)$$

where $a=0.55$ GeV^4 (Ref. 8), $L=(\ln(M^2/\Lambda^2)\ln^{-1}(\Lambda^2/\mu^2))^{4/9}$, $\Lambda=0.5$ GeV, $\mu=0.15$ GeV, and

$$E_0(M^2)=1-e^{-W^2/M^2},\quad E_1(M^2)=1-\left(1+\frac{W^2}{M^2}\right)e^{-W^2/M^2}. \qquad (47)$$

The quantities $\varkappa_0$ and $\langle N|\alpha_s\pi^{-1}G^2|N\rangle$ are connected by the $SU(3)$ relation

$$\left\langle N\left|-\frac{\alpha_s}{\pi}G^2\right|N\right\rangle=\frac{8}{9}\left[m-m_s\left(\frac{\varkappa_0}{2}-2{,}1\right)\right]. \qquad (48)$$

The quantity $\varkappa_0$ can itself be expressed in terms of the pion-nucleon σ-term:

$$\varkappa_0=2\sigma/(m_u+m_d), \qquad (49)$$

where $m_{u(d)}$ is the mass of the $u(d)$ quark. For the σ-term the authors of Ref. 15 obtained

$$\sigma=(60\pm10)\ \text{MeV},\quad \varkappa_0=11\pm2, \qquad (50)$$

while in Ref. 16 the following estimate is given:

$$\sigma=(40\pm10)\ \text{MeV},\quad \varkappa_0=7{,}3\pm1{,}8. \qquad (51)$$

Using also the relation

$$\langle\pi|\bar{q}q|\pi\rangle/m_\pi^2=(m_u+m_d)^{-1}, \qquad (52)$$

we find the unknown quantities Δm, $\Delta\lambda^2$, and ΔW^2 by minimizing the relative difference of the left- and right-hand sides of Eqs. (45) and (46) by the method of least squares:

$$\Delta m=[(-34-9{,}4\varkappa_0)\xi+54\xi^{4/3}]\ \text{MeV}, \qquad (53)$$

$$\Delta\lambda^2=[(-0{,}12-0{,}062\varkappa_0)\xi+0{,}34\xi^{4/3}]\ \text{GeV}^6, \qquad (54)$$

$$\Delta W^2=[(-0{,}13-0{,}045\varkappa_0)\xi+0{,}25\xi^{4/3}]\ \text{GeV}^2, \qquad (55)$$

where we have introduced the notation $\xi=\rho/\rho_{\text{ph}}$, where $\rho_{\text{ph}}=0.17$ fm^{-3} is the phenomenological equilibrium density. It can be seen that the solutions (53)–(55) are sensitive to the value of $\varkappa_0$. The equilibrium density ρ_0 is determined from the equation[6]

$$T(\rho_0)+U(\rho_0)=\frac{1}{\rho_0}\int\limits^{\rho_0}(T(\rho)+U(\rho))\,d\rho, \qquad (56)$$

where

$$T=p_F^2/2m \qquad (57)$$

is the kinetic energy at the Fermi surface. When (53) and (57) are taken into account, Eq. (56) is transformed into a simple algebraic equation

$${}^4/_7by^2+{}^1/_2ay+15=0 \qquad (58)$$

for the ratio $y=\xi^{1/3}$ of the Fermi momenta, with a and b given by (44) and (53). For $\sigma=40$ MeV we obtain

$$y=1{,}26,\quad \xi=2{,}0,\quad \rho_0=2\rho_{ph}, \qquad (59)$$

and for the potential energy U, the binding energy ε, and the bulk modulus K we obtain

$$U=-69\ \text{MeV},\quad \varepsilon=-7\ \text{MeV},\quad K=162\ \text{MeV}. \qquad (60)$$

The dependence of these quantities on $\varkappa_0$ is shown in Fig. 4. For $\sigma=40$ MeV we obtain

$$\Delta\lambda^2=-0{,}29\ \text{GeV}^6,\quad \Delta W^2=-0{,}29\ \text{GeV}^2. \qquad (61)$$

