Multipole electromagnetic moments of a neutrino in a dispersive medium

V. B. Semikoz and Ya. A. Smorodinskiĭ

Institute of Terrestrial Magnetism, The Ionosphere, and Radio-wave Propagation, Academy of Sciences of the USSR, and I. V. Kurchatov Institute of Atomic Energy
(Submitted 9 June 1988)
Zh. Eksp. Teor. Fiz. **95**, 35–46 (January 1989)

Four multipole moments of Dirac and Majorana neutrinos in a dispersive medium are calculated: electric monopole (charge), electric dipole, magnetic dipole, and anapole dipole moment. The same quantities are given in vacuum for comparison. In an isotropic medium neutrinos have no (induced) anapole moment, but in a ferromagnetic material such a moment appears, and for a Majorana neutrino it is the only electromagnetic characteristic. The cross section for elastic scattering of Majorana neutrinos by nuclei in an isotropic plasma is calculated as an illustration.

1. INTRODUCTION

A neutrino which interacts with a vacuum of vector bosons and leptons has vacuum electromagnetic characteristics, for example, an anomalous magnetic moment $\Delta\mu_{\rm vac}$ and a charge-distribution rms radius $\langle r_\nu^2\rangle^{1/2}$. As a result a neutrino, like a neutron, interacts with an external electromagnetic field. However, all electromagnetic characteristics of neutrinos are proportional to the Fermi weak-interaction constant $G_F = 10^{-5}/m_p^2$ (m_p is the proton mass),[1)] and therefore in vacuum the role of an additional electromagnetic interaction reduces only to radiative corrections to the principal Born contribution to the cross sections of weak processes.

In a dispersive medium the electromagnetic interaction of neutrinos with particles and with external electromagnetic fields is greatly enhanced. It becomes particularly important to take into account the electromagnetic structure of the neutrino in a medium, since the cross section for the electromagnetic channel of reactions (for example, scattering of neutrinos by charged particles through exchange of a longitudinal plasmon) is many orders of magnitude greater than the Born cross section.[1]

This means that in a dispersive medium, in contrast to a vacuum, the electromagnetic contributions to the cross sections of weak processes do not reduce to small corrections. Their magnitude is determined completely by the electromagnetic form factors of the neutrino in the medium. In the present work we shall be interested in the normalizations of these form factors, which determine the so-called intrinsic multiple electromagnetic moments of a particle.

In order to see the differences and similarities of the electromagnetic properties of a neutrino in a medium and in vacuum, in Section 2 we shall repeat the well known formulas for the electromagnetic current of Dirac and Majorana neutrinos in vacuum,[2] by means of which general formulas are derived for the four lowest multipole moments: electric monopole (charge) and dipole, magnetic dipole and anapole (toroidal dipole) moments. The densities of the vacuum moments are calculated in the Breit system, and the moments themselves (which correspond to the limit $\mathbf{k} = -2\mathbf{p} = 0$) are calculated in the rest system of the particle.

In Sec. 3 we shall give first the electromagnetic vertex of a Dirac neutrino in an isotropic dispersive medium,[3] which is easily transformed by means of C conjugation into the electromagnetic vertex of a Majorana neutrino. Then we shall calculate the effective coupling constants, which depend on the velocity in the medium, which is at rest as a whole. Superposing this system of reference on the rest system of the neutrino, we obtain characteristics which it is appropriate to call the intrinsic multipole moments of the neutrino in the medium. The difference of the intrinsic induced moments of Dirac and Majorana neutrinos is similar to the vacuum case, but there is a dependence on the choice of the medium. In particular, the induced anapole moment of a neutrino is absent in an isotropic dispersive medium but is nonzero in an anisotropic medium such as a ferromagnetic material, in which the anapole is the only intrinsic moment of the Majorana neutrino.[4] At the end of Sec. 3, using the electromagnetic vertex of a neutrino in a ferromagnetic material obtained by the authors of Ref. 5, we calculate the neutrino anapole moment.[4]

As an example illustrating the predominant contribution of the calculated characteristics of a neutrino in a medium in comparison with the vacuum characteristics, in Sec. 4 we give the cross section for elastic scattering of a Majorana neutrino by nuclei in a dense plasma, comparing it with similar calculations for scattering of Dirac neutrinos.[6]

2. MULTIPOLE MOMENTS OF A NEUTRINO IN VACUUM

We shall consider the diagonal[2)] multipole moments of a spinor particle, whose interaction with an electromagnetic field A_μ (of the form $\propto J_\mu A^\mu$) is determined by the electromagnetic 4-current

$$J_\mu^{fi}(\omega,\mathbf{k}) = \frac{e}{2(E_pE_{p'})^{1/2}}\bar{u}_f(\mathbf{p}')\Gamma_\mu(\omega,\mathbf{k})u_i(\mathbf{p}), \qquad (1)$$

where e is the electric charge of the electron ($\alpha = e^2 = 137^{-1}$), $q_\mu = (\omega,\mathbf{k})$ is the transferred 4-momentum $q_\mu = p'_\mu - p_\mu$, $\Gamma_\mu(\omega,\mathbf{k})$ is the electromagnetic vertex of a fermion with spin $s = \frac{1}{2}$, and $u(p)$ are bispinors normalized to the rest mass $(\bar{u}(\mathbf{p})u(\mathbf{p}) = 2M)$. It is well known (see for example Ref. 2) that a Dirac neutrino in vacuum is characterized by four form factors which determine the Lorentz structure of the electromagnetic vertex:

$$\Gamma_\mu^{(D)}(q) = F_{vac}^{(D)}(q^2)\gamma_\mu + iM_{vac}^{(D)}(q^2)\sigma_{\mu\nu}q^\nu + G_{vac}^{(D)}(q^2)[q^2\gamma_\mu - q_\mu\hat{q}]\gamma_5 + E_{vac}^{(D)}(q^2)\sigma_{\mu\nu}q^\nu\gamma_5, \quad \sigma_{\mu\nu} = {}^1/_2 i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu). \qquad (2)$$

