The thermodynamic and magnetic properties of a system of superconducting twinning planes

A. A. Abrikosov, A. I. Buzdin, M. L. Kulić and D. A. Kuptsov

Moscow State University

(Submitted 9 June 1988)
Zh. Eksp. Teor. Fiz. **95**, 371–383 (January 1989)

The influence of a system of twinning boundaries with local enhancement of superconducting pairing on the thermodynamic characteristics of the superconducting transition is examined. The temperature dependence of the specific heat of such a system is calculated. The results are compared to specific-heat measurements of single crystals of the high-temperature superconductor $YBa_2Cu_3O_{7-\delta}$. Josephson interaction of the superconducting twinning planes occurs above the bulk transition critical temperature. The behavior of the upper and lower critical fields is investigated in this region. The force of interaction between the superconducting vortex and the twinning plane is determined and the role of this interaction in vortex pinning is discussed.

1. INTRODUCTION

A characteristic feature of $YBa_2Cu_3O_{7-\delta}$ high-temperature superconductors is a developed twinned structure with a (110) twinning plane. The twinning boundaries form a regular sequence with a characteristic interboundary distance $L\sim 200$ Å–2000 Å (see, for example, Refs. 1–3). Twinning is caused by a structural transition at ~ 700 °C from the tetragonal to the orthorhombic phase.

Clearly, specific conditions for the superconducting transition can arise near the twinning plane (TP): both local enhancement and suppression of superconductivity. The influence of TP on the properties of regular superconductors has been noted already in Ref. 4. Experiments conducted by Khaikin and Khlyustikov[5] have demonstrated that a twinning plane in tin will produce superconductivity with a critical temperature T_c higher than the bulk critical temperature T_{c0} localized at the TP. TP-enhanced superconductivity also occurs in niobium[6,7] and a number of other simple metals.[5] A theory of twinning-plane superconductivity (TPS) based on a modified Ginzburg-Landau functional was developed in Refs. 8–10. This theory has made possible a rather comprehensive description of the properties of TPS in simple metals. A survey of experimental and theoretical research of this stage of investigation of TPS is given in Ref. 11.

Evidently the presence of twinning planes in $YBa_2Cu_3O_{7-\delta}$ high-temperature superconductors can also lead to local superconductivity enhancement and to an increase of T_c by a few degrees over C_{c0} ($T_c - T_{c0} \approx 4$–5 K). This is indicated by precision measurements of the specific heat of a $YBa_2Cu_3O_{7-\delta}$ single crystal,[12] revealing two specific-heat anomalies: a small anomaly at 93 K and a substantially greater anomaly at 89 K.[1)] Observation of the radical temperature dependence characteristic of TPS, of a critical field parallel to the twinning plane in oriented $YBa_2Cu_3O_{7-\delta}$ crystals has also been reported.[14] These results are naturally explained within the framework of twinning-plane superconductivity.[15]

The nature of the boundary which is the twinning plane in high-temperature superconductors is not yet clear. In Ref. 16 it is proposed that the TP has weak coupling properties, i.e., it is in fact an insulating interlayer making electron transitions between the twins difficult. In this case, however, superconductivity enhancement can also occur within each of the twins near the TP, and the thermodynamic properties of such a system will be identical to those in a model with a superconductivity order parameter that is continuous at the TP.[11,15] Finally, TP in high-temperature superconductors is also related to the interesting possibility of exotic superconductivity[17] where the phase of the order parameter changes by π in the transition through the TP.

In the present study we assume for definiteness the local superconductivity enhancement occurs near the TP and that the order parameter is continuous at the TP. We also briefly discuss the situation when the TP is impermeable to electrons.

In Sec. 2 is considered the temperature dependence of the specific heat for a twinning-boundary lattice. These results were partially published in short form in Refs. 15 and 18.

Near T_c the superconductivity is largely localized in the vicinity of the twinning planes, and their interaction is of a Josephson nature. This case is analyzed in Sec. 3 which also contains an exact calculation of the Josephson current between neighboring twinning planes.

In Sec. 4 is analyzed the vortex structure in a TP lattice in conditions of Josephson interaction, as are also the upper and lower critical fields of this system.

Finally in Sec. 5 we investigate vortex interaction with the TP at a temperature below the bulk superconducting transition temperature T_{c0}.

We note that our analysis also provides a description of superconducting superlattices[19] fabricated by layered sputtering of two superconductors when the thickness of the superconductor layers with the higher critical temperature is less than the superconductor correlation length ξ_0 yet the superlattice period exceeds ξ_0.

2. TEMPERATURE DEPENDENCE OF THE SPECIFIC HEAT FOR A PERIODIC TWINNED STRUCTURE

It is convenient to use for the description of TP superconductivity the modified Ginzburg-Landau functional[11]

$$F=a|\psi|^2+\frac{b}{2}|\psi|^4+\frac{1}{4m}\left|\left(\nabla-\frac{2ie}{c}\mathbf{A}\right)\psi\right|^2 -\gamma\sum_n\delta(x-nL)|\psi|^2; \qquad (1)$$

 0038-5646/89/010210-07$04.00

the x axis is assumed to be perpendicular to the TP, $\alpha = \tau/\eta$, $\tau = (T - T_{c0})/T_{c0}$ and standard notation is employed. The ony terms requiring special mention are the δ-function terms describing superconductivity enhancement near the TP ($x = nL$, $n = 0, \pm 1, \pm 2, \ldots$). The coefficient γ is directly related to the increasing critical temperature for a solitary TP ($L = \infty$):

$$[T_c(\infty) - T_{c0}]/T_{c0} \equiv \tau_0 = m\eta\gamma^2. \tag{2}$$

τ_0 represents the characteristic dimensionless temperature for TPS, and hence it is convenient to introduce the variable $t = \tau/\tau_0 = (T - T_{c0})/(T_c - T_{c0})$ in place of temperature. $t_c = 1$ corresponds to the transition temperature of a solitary TP, while the transition temperature for a periodic sequence of twinning planes is determined by the expression[9,11]

$$\frac{1}{t_c^{1/2}} \ln\left(\frac{t_c^{1/2}+1}{t_c^{1/2}-1}\right) = \frac{L}{\xi(\tau_0)}, \tag{3}$$

where $\xi(\tau_0) = (\eta/4m\tau_0)^{1/2} = \xi_0/\tau_0^{1/2}$ is the characteristic dimension of the region where superconductivity is localized near the TP.