Thus, taking into account only the terms $\sim\rho$ and $\sim\rho^{4/3}$ we obtain a qualitative description of nuclear matter. For the values of σ given by (50) and (51) nuclear matter is in a bound state. For $\sigma=40$ MeV the Fermi momentum is 20–25% greater than the phenomenological value, and this leads to an overestimate of the density by a factor of two. The

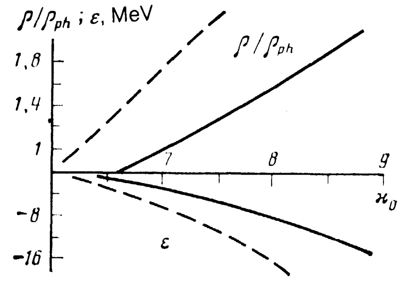

FIG. 4. Dependence of the equilibrium density and binding energy on the quantity $\varkappa_0$. The solid traces are obtained when terms $\sim\rho^{5/3}$ are taken into account. The broken traces are obtained when only the terms $\sim\rho$ and $\sim\rho^{4/3}$ are taken into account.

magnitude of the binding energy is then strongly underestimated, while the potential energy and bulk modulus are close to those that can be obtained in nuclear models.

6. VELOCITY-DEPENDENT FORCES; THE EFFECTIVE MASS

The function Π_1 in Eq. (29) depends explicitly on the quantity s, which, in its turn, depends on the three-dimensional momenta of the nuclei:

$$s=4m^2+8mU+2\left(\frac{p^2}{2m}+\frac{q^2}{2m}\right)+2(\mathbf{pq}). \tag{62}$$

After averaging over the directions and magnitudes of the momenta p and q, we obtain the corrections

$$\delta m(\lambda^2, W^2)=\frac{\partial\Delta m(\Delta\lambda^2,\Delta W^2)}{\partial s}\left(\frac{6}{5}p_F{}^2+8mU\right) \tag{63}$$

to the solutions (53)–(55). The first term in (63) gives corrections $\sim\rho^{5/3}$, which will be calculated in this section. The second term in (63) leads to contributions $\sim\rho^2$, which will be calculated in Sec. 7. Using Eq. (24), which sums the s-dependent contributions to Π_1, we obtain

$$\frac{\partial\Pi_1}{\partial s}=-2\pi q^2\ln q^2\cdot\rho\int_0^1\frac{f(\alpha)}{1+\alpha}\exp\left\{-\frac{\alpha(s-m^2)}{M^2(1+\alpha)}\right\}d\alpha, \tag{64}$$

where $f(\alpha)$ is given by the formula (25). In the second sum rule the dependence on s appears in higher terms of the operator expansion (Fig. 3b). An analogous calculation gives

$$\frac{\partial\Pi_2}{\partial s}=-2\pi q^2\ln q^2\cdot 2a\rho\int_0^1\frac{f(\alpha)}{(1+\alpha)^2}\exp\left\{-\frac{\alpha(s-m^2)}{M^2(1+\alpha)}\right\}d\alpha. \tag{65}$$

Solving the sum-rule equations with allowance for (64) and (65) we obtain

$$\partial\Delta m/\partial s=1.8\xi^{5/3}\ \text{MeV}^{-1},\ \partial\Delta\lambda^2/\partial s=0.11\xi^{5/3}\ \text{GeV}^4,$$
$$\partial\Delta W^2/\partial s=0.082\xi^{5/3}, \tag{66}$$

which leads to the solutions

$$\Delta m=[(-34-9.4\varkappa_0)\xi+54\xi^{4/3}+1.6\xi^{5/3}]\ \text{MeV}, \tag{67}$$

$$\Delta\lambda^2=[(-0.12-0.062\varkappa_0)\xi+0.34\xi^{4/3}+0.01\xi^{5/3}]\ \text{GeV}^6, \tag{68}$$

$$\Delta W^2=[(-0.13-0.045\varkappa_0)\xi+0.25\xi^{4/3}+0.01\xi^{5/3}]\ \text{GeV}^2: \tag{69}$$

