Possibility of nonthreshold γ-ray channeling in crystals

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The propagation of radiation both in ideal single crystals and in crystals with microchannels is analyzed on the basis of the general idea of particle channeling. The problem is approached by the route of the exact (unaveraged) equations of electrodynamics for crystals consisting of a system of thin planes. It is shown that in a strictly periodic lattice with a single plane per period there is no radiation channeling (i.e., there are no modes which are localized along the transverse coordinate), regardless of the relation between the wavelength of the radiation and the period of the lattice. This is a universal result. It stems from the circumstance that the interaction of the radiation with an individual plane is weak. If there is a planar microchannel with a width exceeding the lattice period, there is always a channeled mode with an absorption coefficient which differs from that in the case of motion in a continuous medium or in a uniform lattice. For narrow microchannels, with a width comparable to the lattice period, the spatial dimension of the mode in the transverse direction lies between tens of angstroms (for Mössbauer radiation) to a few microns (for nonresonant x radiation). The probability for the excitation of a nonthreshold mode of this sort, which exists regardless of the relation between the radiation wavelength and the width of the microchannel, can reach unity for beams which have a transverse dimension comparable to the width of the mode. Some general conclusions are also reached regarding the propagation of radiation in single crystals which lack microchannels. These predictions distinguish this regime of motion in a substantial way from that of propagation in a continuous medium of the same optical thickness. In particular, it is shown that during small-angle motion along planes in crystals the absorption coefficient and the structure of the field are different from those in an isotropic medium. This regime is called "nonthreshold quasichanneling."

It is generally believed that the well-known phenomenon of the channeling of charged particles has no analog in the case of electromagnetic radiation which is passing through a single crystal.[1-4] It is furthermore believed that no radiation-transport regime of this type prevails even in any type of individual channel with a width a less than a threshold value $\Delta x_{\min}$, which is some hundreds of angstroms.

For radiation with a wavelength $\lambda = 2\pi/k$ exceeding the lattice period d and the channel width a, that thesis seems obvious and provokes no debate. In the other case, that of short-wave radiation, with $\lambda < a, d$, that assertion is customarily justified in the following way. For a wave which is localized in the transverse coordinate x (transverse with respect to the channeling direction) in a region Δx there will unavoidably be some uncertainty in the transverse wave number, $\Delta k_x \gtrsim 2\pi/\Delta x$. Under the assumption that channeling is possible only if the uncertainty (related to Δk_x) in the direction in which the wave is moving $\Delta\Theta \approx \Delta k_x/k$ is smaller than the angle of total external reflection, $\Theta_0 = (2|\chi|)^{1/2}$ [$\chi \approx -\omega_e^2/2\omega^2$ is the nonresonant susceptibility of the medium in the x-ray range, and $\omega_e = (4\pi N_e^2/m)^{1/2}$ is the plasma frequency], we find the minimum localization region to be $\Delta x_{\min} \gtrsim 2\pi c/\omega_e$. For the typical values $\omega_e \approx (3\text{–}6)\cdot 10^{16}\ \text{s}^{-1}$ we have $\Delta x_{\min} \approx 300$–600 Å. This figure is more than two orders of magnitude greater than the distance between the planes in a crystal. Even in the case of resonant Mössbauer radiation with $|\chi_{\max}| \approx 10^{-4}$ we would have $\Delta x_{\min} \approx 100$ Å. It is concluded on this basis that there can be no nondiffractive orientational motion, strongly coupled with the crystal, of x radiation or γ radiation. This motion would be impossible in natural homogeneous crystals, in crystals with microchannels of width $a < \Delta x_{\min}$, and in superlattices with a period $d < \Delta x_{\min}$.