The characteristic order parameter for describing TPS is $\psi_0 = (\tau_0/\eta b)^{1/2} = \gamma(m/b)^{1/2}$.[11] It is therefore convenient to use the dimensionless variables $\mathbf{r}' = \mathbf{r}/\xi(\tau_0)$ and $\psi' = \psi/\psi_0$. We shall henceforth drop the primes. The equation that follows for the order parameter from (1) takes in this notation the form

$$-\left(\nabla - \frac{2ie}{c}\xi(\tau_0)\mathbf{A}\right)^2\psi + t\psi + \psi|\psi|^2 = 2\sum_n \delta(x - nL)\psi. \tag{4}$$

It is necessary to limit the analysis of the specific heat of the TP system in this section to a solution of (4) that depends only on x, $\psi = \psi(x)$, and to set $\mathbf{A} = 0$. The solution $\psi(x)$ is periodic and it is sufficient to consider the interval $-\tilde{L}/2 \leqslant x \leqslant \tilde{L}/2$, where $\tilde{L} = L/\xi(\tau_0)$, and the role of the δ-function in (4) reduces to the boundary condition $\psi'|_{x=+0} = -\psi(0)(\psi'|_{x=-0} = \psi(0))$. The maximum order parameter $\psi = \psi_0$ is achieved at a TP for $x = 0$, while the minimum order parameter $\psi = \psi_1$ is achieved halfway between the twinning planes, for $x = \pm\tilde{L}/2$. The first integral of Eq. (4) is

$$-(\psi')^2 + t\psi^2 + \psi^4/2 = I, \tag{5}$$

and I is related to ψ_0 and ψ_1:

$$I = (t-1)\psi_0^2 + \psi_0^4/2 = t\psi_1^2 + \psi_1^4/2. \tag{6}$$

Taking advantage of the fact that the solution of Eq. (4) satisfies the condition $\delta F/\delta\psi = 0$, we can write the expression for the specific heat of an inhomogeneous superconducting system as

$$\frac{C}{\Delta C_0} = -\frac{2\xi(\tau_0)}{L}\frac{T}{T_{c0}}\frac{d}{dt}\left(\int_0^{\tilde{L}/2}\psi^2(x)\,dx\right), \tag{7}$$

where $\Delta C_0 = 1/T_{c0}\eta b$ is the jump of the specific heat in the transition to the superconducting state when $T = T_{c0}$ in a homogeneous superconductor (in the absence of TP).

Knowledge of the first integral (5) makes it possible to find the solution for $\psi(x)$ by quadratures and to go from integration with respect to dx to integration with respect to $d\psi$. We then arrive at the equation set

$$\frac{L}{2\xi(\tau_0)} = \int_{\psi_1}^{\psi_0}\frac{2^{1/2}\,d\psi}{[(\psi^2-\psi_1^2)(2t+\psi^2+\psi_1^2)]^{1/2}},$$
$$\psi_1^4 + 2t\psi_1^2 = \psi_0^4 + 2(t-1)\psi_0^2, \tag{8}$$
$$\frac{C}{\Delta C_0} = -\frac{T}{T_{c0}}\frac{2\xi(\tau_0)}{L}\frac{d}{dt}\left\{\int_{\psi_1}^{\psi_0}\frac{2^{1/2}\psi^2\,d\psi}{[(\psi^2-\psi_1^2)(2t+\psi^2+\psi_1^2)]^{1/2}}\right\}.$$

It is also possible to write (8) in terms of the incomplete elliptic integrals $F(\varphi,k)$ and $E(\varphi,k)$:

$$\frac{L}{2\xi(\tau_0)} = \frac{1}{[t+\psi_1^2]^{1/2}}F(\varphi,k),$$
$$\frac{C}{\Delta C_0} = -\frac{T}{T_{c0}}\frac{2\xi(\tau_0)}{L}\frac{d}{dt}\left\{\psi_1^2\frac{L}{2\xi(\tau_0)} - 2(t+\psi_1^2)^{1/2}E(\varphi,k)\right\},$$
$$\varphi = \arccos\left(\frac{\psi_1}{\psi_0}\right), \quad k = \left(\frac{2t+\psi_1^2}{2t+2\psi_1^2}\right)^{1/2}. \tag{9}$$

The temperature dependences of the specific heat for various $\tilde{L}$ calculated by formulae (8) are shown in Fig. 1.

In the case of a large distance between twinning planes $L \gg \xi(\tau_0)$ and $t \ll 1$ we obtain by expanding all integrals in (8), (9) in the parameter $t/\psi_1^2 \ll 1$

$$\frac{L}{2\xi(\tau_0)} = \frac{1}{\psi_1}\left[K - \frac{t}{\psi_1^2}(K-E)\right],$$
$$\frac{C(t=0)}{\Delta C_0} = \frac{(K-E)^2+E^2}{K^2} \approx 0.61, \tag{10}$$

where the complete elliptic integrals are $K = K(2^{-1/2})$ and $E = E(2^{-1/2})$, and the result is satisfaction of the following universal relation: The specific heat of the system is $C \to 0.61\Delta C_0$ when $T = T_{c0}$. When $T = T_c$ the jump in specific heat is not substantial: $\Delta C/\Delta C_0 = 2\xi(\tau_0)/L$.

Experimental investigations of the specific heat of a $YBa_2Cu_3O_{7-\delta}$ single crystal[12] have revealed an anomalous temperature dependence $C(T)$ similar to ours. A comparison with experiment[12] reveals that $\Delta C/\Delta C_0 \sim 1/6$ and therefore the period $L \sim 12\xi(\tau_0) \sim 10^3$ Å. This corresponds to the period of the usually observed twinned structure in $YBa_2Cu_3O_{7-\delta}$. The temperature dependence of the specific heat for $L/2\xi(\tau_0) = 6$, calculated from (8), is given togeth-

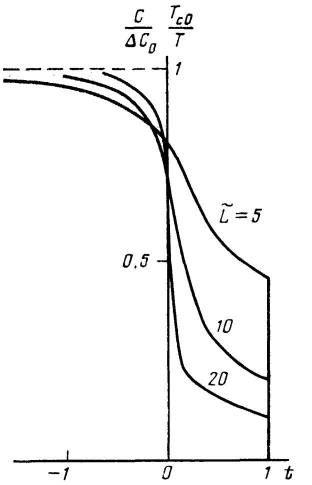

FIG. 1. Temperature dependences of the specific heat in the superconducting phase for a periodic system of twinning boundaries for various distances between the TP $\tilde{L} = L/\xi(\tau_0)$, $t = (T - T_{c0})/(T_c - T_{c0})$.