Neglecting small terms of order ΔmT^2 we have

$$U=\Delta m/(1+T/m). \tag{70}$$

When (67) and (70) are taken into account the equilibrium equation can be written in the form

$$(2+0.25\varkappa_0)y^3+31y^2-(17+4.7\varkappa_0)y+15=0, \tag{71}$$

which, for $\sigma=40$ MeV, gives

$$y=1.09;\quad \xi=1.29. \tag{72}$$

In the form conventionally empolyed for nuclear physics,

$$\varepsilon=U+p_F{}^2/2m^*, \tag{73}$$

where U is the potential energy, determined by Eq. (53). Under the condition (71), at the point $\sigma=40$ MeV we find

$$U=-56\ \text{MeV}. \tag{74}$$

The velocity-dependent contribution $\sim\rho^{5/3}$ is taken into account in the second term of (74) by means of the effective mass

$$m^*=m(1-(0.078+0.010\varkappa_0)\xi). \tag{75}$$

Thus, for $\sigma=40$ MeV, we have

$$\varepsilon=-2\ \text{MeV},\quad m^*=0.84m. \tag{76}$$

The value of the equilibrium density has a substantial dependence on the quantity σ. For $\sigma=50$ MeV,

$$\xi=2.25,\ \varepsilon=-16\ \text{MeV},\ K=210\ \text{MeV}. \tag{77}$$

The dependence of the quantities ξ and ε on σ is shown in Fig. 5.

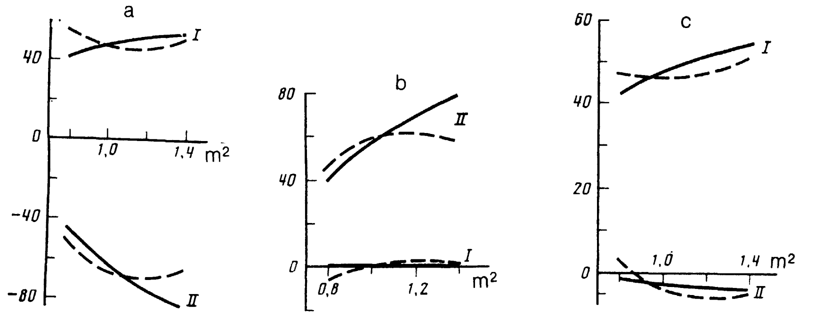

FIG. 5. Consistency of the left-hand side (the solid curve) and right-hand side (the broken trace) of Eq. (45) (the curves I) and Eq. (46) (the curves II) for $\sigma=40$ MeV. Figures a and b show the terms $\sim\rho$ and $\sim\rho^{4/3}$, and Fig. c shows their sum.

7. FURTHER TERMS OF THE EXPANSION IN POWERS OF p_F^2

To take further terms of the expansion in powers of p_F^2 into account we write the sum rules [the difference of Eqs. (41) and the corresponding equations for the vacuum] in the form

$$\mathscr{L}(\rho)-\mathscr{L}(0)=R(x_{im})-R(x_i), \tag{78}$$

where $\mathscr{L}(R)$ is the left-hand (right-hand) side of the sum rules, and x_i $(i=1,2,3)$ are the quantities m, λ^2, W^2. The solutions (67)–(69) correspond to the first terms of the expansion

$$\mathscr{L}(\rho)-\mathscr{L}(0)=l_1\rho+l_2\rho^{4/3}+l_3\rho^{5/3}, \tag{79}$$

where $l_{1,2,3}$ are expressed by means of the formulas (29), (36), (64), and (65). Then the changes Δx^i of the quantities are found from the equation

$$l_1\rho+l_2\rho^{4/3}+l_3\rho^{5/3}=\sum_i\frac{\partial R}{\partial x_i}\Delta x_i, \tag{80}$$

and it is this which gave the solutions (67)–(69).

To take the next terms of the expansion in powers of ρ into account, we expand the right-hand side of (78) to terms of second order in Δx_i. Then from (79) we obtain

$$l_4\rho^2+l_5\rho^{7/3}+l_6\rho^{8/3}=\sum_i\frac{\partial R}{\partial x_i}\Delta x_i^{(1)}+\frac{1}{2}\sum_{i,j}\frac{\partial^2 R}{\partial x_i x_j}\Delta x_i^{(0)}\Delta x_j^{(0)}, \tag{81}$$

where $\Delta x_i^{(1)}=\alpha_i\rho^2+\beta_i\rho^{7/3}+\gamma_i\rho^{8/3}$, and $l_{4,5,6}$ are unknown constants. As shown in Sec. 4, in the first sum rule we have $l_{4,5,6}=0$. In the second sum rule these constants can be expressed in terms of the unknown functions $f_{1,2}$ appearing in Eq. (37).