On the other hand, it is well known from the theory of plane dielectric optical-range waveguides (Ref. 5, for example) that there exists a rich spectrum of natural modes in waveguides for which the width of a central layer, with a susceptibility higher than that of its surroundings, satisfies $a \gg \lambda$. As the ratio a/λ is reduced, the mode spectrum thins out, and at $a/\lambda < 1$ only a single mode can propagate through the waveguide (a "mode without a cutoff"). The propagation constant (longitudinal wave number) for it,

$$\beta \approx k(1+\chi/2+k^2a^2\chi^2/8), \tag{1}$$

is found from the dispersion relation

$$\operatorname{tg}[a(k^2-\beta^2)^{1/2}/2] = [(\beta^2-k^2\varepsilon)/(k^2-\beta^2)]^{1/2}, \quad \varepsilon = 1+\chi$$

and differs by an amount $\Delta\beta = k^3a^2\chi^2/8$ from that in the case of a continuous medium without a channel. A characteristic circumstance for this mode is that, in opposition to the qualitative estimates made above, this mode exists for any value of a, no matter how small. At a purely formal level, such a mode should exist even if the value of a approaches the crystal period d, and even if it vanishes. Since the entire standard theory of dielectric waveguides is built on the foundation of the macroscopically averaged equations of electrodynamics (i.e., this theory uses an average susceptibility $\chi = \langle\chi(x)\rangle$ as a starting point and ignores the atomic structure of the channel walls), both a direct derivation from (1) in the case $a = d$ and the qualitative estimates of $\Delta x_{\min}$ made above are generally incorrect, and they do not resolve the question of whether radiation can be channeled in single crystals.

In the present we work from an exact solution of the

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rigorous electrodynamic equations—equations which have not been averaged over the distance between planes—to analyze the problem of the mode structure and channeled motion of photons in a crystal and the macroscopic manifestations of this effect. This is the first such analysis. In particular, we predict that a channeled regime of photon motion occurs for radiation of arbitrary wavelength (including $\lambda \gg a,d$), even for arbitrarily narrow microchannels (down to $a \to d$, but with $a \neq d$). In a sense (without transverse localization of the wave, but with a reduced coefficient over a narrow angular interval near $\Theta \approx 0$), this channeled regime prevails even at $a = d$, i.e., in a uniform lattice without a channel. Such a regime, which is not limited by a threshold $a,d > \lambda$, $\Delta x_{\min}$; $a \gg d$, could reasonably be called "nonthreshold channeling."

For the analysis we first consider a crystal which consists of two parts separated by a distance a. Each part of the crystals constitutes a system of thin planes with a period d, which are oriented parallel to the interface. The middle of the space between the parts of the crystal, a, coincides with the yz plane. In the course of the analysis, a may vary all the way to $a = d$, giving us the case corresponding to a common single crystal.

It is a simple matter to justify the use of continuous planes which are averaged along the longitudinal direction z by making use of the wave nature of radiation. Specifically, the change $\Delta\beta$ in the wave number in the course of the internal rereflection of waves which would be required for channeling is related to the effective reflection angle Θ by $\Delta\beta_{\max} = k - \beta_{\min} \approx k\Theta^2/2$. Using $\Theta_{\max} \lesssim (2|\chi|)^{1/2}$, we find that the minimum coherence length along the scattering planes, $\Delta z_{\min} \gtrsim 2\pi/\Delta\beta_{\max}$, within which the structure of the scatterer is in principle indistinguishable, is determined by $\Delta z_{\min} \gtrsim 2\pi/(k|\chi|)$. The latter relation leads us to the estimate $\Delta z_{\min} \gg d_z e$ for all possible values of k and of the longitudinal period of the lattice, d_z.

As a sinusoidal wave is incident along the positive z direction near crystal planes which are infinite along the y axis, there is no variation along y. This circumstance is expressed formally by the expression $\partial E_y/\partial y = 0$. Maxwell's equations then reduce to the wave equation

$$\partial^2 E_y/\partial x^2 + \partial^2 E_y/\partial z^2 + k^2 \varepsilon(x) E_y = 0, \quad k = \omega/c. \tag{2}$$

For a system of uniform planes the dielectric constant can be written

$$\varepsilon(x) = 1 + \chi(x) = 1 + \chi d \sum_{n=0}^{\infty} \delta(|x| - a/2 - nd). \tag{3}$$