The particle electric charge Ze is determined in terms of its density $J_0^{fi}(r)$, which is obtained from (1) for $\mu = 0$ by the simple relation

 0038-5646/89/010020-07$04.00

$$Ze=\int d^3r\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{kr}}J_0^{fi}(0,\mathbf{k}),\tag{3}$$

in which we shall separate the arguments ω and $\mathbf{k}$ to generalize the formulas obtained here to the case of a medium. The difference of the vacuum form factors $F_{\rm vac}(q^2)$ and the form factors in a medium $F_{\rm med}(\omega,\mathbf{k})$ consists first of all of the number of arguments, which in a medium do not reduce to the simple combination $q^2=\omega^2-\mathbf{k}^2$. Covariance of the description is achieved in a medium by the transition to a moving frame of reference with use of the Doppler formulas $\omega'=q\Omega$, $k'=[q^2-(q\Omega)^2]^{1/2}$, where Ω_μ is the unit vector $(\Omega^2=1)$ of the 4-velocity of the medium as a whole.

The vacuum form factors of the vertex (2) depend on one argument $q^2=\omega^2-\mathbf{k}^2$, and the three-dimensional description introduced in (3) is achieved in the Breit system $(\mathbf{p}+\mathbf{p}'=0)$, where there is no time component of the momentum transfer, $\omega=0$. It is not difficult to see that the intrinsic moments of a particle (in its rest system) are obtained automatically as the result of integration of the corresponding densities of the moments over d^3r with a limiting transition to the value $\mathbf{k}=-2\mathbf{p}=0$. We shall give the obvious answers. The electric charge (3) if we take into account (1) is

$$Ze=(e/2M)\bar{u}(0)(\lim_{k\to 0}\Gamma_0(0,\mathbf{k}))u(0).\tag{3'}$$

The electric dipole moment of a particle in accordance with (1) and with the standard definition[7]

$$Q_m^{fi}=\left(\frac{4\pi}{3}\right)^{1/2}\int d^3r\,J_0^{fi}(\mathbf{r})\,rY_{1m}\left(\frac{\mathbf{r}}{r}\right)$$

($Y_{1m}(x)$ are spherical harmonics) is

$$Q_m=-\frac{e}{2M}\bar{u}(0)\left[\frac{\partial\Gamma_0(0,\mathbf{k})}{\partial k^m}\right]_{\mathbf{k}=0}u(0),\tag{4}$$

where $m=\pm,0$; $k_\pm=\pm(k_x\pm ik_y)/\sqrt{2}$, and $k_0=k_z$. The magnetic dipole moment

$$\mu_k^{fi}=\frac{1}{2}\int d^3r[\mathbf{r}\mathbf{J}^{fi}]_k$$

is

$$\mu_k=\frac{ie}{4M}e_{kni}\bar{u}(0)\left[\frac{\partial\Gamma_i(0,\mathbf{k})}{\partial k^n}\right]_{\mathbf{k}=0}u(0).\tag{5}$$

The vector part of the electromagnetic vertex (2) (the first two terms) makes a contribution to interactions with parity conservation $V_{\rm int}\propto ZeA_0$, namely $\boldsymbol{\mu}\cdot\mathbf{B}$. Substituting the vertex (2) into the formulas (3) and (5), we obtain the well known definitions of the electric charge $Z=F_{\rm vac}(0)$ and the magnetic moment

$$\mu_k=e\varphi^+\sigma_k\varphi[M_{vac}(0)+F_{vac}(0)/2M]$$

(φ is a spinor in the rest system), which are common for quantum electrodynamics and the electroweak standard model.

Calculations in the one-loop approximation of the standard model permit verification of the requirement $F_{\rm vac}(0)=0$, i.e., absence in a neutrino of the electric charge (3) in vacuum.[8] Here the remaining anomalous (Pauli) magnetic moment $\Delta\boldsymbol{\mu}_{\rm vac}=eM_{\rm vac}(0)\varphi^+\boldsymbol{\sigma}\varphi$ is determined in the minimal model $SU_L(2)\otimes U(1)$ in the same one-loop approximation by the quantity[9]

$$eM_{vac}(0)=\frac{3G_Fm_em_\nu}{2^{5/2}\pi^2}\mu_B,\tag{6}$$

where m_e and m_ν are the electron and Dirac neutrino masses and $\mu_B=e/2m_e$ is the Bohr magneton.

An intrinsic electric dipole moment exists only in a model with T-noninvariance of the interaction. (Thus, since $T\boldsymbol{\sigma}T^{-1}=-\boldsymbol{\sigma}$ and $T\mathbf{E}T^{-1}=\mathbf{E}$, then an interaction $\propto(\boldsymbol{\sigma}\cdot\mathbf{E})$ is not invariant to time reversal). We shall restrict the discussion to the general formula for an electric dipole obtained by substitution of the vertex (2) into (4):

$$Q_m=iE_{vac}(0)\varphi^+\sigma_m\varphi.\tag{7}$$

It is obvious also that an interaction $\mathbf{Q}\cdot\mathbf{E}$, where $\mathbf{E}$ is the electric field strength, does not conserve P-parity. If an intrinsic (diagonal) electric dipole moment is impossible, then it is possible to have a nondiagonal matrix element of the transition current, which corresponds to the radiation of an electric dipole, which is not discussed here.

We do not calculate here the rms radii for the distributions of densities of multipoles (3')–(5). The fourth and last of the intrinsic multipoles is the anapole moment of a spinor particle, which was first introduced by Zel'dovich[10] for a T-invariant interaction $V_{\rm int}\propto\boldsymbol{\sigma}\cdot\mathbf{j}$ which does not conserve P-parity and C-parity individually. Here $\mathbf{j}$ is the polar vector of the electromagnetic current

$$\mathbf{j}=(\mathrm{rot}\,\mathbf{H}+\dot{\mathbf{E}})/4\pi.\tag{8}$$

The presence in particles of an anapole moment does not lead to a transition current or to radiation of electromagnetic waves in vacuum. This is obvious from the form of the third term in (3) when one takes into account that the Lorentz condition $(\partial A_\mu/\partial x_\mu=0)$ is satisfied also in the case of absence of external currents $(\Box A_\mu=0)$.