Substituting the solutions $\Delta x_i^{(0)}$ from (67)–(69) into Eq. (81) and equating the coefficients of equal powers of ρ, we obtain equations for l_R, α_i, β_i, and γ_i:

$$\frac{e^{-m^2/M^2}}{M^2}\left(1-\frac{2m^2}{M^2}\right)\lambda^2(34+9.4\varkappa_0)^2\cdot 10^{-3}+\frac{W^2}{2L}\left(1-\frac{W^2}{2M^2}\right)\cdot$$
$$\times e^{-W^2/M^2}(130+45\varkappa_0)^2\cdot 10^{-3}$$
$$+\frac{me^{-m^2/M^2}}{M^2}(34+9.4\varkappa_0)(0.12+0.062\varkappa_0)$$
$$=-\frac{2m\lambda^2e^{-m^2/M^2}}{M^2}\alpha_1+e^{-m^2/M^2}\alpha_2-\frac{W^4}{2}e^{-W^2/M^2}\frac{\alpha_3}{L}, \tag{82}$$

$$\frac{6e^{-m^2/M^2}}{M^2}\left(1-\frac{2m^2}{M^2}\right)(34+9.4\varkappa_0)^2\cdot 10^{-3}$$
$$-\frac{W^2}{4}e^{-W^2/M^2}\left(\frac{W^2}{M^2}-1\right)$$
$$\times(130+45\varkappa_0)^2\cdot 10^{-3}$$
$$+\frac{m}{2}e^{-m^2/M^2}(34+9.4\varkappa_0)(0.12+0.062\varkappa_0)$$
$$=\left(1-\frac{2m^2}{M^2}\right)m\lambda^2e^{-m^2/M^2}\alpha_1+me^{-m^2/M^2}\alpha_2$$
$$-2aW^2e^{-W^2/M^2}\alpha_3+M^4E_1(M^2)\,l_4, \tag{83}$$

$$-\frac{e^{-m^2/M^2}}{M^2}\left(1-\frac{2m^2}{M^2}\right)\lambda^2(1.8+0.31\varkappa_0)$$
$$-\frac{W^2}{2L}\left(1-\frac{W^2}{2M^2}\right)e^{-W^2/M^2}(3.2+1.1\varkappa_0)$$
$$-\frac{me^{-m^2/M^2}}{M^2}(2.2+3.7\varkappa_0)$$
$$=-\frac{2m\lambda^2}{M^2}e^{-m^2/M^2}\beta_1+e^{-m^2/M^2}\beta_2-\frac{W^4}{2}\frac{e^{-W^2/M^2}\beta_3}{L}, \tag{84}$$

$$-\frac{3e^{-m^2/M^2}}{M^2}\lambda^2\left(2-\frac{4m^2}{M^2}\right)(1.8+0.31\varkappa_0)$$
$$+\frac{W^2}{4}e^{-W^2/M^2}\left(\frac{W^2}{M^2}-1\right)$$
$$\times(3.2+1.1\varkappa_0)-\frac{m}{2}e^{-m^2/M^2}\frac{1}{M^2}(2.2+3.7\varkappa_0)$$
$$=\left(1-\frac{2m^2}{M^2}\right)m\lambda^2e^{-m^2/M^2}\beta_1$$
$$+me^{-m^2/M^2}\beta_2-2aW^2e^{-W^2/M^2}\beta_3+M^4E_1(M^2)\,l_5, \tag{85}$$

$$2.9e^{-m^2/M^2}\left(1-\frac{2m^2}{M^2}\right)\lambda^2+\frac{6.2W^2}{2L}\left(1-\frac{W^2}{2M^2}\right)e^{-W^2/M^2}$$
$$+\frac{18.4me^{-m^2/M^2}}{M^2}=$$
$$=-\frac{2m\lambda^2}{M^2}e^{-m^2/M^2}\gamma_1+e^{-m^2/M^2}\gamma_2-\frac{W^4}{2}\frac{e^{-W^2/M^2}}{L}\gamma_3, \tag{86}$$

