Resonance tunneling and destruction of the quantum Hall effect in strong electric fields

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In the plateau regions in which the quantum Hall effect is manifested, the dissipative current depends nonlinearly on the electric field. This current varies strongly as a function of the occupation of the Landau level. A decisive role in the formation of the dissipative current is played by resonance tunneling via bound states of electrons at impurities. The conditions for which the largest contribution to the current comes from scattering of electrons by just one impurity are found. For these conditions the dependence of the dissipative current on the electric field and on the total electron density is found.

1. INTRODUCTION

In the conditions of the quantum Hall effect the longitudinal conductivity vanishes at zero temperature. Nevertheless, a longitudinal current with a nonlinear dependence on the electric field E exists. This dissipative current is induced by the interaction of electrons with impurities and phonons.

The experiments of Refs. 1 and 2 showed that the longitudinal current increases when E exceeds a certain critical value E_c. The electric field is related uniquely to the electron drift velocity $v_D = cE/H$. In Refs. 1 and 2 it was discovered that the v_D corresponding to E_c is close to the sound velocity s. As a consequence theories have appeared[3–5] that have explained the dissipation by Čerenkov emission of phonons. It is difficult to compare results obtained in Refs. 3 and 4 with experiment because the current distribution in the samples is highly nonuniform.[6] Moreover, in small samples[7–9] the drift velocity v_D corresponding to E_c exceeds the sound velocity by at least an order of magnitude. It is clear that Čerenkov emission alone does not determine the dissipation.

In the present paper it is shown that under certain conditions the leading role in creating the dissipative current is played by tunneling processes with the participation of impurities. In the process of tunneling the energy is conserved. The dissipation is ensured by the emission of phonons. We assume the temperature to be zero. This enables us to disregard electron scattering by phonons and consider only spontaneous emission of phonons by electrons. It is assumed that the phonons have the possibility of escaping freely from the system, thereby protecting the lattice from being heated.

The paper is organized as follows. In Sec. 2 we consider the elementary act of tunneling through a resonance impurity. The conditions under which this process is dominant in comparison with tunneling through chains of impurities are elucidated. In Sec. 3 we consider the kinetics of a system of electrons and holes with the participation of processes of interband tunneling, tunneling from impurities into a band, intraband and interband transitions, and also transitions from a band to an impurity with the emission of phonons. The dependence of the current on the electric field and on the occupation of the Landau level is found. In Sec. 4 the current fluctuations are considered.

2. INTERBAND RESONANCE TUNNELING

We shall consider an electron moving in a plane under the action of a magnetic field H perpendicular to the plane and an electric field E lying in the plane. We choose the x axis along the electric field and fix the Landau gauge $A_y = Hx$. The electrons are characterized by a conserved momentum component p along the y axis. The wavefunctions $\psi^0_{np}(r)$ of the electrons in the presence of the electric field have the same form as in the absence of the field, and the energy levels depend linearly on p:

$$\varepsilon_{np}=\hbar\omega_c(n+{}^1/_2)-eE(cp/eH), \tag{1}$$

where $\omega_c = eH/m^*c$ is the cyclotron frequency. We recall that the quantity $x = cp/eH$ is the average electron coordinate. Direct transitions from one Landau band to another are allowed by energy conservation but forbidden by momentum conservation. This prohibition can be lifted by virtue of scattering by impurities or phonons.

We shall start from an interband transition with scattering by an impurity. Every impurity, including a repulsive impurity, has bound states in the gaps between the Landau levels. Resonance tunneling via these bound states is always much more effective than nonresonance tunneling. Since this is so, we need to know the exact wavefunctions of the electrons near the impurities. We take a model of a point impurity with potential

$$V(\mathbf{r})=\lambda\delta(\mathbf{r}). \tag{2}$$

The energy levels of an electron in the field of such an impurity have been found by Prange.[10]

We shall find the exact wavefunctions of the stationary states of a particle in the field of a point impurity. It is convenient to consider this problem in a more general form for a certain Hamiltonian $H = H_0 + V(\mathbf{r})$ with potential $V(\mathbf{r})$ defined by Eq. (2). The Green's function has the form

$$G_\varepsilon(\mathbf{r},\mathbf{r}')=G_\varepsilon{}^0(\mathbf{r},\mathbf{r}')+\frac{G_\varepsilon{}^0(\mathbf{r},0)\lambda G_\varepsilon{}^0(0,\mathbf{r}')}{1-\lambda G_\varepsilon{}^0(0,0)}, \tag{3}$$

where

$$G_\varepsilon{}^0(\mathbf{r},\mathbf{r}')=\sum_\alpha\frac{\psi_\alpha{}^0(\mathbf{r})\psi_\alpha{}^{0*}(\mathbf{r}')}{\varepsilon-\varepsilon_\alpha+i0} \tag{4}$$

is the unperturbed Green's function of the Hamiltonian H_0, and α labels the states with energy ε_α. By the usual procedure we obtain the wavefunctions of the states in the field of the impurity:

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$$\psi_\alpha(\mathbf{r}, t) = \lim_{t'\to-\infty} \int G_\varepsilon(\mathbf{r}, \mathbf{r}', t-t')\psi_\alpha{}^0(\mathbf{r}', t')\,d\mathbf{r}'$$
$$= \left[\psi_\alpha{}^0(\mathbf{r}) + \frac{\lambda G^0_{\varepsilon_\alpha}(\mathbf{r}, 0)}{1-\lambda G^0_{\varepsilon_\alpha}(0, 0)}\psi_\alpha{}^0(0)\right]\exp(-i\varepsilon_\alpha t). \quad (5)$$

The scattering amplitude is equal to the projection of ψ_α on the unperturbed state $\psi^0_{\alpha'}$:

$$M_{\alpha\alpha'} = \frac{\lambda\psi_\alpha{}^0(0)\psi_{\alpha'}^{0*}(0)}{(\varepsilon_\alpha-\varepsilon_{\alpha'}+i0)(1-\lambda G_{\varepsilon_\alpha}(0,0))}. \quad (6)$$

Hence, for the transition probability per unit time we find

$$W_{\alpha\alpha'} = \frac{2\pi}{\hbar}\frac{\delta(\varepsilon_\alpha-\varepsilon_{\alpha'})|\phi_\alpha|^2|\phi_{\alpha'}|^2}{|1/\lambda-K(\varepsilon_\alpha)|^2}, \quad (7)$$

where

$$\phi_\alpha \equiv \psi_\alpha{}^0(0), \quad K(\varepsilon) = G_\varepsilon{}^0(0,0).$$

Equation (7) has the typical form of the Breit-Wigner probability of scattering via a bound state. The energy ε_0 of the bound state is determined by the equation

$$1/\lambda - \operatorname{Re} K(\varepsilon_0) = 0.$$

The two factors in the numerator of the right-hand side of Eq. (7) determine the probabilities of arrival in and departure from the bound state, and the imaginary part

$$\operatorname{Im} K(\varepsilon_0) = \pi \sum_\alpha \delta(\varepsilon_0-\varepsilon_\alpha)|\phi_\alpha|^2 \quad (8)$$

is determined by all the possible decay channels of the bound state.

In our specific problem α is the set consisting of the integer index n labeling the Landau levels and the y-component p of the momentum. We are particularly interested in the case when $n = 0$ in the initial state and $n = 1$ in the final state. In this situation α and α' are represented entirely by the initial values and final values of the momenta p and p', or by the coordinates x and x' corresponding to them. The law of conservation of energy establishes a relation between the difference $x' - x$ and the electric field:

$$x' - x \equiv \Delta = \hbar\omega_c/eE. \quad (9)$$

The geometrical meaning of Δ is clarified in Fig. 1. It is the distance through which the electron should hop in tunneling from a "valence" Landau band to a "conduction" Landau band.

For small electric fields the magnitude of Δ substantially exceeds the magnetic length. For this reason an electron executing tunneling can be regarded as quasiclassical. Then the given impurity creates an electric current

$$I = e\sum_{x_1x_2} W_{x_1x_2}. \quad (10)$$

Here it is assumed that the first band is almost filled and the second is almost empty. For small electric fields this assumption is fulfilled by virtue of the exponential smallness of the tunneling current (10).