In the Breit system the density of a point anapole moment

$$\mathbf{g}(\mathbf{r})=\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{kr}}\mathbf{G}(-k^2)$$

is determined by the pseudovector $\mathbf{G}$, which does not depend on the magnitude of the wave vector k and which is directed in spin—the only vector characteristic of the particle in its own system of reference[10]:

$$\mathbf{G}=eG_{vac}(0)\varphi^+\boldsymbol{\sigma}\varphi.$$

The most general desired form of coupling of the vector form factor $G_i(-k^2)$ with the current (1) has in the Breit system the form

$$G_i(-k^2)=a\frac{\partial^2J_k(\mathbf{k})}{\partial k^i\,\partial k_k}+b\frac{\partial^2J_i(\mathbf{k})}{\partial k_n\,\partial k_r},\tag{9}$$

where in the current $J(\mathbf{k})$ it is possible to omit the dependence on the argument $\mathbf{k}$ in the bispinors and in the normalization, taking into account the limiting transition to the vector $\mathbf{G}=\lim_{k\to 0}\mathbf{G}(-k^2)$. The vector $\mathbf{G}$ can depend only on the normalization of the form factor $G_{\rm vac}(0)$ in the third term of (2), which corresponds to the necessary C, P, and T properties.[10] The first condition for determination of the two coefficients in (9) is due to the requirement that the vector $\mathbf{G}$, which does not depend on k, have a standard normalization $\mathbf{G} = eG_{\text{vac}}(0)\varphi^{+}\sigma\varphi$, i.e., be equal to

$$G_i=(e/2m_\nu)G_{vac}(0)\bar{u}(0)\gamma_i\gamma_5 u(0). \qquad (10)$$

Equality of the quantities (9) and (10) gives the equation $2a - 4b = 1$. A second condition for determination of the coefficients a and b can be obtained by various means. One can make use of the absence of a contribution from the longitudinal current $\mathbf{J}_{\parallel} = \nabla\varphi$—a source of electric multipoles—to the anapole moment $\mathbf{G} = \int d^3r\mathbf{g}(\mathbf{r})$ with a density $G_i(r) = -[ar_ir_k + b\delta_{ik}r^2]J_k(r)$ corresponding to (9). Taking into account the explicit form of the 3-current

$$\mathbf{J}(\mathbf{k})=(e/2m_\nu)G_{vac}(0)\bar{u}(0)[-k^2\boldsymbol{\gamma}+(\mathbf{k}\boldsymbol{\gamma})\mathbf{k}]\gamma_5 u(0),$$

it is easy to obtain a second equation $2a + b = 0$, which permits determination of the coefficients in (9): $a = 1/10$, $b = -1/5$.

Another means of obtaining the last equation ($2a + b = 0$) is transition to the variable ξ and $\dot{\xi}$ in the formula

$$g_i(\xi)=[a\xi_i\xi_k+b\xi^2\delta_{ik}]\,\dot{\xi}_k$$

with use of the condition $\partial g_i(\xi)/\partial\xi_i = 0$ that the matrix in the square brackets be transverse. Finally, in the case of an isotropic medium when one takes into account spatial dispersion in the polarization tensor of the medium and of the vacuum, the equation $2a + b = 0$ is obtained automatically (see the beginning of Sec. 3c).

Therefore, the anapole moment of a Dirac neutrino is a toroidal dipole moment

$$G_i=\frac{eG_{vac}^{(D)}(0)}{20m_\nu}\bar{u}(0)\left[\frac{\partial^2\Gamma_n(0,\mathbf{k})}{\partial k_i\,\partial k_n}-2\frac{\partial^2\Gamma_i(0,\mathbf{k})}{\partial k_n\,\partial k_n}\right]_{\mathbf{k}=0}u(0), \qquad (11)$$

which in the coordinate representation has a density[11]

$$-\mathbf{g}(\mathbf{r})=[\mathbf{r}(\mathbf{J}\mathbf{r})-2r^2\mathbf{J}]/10. \qquad (11')$$

Calculation of the form factor $G_{\text{vac}}(q^2)$ in the one-loop approximation of the standard vacuum model of electroweak interactions is a problem which apparently has not been solved up to the present time.

A Majorana neutrino as a truly neutral particle does not have electric charge or magnetic moment, and its only characteristic is an anapole moment of the type (11) obtained from the vertex $\Gamma_\mu^{(M)}(q)$:

$$\Gamma_\mu^{(M)}(q)=G_{vac}^{(M)}(q^2)[q^2\gamma_\mu-q_\mu\hat{q}]\gamma_5. \qquad (12)$$

The vertex (12) is obtained from (2) by adding to the amplitude of the interaction with the external electromagnetic field $M^{fi} \propto J_\mu^{fi}A^\mu$ a contribution corresponding to an antiparticle, whose current

$$J_\mu^{fi}=e\bar{v}_i(\mathbf{p})\Gamma_\mu v_f(\mathbf{p}')/2(E_pE_{p'})^{1/2}$$

is multiplied by a field $CA_\mu C^{-1} = -A$ in accordance with the change of sign of the electric charge under charge conjugation. We shall use the representation $U_C = i\gamma^2\gamma^0$ for a unitary C-transformation, carrying out the substitution

$$\bar{v}_i(\mathbf{p})\Gamma_\mu v_f(\mathbf{p}')=-\bar{u}_f(\mathbf{p}')[U_C^{-1}\Gamma_\mu U_C]^T u_i(\mathbf{p}),$$

as the result of which all contributions except the anapole cancel on addition of the amplitudes. For example, for the first term in (2) we have the equality

$$\bar{v}_i(\mathbf{p})\gamma_\mu v_f(\mathbf{p}')=\bar{u}_f(\mathbf{p}')\gamma_\mu u_i(\mathbf{p}),$$

and when we take into account the indicated change of sign of the electric charge in the antiparticle, the term disappears on summation of the contributions of the particle and its antiparticle.