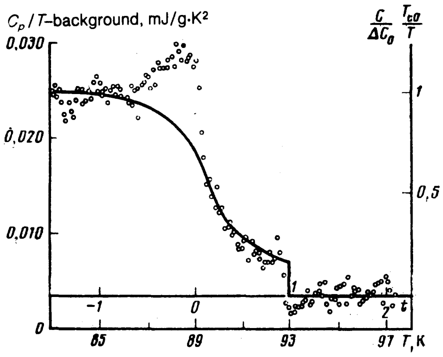

FIG. 2. Temperature dependence of the specific heat for a periodic TP system with $L/\xi(\tau_0) = 12$ together with measurement data on the specific heat of a $YBa_2Cu_3O_{7-\delta}$ single crystal[12]; $t = (T - T_{c0})/(T_c - T_{c0})$.

er with experimental data[12] in Fig. 2. The experimental data[12] clearly indicate that a small specific-heat peak occurs near T_{c0} and is apparently related to fluctuation effects neglected in our analysis.

3. JOSEPHSON INTERACTION BETWEEN SUPERCONDUCTING TWINNING PLANES

If the distance L between twinning planes exceeds the width of the localization region $\xi(\tau_0)$ of $\psi(x)$ near the twinning plane neighboring planes will have little mutual influence and the critical temperature will only slightly exceed the critical temperature of a solitary plane[9]:

$$t_c = 1 + 4\exp(-L/\xi(\tau_0)). \tag{11}$$

This formula is obtained from (3) in the limit $L/\xi(\tau_0) \gg 1$. In this case Josephson interaction between neighboring twinning planes should occur at a temperature $0 < t < t_c$. Using a description based on the functional (1) it is possible to obtain a complete solution to the problem of Josephson interaction of TP.

We will find the critical current between neighboring twinning planes. Assume a zero phase of the superconductivity parameter ψ at one plane ($x = 0$), and a phase φ at a neighboring plane ($x = L$). We will write the order parameter as $\psi(x) = fe^{i\varphi}$, where f is the amplitude and φ the phase. Then the equations that follow from (4) for f and φ take the form

$$-f'' + (\varphi')^2 f + tf + f^3 = 0, \tag{12}$$

$$2\varphi' f' + \varphi'' f = 0, \tag{12'}$$

and $f'(x = +0) = -f(0)$ at the twinning plane, while the density of the current J flowing between the planes is

$$J = \psi_0^2 \frac{e}{m} \frac{1}{\xi(\tau_0)} j, \quad j = f^2 \varphi'. \tag{13}$$

Eliminating the phase φ from (12) we obtain an equation for f:

$$-f'' + j^2/f^3 + tf + f^3 = 0, \tag{14}$$

which has a first integral

$$-(f')^2 + tf^2 + f^4/2 - j^2/f^2 = I_1. \tag{15}$$

The maximum value $f(0) = f_0$ of the order-parameter modulus f is achieved at the TP, while the minimum value $f(L/2) = f_1$ is achieved between the twinning planes. Taking this into account, as well as the boundary condition on the TP, we write

$$I_1 = (t-1) f_0^2 + \frac{f_0^4}{2} - \frac{j^2}{f_0^2} = t f_1^2 + \frac{f_1^4}{2} - \frac{j^2}{f_1^2}. \tag{16}$$

Using the first integral (15) and relation (13) and going from integration with respect to x to integration with respect to the new variable $u = f^2/f_1^2$ we can obtain a solution to our problem by quadratures:

$$\frac{L}{\xi(\tau_0)} = \int_1^{(f_0/f_1)^2} \frac{du}{\{(u-1)(tu + f_1^2 u(u+1)/2 + j^2/f_1^4)\}^{1/2}}, \tag{17}$$

$$\varphi = \frac{j}{f_1^2} \int_1^{(f_0/f_1)^2} \frac{du}{u\{(u-1)[tu + f_1^2 u(u+1)/2 + j^2/f_1^4]\}^{1/2}}. \tag{18}$$

In the case of Josephson interaction $f_0 \gg f_1$ and we break up the integration range in (17) into two ranges: from 1 to f_0/f_1 and from f_0/f_1 to $(f_0/f_1)^2$, where it is possible to ignore $f_1^2 u(u+1)/2$ in the radicand of the first integral, while in the second integral we assume $f_1^2 u(u+1)/2 \approx f_1^2 u^2/2$ and ignore the contribution from j^2/f_1^4.

Carrying out the integrations, we obtain

$$\left(\frac{f_0}{f_1}\right)^2 = \frac{\exp(\mathcal{L} t^{1/2})}{2 j_0^2 F^2} [1 - (1 - j_0^2)^{1/2}], \tag{19}$$

$$j_0 = j \frac{\exp(\mathcal{L} t^{1/2})}{2 f_0^2 F^2 t^{1/2}}, \quad F = \frac{2}{1 + (1 + f_0^2/2t)^{1/2}}. \tag{19'}$$

It is possible to extend integration in the integral (18) to ∞, and since the primary contribution comes from the region $u \ll f_0/f_1$ we can ignore $f_1^2 u(u+1)/2$. Integrating and taking (19) into account we find that

$$\sin\varphi = j_0, \tag{20}$$

i.e., the superconducting current (in dimensionless units) is

$$j = 2 f_0^2 F^2 t^{1/2} \exp(-\mathcal{L} t^{1/2}) \sin\varphi, \tag{21}$$

or

$$J = J_c \sin\varphi, \quad J_c = \frac{8\psi_0^2 e f_0^2 t^{1/2} \exp(-\mathcal{L} t^{1/2})}{m\xi(\tau_0)[1 + (1 + f_0^2/2t)^{1/2}]^2}. \tag{22}$$

As regards f_0^2, if follows from (16) and (19) that, it is possible to assume $f_0^2 = 2(1 - t)$.

The applicability condition of our approach is the requirement $\mathcal{L} t^{1/2} \gg 1$ ($L \gg \xi(T)$), since it is this requirement that will satisfy [see (19)] the inequality $f_0/f_1 \gg 1$, i.e., Josephson interaction between the planes. As seen from (22), in the case of TPS the current and phase are related by the regular Josephson relation: a sinusoidal dependence of the current on the phase.

A twinned structure of the type formed in $YBa_2Cu_3O_{7-\delta}$ therefore consists of a system of multiple Josephson junctions whose critical current is determined by relation (22) when $T_{c0} < T < T_c$.