The expression (7) has a sharp maximum at $\varepsilon_\alpha = \varepsilon_0$. Therefore, in the asymptotic limit as $E\to 0$, we have

$$I = \frac{e\Delta^2}{\hbar}\frac{|\phi_{0,p_0}|^2|\phi_{1,p_1}|^2}{\partial K/\partial\varepsilon|_{\varepsilon=\varepsilon_0} 2\operatorname{Im} K(\varepsilon_0)}, \quad p_n = \frac{\hbar\Delta}{l_H^2}\left(\xi + \frac{1}{2} - n\right), \quad (11)$$

where the dimensionless parameter $\xi = \varepsilon_0/\hbar\omega_c - 1$ characterizes the deviation of the bound-state energy from the middle of the gap between the Landau levels. The expression (11) can be rewritten in the form

$$I = e\frac{W_0W_1}{W_0+W_1}, \quad (12)$$

where

$$W_n = \frac{\Delta}{\hbar}\frac{|\phi_{n,p_n}|^2}{\partial K/\partial\varepsilon|_{\varepsilon=\varepsilon_0}} \quad (13)$$

is the probability of departure of the electron from the impurity to the nth Landau level.

The current (12) is a maximum when the bound state in the absence of the electric field lies halfway between the Landau levels. Near the maximum the current can be written in the form

$$I = \frac{\pi^{1/2}}{4(1+\pi^2/8)}(e\omega_c)\left(\frac{\Delta}{l_H}\right)^3$$
$$\times \frac{\exp(-\Delta^2/4l_H^2)}{\exp[\xi(\Delta/l_H)^2] + (\Delta^2/2l_H^2)\exp[-\xi(\Delta/l_H)^2]}. \quad (14)$$

In sufficiently small samples the electron and hole that have been formed do not have time to recombine before emerging from the system. In such samples the total current is equal to the sum of the currents (14) generated by separate impurities.[16] In large samples, when there is time for recombination to occur (see Sec. 3), the current decreases. Therefore the current (14) that we have obtained determines an upper bound on the contribution from an individual impurity. In practice, this current is accessible to measurement only when the ratio Δ/l_H is not too large. When the quantity Δ/l_H approaches unity, the current I is equal in order of magnitude to $e\omega_c$, i.e., is $\sim 1\,\mu$A when $H = 10$ T.

The current $I(\xi)$ decreases exponentially with increase of $|\xi|$. Assuming that the dimensionless energy density of bound states $\rho(\xi + \frac{1}{2})$ depends weakly on ξ near the middle of the gap between the Landau levels, we obtain the average value of the current produced by one impurity:

$$\bar{I} = \frac{\pi(2\pi)^{1/2}}{\pi^2+8}\rho(1/2)(e\omega_c)\exp\left(-\frac{\Delta^2}{4l_H^2}\right). \quad (15)$$

The problem of tunneling from a filled Landau level to an empty one was first considered by Tavger and Erukhimov.[11] They studied a nonresonance transition and obtained a

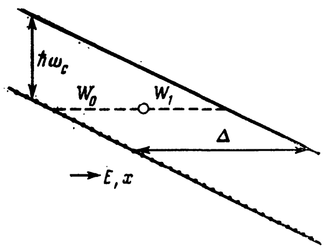

FIG. 1.

much smaller exponential: $-\exp[\frac{1}{2}(\Delta/l_H)^2]$. Chaplik and Éntin,[12] and then Lifshits and Kirpichenkov,[13] considered resonance tunneling via bound states without a magnetic field.

Tunneling transitions with the participation of one impurity are accompanied by a change of the electron coordinates by an amount of order Δ. The influence of other impurities is unimportant only in the case when they are at a distance greater than Δ from the given impurity. For a small concentration of impurities this is the case for most of the impurities.

To estimate how small the impurity concentration needs to be we shall consider tunneling through a pair of point impurities, with strengths λ_1 and λ_2, placed at the points $\mathbf{r}_1$ and $\mathbf{r}_2$. The electron Green's function in the field of these impurities has the form

$$G_\varepsilon(\mathbf{r},\mathbf{r}')=G_\varepsilon^0(\mathbf{r},\mathbf{r}')+\sum_{ij=1,2} G_\varepsilon^0(\mathbf{r},\mathbf{r}_i)[M^{-1}]_{ij}G_\varepsilon^0(\mathbf{r}_j,\mathbf{r}'), \quad (16)$$

where

$$M=\begin{pmatrix} 1/\lambda_1-G_\varepsilon^0(\mathbf{r}_1,\mathbf{r}_1) & -G_\varepsilon^0(\mathbf{r}_1,\mathbf{r}_2) \\ -G_\varepsilon^0(\mathbf{r}_2,\mathbf{r}_1) & 1/\lambda_2-G_\varepsilon^0(\mathbf{r}_2,\mathbf{r}_2) \end{pmatrix}. \quad (17)$$

Hence the probability of a transition between the Landau levels per unit time is equal to

$$W_{pp'}=\frac{2\pi}{\hbar}\delta(\varepsilon_{0p}-\varepsilon_{1p'})\left|\sum_{ij}\phi_{0p}^*(\mathbf{r}_i)[M^{-1}]_{ij}\phi_{1p'}(\mathbf{r}_j)\right|^2. \quad (18)$$

The probability (18) is a maximum when the coordinates y and the bound-states energies in the given electric field are nearly the same for both impurities.