Thus, Eqs. (3')–(5) and (11) determine the four lowest multipole moments of a spinor particle in the electroweak model. In vacuum a direct neutrino does not have electric charge: $F_{\text{vac}}(0) = 0$, but in the presence of rest mass $m_\nu \neq 0$ it may have magnetic (6) and electric (7) dipole moments. A Majorana neutrino does not have these electromagnetic characteristics but, like a Dirac neutrino, can have an anapole moment (11) (with the substitution $G_{\text{vac}}^{(D)} \to G_{\text{vac}}^{(M)}$). It is easy to show that in the massless case ($m_\nu = 0$) the electromagnetic vertices (2) and (12) coincide.[2]

3. MULTIPOLE MOMENTS OF NEUTRINOS IN A DISPERSIVE MEDIUM

a. Electromagnetic structure of a neutrino in an isotropic dispersive medium

For an isotropic dispersive medium the electromagnetic vertex of a Dirac neutrino was obtained in Refs. 1 and 3, where the authors erroneously omitted the contribution of the pseudovector current in the electron-hole loop corresponding to a polarization diagram with exchange of a Z^0 boson (the diagram of Fig. 1a of Ref. 1). Addition to the formula (2) of Ref. 1 of the corresponding current

$$I_\mu{}^a(PS)=\frac{ie(g^{\rho\sigma}-q^\rho q^\sigma/M_Z{}^2)}{2\sin^2 2\theta_W(q^2-M_Z{}^2+i\varepsilon)}$$

$$\times\int\frac{d^4p''}{(2\pi)^4}\bar{\nu}_L(\mathbf{p}')\gamma_\rho\,\mathrm{Sp}[\gamma_5\gamma_\sigma G(\mathbf{p}'',p_0'')$$

$$\times\gamma_\mu G(\mathbf{p}''-\mathbf{k},p_0''-\omega)]\gamma_L(\mathbf{p})$$

adds to the formula (6) of the same work a term

$$I_\mu{}^a(PS)=-\frac{iG_Fe}{2^{1/2}e^2}\varepsilon_{\mu\nu\rho\sigma}\bar{\nu}_L(\mathbf{p}')\gamma^\nu\nu_L(\mathbf{p})A^{\rho\sigma}(\omega,\mathbf{k}),$$

where the antisymmetric tensor $A^{\rho\sigma}$ is defined in (10) of Ref. 1. As a result the coefficient in front of the second term in square brackets of the final formula for the electromagnetic vertex (14) in Ref. 1 decreases by a factor of two [see Eq. (13) below]. All subsequent results in Refs. 1 and 3 are not changed.

We give the complete expression for the electromagnetic vertex of a Dirac neutrino $\Gamma_\mu^{(D)}$ in an isotropic dispersive medium in the statistical minimal electroweak model:

$$\Gamma_\mu^{(D)}(\omega,\mathbf{k})=\frac{(1+\gamma_5)\gamma^\rho}{2^{1/2}\pi\alpha}[G_V\Pi_{\mu\rho}(\omega,\mathbf{k})$$

$$+iG_F\varepsilon_{\mu\rho\nu\sigma}A(\omega,k)q^\nu\Omega^\sigma]+\Gamma_\mu^{(D)}(q). \qquad (13)$$

Here $G_V = G_F(1 + 4\xi)$ is the vector constant of the weak interaction, $\xi = \sin^2\theta_W$ is the parameter of the electroweak model, θ_W is the Weinberg angle, $\Pi_{\mu0}(\omega,\mathbf{k})$ is the symmetric polarization tensor of statistical quantum electrodynamics,[12] $\varepsilon_{\mu\rho\nu\sigma}$ is the completely antisymmetric Levi-Civita unit tensor, the quantity $A(\omega,k)$ determines the induced magnetic moment of the neutrino in a medium and has been calculated in Ref. 3, and finally the last term in (13) is the vacuum vertex (2).

The terms in square brackets in Eq. (13) can be rewritten in the rest system of the medium as a whole ($\mathbf{V} = \mathbf{\Omega} = 0, \Omega^\sigma = \delta_{\sigma 0}$) in the form of a sum which depends on three additional form factors[1] on top of the already existing four in the vacuum contribution $\Gamma_\mu^{(D)}(q)$:

$$\pi_{\mu\rho}^{med}(\omega,\mathbf{k}) = -\{F_\parallel(\omega,k)e_\mu e_\rho + F_\perp(\omega,k)\delta_{\mu k}\delta_{\rho i}(\delta_{ik} - \hat{k}_i\hat{k}_k) + iM(\omega,k)\delta_{\mu i}\delta_{\rho l}e_{ikl}k_k\}. \tag{14}$$

Here the longitudinal form factor is[13]

$$F_\parallel(\omega,k) = G_V q^2[\varepsilon_l(\omega,k)-1]/2^{5/2}\pi\alpha, \tag{15}$$

the transverse form factor has the form[1]

$$F_\perp(\omega,k) = G_V\omega^2[\varepsilon_{tr}(\omega,k)-1]/2^{5/2}\pi\alpha, \tag{16}$$

and the analog of the magnetic form factor is[3]

$$M(\omega,k) = G_F A(\omega,k)/2^{5/2}\pi\alpha. \tag{17}$$

The form factor (15) is determined by the longitudinal permittivity $\varepsilon_l(\omega,k)$ of the isotropic medium and corresponds to interaction of the neutrino with a longitudinal electric field ($e_\mu = (k,\omega\hat{\mathbf{k}})(q^2)^{-1/2}$ is the polarization vector, $e^2 = -1$). The form factors (16) and (17) are determined respectively by the transverse permittivity $\varepsilon_{tr}(\omega,k)$ and by the third dispersion characteristic $A(\omega,k)$, which is due to pseudovector currents and which is absent in quantum electrodynamics (see Ref. 3).

The increase of the number of form factors in (13) in comparison with the vacuum case (2) is explained by the smaller number of symmetries of the medium. In particular, in a medium, translation invariance is destroyed as the result of existence of a selected reference system—the rest system of the medium as a whole.