When $T < T_{c0}$ the critical current will already be determined by vortex-pinning effects. An entirely different temperature dependence of the critical current of the specimen can therefore be expected when $T < T_{c0}$ and $T_{c0} < T < T_c$. Our analysis has focused on a critical current perpendicular to the TP. An analysis of critical current along a TP is given in Ref. 11. An experimental analysis of the critical current of TP in single crystals is likely to encounter difficulties both due to the imperfection of the twinning planes themselves and due to twinning-boundary systems having two different orientations.[1–3]

4. THE MAGNETIC PROPERTIES OF A SYSTEM OF SUPERCONDUCTING TWINNING PLANES

If we know the dependence of the current on the phase difference of the order parameter at neighboring planes it is possible to write the free-energy functional of our system for the case of Josephson interaction of layers:

$$F=\frac{2\tau_0^2}{\eta^2 b}\xi(\tau_0)\sum_n\left\{\frac{t-1}{2}|\psi_n|^2+\frac{1}{2}|\nabla_\perp\psi_n|^2\right.$$
$$\left.+\frac{1}{8}|\psi_n|^2-(\psi_n\psi_{n+1}^*+\psi_n^*\psi_{n+1})\frac{4t^{1/2}\exp(-\tilde{L}t^{1/2})}{(1+t^{1/2})^2}\right\}, \qquad (23)$$

where[2)] ψn is the order parameter of the nth TP, while $\nabla_\perp$ is the gradient in the yz plane. Josephson interaction is also possible in quasi-two-dimensional intercalated superconductors. This situation was considered in Refs. 20–22 whose approach can also be used in describing the magnetic properties of a TPS system.

It is also interesting to consider the behavior of the upper critical field $H_{c2}^{\parallel}$ parallel to the TP (the case of a critical field perpendicular to the TP is trivial: the $H_{c2}^{\perp}$ plot has the same slope as the case where there are no twinning planes, although it begins at T_c rather than T_{c0}). Selecting a field gauge in the form $A\equiv A_x(y) = -H_y\xi(\tau_0)$ for the case $H\parallel z$ (the dimensionless variable $y\to y/\xi(\tau_0)$ is used for y, see Sec. 2) and introducing into (23) in gauge-invariant form the vector potential

$$(\psi_n\psi_{n+1}^*+\psi_n^*\psi_{n+1})\to\psi_n\psi_{n+1}^*\exp\left(i\frac{2\pi}{\Phi_0}A_x(y)L\right)$$
$$+\psi_n^*\psi_{n+1}\exp\left(-i\frac{2\pi}{\Phi_0}A_x(y)L\right),$$

we obtain the following linear equation for the order parameter $\psi_n=\psi$ (Ref. 20):

$$-\frac{d^2\psi}{dy^2}+(t-1)\psi-4\psi\exp(-\tilde{L}t^{1/2})\cos\left(\frac{2\pi}{\Phi_0}HL\xi(\tau_0)y\right)=0. \qquad (24)$$

It follows from (24) that in the absence of a magnetic field the transition temperature is $t_c=1+4\exp(-\tilde{L})$, in complete agreement with (11). The critical field is small in the immediate vicinity of t_c and it is possible to expand the cosine in (24). As a result we obtain for ψ the usual oscillator equation

$$-\frac{d^2\psi}{dy^2}+2\psi\exp(-\tilde{L})\left(\frac{2\pi HL\xi(\tau_0)y}{\Phi_0}\right)^2=(t_c-t)\psi, \qquad (25)$$

from which we find directly the temperature dependence of the parallel critical field as $t\to t_c$:

$$H_{c2}^{\parallel}=\frac{\Phi_0\exp(\tilde{L}/2)}{2\cdot 2^{1/2}\pi\xi(\tau_0)L}(t_c-t). \qquad (26)$$

The parallel critical field for a solitary twinning plane near the critical temperature $t=1$ is[11]

$$H_{c2}^0=\frac{\Phi_0}{2\pi\xi^{(2)}(\tau_0)}[2(1-t)]^{1/2}. \qquad (27)$$

$H_{c2}^{\parallel}(t)$ will approach $H_{c2}^0(t)$ with diminishing temperature. Here, however, the magnetic field has already a substantial effect on the distribution of the order parameter $\psi(x)$ near the TP, and our approach based on the functional (23) becomes inapplicable. Nonetheless even a simple comparison of relations (26) and (27) reveals that a segment of positive slope appears on the $H_{c2}^{\parallel}(t)$ curve (see Fig. 3). We note that this region can be observed only for systems with a period $L\sim\xi(\tau_0)$ (100–200 Å for the $YBa_2Cu_3O_{7-\delta}$ compound), while the temperature interval $(1,t_c)$ where a positive curvature already exists is exponentially small for $L\gg\xi(\tau_0)$, and the square-root dependence (27) should in fact be observed.

The weak Josephson current between the layers leads also to a specific feature of the magnetic-field screening in the TPS system. We will first consider the case of a parallel field $B=B_z$. Maxwell's equations take here the form (using the regular dimensional coordinates x and y):

$$\frac{\partial B}{\partial x}=\frac{4\pi}{c}j_y=\frac{1}{\lambda^2(\tau_0)}\psi^2(x)\left(\frac{c}{2e}\frac{\partial\varphi}{\partial y}-A_y\right), \qquad (28)$$

$$\frac{\partial B}{\partial y}=-\frac{4\pi}{c}j_x=-\frac{4\pi}{c}J_c\sin\left(\frac{\partial\varphi}{\partial x}L-\frac{2\pi L}{\Phi_0}A_x\right), \qquad (29)$$

where

$$\lambda^{-2}(\tau_0)=8\pi e^2\psi_0^2/mc^2=8\pi e^2\tau_0/\eta bmc^2=[\varkappa\xi(\tau_0)]^{-2}.$$

When the magnetic field varies slowly over the period of the system we can replace $\psi^2(x)$ in (28) with

$$\overline{\psi^2(x)}=4(1-t^{1/2})\xi(\tau_0)/L,$$

and in an approximation linear in the field we can expand the sine in (29). As a result we obtain

$$\frac{\partial B}{\partial x}=\frac{1}{\lambda^2}\left(\frac{c}{2e}\frac{\partial\varphi}{\partial y}-A_y\right),$$
$$\lambda^2=\lambda^2(\tau_0)\frac{L}{4\xi(\tau_0)(1-t^{1/2})}, \qquad (30)$$
$$\frac{\partial B}{\partial y}=-\frac{1}{\lambda_J^2}\left(\frac{c}{2e}\frac{\partial\varphi}{\partial y}-A_x\right),$$
$$\lambda_J^2=\frac{c\Phi_0}{8\pi^2J_cL},\quad\lambda_J\gg\lambda. \qquad (31)$$

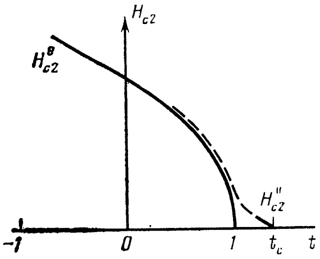

FIG. 3. Schematic representation of the temperature dependence of the parallel upper critical field $H_{c2}\parallel$ for a TP system, H_{c2} is the critical field of an isolated TP; the dashed curve represents the $H_{c2}\parallel(t)$ relation in the region where the approximation employed is not valid.