The tunneling current produced by the pair of impurities is the sum of the probabilities $W_{pp'}$ over all initial and final states. It is equal to

$$I=\int\frac{dp\,dp'}{(2\pi\hbar)^2}\frac{2\pi}{\hbar}\frac{\delta(\varepsilon_{0p}-\varepsilon_{1p'})|\phi_{0p}^*(\mathbf{r}_1)G_\varepsilon^0(\mathbf{r}_1,\mathbf{r}_2)\phi_{1p'}(\mathbf{r}_2)|^2}{|\det M|^2}. \quad (19)$$

The denominator of the integrand in (19) is a minimum when the energy of the electron is close to the energies of the bound states, i.e., when the equalities

$$1/\lambda_1=\mathrm{Re}\,G_\varepsilon^0(\mathbf{r}_1,\mathbf{r}_1),\quad 1/\lambda_2=\mathrm{Re}\,G_\varepsilon^0(\mathbf{r}_2,\mathbf{r}_2) \quad (20)$$

are fulfilled. Since the system is placed in an electric field, the Green function $G_\varepsilon(r,r)$ depends explicitly on r. In Eqs. (19) and (20) all the $G_\varepsilon(\mathbf{r},\mathbf{r}')$ are taken at the value $\varepsilon=\varepsilon_{0p}\equiv\varepsilon_p$.

We shall average the current (19) over the energies of the states bound at the impurities. We obtain

$$I=\frac{\pi}{\Delta\hbar^3}\int dp\,dp'\frac{\delta(\varepsilon_{0p}-\varepsilon_{1p'})\rho_1(\varepsilon_p)\rho_2(\varepsilon_p)}{\partial G_\varepsilon^0(\mathbf{r}_1,\mathbf{r}_1)/\partial\varepsilon|_{\varepsilon=\varepsilon_p}\partial G_\varepsilon^0(\mathbf{r}_2,\mathbf{r}_2)/\partial\varepsilon|_{\varepsilon=\varepsilon_p}}$$
$$\times\frac{|\phi_{0p}(\mathbf{r}_1)||G_{\varepsilon_p}^0(\mathbf{r}_1,\mathbf{r}_2)|^2|\phi_{1p'}(\mathbf{r}_2)|}{\{|{}^1/{}_2\Delta\phi_{0p}(\mathbf{r}_1)\phi_{1p'}(\mathbf{r}_2)|^2+|G_{\varepsilon_p}^0(\mathbf{r}_1,\mathbf{r}_2)|^2\}^{1/2}}, \quad (21)$$

where $\rho_i(\varepsilon)=\rho(\varepsilon/\hbar\omega_c+x_i/\Delta-1/2)$.

The expression (21) is a maximum when the distance between the impurities is equal to $\Delta/2$. Averaging over the positions of the impurities, we obtain

$$I\sim c\rho({}^1/{}_4)\rho({}^3/{}_4)\exp[-{}^1/{}_8(\Delta/l_H)^2], \quad (22)$$

where c is the impurity concentration. Comparing this current with the current (15) produced by one impurity, we find that the contribution of pairs of impurities can be neglected when

$$\rho({}^1/{}_4)\rho({}^3/{}_4)cl_H^2\ll\exp[-{}^1/{}_8(\Delta/l_H)^2]\rho({}^1/{}_2). \quad (23)$$

For a given impurity concentration it is always possible to violate the inequality (23) by decreasing the electric field. Interband tunneling will then proceed through chains of impurities. Such a situation was described by Shklovskiĭ.[14] Resonance tunneling through two impurities in the absence of a magnetic field was considered in Ref. 17.

3. THE DISSIPATIVE CURRENT AS A FUNCTION OF THE OCCUPATION OF THE LEVEL

The interaction of electrons with impurities cannot change their energies. In order to find the dissipative current, it is necessary to take into account the interaction with phonons. Since the temperature is assumed to be zero, only emission of phonons is possible. For $v_D>s$, the laws of energy and momentum conservation permit electrons to move along the upper Landau level while emitting phonons.[4] But for $v_D<s$, this is impossible in an impurity-free system. Impurities, however, by accepting the excess momentum, allow transitions along one Landau level in this case as well.