$$\frac{8.7e^{-m^2/M^2}}{M^2}\left(2-\frac{4m^2}{M^2}\right)\lambda^2+\frac{6.2W^2}{4}e^{-W^2/M^2}\left(\frac{W^2}{M^2}-1\right)$$
$$+\frac{18.4me^{-m^2/M^2}}{2M^2}=$$
$$=\left(1-\frac{2m^2}{M^2}\right)m\lambda^2e^{-m^2/M^2}\gamma_1+me^{-m^2/M^2}\gamma_2$$
$$-2aW^2e^{-W^2/M^2}\gamma_3+M^4E_1(M^2)\,l_6. \tag{87}$$

We note that in the case of the vacuum the sum rules[8] have been used to determine the condensates that appear in the higher orders of the operator expansion. Equations (82)–(87) are used to determine the unknown condensates in the higher orders of the expansion in powers of the density. Minimizing the difference of the left- and right-hand sides in Eqs. (82)–(87) by the method of least squares, we obtain for $\Delta x_i^{(1)}$

$$\Delta m^{(1)}=[(2.3\varkappa_0+1)\xi^2-(0.2\varkappa_0+7)\xi^{7/3}+3\xi^{8/3}]\ \text{MeV}, \tag{88}$$

$$\Delta\lambda^{2(1)}=[(0.025\varkappa_0-0.05)\xi^2$$
$$-(0.004\varkappa_0+0.036)\xi^{7/3}+0.03\xi^{8/3}]\ \text{GeV}^6, \tag{89}$$

$$\Delta W^{2(1)}=[(0.010\varkappa_0-0.01)\xi^2$$
$$-(0.002\varkappa_0+0.01)\xi^{7/3}+0.01\xi^{8/3}]\ \text{GeV}^2. \tag{90}$$

and for the unknown condensates

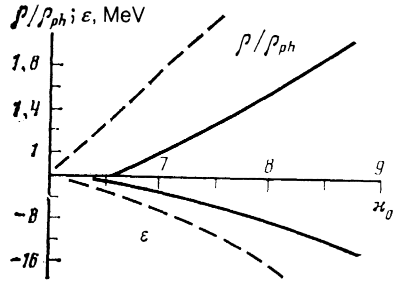

FIG. 6. Dependence of the equilibrium density and binding energy on the quantity $\varkappa_0$ with allowance for terms up to $\sim\rho^{8/3}$.

$$l_4=(6\varkappa_0-13)\ \text{MeV}\sim 50\ \text{MeV}, \tag{91}$$

$$l_5=-(1.2\varkappa_0+5)\ \text{MeV}\sim -20\ \text{MeV}, \tag{92}$$

$$l_6=8\ \text{MeV}, \tag{93}$$

(the terms $\sim\varkappa_0^2$ in $\Delta x_i^{(1)}$ and l_4 are negligibly small). In the equation for $\Delta x_i^{(0)}$, $l_1\sim -100$ MeV and $l_2\sim 50$ MeV.

Finally, we obtain $\Delta x_i = \Delta x_i^{(0)} + \Delta x_i^{(1)}$:

$$\Delta m=[(-34-9.4\varkappa_0)\xi+54\xi^{4/3}+1.6\xi^{5/3} + (2.3\varkappa_0+1)\xi^2-(0.2\varkappa_0+7)\xi^{7/3}+3\xi^{8/3}]\ \text{MeV}, \tag{94}$$

$$\Delta\lambda^2=[(-0.12-0.062\varkappa_0)\xi+0.34\xi^{4/3}+0.01\xi^{5/3} + (0.025\varkappa_0-0.05)\xi^2-(0.004\varkappa_0+0.03)\xi^{7/3}+0.03\xi^{8/3}]\ \text{GeV}^6, \tag{95}$$

$$\Delta W^2=[(-0.13-0.045\varkappa_0)\xi+0.25\xi^{4/3}+0.01\xi^{5/3} + (0.010\varkappa_0-0.01)\xi^2-(0.002\varkappa_0+0.01)\xi^{7/3}+0.01\xi^{8/3}]\ \text{GeV}^2. \tag{96}$$