The equation for the field distribution of a vortex filament

$$\lambda^2\frac{\partial^2 B}{\partial x^2}+\lambda_J{}^2\frac{\partial^2 B}{\partial y^2}=B \tag{32}$$

has the usual solution[23] characterized solely by small scales on the x and y axes. The vortex is elliptical in shape and is strongly elongated along the y axis:

$$B=\frac{\Phi_0}{2\pi\lambda\lambda_J}K_0\left(\left(\left(\frac{x}{\lambda}\right)^2+\left(\frac{y}{\lambda_J}\right)^2\right)^{1/2}\right). \tag{33}$$

The core contribution to the vortex energy (core diameter $\sim L$) is, as in the regular case, small and the parallel lower critical field is[20]

$$H_{c1}{}^{\parallel}\approx\frac{\Phi_0}{4\pi\lambda\lambda_J}\ln\left(\frac{\lambda}{L}\right). \tag{34}$$

$H^{\parallel}_{cl}$ is exponentially small and its temperature dependence $H^{\parallel}_{cl}(T)$ is substantially nonlinear near T_c due to the sharp reduction in the Josephson depth λ_J with diminishing temperature.

As follows from (30) and (31), the screening of the weak magnetic field parallel to the TP along the x axis is characterized by the London depth λ_l [see (30)], while on the y axis it is substantially weaker and is determined by the Josephson depth λ_J [see (31)], and $\lambda_J\gg\lambda$.

When the field is perpendicular to the TP and the distance between the twinning planes is small compared to the characteristic scale of field variation, the field distribution is described by ordinary Maxwell equations for a superconductor, where the London depth of penetration λ_L is determined by the function $\overline{\psi^2(x)}$, i.e., $\lambda_L=\lambda$ [see (30)]. The lower critical field in this case is given by the regular expression[23]

$$H_{c1}^{\perp}=\frac{\Phi_0}{4\pi\lambda^2}\ln\frac{\lambda}{\xi}. \tag{35}$$

The applicability condition of this expression is $L\ll\lambda$, i.e., $L\ll\lambda^2(\tau_0)/[\xi(\tau_0)(1-t)]$. We note that although superconductivity actually breaks down between the twinning planes the vortices of the various layers interact through the magnetic field.

In the inverse limiting case, $L\gg\lambda^2(\tau_0)/[\xi(\tau_0)(1-t)]$, the vortex on the TP has a two-dimensional nature analogous to the case of a thin film,[24] while there is virtually no vortex interaction at the neighboring twinning planes (this interaction is exponentially small). The effective London depth is here $\lambda_{\text{eff}}=\lambda^2(\tau_0)/[4\xi(\tau_0)(1-t^{1/2})]$, while we can easily determine the lower critical field as

$$H_{c1}^{\perp}=\frac{\Phi_0}{4\pi L\lambda_{eff}}\ln\left(\frac{\lambda_{eff}}{\xi}\right)=\frac{\Phi_0}{4\pi\lambda^2}\ln\left(\frac{\lambda_{eff}}{\xi}\right). \tag{36}$$

This expression for $H^{\perp}_{cl}$ differs from (35) solely by the substitution $\lambda\to\lambda_{\text{eff}}$ under the logarithm sign.

The transition from a system of weakly coupled superconducting twinning planes to three-dimensional superconductivity occurs at $T\approx T_{c0}$, which should be manifested as a sharp change in the nature of the temperature dependence of the field H_{cl}.

The onset of a system of Josephson junctions due to the twinned structure for $T_{c0}<T<T_c$ could also be manifested as a nonstationary Josephson effect: generation of an alternating current and radiation of characteristic frequency $\omega\approx 2eU/(a/L)$, where U is the dc voltage applied to the specimen, while a is its thickness.

5. VORTEX PINNING AT THE TWINNING PLANES

Local superconductivity enhancement near the TP can cause a specific vortex pinning mechanism to appear at the TP lattice below the bulk critical temperature T_{c0}. We will therefore consider the interaction of an Abrikosov vortex with the TP in a superconductor with $\chi\gg 1$. In addition to the dimensionless variables $\mathbf{r}\to\mathbf{r}/\xi(\tau_0)$ and $\psi\to\psi/\psi_0$ introduced in Sec. 2 we define also a dimensionless magnetic field $B\to B/(2^{1/2}H_0)$ and a vector potential $\mathbf{A}\to\mathbf{A}/(2^{1/2}A_0)$, where $H_0=H_c(-\tau_0)=(2\tau_0/\eta)(\pi/b)^{1/2}$ and $A_0=H_0\lambda(\tau_0)=\chi\gamma(\pi/b)^{1/2}$.

In these variables the equations for the order parameter and the field take in our case the form

$$-\nabla^2 f+\mathbf{v}_s{}^2 f+tf+f^3=2\delta(x-D)f, \tag{37}$$

$$\text{rot rot}\,\mathbf{A}=\frac{f^2}{\varkappa^2}\mathbf{v}_s, \tag{38}$$

where $\mathbf{v}_s=\nabla-\mathbf{A}$ and the field $\mathbf{B}=-\chi\,\text{curl}\,\mathbf{v}_s$. We will assume that the vortex axis coincides with the z axis while the twinning plane corresponds to the $x=D$ plane, i.e., it is located at a distance D from the vortex.