Tunneling processes lead to the appearance of holes in the first Landau level and electrons in the second Landau level. The electrons and holes can recombine with the emission of phonons. Recombination is possible without change of the electron coordinates. Therefore, its probability does not become exponentially small in a small electric field.

Recombination processes open up additional decay channels of a state bound to an impurity. This leads to broadening of the resonance level, and weakens the resonance-tunneling effect even for an exponentially small number of electrons in the second Landau level or holes in the first Landau level.

Tunneling transitions with the participation of phonons occur much more slowly than the analogous elastic tunneling processes. This is due to the smallness of the dimensionless electron-phonon coupling constant. In the kinetic equations we neglect the process of impurity-free interband tunneling. The probability of this process is exponentially small in the electric field in comparison with tunneling through the impurity. To simplify the analysis we also neglect transitions between states localized at impurities, assuming the impurities to be rare. This implies that we shall not consider effects associated with hopping conduction. The low-impurity-density approximation limits the applicability of the given theory to a region of sufficiently strong electric fields. We note that the nonlinear dissipative current is accessible to measurement only in this region.

The electron density N_e in the upper Landau level and hole density N_h in the lower Landau level obey the following kinetic equations:

$$\frac{\partial N_e(\mathbf{r})}{\partial t}=\sum_{r'}\mathscr{P}_e(\mathbf{r}',\mathbf{r})N_e(\mathbf{r}')-N_e(\mathbf{r})\sum_{r'}\mathscr{P}_e(\mathbf{r},\mathbf{r}')-v_D\frac{\partial N_e}{\partial y}+\sum_i n_iW_e(i,\mathbf{r})$$
$$-N_e(\mathbf{r})\sum_i(1-n_i)Q_e(i,\mathbf{r})-N_e(\mathbf{r})\sum_{r'}N_h(\mathbf{r}')Q(\mathbf{r},\mathbf{r}'), \quad (24)$$

$$\frac{\partial N_h(\mathbf{r})}{\partial t}=\sum_{r'}\mathscr{P}_h(\mathbf{r}',\mathbf{r})N_h(\mathbf{r}')-N_h(\mathbf{r})\sum_{r'}\mathscr{P}_h(\mathbf{r},\mathbf{r}')-v_D\frac{\partial N_h}{\partial y}$$
$$+\sum_i(1-n_i)W_h(i,\mathbf{r})-N_h(\mathbf{r})\sum_i n_iQ_h(i,\mathbf{r})$$
$$-N_h(\mathbf{r})\sum_{r'}N_e(\mathbf{r}')Q(\mathbf{r}',\mathbf{r}). \quad (25)$$

Here $\mathscr{P}_e(\mathbf{r}',\mathbf{r})$ is the probability of scattering of an electron in the upper Landau level. The quantity $W_e(i,\mathbf{r})$ is the probability of a tunneling transition from the ith impurity to the upper Landau level, and $Q_e(i,\mathbf{r})$ is the probability that an electron falls from the upper Landau level to the impurity with emission of a phonon. The analogous notation with the subscript h refers to holes in the lower Landau level. The equation also contains the probability $Q(\mathbf{r},\mathbf{r}')$ of recombination of electrons and holes. The quantity n_i is the average number of electrons at the ith impurity. The population of each impurity is also determined from the kinetic equation

$$\frac{\partial n_i}{\partial t}=-n_i\sum_r W_e(i,\mathbf{r})+(1-n_i)\sum_r W_h(i,\mathbf{r})$$
$$-n_i\sum_r N_h(\mathbf{r})Q_h(i,\mathbf{r})+(1-n_i)\sum_r N_e(\mathbf{r})Q_e(i,\mathbf{r}). \quad (26)$$