Here $\chi \equiv \langle \chi(x) \rangle$ is the macroscopic susceptibility of the crystal which has been averaged over the period d and which corresponds to an average dielectric constant $\varepsilon = 1 + \chi$. The replacement of the actual susceptibility $\chi(x)$ for planes of nonzero "thickness" $\alpha \approx 0.1$–0.3 Å by relation (3), which holds for infinitely thin planes, can be justified quite easily both by the natural conditions a, $d \gg \alpha$ and by arguments similar to those advanced above in the estimate of $\Delta x_{\min}$. Since the relation $\Delta x_{\min} \gg \alpha$ holds for any crystal, the actual structure of $\chi(x)$ within a is unimportant and can be approximated by expression (3). This replacement is not possible in the case of superlattices with $\alpha \gtrsim \Delta x_{\min}$.

Writing $E_y = u(x)v(z)$ we find $v(z) = \exp(i\beta z)$, where the separation parameter β has the meaning of a longitudinal wave number.

As a result, we arrive at an equation for the transverse modes:

$$\frac{d^2u}{dx^2} - G \sum_{n=0}^{\infty} \delta(|x| - a/2 - nd)\, u + \varkappa^2 u = 0,$$

$$G = -k^2 d\chi, \quad \varkappa^2 = k^2 - \beta^2. \tag{4}$$

The eigenvalues $\varkappa$ and the modes u can be found from the boundary conditions. Making use of the symmetry of the problem, we can characterize the transverse structure of a mode either by an even solution

$$u_a = A\cos(\varkappa x) \tag{5}$$

or by an odd solution

$$u_a = A\sin(\varkappa x). \tag{6}$$

We will show below that nonthreshold channeling can occur in a crystal only for an even mode, (5).

In the region $a/2 \leqslant |x| \leqslant a/2 + d$, a solution of Eq. (4) is

$$u = B\exp(i\varkappa x) + C\exp(-i\varkappa x). \tag{7}$$

Since the susceptibility is periodic, $\chi(x) = \chi(x + d)$, solution (7) must obey the condition of the Bloch theorem, $u(x + d) = fu(x)$, where $f = \exp(ivd)$ is the Bloch parameter, and v the quasimomentum. Making use of that circumstance and also the continuity, $u(a/2 + d - 0) = u(a/2 + d + 0)$, we find

$$u(a/2 + d) = fu(a/2). \tag{8}$$

Correspondingly, since we have $u'(a/2 + d) = fu'(a/2)$ and, simultaneously,

$$u'(a/2 + d - 0) = u'(a/2 + d + 0) - Gu(a/2 + d)$$

[this relation follows directly from (4) and includes the increment in the jump in the derivative which is characteristic of singular susceptibilities and also, in quantum mechanics, characteristic of singular potentials], we find

$$u'(a/2 + d) = fu'(a/2) + Gu(a/2 + d). \tag{9}$$

The solution of the homogeneous system (8), (9) has a nontrivial value for the amplitudes B and C only if the determinant of this system vanishes. The latter requirement leads us to an equation for the Bloch parameter:

$$f^2 - f[2\cos(\varkappa d) + (2G/\varkappa)\sin(\varkappa d)] + 1 = 0. \tag{10}$$

To find the spectrum of transverse wave numbers $\varkappa$, we make use of the boundary conditions for the solutions (5), (6), and (7) in the plane $|x| = a/2$, i.e., at the boundary of the channel:

$$u_a(a/2) = u(a/2), \quad u_a'(a/2) = u'(a/2) - Gu_a(a/2). \tag{11}$$

Once the amplitude C has been eliminated with the help of the relation

$$C = B\exp(i\varkappa a)[f - \exp(i\varkappa d)]/[\exp(-i\varkappa d) - f],$$

which follows from (8), we can reduce the system (10) to the dispersion relation

$$\mathrm{tg}(\varkappa a/2)=(G/\varkappa)+[\cos(\varkappa d)-f]/\sin(\varkappa d) \qquad (12)$$

for the even solution (5) or

$$\mathrm{ctg}(\varkappa a/2)=-(G/\varkappa)-[\cos(\varkappa d)-f]/\sin(\varkappa d) \qquad (13)$$

for the odd solution (6). The resulting system of equations, (10), (11), (13), contains the complete solution of the problem.