The potential energy, when Eq. (70) is taken into account, is

$$U(\xi)=[(-34-9.4\varkappa_0)\xi+54\xi^{4/3}+(2.3\varkappa_0-1)\xi^2+(-0.2\varkappa_0-5)\xi^{7/3} + (3-0.1\varkappa_0)\xi^{8/3}]\ \text{MeV},$$
$$\xi=\rho/\rho_{ph},\ \rho_{ph}=0.17\ \text{fm}^{-3}. \tag{97}$$

We note that the series in ξ converges rapidly. Using (56), (73), and (75), we obtain the equilibrium equation of nuclear matter:

$$(2.2-0.07\varkappa_0)y^6+(-3.5-0.14\varkappa_0)y^5+(-0.7+1.7\varkappa_0)y^4 + (1.9+0.25\varkappa_0)y^3+31y^2+(-17-4.7\varkappa_0)y+15=0. \tag{98}$$

The values of the equilibrium density ρ_0 and binding energy ε as functions of the quantity $\varkappa_0$ are given in Fig. 6. For $\sigma = 60$ MeV and $\varkappa_0 = 10.9$,

$$y=1.05,\ \rho_0=0.197\ \text{fm}^{-3},\ \varepsilon=-13\ \text{MeV}; \tag{99}$$

then

$$U=-67\ \text{MeV},\ K=154\ \text{MeV}, \tag{100}$$

(see also Fig. 7), and

$$\Delta\lambda^2=-0.26\ \text{GeV}^6,\ \Delta W^2=-0.30\ \text{GeV}^2. \tag{101}$$

8. CONCLUSION

Thus, we have attempted to obtain the basic properties of nuclear matter starting from the QCD sum rules. We have not used any phenomenological assumptions about the nucleon-nucleon interactions. The values of the change of the nucleon mass in the medium, the change of the residue at the nucleon pole, and the change of the threshold of the continuum have been represented in the form of an expansion in powers of the nucleon density.

In the lowest terms of the operator expansion the left-hand side of the sum rules, as shown in the paper, does not contain an infinite series of operators but can be expressed in terms of the values of several operators averaged over the states of the medium. In their turn, in the lowest orders in ρ these are expressed in terms of known or calculable expectation values over the nucleons.

The right-hand sides of the dispersion relations contain not only singularities in q^2 but also singularities in the u-channel. In this paper it is shown that in the bound system with a large number of nucleons ($A\gg 1$) the latter are shifted far to the right along the axis of the variable q^2 and give an exponentially small contribution after a Borel transforma-

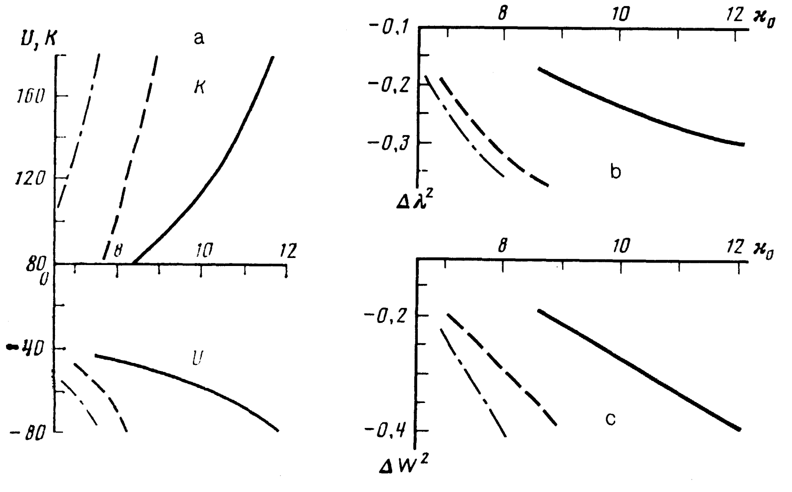

FIG. 7. Dependence of the quantities U and K (Fig. a) and $\Delta\lambda^2$ and ΔW^2 (Figs. b and c) on the quantity $\varkappa_0$ for equilibrium values of the density. The dashed-dotted curve takes only terms $\sim\rho$ and $\sim\rho^{4/3}$ into account; the broken trace corresponds to inclusion of terms $\sim\rho^{5/3}$; the solid trace takes terms up to $\sim\rho^{8/3}$ into account.