We shall assume that the system is spatially uniform. We introduce the average density $n(\varepsilon)$ of electrons occupying impurity levels with energy ε, the latter being measured in units of $\hbar\omega_c$ and reckoned from the lower Landau level. In a large sample the stationary densities of electrons and holes do not depend on the coordinates and satisfy the equations

$$c\int n(\varepsilon)\rho(\varepsilon)W_e(\varepsilon)d\varepsilon$$
$$-cN_e\int(1-n(\varepsilon))\rho(\varepsilon)Q_e(\varepsilon)d\varepsilon-N_eN_hQ=0, \quad (27)$$
$$c\int[1-n(\varepsilon)]\rho(\varepsilon)W_h(\varepsilon)d\varepsilon$$
$$-cN_h\int n(\varepsilon)\rho(\varepsilon)Q_h(\varepsilon)d\varepsilon-N_eN_hQ=0, \quad (28)$$
$$n(\varepsilon)=\frac{W_h(\varepsilon)+N_eQ_e(\varepsilon)}{W_e(\varepsilon)+W_h(\varepsilon)+N_eQ_e(\varepsilon)+N_hQ_h(\varepsilon)}, \quad (29)$$

which follow from (24)–(26). Here $W_e(\varepsilon)$ is the total probability of tunneling from the impurity to the upper Landau level, and $W_h(\varepsilon)$ is the total probability of tunneling from the lower Landau level to the impurity. These probabilities are determined with good accuracy by the expression (13) with $n=1$ and $n=0$. The quantity $Q_e(\varepsilon)$ is the probability of transition of an electron from the upper Landau level to the impurity with emission of a phonon, and $Q_h(\varepsilon)$ is the probability of transition of an electron from the impurity to the lower Landau level with emission of a phonon. The scheme of the processes is depicted in Fig. 2.

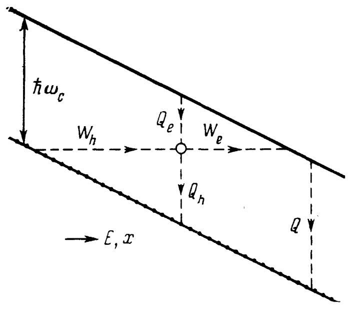

FIG. 2.

Knowing the transition probabilities, it is not difficult to find an expression for the dissipative-current density:

$$I=\eta_eN_e+\eta_hN_h+c\Delta\int[n(\varepsilon)(1-\varepsilon)W_e(\varepsilon)$$
$$+[1-n(\varepsilon)]\varepsilon W_h(\varepsilon)]\rho(\varepsilon)d\varepsilon. \quad (30)$$

Here

$$\eta_e=-\sum_r x\mathscr{P}_e(\mathbf{r},0),\qquad \eta_h=-\sum_r x\mathscr{P}_h(0,\mathbf{r}).$$

Of the equations (27)–(29) only two are independent. Therefore, in order to find N_e, N_h, and $n(\varepsilon)$, it is necessary to use an additional condition. In experiment one usually specifies the occupation ν, or, equivalently, the total electron density N:

$$N=N_e-N_h+c\int\rho(\varepsilon)n(\varepsilon)d\varepsilon. \quad (31)$$

The equations (28)–(31) determine the parametric dependence of the dissipative-current density on the total electron density N. The parameters are N_e and N_h.

In Ref. 16 we considered the problem of the destruction of the quantum Hall effect in a somewhat different formulation. The distribution of electrons and holes obtained in Ref. 16 depended on the coordinate x. This was connected with the small size of the system considered there. In the problem there are two characteristic lengths. One of them, L_0, is defined as the distance at which the growth of the number of carriers is limited by the recombination of electrons and holes. In order of magnitude, $L_0\sim\eta/cQ_e(1/2)$. The other characteristic length is the electric-charge screening length r_0. A satisfactory theory of screening for the quantum Hall effect does not exist at present. In Ref. 16 we assumed that L_x is much smaller than both L_0 and r_0. Here, on the contrary, it is assumed $L_x\gg L_0$, r_0.

Let the number N of electrons be smaller than the number of levels at the impurities. It is natural to expect that in weak fields the distribution of electrons over the impurities will be almost the same as in zero field, i.e., impurity levels with $\varepsilon<\varepsilon_0(N)$ will be filled. If ε_0 is not too close to one of the Landau levels, N_e and N_h are exponentially small. This assumption is consistent with Eqs. (27)–(29), and it follows from (29) that the smearing of the energy ε_0 has magnitude $\sim l_h^2/\Delta^2$.