We will first show that, in contrast to the direct analogy with the limiting case $a = d$ of a microchannel in an ordinary dielectric waveguide with "continuous" walls,[1] no channeling will occur in a uniform lattice with one plane per period d (no channeling will occur in this case in the sense that there will be no mode which is localized along the transverse coordinate). To demonstrate this point, we eliminate $G/\varkappa$ from Eqs. (10) and (12) and also from (10) and (13). We find expressions for f in the cases of even and odd modes:

$$f=[\cos(\varkappa d)+\mathrm{tg}(\varkappa a/2)\sin(\varkappa d)]^{-1},$$
$$f=[\cos(\varkappa d)-\mathrm{ctg}(\varkappa a/2)\sin(\varkappa d)]^{-1}. \qquad (14)$$

In the limit $a = d$, simple trigonometric conversions lead to the result $f = \pm 1$, with a purely real quasimomentum v. Using (8), we see that this result is evidence that the even and odd modes are completely periodic (with periods of d and $2d$, respectively). These modes are therefore completely unlocalized, regardless of the relation between d and λ! We recall that this result was derived within the framework of the original restrictions $\alpha \ll d$, $\Delta x_{\min}$ and is valid only for single crystals. The effect is essentially related to the very weak reflection from an individual plane; this situation may change in artificial structures, where channeling is of course also possible within a single layer in a periodic superstructure.

Let us examine the solution at $a > d$. Eliminating the parameter f from system (10), (12), (13), we find the following dispersion relations:

$$2\varkappa/G=\sin(\varkappa a)+2\cos^2(\varkappa a/2)\,\mathrm{ctg}(\varkappa d) \qquad (15)$$

for an even solution and

$$2\varkappa/G=-\sin(\varkappa a)+2\sin^2(\varkappa a/2)\,\mathrm{tg}(\varkappa d) \qquad (16)$$

for an odd solution. We will analyze these relations only for the most important and most debatable case, that of a thin microchannel with $\varkappa a$, $\varkappa d \ll 1$.

A direct expansion of (15) leads to a wave number

$$\varkappa=\{(G/d)-G^2[1+3(a-d)^2/d^2]/12\}^{1/2}, \qquad (17)$$

which characterizes a unique even nonthreshold mode $u(x)$. Correspondingly, the expression for $f \approx 1 + ivd$ in this case becomes

$$f\approx 1-G(a-d)/2=1+k^2d(a-d)\chi/2. \qquad (18)$$

The imaginary part $\nu'' = -k^2(a-d)\chi'/2$ of the complex quasimomentum $\nu = \nu' + i\nu''$ for the solution (17) characterizes the decrease in the mode amplitude with distance from the channel in the transverse direction in the region $|x| > a/2$. It can thus be seen from the f structure that the total spatial width of the mode is roughly equal to $\Delta = a + 2/|\nu''|$. Finally, we can write the overall structure of a nonthreshold channeling mode:

$$E_\nu[a/2+nd\leqslant|x|\leqslant a/2+(n+1)d, z]=u(x)\cdot\exp[i(\nu nd+\beta z)],$$
$$u(x)=A\cos(\varkappa a/2)\{f\sin[\varkappa(|x|-a/2-nd)]$$
$$-\sin[\varkappa(|x|-a/2-(n+1)d)]\}/\sin(\varkappa d);$$
$$E_\nu(0\leqslant|x|\leqslant a/2)=A\cos(\varkappa x)\exp(i\beta z). \qquad (19)$$

This result is illustrated by Fig. 1.

The dispersion relation (16) has no solution for odd modes with $\varkappa a$, $\varkappa d \ll 1$, so there are no odd modes in a narrow channel.