tion. Thus, the direct application of the method to the calculation of the amplitude of the nucleon-nucleon interaction is not possible. We note that, for the same reasons, it is not possible to use the method to calculate the πN-scattering amplitude, which vanishes in the chiral limit. The left-hand sides of the sum rules do not vanish as $m_\pi \to 0$. At the present time we cannot answer the question as to whether the πN-scattering amplitude vanishes on account of the u-channel or on account of the contribution of graphs of higher orders. We cannot rule out the latter possibility, which would suggest poor convergence of the operator-expansion series.

Taking into account only terms $\sim\rho$ and $\sim\rho^{4/3}$, we have obtained the basic properties of nuclear matter. We have shown that the matter has a state of stable equilibrium. The density depends strongly on the magnitude of the πN σ-term. For $\sigma = 40$ MeV the Fermi momentum, for which the equilibrium equation is solved, turns out to be 25% greater than the known phenomenological value. This leads to a density $\rho = 0.34\ \mathrm{fm}^{-3}$ for the given value of σ. The value of the binding energy turns out to be strongly underestimated, although the potential energy and compressibility agree well with the known values.

In the next order ($\rho^{5/3}$) velocity-dependent forces arise naturally. Allowance for these leads to a smaller value of the equilibrium density ($\rho = 0.19\ \mathrm{fm}^{-3}$ for $\sigma = 40$ MeV), but for values of U close to those obtained in nuclear physics the binding energy is sharply underestimated.

The calculation of the next terms of the expansion in powers of $\rho^{1/3}$ includes the unknown condensates. The latter are also determined from the sum rules, just as, in the case of the vacuum, the condensates appearing in higher orders of the operator expansion have been determined from the sum rules. Allowance for terms up to $\rho^{8/3}$ leads to a smoother dependence of the quantities sought on the magnitude of the σ-term (Fig. 6). For $\sigma = 60$ MeV the values given in Eq. (1) are realized.

We note that the magnitude of the potential energy can be estimated easily. In the case of the vacuum,[8]

$$m=[-8\pi^2\langle 0|\bar{u}u+\bar{d}d|0\rangle]^{1/3}=1\ \mathrm{GeV},$$

while in a medium

$$m_m=[-8\pi^2\langle\mathcal{M}|\bar{u}u+\bar{d}d|\mathcal{M}\rangle]^{1/3}$$

$$\sim m\left\{1+\frac{\rho}{3}\frac{\langle N|\bar{u}u+\bar{d}d|N\rangle}{\langle 0|\bar{u}u+\bar{d}d|0\rangle}\right\}\sim 0.9m, \qquad (102)$$

which determines the scale of the phenomenon.

We recall one further result of the paper. Since the quantity $\Delta\lambda^2$ has the meaning of the quark wavefunction at the origin of the coordinate space, the estimate $\Delta\lambda^2/\lambda^2 \sim -0.1$, which follows from formula (95), gives the scale of the nucleon swelling in the nucleus. In terms of the increase of the radius, this scale corresponds to $\Delta r/r \sim 3\%$.

Improvement of the method should involve allowance for the next terms of the operator expansion, and this requires the calculation of expectation values (over the nucleus) of several more operators. The results obtained in the paper permit us to hope that it will be possible to use the method to give a simultaneous description both of the structure of nuclear matter and of hard processes in it.