Explicit solutions of the system (27)–(29), (31) can be obtained in two limiting cases—near the middle and the edge of the Hall plateau. We start from the first of these cases, corresponding to $\varepsilon_0\approx 1/2$. In Eqs. (27)–(29) the quantities that vary most rapidly with energy are the tunneling-transition probabilities $W_e(\varepsilon)$ and $W_h(\varepsilon)$, which contain the exponential factors $\exp[-(1-\varepsilon)^2\Delta^2/l_H{}^2]$ and $\exp(-\varepsilon^2\Delta^2/l_H{}^2)$. Near $\varepsilon=1/2$ these probabilities can be represented in the following form:

$$W_e=\gamma(\Delta^2/2l_H{}^2)\exp[-(\Delta^2/4l_H{}^2)(1-4\xi)], \quad (32)$$
$$W_h=\gamma\exp[-(\Delta^2/4l_H{}^2)(1+4\xi)], \quad (33)$$

where, as before, $\varepsilon = 1/2 + \xi$, and γ is a constant which can be found from Eq. (13). We shall regard the quantities $\rho(\varepsilon)$, $Q_e(\varepsilon)$, and $Q_h(\varepsilon)$ as slowly varying functions of ε and replace them by the corresponding values at the point $\varepsilon = 1/2$.

In order that, on the one hand, the impurity levels be largely filled upon change of N, and, on the other hand, it be possible to neglect tunneling through chains of impurities, it is necessary that the inequalities

$$\exp(-\Delta^2/4l_H^2) \ll c l_H^2 \ll \exp(-\Delta^2/8l_H^2). \quad (34)$$

be fulfilled. We introduce the notation

$$N_e Q_e(1/2) = \alpha\gamma \frac{\Delta}{l_H\sqrt{2}} \exp(-\Delta^2/4l_H^2),$$
$$N_h Q_h(1/2) = \beta\gamma \frac{\Delta}{l_H\sqrt{2}} \exp(-\Delta^2/4l_H^2). \quad (35)$$

Then the system (27)–(29) reduces to the equation

$$g(1-\alpha\beta) \int_0^\infty \frac{dy}{y^2 + y(\alpha+\beta) + 1} = \alpha\beta, \quad (36)$$

where

$$g = c\rho(1/2) \frac{Q_e(1/2) Q_h(1/2)}{Q} \frac{\sqrt{2}}{\gamma} \frac{\exp(\Delta^2/4l_H^2)}{(\Delta/l_H)^3}. \quad (37)$$

We shall assume that $g \gg 1$. Then near the center of the plateau it follows from (36) that $\alpha\beta = 1$. In this case Eq. (31) reduces to the relation

$$N = c\int_0^{1/2} \rho(\varepsilon)\, d\varepsilon + c\rho(1/2) \frac{l_H^2}{\Delta^2} \ln\left(\frac{\alpha l_H \sqrt{2}}{\Delta}\right). \quad (38)$$

The first term of the right-hand side of (38) corresponds to filling of the impurity levels up to exactly the middle of the gap between the Landau levels. The second term takes into account small deviations from this filling. In the approximations made the dissipative current is equal to

$$I = \eta_e N_e + \eta_h N_h$$
$$= \gamma \frac{\Delta}{l_H\sqrt{2}} \exp\left(-\frac{\Delta^2}{4l_H^2}\right)\left(\frac{\eta_e}{Q_e(1/2)}\alpha + \frac{\eta_h}{Q_h(1/2)}\frac{1}{\alpha}\right). \quad (39)$$

The dissipative current as a function of the occupation reaches a minimum

$$I_{min} = \gamma \frac{\Delta}{l_H\sqrt{2}} \exp\left(-\frac{\Delta^2}{4l_H^2}\right) \cdot 2\left(\frac{\eta_e \eta_h}{Q_e(1/2) Q_h(1/2)}\right)^{1/2} \quad (40)$$

at a value $N_{\min}$ that depends on the electric field:

$$N_{min} = c\int_0^{1/2} \rho(\varepsilon)\, d\varepsilon + c\rho(1/2) \frac{l_H^2}{2\Delta^2} \ln\left(\frac{2l_H^2}{\Delta^2} \frac{\eta_h Q_e}{\eta_e Q_h}\right). \quad (41)$$

We now consider the occupation of the impurity levels near the edge of the Hall plateau. When the condition

$$c l_H^2 \gg \exp(-\Delta^2/4l_H^2) \quad (42)$$

is fulfilled, we can neglect the occupation numbers of the hole states and set the quantities N_h and W_h equal to zero in (29). Then the occupation of the impurity levels is determined by the formula

$$n(\varepsilon) = \frac{N_e Q_e(\varepsilon)}{N_e Q_e(\varepsilon) + W_e}. \quad (43)$$