Analysis of the expression for the longitudinal wave number,

$$\beta\equiv\beta'+i\beta''=(k^2-\varkappa^2)^{1/2}$$
$$\approx k-(G/2kd)\{1-Gd[1+3(a-d)^2/d^2]/12\};$$
$$\beta'=k\{1+\chi'/2+k^2d^2[(\chi')^2-(\chi'')^2][1+3(a-d)^2/d^2]/24\},$$
$$\beta''=k\chi''\{1+k^2d^2\chi'[1+3(a-d)^2/d^2]/6\}/2, \qquad (20)$$

and the relation for ν'' together shows that true channeling [by which we mean the existence of a mode which is localized

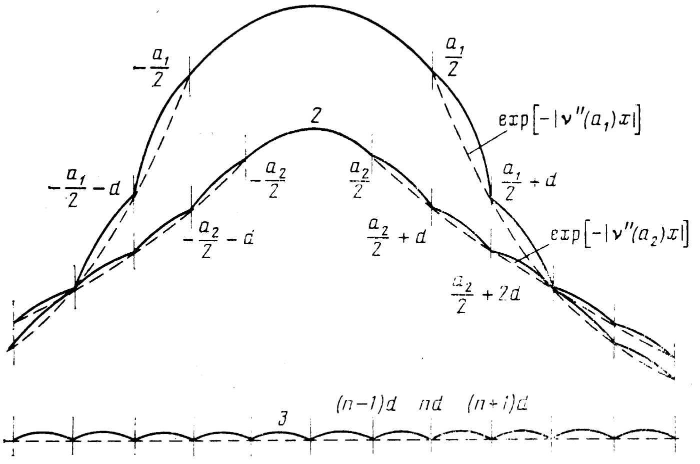

FIG. 1. 1—Structure of a nonthreshold mode in the presence of a microchannel of width $\alpha = \alpha_1 > d$; 2—the same, for $\alpha = \alpha_2 > d$, $\alpha_2 < \alpha_1$; 3—structure in the case of quasichanneling in a uniform lattice with $\alpha = d$. The vertical bars show the positions of crystal planes.

in the transverse direction, whose amplitude falls off on both sides of the channel, and for which the longitudinal attenuation coefficient is lower by an amount $\delta\beta'_a = k^3\chi''\chi'(a-d)^{2/4}$ than that for a uniform lattice] occurs for an arbitrary value $a>d$ and under the condition $\chi'<0$.

Interestingly, although localized modes are not found in a uniform lattice without a channel ($a=d$, $v''=0$), the values of β' and β'' and the wave structure (line 3 in Fig. 1) nevertheless differ from the results $\beta' = k(1+\chi'/2)$, $\beta'' = k\chi''/2$, which correspond to a plane wave and which are found from the average electrodynamic equations. The differences are by amounts

$$\delta\beta_d' = k^3d^2[(\chi')^2-(\chi'')^2]/24, \quad \delta\beta_d'' = k^3d^2\chi''\chi'/12.$$

This regime of orientational motion of a wave in an ideal lattice with $a=d$ could reasonably be called "nonthreshold quasichanneling." It has some interesting features. In particular, along with an absolute change $\delta\beta'_d(\chi',\chi'')$ in the longitudinal attenuation coefficient, there is an asymmetry of this coefficient as a function of the frequency on the two sides of the absorption line. This situation is not found in an isotropic medium of the same composition.

The efficiency with which a nonthreshold mode is excited can be characterized by an amplitude K, which, as in the case of charged particles, can be found from the following condition: At the entrance surface of the crystal ($z=0$), an incident plane wave $E_{y0} = \exp[i(k_{0x}x + k_{0z}z)]/L^{1/2}$, which is normalized to the transverse dimension of the beam or the crystal, L, and the channeled mode E_y are continuous. We find the expression

$$K = \int E_{y0}(x,0)E_y^*(x,0)\,dx.$$

Since the transverse dimensions of the mode satisfies $\Delta \approx a + 2|v''| \gg a, d$ and the oscillation amplitude of the field E_y over a period d is very small, we can write an approximate expression for the field E_y:

$$E_y(x,z) \approx |v''|^{1/2}\exp(i\beta z - |v''x|).$$

For such a field E_y, the total probability for excitation of a nonthreshold mode is

$$|K|^2 \approx 4/\{L|v''|[1+(k_{0x}/|v''|)^2]\}, \quad \text{if} \quad L>2/|v''|.$$

In particular, when radiation is incident exactly along a channel axis ($k_{0x}=0$), the excitation probability $|K|^2 \approx 4/(L|v''|)$ will actually be determined by the ratio of the transverse dimension of the mode and the width of the crystal (or the width of the beam).