In a zero magnetic field Eq. (37) for $f(x)$ has an exact solution also for $t<0$ ($T<T_{c0}$)

$$\begin{aligned} f(x)&=|t|^{1/2}\,\text{cth}[(|t|/2)^{1/2}|x-D|+p],\\ p&=\tfrac{1}{2}\,\text{Arsh}(2|t|)^{1/2}=\tfrac{1}{2}\ln[(2|t|)^{1/2}+(2|t|+1)^{1/2}]. \end{aligned} \tag{39}$$

In the presence of a vortex parallel to the TP the magnetic-field-induced change in f will be small everywhere aside from the vortex core and expression (39) can be used for f. We will consider the case $D\gg|t|^{-1/2}$ when the distance from the vortex to the TP is large compared to the superconducting correlation length, and vortex interaction with the TP will occur solely through the magnetic field. The contribution of the vortex to the free energy takes the form[25,26]

$$\Delta F=\frac{\Upsilon}{8mb}\int(\mathbf{B}^2+f^2\mathbf{v}_s{}^2)\,d^3r. \tag{40}$$

Integration in (40) is carried out over the region outside the vortex base. Transforming volume integral (40) into an integral over the surface of the vortex base in the standard manner using equation (38), we write

$$\Delta F=\frac{H_0{}^2}{4\pi}\xi^3(\tau_0)\varkappa^2\oint[\text{rot}\,\mathbf{v}_s\cdot\mathbf{v}_s]\,d\mathbf{S}, \tag{41}$$

where $d\mathbf{S}=\mathbf{n}dS$, while $\mathbf{n}$ is the outward normal to the base surface. The presence of TP alters $\mathbf{v}_s$:

$$\mathbf{v}_s=\mathbf{v}_s{}^{(0)}+\mathbf{v}_s{}^{(1)}, \tag{42}$$

where $\mathbf{v}_s^{(0)}$ is the velocity distribution in the absence of TP[26]: $v_s^{(0)}=(|t|^{1/2}/\chi)K_1(|t|^{1/2}\rho/\chi)$ and $v_s^{(1)}\ll v_s^{(0)}$ (the criterion for satisfaction of this condition will be given below). The force of interaction between the vortex and the TP is determined by the contribution made to free energy and dependent on the vortex position with respect to the TP. The corresponding contribution in first approximation, is equal to

$$\Delta F_i=\frac{\varkappa^2H_0^2}{4\pi}\xi^3(\tau_0)\oint\{[\mathrm{rot}\,\mathbf{v}_s^{(1)}\mathbf{v}_s^{(0)}]+[\mathrm{rot}\,\mathbf{v}_s^{(0)}\mathbf{v}_s^{(1)}]\}d\mathbf{S}. \tag{43}$$

We can assume $f^2=|t|$ far from the TP (aside from the vortex base region), and, subject to (38), the following estimate is valid for the terms in the integrand of (43):

$$|[\mathrm{rot}\,\mathbf{v}_s^{(1)}\ \mathbf{v}_s^{(0)}]|\sim B^{(1)}/\varkappa\rho,$$

$$|[\mathrm{rot}\,\mathbf{v}_s^{(0)}\ \mathbf{v}_s^{(1)}]|\sim\frac{1}{\varkappa}\ln\left(\frac{\varkappa}{|t|^{1/2}\rho}\right)|\mathrm{rot}\,\mathbf{B}^{(1)}|.$$

Here $\mathbf{B}^{(1)}$ is the supplementary magnetic field that arises near the vortex base due to TP presence, while $\rho\sim 1/|t|^{1/2}$. The characteristic scale of variation of $B^{(1)}$ is the London depth $\chi/|t|^{1/2}$ or the distance D (if $D\ll\chi/|t|^{1/2}$) and hence $|\mathrm{curl}\,\mathbf{B}^{(1)}|\sim B^{(1)}|t|^{1/2}/\chi$ or $B^{(1)}/D$ and the second term in (43) can be ignored in terms of the parameter $1/\chi$ or $1/D|t|^{1/2}$. As a result

$$\Delta F_i=\frac{\varkappa^2H_0^2}{4\pi}\xi^3(\tau_0)\oint[\mathrm{rot}\,\mathbf{v}_s^{(1)}\,\mathbf{v}_s^{(0)}]d\mathbf{S}$$

$$=\frac{\varkappa^2H_0^2}{4\pi}\xi^3(\tau_0)[\mathrm{rot}\,\mathbf{v}_s^{(1)}]_{\rho=0}\oint[\mathbf{v}_s^{(0)}\,d\mathbf{S}]=\frac{\varkappa lH_0^2\xi^3(\tau_0)}{2}B_1(0). \tag{44}$$

Here l is the vortex length (in dimensionless units). An explicit form of the expression for $\mathbf{v}_s^{(0)}$ was used in obtaining (44).

Applying the curl operation to Eq. (38) describing the magnetic field distribution, we obtain

$$\Delta\mathbf{B}-\frac{|t|}{\varkappa^2}\mathbf{B}=-\frac{1}{\varkappa}\mathrm{rot}[(f^2-|t|)\mathbf{v}_s]. \tag{45}$$

Using the Green's function

$$G(\rho)=-\frac{1}{2\pi}K_0\left(\frac{|t|^{1/2}}{\varkappa}\rho\right),$$

satisfying the equation $\Delta G-(|t|\chi^2)G=\delta(\rho)$ we write the solution of (45):

$$\mathbf{B}(\rho)=\mathbf{B}^{(0)}(\rho)+\frac{|t|^{1/2}}{2\pi\varkappa^2}\int K_1\left(\frac{|t|^{1/2}}{\varkappa}|\rho-\rho'|\right)$$

$$\times\frac{[(\rho'-\rho)\mathbf{v}_s(\rho')]}{|\rho'-\rho|}[f^2(\rho')-|t|]d^2\rho'. \tag{46}$$

It is possible to use the known expression for $\mathbf{v}_s^{(0)}$ in place of $\mathbf{v}_s$ in (46), i.e., to consider $v_s^{(1)}\ll v_s^{(0)}$. Analysis reveals that this is valid for $t<0$ across virtually the entire range of applicability of the approach based on Ginzburg-Landau theory, at least when $|t|\gg 1/\chi^2$.