For a Majorana neutrino we obtain instead of (13) the electromagnetic vertex in a medium in the form

$$\Gamma_\mu^{(M)}(\omega,\mathbf{k}) = [\gamma_5\gamma^\rho G_V\Pi_{\mu\rho}(\omega,\mathbf{k}) + i\varepsilon_{\mu\rho\nu\sigma}G_F A(\omega,k)\gamma^\rho q^\nu\Omega^\sigma]/2^{5/2}\pi\alpha + \Gamma_\mu^{(M)}(q), \tag{13'}$$

where the last term is determined by Eq. (12). Again in the massless case ($m_\nu = 0$) we have, using the condition of chirality for the 4-current $J_\mu \approx \bar{u}\Gamma_\mu u$ in the form $\frac{1}{2}(1-\gamma_5) \times u\mathbf{p}) = u(\mathbf{p})$, agreement of the structures (13) and (13').

Obtaining (13') from (13) by charge-conjugation is similar to the vacuum case which was discussed at the end of the preceding section. In addition in the case of a medium we take into account the difference in the signs ($\pm$) of the transformation of the two bilinear forms under the action of C-conjugation: the tensor $\Pi_{\mu\rho}(\omega,\mathbf{k})$ which stems from statistical averaging of the vector of the electron current $\langle\bar{\psi}_e\gamma_\mu\psi_e\rangle$ and the pseudotensor $\varepsilon_{\mu\rho\nu\sigma}A(\omega,k)q^\nu\Omega^\sigma$, which stems from the same averaging of the pseudovector $\langle\bar{\psi}_e\gamma_\mu\gamma_5\psi_e$.

Therefore, in addition to the four independent form factors in vacuum ($m_\nu \neq 0$), in a medium three additional form factors (14) appear in a Dirac neutrino. The electromagnetic structure of a Majorana neutrino (13') also becomes more complicated.

b. Multipole electromagnetic moments of a neutrino in an isotropic dispersive medium

We shall now use the general formulas (3')–(5) and (11) to discuss the interaction of neutrinos with external quasistatic fields in a medium. For this purpose instead of the vacuum vertices (2) and (12) we must substitute into the indicated formulas the electromagnetic vertices in a medium (13) and (13'). In addition it is necessary, generally speaking, to conserve the momentum $\mathbf{p}\neq 0$ in the bispinors $u(\mathbf{p})$ in these general formulas.

Actually, combining the rest system of the medium as a whole ($\Omega_\mu = \delta_{\mu 0}$) and the rest system of the particle ($\mathbf{p} = 0$) is an additional requirement. This requirement no longer follows from the limiting transition as $k\to 0$, which corresponds to forward scattering, $k = 2p\sin(\theta/2) = 0$, of a moving ($\mathbf{p}\neq 0$) particle. We recall that in the vacuum we chose the Breit system ($\mathbf{k} = -2\mathbf{p}$) and the transition to the rest system of the particle was carried out in the formulas (3')–(5) and (11) automatically.

The multipole moments obtained here can be considered to be effective coupling constants with external static fields for a moving neutrino in a medium which is stationary as a whole (the laboratory system of the medium). Such coupling constants will depend on the neutrino velocity. For example, instead of (3') we have the electric moment $l = m = 0$

$$Q_{00} = e\bar{u}_{r'}(\mathbf{p})\lim_{k\to 0}\Gamma_0(0,\mathbf{k})u_r(\mathbf{p})/2E_\mathbf{p}. \tag{3''}$$

The effective constant of coupling with an electric field for a Dirac neutrino with inclusion of (13) and (3'') is

$$Q_{00}^{(D)} = {}^1/_2 e_\nu^{\text{ind}}\,\delta_{rr'}(1 - rv/c). \tag{18}$$

For a Majorana neutrino, using (13'), we obtain from (3'')

$$Q_{00}^{(M)} = -e_\nu^{\text{ind}}\,\delta_{rr'}rv/c. \tag{19}$$

Here $r = \pm 1$ is the helicity of the neutrino ($(\boldsymbol{\sigma}\mathbf{p})\varphi_r = pr\varphi_r$), and

$$e_\nu^{\text{ind}} = -eG_V/2^{5/2}\pi\alpha r_D^2 \tag{20}$$

is the quasistatic induced electric charge of the neutrino,[14] which is determined by normalization of the longitudinal charge form factor (15) ($\lim_{k\to 0}F_\parallel(0,k) = e_\nu^{\text{ind}}/e$; see Refs. 13 and 14); r_D is the Debye radius of the plasma.

The charge e_ν^{ind} behaves as a point charge in the limit of low energies $|q|^2/M_W^2 \to 0$ (M_W is the mass of the W boson), is due to the weak ($\sim G_F$) attraction of the electrons (a δ-shaped inhomogeneity at the place where the particle is located), and is compensated by the charge of the ions (holes) at distances of the order of r_D. The system as a whole remains electrically neutral when neutrinos are introduced into it, i.e., on taking into account all components of a dispersive medium $k = e, i$ (electrons, ions), the electric charge is equal to zero:

$$\sum_k \oint \rho_k(r)\,d^3r = 0,$$

where the integration of the charge density $\rho_k(r)$ includes the boundary of the system.

The moments (18) and (19) have the meaning of charges only when the rest system of the particle (velocity $v=0$) coincides with the rest system of the medium as a whole, i.e., if the particle stops in the medium. From (18) one obtains the following definition: the electric charge of a Dirac neutrino exists and is equal to $Q_{00}^{(D)}=e_\nu^{\text{ind}}/2$ when $v=0$. A Majorana neutrino does not have an electric charge, $Q_{00}^{(M)}=0$ $(v=0)$, although a moving Majorana neutrino can interact with an external electric field [see Eq. (19)] and can radiate longitudinal plasmons without having intrinsic multipole moments in a medium for $\mathbf{p}=0$, except possibly an anapole moment (see below).