The quantity $W_e(\varepsilon) \sim \exp[-(\Delta^2/l_H^2)(1-\varepsilon)^2]$ depends exponentially on ε. For $W_e(\varepsilon) < N_e Q_e(\varepsilon)$ the impurity states are practically completely filled, while for $W_e(\varepsilon) > N_e Q_e(\varepsilon)$ they are empty. We specify the energy ε_0 by the relation

$$W_e(\varepsilon_0) = N_e Q_e(\varepsilon_0). \quad (44)$$

The width of the energy range over which $n(\varepsilon)$ changes from unity to zero is, in order of magnitude,

$$l_H^2/\Delta^2(1-\varepsilon_0). \quad (45)$$

The total electron density is equal to

$$N = c\int_0^{\varepsilon_0} \rho(\varepsilon)\, d\varepsilon. \quad (46)$$

Here we have neglected the occupation of the upper Landau level. It follows from (44) that this is valid under the condition

$$\rho(\varepsilon_0) l_H^2 c \gg \exp(-\Delta^2(1-\varepsilon_0)^2/l_H^2). \quad (47)$$

Near the edge of the plateau we take account of transitions only between the upper Landau level and the impurity levels. We neglect transitions from impurity to impurity and scattering by several impurities; this is valid when

$$\rho_0(\varepsilon_0) l_H^2 c \ll \exp(-\Delta^2(1-\varepsilon_0)^2/2l_H^2). \quad (48)$$

The conditions (47) and (48) admit a larger $\rho(\varepsilon_0)$ than (34). This is connected with the fact that near the edge of the plateau the necessary tunneling length is smaller than at the center.

The dissipative current is determined by the first term of expression (30). It is equal to

$$I = \eta_e W_e(\varepsilon_0)/Q_e(\varepsilon_0). \quad (49)$$

With increase of N the energy ε_0 increases, according to (46), until the inequality (47) is violated. With further increase of N, delocalized states in the upper Landau level will be populated. This implies that the Hall plateau terminates after the impurity levels with energy

$$1-\varepsilon_0 \sim \frac{l_H}{\Delta}\left[\ln\left(\frac{1}{l_H^2 c}\right)\right]^{1/2} \quad (50)$$

are filled. Thus, the width of the Hall plateau decreases linearly as a function of the electric field. The linear dependence of the width of the plateau on the electric-field intensity has an extremely general character and is due entirely to the sharp dependence of the tunneling probability on the length of the jump.

4. FLUCTUATIONS

In experiments with narrow Hall bridges[7–9] samples with a characteristic size of 1 μm have been used. This size is substantially greater than the screening radius (which is about 400 Å, according to the estimate of Kane *et al.*[15]). Therefore, the total electron density in the sample can be regarded as given. The number of electrons in the sample is of the order of 10^4. The number of impurities, especially with deep bound levels, is substantially smaller. One may expect considerable fluctuations of the density of states, and associated fluctuations of the dependence of the current on the occupation. These fluctuations are described by the formula

$$d\ln I/dN = 2\Delta^2(1-\varepsilon_0)/l_H^2 c\rho(\varepsilon_0), \tag{51}$$

and, because of the smearing of the step in the electron distribution function $n(\varepsilon)$, the quantity $\rho(\varepsilon_0)$ appearing in (51) should be smoothed over an energy range $\delta\varepsilon \sim l_H{}^2/\Delta^2(1-\varepsilon_0)$.

The equation (51) shows that fluctuations of the density of states $\rho(\varepsilon)$ lead to fluctuations of the slope of the curve $I(N)$. Indeed, suppose that $\rho(\varepsilon)$ has a peak at a certain ε_1. Then in a certain range of variation of N only states with energy ε_1 will be occupied. In other words, as a function of N the quantity $\varepsilon_0(N)$ does not change, and therefore the current determined by Eq. (49) also does not grow. A step appears on the curve $I(N)$.

We shall consider the change of the position of the step as the electric field increases. For this we calculate the correction (associated with the smearing out of the distribution function) to the formula (46) determining the relationship between the number N of electrons and the quasi-Fermi level ε_0:

$$N = c\int_0^{\varepsilon_0} \rho(\varepsilon)\,d\varepsilon + c\rho(\varepsilon_0)\frac{\pi^2/24}{(1-\varepsilon_0)^3(\Delta/l_H)^4}. \tag{52}$$

It follows from Eq. (51) that the step moves in accordance with the law

$$\delta N \sim E^4. \tag{53}$$

Qualitatively, this behavior corresponds to the step motion that was observed in Ref. 8.

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