The f peak near the TP appears at a distance of the order of the correlation length $1/|t|^{1/2}$. At the same time, we are interested in $B_1(0)$ on the axis of the vortex at a distance $D\gg 1/|t|^{1/2}$ from the TP, it is possible to carry out a separate integration of the function $(f^2(x')-|t|)$ in (46), which is essentially a δ-function near the TP, and in the remaining integrand we can set $x'=D$. $B_1(0)$ is given by the second term in (46). As a result we have the following representation for the energy ΔF_i

$$\Delta F_i=\frac{2^{1/2}H_0^2\xi^3(\tau_0)l|t|}{\pi\varkappa[(2|t|)^{1/2}+(2|t|+1)^{1/2}-1]}$$

$$\times\int_{-\infty}^{\infty}K_1^2\left(\left(y^2+\frac{D^2|t|}{\varkappa^2}\right)^{1/2}\right)dy. \tag{47}$$

When $D\ll\chi/|t|^{1/2}$ (but $D\gg 1/|t|^{1/2}$) the repulsive force of the vortex from the TP

$$f_r=-\frac{1}{\xi(\tau_0)}\frac{\partial(\Delta F_i)}{\partial D} \tag{47a}$$

is given by the expression

$$f_r=\frac{H_0^2\xi^2(\tau_0)(2|t|)^{1/2}l}{[(2|t|)^{1/2}+(2|t|+1)^{1/2}-1]D^2}. \tag{48}$$

When $D\gg\chi|t|^{1/2}$ the repulsive force is exponentially small:

$$f_r=\frac{(2\pi)^{1/2}H_0^2\xi^2(\tau_0)|t|^{5/4}l\exp(-2(|t|)^{1/2}D/\varkappa)}{\varkappa^{1/2}D^{1/2}[(2|t|)^{1/2}+(2|t|+1)^{1/2}-1]}. \tag{49}$$

The maximum repulsive force is achieved when the vortex reaches a distance of the order of the correlation length $1/|t|^{1/2}$ to the TP. The change in the condensation energy in the vortex base due to the presence of the TP already plays an important role here.

Especially strong pinning will occur near T_{c0}. A special situation occurs as $t\to 0$: In this case the order parameter at the TP is finite and $f^2(x=D)\to 2$, whereas $f^2=|t|$ in the bulk and is close to zero. In this case the local London screening length near the TP $\lambda_{L,\mathrm{loc}}\sim\chi/2^{1/2}$ and does not tend to infinity as $t\to 0$. Therefore the superconductivity is effectively of type-I near the TP. As a result anomalously strong vortex pinning will be observed as $t\to 0$. The change in the condensation energy $\Delta\varepsilon$ as the vortex core reaches the TP is within an order of magnitude of $H_0^2\xi^3(\tau_0)l/|t|^{1/2}$, while the pinning force $f_{\mathrm{pin}}\sim\Delta\varepsilon/\xi(t)\sim H_0^2\xi^2(\tau_0)l$. We note that the pinning force remains constant with increasing temperature ($T\to T_{c0}$, but $|t|\gg 1/\chi^2$).

If f_{pin} is known it is possible to estimate the critical current density j_c. If we consider the external field to be sufficiently weak to allow neglecting the vortex interaction, we can obtain the vortex equilibrium condition as equality of the Lorentz force f_L to the pinning force f_{pin} acting on the vortex. Since $f_L\approx\Phi_0c^{-1}j_cl\xi(\tau_0)$[26] where $\Phi_0=\pi\hbar c/e$ is the flux quantum, we obtain

$$j_c\sim cH_0^2\xi(\tau_0)/\Phi_0 \tag{50}$$

in the most favorable case when the vortex filaments are parallel to the c axis (and to the TP), and current flows in the ab plane parallel to the TP. However the vortices are hardly likely to remain parallel to the TP. It is more likely that vortex transit through the TP will begin at one of the ends of the boundary of the specimen; the curved segment will then travel along the vortex, causing a drop in the critical current. A system of parallel twinning planes in high-temperature superconductors should cause the vortices to end up between the twinning planes. Since the distance between the twinning boundaries is less than λ_L, the dependence of vortex energy on the core coordinate is easily determined by using expression (48) for the force of interaction between the vortex and the twinning planes.

This specific vortex pinning mechanism at TP is, *naturally*, most effective when the vortices are oriented parallel to the TP and vanishes in the case of perpendicular orientation. This is consistent with the results of Ref. 27 where observation of strong pinning anisotroy in $YBa_2CU_3O_{7-\delta}$ single crystals was reported.

We have considered the case where the electrons in fact freely penetrate the TP. Another situation is also possible, as noted in Sec. 1, when the twinning plane is impermeable to electrons. In this case the nature of interaction between the vortex and the TP changes: the vortex is attracted to the boundary which in this case coincides with the twinning plane (in analogy with the problem of the Bean-Livingston surface barrier[26]). In order to calculate the force acting on the vortex we will use formula (41), where integration is now carried out not only over the surface of the vortex base but also over the twinning plane. Since the TP is now an insulating interlayer, while the current $j \sim f^2 \mathbf{v}_s$, the component of $\mathbf{v}_s$ normal to the twinning plane vanishes on the twinning plane. This condition can be satisfied if we add to the vortex its mirror image with respect to the TP with opposite current and field direction. Since in this case the magnetic field $\mathbf{B} = -\chi \operatorname{curl} \mathbf{v}_s = 0$ at the TP, due to symmetry, the integral over the TP drops out of (41). Consequently the vortex interaction with the TP is described by Eq. (44) in which $B_1(0)$ can be represented as the sum of the field $-(|t|/\varkappa)K_0(2|t|^{1/2}D/\chi)$ generated by the vortex-image ignoring superconductivity enhancement near the TP and the term associated with the increase in f near the TP. This latter term is given by the second term in formula (46), in which we assume $\mathbf{B}^{(0)}$ and $\mathbf{v}_s^{(0)}$ to be the sums of the corresponding quantities for the vortex and its image. It then turns out that when $1/|t|^{1/2} \ll D \ll \chi/|t|^{1/2}$ the force repelling the vortex away from the region near the TP with the larger value of f is proportional to D^{-2} and is given by Eq. (48), while the attractive force due to the image is inversely proportional to the distance: $f_a = H_0^2 \xi^2(\tau_0) l |t|/2D$. Consequently the equilibrium position of the vortex is localized near the TP. The vortex will be at a distance $D \approx 2\xi(\tau_0)/|t|$ from the TP in immediate proximity to the transition temperature T_{c0} (for $|t| \ll 1$) while at lower T the equilibrium position virtually coincides with the TP.