As should be the case, there is no difference between interactions of massless ($m_\nu=0$, $v=1$) Dirac and Majorana neutrinos with an external electrostatic field. For left-polarized ($r=-1$) neutrinos in the massless case ($m_\nu=0$) the effective coupling constants (18) and (19) are equal, $Q_{00}^{(D)}=Q_{00}^{(M)}=e_\nu^{\text{ind}}$. However, whereas right-polarized massless Majorana neutrinos ($r=+1$) have a coupling constant $Q_{00}^{(M)}=-e_\nu^{\text{ind}}$ [see Eq. (19)], to obtain the same constant for right-polarized Dirac antineutrinos ($r=+1$) formula (18) must be replaced by

$$\tilde{Q}_{00}^{(D)}=-{}^1/_2 e_\nu^{\text{ind}}\delta_{rr'}(1+rv/c). \tag{18'}$$

Here we have taken into account that the bispinor of a massive ($m_\nu\neq 0$) antineutrino has the form

$$v_r(\mathbf{p})=(E_p+m_\nu)^{1/2}\begin{pmatrix}-|\mathbf{p}|r\chi_r/(E_p+m_\nu)\\ \chi_r\end{pmatrix},$$

i.e., in the massless case ($m_\nu=0$) the left-handed currents in the matrix elements of the interaction automatically separate in (18) the contribution of right-polarized antineutrinos: $1/2(1-\gamma_5)v_1(p)=v_1(p)$, but $1/2(1-\gamma_5)v_{-1}(p)=0$. The overall minus sign in the formula (18′) appears as the result of the change of sign in the operator of the 4-current $\hat{J}_\mu=N(\hat{\bar{\psi}}\Gamma_\mu\hat{\psi})$ in the transition to antiparticles.

It is not difficult to show by means of Eqs. (13) and (13′) that there is no contribution to the static electric dipole moment, i.e., for a Dirac neutrino it coincides with the vacuum value (7), and for a Majorana neutrino it is altogether zero.

We shall obtain magnetic dipole coupling constants with a static transverse field which are analogs of the electric coupling constants (18) and (19) with a longitudinal electric field. For this purpose it is necessary to retain in the general formula (5) the momentum of a neutrino moving in a medium ($\mathbf{p}\neq 0$), rewriting the 3-vector of the magnetic moment μ_k^{fi} in the form

$$\mu_k^{fi}=\frac{iee_{kni}}{4E_p}\bar{u}_f(\mathbf{p})\left[\frac{\partial\Gamma_i(0,\mathbf{k})}{\partial k^n}\right]_{\mathbf{k}=0}u_i(\mathbf{p}). \tag{5'}$$

In the bispinor $\bar{u}(\mathbf{p}+\mathbf{k})$ and in the energy $E_{\mathbf{p}+\mathbf{k}}$ [in the normalization factor in (1)] we have omitted the transferred momentum $\mathbf{k}$, since the static ($\omega=0$) vertex $\Gamma(0,\mathbf{k})$ already depends linearly on the vector $\mathbf{k}$. (The medium as a whole is at rest.) Substituting here the electromagnetic vertex of a Dirac neutrino (13), we obtain an effective vector constant which depends on the particle velocity:

$$\mu_k^{(D)}=-\frac{eM(0,0)}{2E_p}\bar{u}_{r'}(\mathbf{p})\gamma_k\frac{(1-\gamma_5)}{2}u_r(\mathbf{p})$$

$$+\frac{eM_{\text{vac}}^{(D)}(0)}{2E_p}\bar{u}_{r'}(\mathbf{p})\Sigma_k u_r(\mathbf{p})-\frac{ieE_{\text{vac}}^{(D)}(0)}{2E_p}\bar{u}_{r'}(\mathbf{p})\alpha_k u_r(\mathbf{p}), \tag{21}$$

where $\Sigma_k=\gamma_5\gamma_0\gamma_k$, $\boldsymbol{\alpha}=\gamma_0\boldsymbol{\gamma}$, and in addition to the two magnetic form factors, the induced form factor $M(0,0)$ [see Eq. (17)] and the vacuum form factor $M_{\text{vac}}^{(D)}(0)$ [see Eq. (6)], there remains the contribution of the vacuum electric dipole moment $d=eE_{\text{vac}}^{(D)}(0)$.

In the case of a Majorana neutrino the magnetic dipole coupling constant

$$\mu_k^{(M)}=-\frac{eM(0,0)}{2E_p}\bar{u}_{r'}(\mathbf{p})\gamma_k u_r(\mathbf{p})$$

depends directly on the velocity v_k and is equal to

$$\mu_k^{(M)}=-eM(0,0)v_k\delta_{r'r}. \tag{22}$$

Only for a stopped neutrino ($\mathbf{p}=0$, $m_\nu\neq 0$), i.e., in the case of coincidence of the rest systems of the medium and the particle, is the total magnetic moment of a Dirac neutrino in a medium equal to the sum

$$\mu_k^{(D)}=e[{}^1/_2M(0,0)+M_{\text{vac}}(0)]\varphi_{r'}^{+}\sigma_k\varphi_r, \tag{23}$$

while in a Majorana neutrino, as in vacuum, there is no magnetic moment, $\mu_k^{(M)}=0$ [see Eq. (22)]. The quantity $M(0,0)$ in Eq. (22) is the induced magnetic moment[3] in units of the charge,

$$\mu_\nu^{\text{ind}}=eM(0,0)=\frac{e_\nu^{\text{ind}}}{2m_e}\frac{2}{(1+4\xi)}, \tag{24}$$

where e_ν^{ind} is the charge (20).

c. Anapole moment of a neutrino in a ferromagnetic material

The anapole moments $G_i^{(D)}$ and $G_i^{(M)}$ calculated by substitution of Eqs. (13) and (13′) into the general formula (11) reduce to only the corresponding vacuum values (10), and in the static case ($\omega=0$) each of the terms in the formula (11) is equal to zero, i.e.,

$$\left[\frac{\partial^2\Gamma_n(0,\mathbf{k})}{\partial k^i\partial k_n}\right]_{k=0}=\left[\frac{\partial^2\Gamma_i(0,\mathbf{k})}{\partial k_n\partial k_n}\right]_{k=0}=0.$$