Let us look at some quantitative characteristics of the channeling. For nonresonant radiation we would have $\chi' \approx -\omega_e^2/2\omega^2$ and $v'' \approx \omega_e^2(a-d)/4c^2$. The initial conditions $\varkappa a$, $\varkappa d \ll 1$ are seen to hold, for this particular $\varkappa$, under the condition $a < a_{\max} \approx 2^{1/2}c/\omega_e \approx 150$–$300$ Å. With $d = 2.5$ Å, $\omega_e \approx (3$–$6)\cdot 10^{16}\ \text{s}^{-1}$, and values $a \approx 5$–30 Å, we then find $v'' \approx (0.5$–$5)\cdot 10^4\ \text{cm}^{-1}$, which corresponds to a total width $\Delta \approx 4$–$0.4\ \mu\text{m}$ for the mode. The mode width decreases sharply near a resonance. For example, near the K edge of the fundamental absorption band of a diamond crystal with $\hbar\omega_0 = 284$ eV we would have $\chi' \approx -4\cdot 10^{-3}$ (Ref. 6) and $\Delta \approx 0.8$–$0.08\ \mu\text{m}$ for the same values $a \approx 5$–30 Å. Still greater localization of a nonthreshold mode would correspond to Mössbauer transitions, for which we would have

$$\chi' = \sigma_0 N\Gamma c(\omega_0-\omega)/\{2\omega_0[(\omega_0-\omega)^2+\Gamma^2/4]\},$$

where σ_0 is the total resonant cross section, which includes the Mössbauer factor, and Γ is the level width. For a tantalum crystal which contains an abundance of 99.99% of the isotope ^{181}Ta in its natural state, with $\hbar\omega_0 = 6.26$ keV, $k \approx 3\cdot 10^8\ \text{cm}^{-1}$ and $\sigma_0 \approx 1.7\cdot 10^{-18}\ \text{cm}^2$, we find $\chi' \approx -10^{-4}$ at the frequency $\omega = \omega_0 + \Gamma$. We find that the size of the mode for a microchannel of width $a \approx 5$–12 Å decreases to 120–40 Å.

The relative decrease in the absorption coefficient during channeling, relative to that for unchanneled motion (in an isotropic medium of the same optical density or in crystals lacking a channel at large angles of incidence), is $\delta\beta''/\beta'' \approx 10^{-4}$–$10^{-2}$ for nonresonant x radiation and $\delta\beta''/\beta'' \approx 0.5(10^{-3}$–$10^{-1})$ for resonant x radiation (in the case of diamond, which we examined above). Correspondingly, for a crystal with ^{181}Ta we would have $\delta\beta''/\beta'' \approx 0.5(10^{-2}$–$10^{-1})$ near a Mössbauer γ transition. All of the effects found here could be observed easily by measuring (for example) the angular dependence of the transmission. An extremely characteristic circumstance is that the relative contrast of the transmission maximum along the channeling direction, $[\exp(\delta\beta''z)-1]/[\exp(\delta\beta''z)+1]$, will increase with the thickness of the medium. Despite the small value of $\delta\beta''$, it may reach a value of some tenths at large values $z \gg 1/|\beta''|$. We might add that for extended samples of this type it should also be easy to observe nonthreshold quasichanneling, which would be seen as a maximum in the transmission in the $\Theta \approx 0$ direction in high-quality single crystals lacking channels.

In conclusion we would like to point out that (for example) voids of width $a \approx 10$–50 Å in intercalated crystals or, possibly, structural defects of crystals such as edge dislocations with $d < a \lesssim 2d$ might serve as microchannels.

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