It is possible that the finite transparency of the TP leads to the result of Ref. 28, where the decoration technique was used to record vortex chains along the TP with a preferred orientation of the vortices along the plane at $T = 4.2$ K in a $YBa_2Cu_3O_{7-\delta}$ single crystal. In this connection we emphasize that the actual localization of the vortices near the TP[28] in no way excludes the local superconductivity enhancement near the plane but rather suggests insufficient transparency of the boundary, which is the twinning plane, to the electrons. At the same time it is necessary to take into account the dynamics of formation of the vortex state in analyzing the experimental results.[28] Indeed, the existence of an energy barrier to the vortex in the case of a transparent twinned boundary impedes its motion through the specimen and may cause localization of the vortices near the TP. It would be interesting to investigate the change in the nature of vortex distribution as the temperature is raised to T_{c0}.

CONCLUSION

The existence of twinning boundaries in $YBa_2Cu_3O_{7-\delta}$ high-temperature superconductors may, similar to the situation in tin and niobium, lead to superconductivity localized at the TP. A number of experiments attest to the appearance of superconductivity in twinning planes in $YBa_2Cu_3O_{7-\delta}$ at a temperature 3–5 K above the critical bulk temperature T_{c0}.[12,14] As demonstrated in the present study, the thermodynamic and magnetic properties of high-temperature superconductors in which a regular twinned structure has been observed is characterized by a number of anomalies in the vicinity of T_{c0}. Precision magnetic and calorimetric measurements on $YBa_2Cu_3O_{7-\delta}$ single crystals with simultaneous control of the period of the twinned structure is of interest in this connection.

In conclusion we emphasize that these results can also be used to describe superconducting superlattices obtained by layered sputtering of various superconductors.

The authors wish to express their gratitude to S. V. Polonskiĭ for his assistance with the numerical calculations.

[1)]In Ref. 13 are reported two specific heat anomalies in single-phase polycrystalline $YBa_2Cu_3O_{7-\delta}$ specimens (see also Ref. 29).

[2)]We emphasize that functional (23) is valid when $\tilde{L}t^{1/2} \gg 1$, i.e., when $L \gg \xi_0/[(T - T_{c0})/T_{c0}]^{1/2}$ and the interaction between neighboring twinning planes is no longer weak near T_{c0}.

[1]Yu. A. Osip'yan, N. S. Afonnikova, G. A. Emel'chenko, *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **46**, 189 (1987) [JETP Letters, **46**, 241 (1987)].
[2]Y. Syono, M. Kikuchi, K. Ohishi, *et al.*, Jpn. J. Appl. Phys. **26**, L498 (1987).
[3]G. Roth, D. Ewert, G. Heyer, *et al.*, Z. Phys. **69**, 21 (1987).
[4]Yu. V. Sharvin and V. F. Gantmakher, Zh. Eksp. Teor. Fiz. **38**, 1456 (1960) [Sov. Phys. JETP **11**, 1052 (1960)].
[5]M. S. Khavkin and I. N. Khlyustikov, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 167 (1981) [JETP Letters **33**, 158 (1981)].
[6]I. A. Gindin, V. I. Sokolenko, Yu. D. Starodubov, and M. B. Lazarev, Fizika nizkikh temperatur **8**, 643 (1982) [Sov. Jour. Low Temp. Physics **8**, 321 (1982)].
[7]V. S. Bobrov and S. N. Zorin, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 345 (1984) [JETP Letters **40**, 1147 (1984)].
[8]A. I. Buzdin and L. N. Bulaevskiv, Pis'ma Zh. Eksp. Teor. Fiz. **34**, 118 (1981) [JETP Letters **34**, 112 (1981)].
[9]V. V. Averin, A. I. Buzdin, and L. N. Bulaevskiv, Zh. Eksp. Teor. Fiz. **84**, 737 (1983) [Sov. Phys. JETP **57**, 426 (1983)].
[10]A. I. Buzdin and N. A. Khvorikov, Zh. Eksp. Teor. Fiz. **89**, 1957 (1985) [Sov. Phys. JETP **62**, 1128 (1985)].
[11]I. N. Khlyustikov and A. I. Buzdin, Adv. Phys. **36**, 271 (1987).
[12]S. E. Inderhees, M. B. Salamon, N. Goldenfeld, *et al.*, Preprint Univ. Illinois (1987). Phys. Rev. Lett. **60**, 1178 (1988).
[13]M. Ishikawa, Y. Nakazawa, T. Takabatake, *et al.*, Preprint 1988. Techn. Rep. of ISSP. Ser. A, No. 1907.
[14]M. M. Fang, V. G. Kogan, D. K. Finnemore, *et al.*, Phys. Rev. B **37**, 2334 (1988).
[15]A. A. Abrikosov and A. I. Buzdin, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 204 (1988) [JETP Letters **47**, 247 (1988)].
[16]G. Deutscher and K. A. Mueller, Phys. Rev. Lett. **59**, 1745 (1987).
[17]A. F. Andreev, Pis'ma Zh. Eksp. Teor. Fiz. **46**, 463 (1987) [JETP Letters **46**, 584 (1987)].
[18]A. I. Buzdin, M. L. Kulic, and S. V. Polonskii, Proc. Conf. on High-T_c Superconductivity, Interlaken (1988).
[19]I. Banerjee, Q. S. Yang, C. M. Falco, and I. K. Schuller, Phys. Rev. B **28**, 5037 (1983).
[20]L. N. Bulaevskiv, Zh. Eksp. Teor. Fiz. **64**, 2241 (1973) [Sov. Phys. JETP **37**, 1133 (1973)].
[21]L. N. Bulaevskiv, Uspekhi Fiz. Nauk **116**, 449 (1975) [Sov. Phys. Uspekhi **18**, 514 (1975)].
[22]W. T. Lawrence and S. Doniach, IEEE Proc. LT-12, 361 (1970).
[23]A. A. Abrikosov, Zh. Eksp. Teor. Fiz. **32**, 1442 (1957) [Sov. Phys. JETP **5**, 1174 (1957)].
[24]J. Pearl, J. Appl. Phys. Lett. **5**, 65 (1964).
[25]E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics*, Part 2, Pergamon.
[26]A. A. Abrikosov, *The Principles of the Theory of Metals* [in Russian], Nauka, 1987.
[27]L. Z. Avdeev, A. B. Bykov, L. N. Dem'yanets, *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **46**, 196 (1987) [JETP Letters **46**, 249 (1987)].
[28]L. Ya. Vinnikov, L. A. Gurevich, G. A. Emel'chenko, and Yu. A. Osip'yan, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 109 (1988) [JETP Letters **47**, 131 (1988)].
[29]R. A. Butera, Phys. Rev. B **37**, 5909 (1988).

Translated by Kevin S. Hendzel