If the neutrino interacts with a high-frequency field $\omega\gg k\langle v\rangle$, where $\langle v\rangle$ is the average velocity of the particles in the medium, then the multipole moments will depend on the time (cf. Ref. 11). The medium cannot be considered completely homogeneous, i.e., in the calculation it is necessary to take into account the small spatial dispersion. In an isotropic dispersive medium with vertices (13) and (13′) it is then possible to obtain the exact equality of quantities which are not equal to zero:

$$\left[\frac{\partial^2\Gamma_n(\omega,\mathbf{k})}{\partial k^i\partial k_n}\right]_{k=0}=2\left[\frac{\partial^2\Gamma_i(\omega,\mathbf{k})}{\partial k_n\partial k_n}\right]_{k=0}\neq 0,$$

which reduces the anapole moment $G_i^{\text{ind}}(t)$ to zero.

Thus, in an isotropic dispersive medium there are no induced anapole moments ($G_i^{\text{ind}}=0$). However, in an anisotropic medium, for example, in a ferromagnetic material, the situation changes. In interaction of a (Dirac) neutrino with a static transverse field it is possible to use the electromagnetic vertex[5]

$$\Gamma_n(0,\mathbf{k}) = \frac{G_V(1+\gamma_5)\gamma_i}{2^{7/2}\pi\alpha} e_{imp}e_{kln}k_m k_l[\mu_{pk}^{-1}(0,\mathbf{k})-\delta_{kp}] + \frac{G_F m_e(1+\gamma_5)\gamma_i}{2^{5/2}\pi\alpha} e_{nkp}k_p[\mu_{ik}^{-1}(0,\mathbf{k})-\delta_{ik}]. \quad (25)$$

The pseudovector part, which originates from the statistical averaging of the pseudovector of the electron current $\langle\bar{\psi}_e\gamma_\mu\gamma_5\psi_e\rangle$, does not contribute to the anapole moment (11) as a result of the even dependence of the magnetic susceptibility tensor $\mu_{ik}(0,\mathbf{k}) \approx \delta_{ik}\mu(0,k)$) on the value of the wave vector k [the second term in (25)]. The first vector term in (25) stems from the part of the averaged electron current $\langle\bar{\psi}_e\gamma_\mu\gamma_e\rangle$, which is due to the electron spin, $j_i^{(e)} = \mathrm{curl}_i\mathbf{M}^{(e)}(\mathbf{x},t)/e$, where

$$\mathbf{M}^{(e)}(\mathbf{x},t) = \mu_B\int d^3p\,\mathrm{Sp}[\boldsymbol{\sigma}\hat{f}^{(e)}(\mathbf{p},\mathbf{x},t)]$$

is the magnetization of the medium, which is described by the single-particle density matrix $\hat{f}^{(e)}(\mathbf{p},\mathbf{x},t)$. The corresponding interaction of a neutrino with a static electric field polarizing the medium, which is proportional to $\int[d^3k/(2\pi)^3]e^{i\mathbf{k}\mathbf{x}}\bar{u}_\nu(p')\Gamma_n(0,\mathbf{k})u_\nu(p)A_n(0,\mathbf{k})/2E_p$, has in the point approximation (current$\times$current) the form

$$-\frac{G_F}{2^{1/2}e}\mathrm{rot}_i\,\mathbf{M}^{(e)}\bar{u}_\nu\gamma_i\frac{(1-\gamma_5)}{2}u_\nu.$$

Substitution of the first term from (25) into the formula (11) on the simplest assumption of diagonality of the tensor $\mu_{ik}{}^{-1} = \mu^{-1}\delta_{ik}$ leads to a nontrivial result for the induced anapole moment $\mathbf{G} = eG_{me}^{\mathrm{ind}}\varphi^+\sigma\varphi$, directed along the spin[4]:

$$G_{med}^{ind} = \frac{G_V[\mu(0,0)-1]}{2^{7/2}\pi\alpha\mu(0,0)}. \quad (26)$$

This moment in a ferromagnetic material ($\mu \gg 1$) has a finite value and corresponds to a toroidal dipole moment with a density

$$\mathbf{g}(\mathbf{r}) = \frac{eG_V\varphi^+\sigma\varphi}{2^{7/2}\pi\alpha}\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}\mathbf{r}}[1-\mu^{-1}(0,k)]. \quad (27)$$

The latter is related to the density of the poloidal current of real electrons of a medium by the equation $\mathbf{J} = \nabla\times\nabla\times\mathbf{g}(\mathbf{r})$, i.e., the current forms loops wound on a torus with axis along the neutrino spin (more precisely a torus composed of densely packed circular currents so that there is no azimuthal current along the ring of the torus and there is no corresponding magnetic moment).

Therefore the elementary magnetic dipoles formed by the current loops in a ferromagnetic material form a closed ring around the neutrino spin (as the result of projection of the induced current $j_i^{(e)}$ on the direction of the spin) in correspondence with the qualitative ideas of Ref. 10. A similar inhomogeneity of the distribution of magnetization does not create a magnetic moment, but leads to appearance of an anapole moment (26).

4. ELASTIC SCATTERING OF MAJORANA NEUTRINOS BY A SPINLESS NUCLEUS IN AN ISOTROPIC PLASMA

As an illustration of the theory of electromagnetic multipole moments of neutrinos in a medium set forth above we shall consider the process of scattering of Majorana neutrinos by nuclei with a number of protons Z and a number of neutrons N in an isotropic plasma. Taking into account the complete electromagnetic vertex (13′) for a Majorana neutrino we obtain a cross section

$$\frac{d\sigma^{(M)}}{d\Omega} = \frac{G_F^2E^2}{8\pi^2}\Big[Z(1-4\xi)-N+\frac{(1+4\xi)Z}{1+(kr_D)^2} + \frac{2^{5/2}G_{vac}^{(M)}(0)\pi Z\alpha(kr_D)^2}{eG_F(1+(kr_D)^2)}\Big]^2 v^2\cos^2(\theta/2), \quad (28)$$

where $v = p/E$ is the neutrino velocity and $G_{\mathrm{vac}}^{(M)}(0)$ is the vacuum anapole moment (in a plasma there is no moment (26), since according to the Bohr-Van Leeuwen theorem for an unpolarized medium in the classical limit we have $\lim_{k\to 0}\mu(0,k) \equiv 1$).