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APPENDIX

We shall compare the space-time pictures of our process and deep inelastic scattering.[17] The latter is described by the matrix element

$$F(q)=\mathrm{Im}\int d^4y\, e^{-i(qy)}\langle N|T\{\bar{J}_\mu(y)J_\nu(0)\}|N\rangle, \qquad (A1)$$

where

$$J_\mu(y)=\bar{u}(y)\gamma_\mu u(y) \qquad (A2)$$

is the electromagnetic current. Choosing as the z axis the direction of $\mathbf{q}$, we denote

$$y^2=y_0^2-y_z^2-y_t^2,\quad y_z^2+y_t^2=y_1^2;\quad z=y_0+y_z. \qquad (A3)$$

In deep inelastic scattering[17],

$$y_1^2\sim 1/q^2\ll z^2;\quad y_t^2\ll z^2 \qquad (A4)$$

and

$$\exp i(qy)=\exp\{iq^2/2mxz+imxz\},\quad x=-q^2/2(pq). \qquad (A5)$$

For the structure proportional to $g_{\mu\nu}$ in F one can obtain[9,17] in the first approximation in y^2

$$T\{\bar{J}_\mu(y)J_\nu(0)\}=g_{\mu\nu}\frac{1}{(2\pi)^2}\frac{1}{y^4}\sum_{\mu_i}y_{\mu_i}\ldots y_{\mu_n}\bar{u}(y)\gamma_{\mu_1}D^{\mu_2\ldots\mu_n}u(0). \qquad (A6)$$

Since, obviously,

$$\langle N|\bar{u}(y)|\gamma_\alpha D^{\mu_2\ldots\mu_n}u(0)|N\rangle\propto p_\alpha,$$

we have, to within terms $\sim y^2$,

$$\langle N|\bar{u}(y)\gamma_\alpha D^{\mu_2\ldots\mu_n}u(0)|N\rangle=p_\alpha\varphi(z), \qquad (A7)$$

where φ is an unknown function. Equation (A1) acquires the form

$$F(q^2,x)=\mathrm{Im}\int\frac{dy_1^2\,dz\,d^2y_t}{(2\pi)^2}\frac{\varphi(z)}{z}(py)\times\exp\left(-i\frac{q^2}{2mx}\frac{y_1^2}{z}-imxz\right), \qquad (A8)$$

which can be transformed to

$$F=(pq)\int dy_1^2\,dz\exp i\left(\frac{q^2}{2mx}\frac{y_1^2}{z}+mxz\right)\varphi(z) \qquad (A9)$$

or

$$F=\Phi(x)=\int dz\varphi(z)\exp(imxz). \qquad (A10)$$

In our case the function Π_1 can be represented in the form

$$\Pi_1(q)=\int\frac{d^4y}{(2\pi)^4}\frac{\gamma_\mu\hat{y}\gamma_\nu}{y^4}\langle N|T\{\bar{j}_\mu'(y)j_\nu'(0)\}|N\rangle e^{-i(qy)} \qquad (A11)$$

where $j_\mu'(y) = u(y)C\gamma_\mu u(y)$. Equation (A11) can be transformed to the form

$$\Pi_1(q)=\int \frac{d^4y}{(2\pi)^3}\frac{1}{y^8}y_\alpha\langle N|\bar{u}(y)\gamma_\alpha u(0)|N\rangle e^{-i(qy)}, \tag{A12}$$

or, with allowance for (A7),

$$\Pi_1(q)=2(pq)\int \frac{dy_1{}^2\,dz\,d^2y_t}{y^4}\exp\left(\frac{iq^2y_1{}^2}{2mxz}+imxz\right)\varphi(z). \tag{A13}$$

After the integration over y_t and $y_1^{1/2}$ we obtain

$$\Pi_1(q^2,x)=-4\pi\frac{q^2}{x}\int\frac{dz}{z}\varphi(z)\ln\frac{q^2}{2mxz}\exp(imxz). \tag{A14}$$

In Eq. (A14) we can set $\ln(q^2/2mxz)=\ln q^2$, since the Borel transformation[6] annihilates the polynomials. Using (A10), we obtain

$$\Pi_1(q^2,x)=-4\pi\frac{q^2}{r}\ln q^2\int_0^x\Phi(\alpha)\,d\alpha. \tag{A15}$$

Since in first order of the expansion in powers of q^2 we have $x=1$ [see Eq. (22)], we arrive at the expression (19) in the same approximation. To take the next terms of the expansion into account requires knowledge of the function $\varphi(z)$, or, equivalently, knowledge of the moments of the structure function of the nucleons.

The above can be extended in an obvious manner to the calculation of expectation values over nuclear matter.

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