We note that over a wide range of neutrino energies, including the ultrarelativistic case $v = 1$, the vacuum anapole contribution calculated in the one-loop approximation should be a correction to the first two terms of the cross section (28), to the Born contribution of the neutral charge of hadrons ($\propto Z(1-4\xi) - N$), and to the polarization contribution ($\propto (1+4\xi)Z[1+(kr_D)^2]^{-1}$ due to the electric constant (19) (cf. Ref. 6).

The electromagnetic interaction of a Majorana neutrino with charged particles in an isotropic plasma is due not to its anapole moment, but to polarization of the medium by the Coulomb field of the target nucleus in the vicinity of the scattered neutrino. As a result the neutrino interacts weakly with the inhomogeneity of the electron concentration in the vicinity of a nucleus—with the virtual electrons and holes in the polarization cloud. The latter are coupled to the nucleus by electromagnetic forces, which leads to an additional channel of scattering through exchange of a longitudinal plasmon.[6]

For small impact parameters or large momentum transfers ($kr_D \gg 1$) this channel is inappreciable: scattering of a neutrino occurs on a bare nucleus having a weak charge proportional to $Z(1-4\xi) - N$. For large impact parameters or small momentum transfers, $kr_D \ll 1$, the scattering, as follows from Eq. (28), is screened completely and occurs already not at the nucleus but as if by a neutral atom.[3)] Thus, the total cross section in the case of electron Majorana neutrinos has in the limit $(pr_D) \ll 1$ the form

$$\sigma_{tot}^{(M)}(E) \approx G_F^2E^2(3Z-A)^2v^2/4\pi,$$

and in the case of μ and τ neutrinos we obtain

$$\sigma_{tot}^{(M)}(E) \approx G_F^2E^2(Z-A)^2v^2/4\pi.$$

With the exception of the factor v^2, these cross sections agree with the result of Ref. 15 for scattering of Dirac neutrinos by a neutral atom.

As in Ref. 6, if we neglect the contribution of neutral currents and corrections from vacuum electromagnetic moments, we obtain from (28) a cross section of the Mott type (Coulomb scattering by a screened potential):

$$\frac{d\sigma_C^{(M)}}{d\Omega} = \frac{Z^2\alpha^2}{p^2v^2}\frac{(e_\nu^{\mathrm{ind}}/e)^2[1-v^2\sin^2(\theta/2)-m_\nu^2/E^2]}{[\sin^2(\theta/2)+(2pr_D)^{-2}]^2}, \quad (29)$$

whose vanishing for a stopped neutrino ($v = 0$) signifies absence in a Majorana neutrino of electric charge [see Eq. (19)], in contrast to a Dirac neutrino.[6]

[1)]We use the system of units $\hbar = c = 1$, a Feynman metric $q^2 = q_\mu q^\mu = \omega^2 - \mathbf{k}^2, \mu = 0, 1, 2, 3$, and a standard representation of the Dirac γ matrices with $\gamma_5 = \gamma_5^+ = -i\gamma^0\gamma^1\gamma^2\gamma^3$; the Latin indices i, k, $m = 1, 2, 3$.

[2)]The masses of the initial and final states are identical, even for a massless neutrino ($M^i = M_f = 0$).

[3)]We recall that we are considering a completely ionized plasma in which there are no neutral atoms but the Coulomb field of each nucleus is screened by free electrons: $\varphi(r) = Ze\exp(-r/r_D)/r$.

[1]V. N. Oraevskiĭ, A. Yu. Plakhov, V. B. Semikoz, and Ya. A. Smorodinskiĭ, Zh. Eksp. Teor. Fiz. **93**, 1557 (1987) [Sov. Phys. JETP **66**, 890 (1987)].

[2]B. Kayser, Phys. Rev. D **26**, 1662 (1982).

[3]V. B. Semikoz, Yad. Fiz. **46**, 1592 (1987) [Sov. J. Nucl. Phys. **46**, 946 (1987)].

[4]V. B. Semikoz and Ya. A. Smorodinskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 361 (1988) [JETP Lett. **48**, 399 (1988)].

[5]L. B. Leinson and V. N. Oraevskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 58 (1988) [JETP Lett. **48**, 60 (1988)].

[6]L. B. Leinson, V. N. Oraevskiĭ, and V. B. Semikoz, Yad. Fiz. **48**, 1513 (1988) [Sov. J. Nucl. Phys. **48**, 963 (1988)].

[7]V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, *Quantum Electrodynamics* (in Russian), Nauka, Moscow, 1980, p. 196 [Pergamon, 1982].

[8]K. Fujikawa and R. E. Shrock, Phys. Rev. D **6**, 2923 (1972).

[9]K. Fujikawa and R. E. Shrock, Phys. Rev. Lett. **45**, 963 (1980).

[10]Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **33**, 1531 (1957) [Sov. Phys. JETP **6**, 1184 (1958)].

[11]B. M. Dubovik and A. A. Cheshkov, Fiz. Elem. Chastits At. Yadra **5**, 791 (1974) [Sov. J. Part. Nucl. **5**, 318 (1975)].

[12]E. S. Fradkin, Trudy FIAN (Proc. of the Lebedev Institute) **29**, 7 (1965).

[13]V. N. Oraevskiĭ, V. B. Semikoz, and Ya. A. Smorodinskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 549 (1986) [JETP Lett. **43**, 709 (1986)].

[14]V. N. Oraevsky and V. B. Semikoz, Physica A **142**, 135 (1987).

[15]R. R. Lewis, Phys. Rev. D **21**, 663 (1980).

Translated by Clark S. Robinson