Laser-induced electron-deformation-thermal instability and a semiconductormetal phase transition involving superstructure formation

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A novel electron-deformation-thermal instability (EDTI) induced on the surface of a semiconductor by short powerful laser pulses is considered. The appearance of the EDTI is due to modulation of the band gap by deformation, by generation of nonequilibrium carriers, and by heating of a surface layer. Under certain conditions there can be three qualitatively different types of EDTI: generation of surface acoustic waves, "softening" of the frequencies of surface acoustic waves, and generation of static ordered surface structures. Development of the EDTI creates, depending on the geometry of the pump field and the symmetry of the surface, coupled surface fields of the strain, nonequilibrium carrier density, and temperature in the form of one-dimensional gratings, concentric rings, and radial-ring structures. The periods of these structures and the depth of their penetration into a medium are determined. It is shown that the EDTI may induce a semiconductor-metal phase transition accompanied by the formation of ordered surface structures of different phases. This theory of the EDTI is used to interpret a whole range of recent experimental data on the formation of ordered structures of different phases and of the surface relief created as a result of the interaction between laser pulses and a semiconductor.

1.INTRODUCTION

The problems of pulsed laser modification of surface layers of semiconductors (laser annealing,¹⁻⁴ amorphization,^{2,5-8} semiconductor-metal phase transitions,⁹⁻¹¹ etc.) are currently attracting much attention. Phase transitions induced by laser pulses are due to a change in the state of the lattice subsystem, whereas laser radiation of photon energy $\hbar\omega > E_g$ (where E_g is the width of the band gap) interacts with the electron subsystem. The electron-phonon interaction and the corresponding transformation of the absorbed energy thus play a key role in the effects mentioned above.

The usual mechanism by which the absorbed energy is transferred to the lattice consists of electron–electron, electron–phonon, and phonon–phonon relaxation stages with relaxation times $\tau_{\rm rel}$ of the order of a picosecond. The resultant spatial distribution of the lattice changes should reflect a monotonic reduction of the radiation intensity away from the center of a laser-illuminated spot and an exponential reduction of the absorbed energy with depth in the investigated medium (with a characteristic scale at least equal to the linear absorption length γ_0^{-1}).

However, some recent experimental results obtained using high-power ultrashort laser pulses cannot be explained on the basis of these standard representations. Thus, instead of a monotonic pattern, it is frequenly found that ordered surface structures are formed. For example, irradiation with picosecond laser pulses of a semiconducting phase of VO₂ creates a spatially periodic surface pattern representing an alternation of metallic and insulating phases, and if the laser field has axial symmetry, it is found that concentric rings appear at the center of the spot and radial rays are formed at the periphery (a "sun" pattern is observed), whereas when the field has slab geometry it is found that one-dimensional gratings are obtained (Fig. 1).¹¹ Pulsed laser irradiation of the surface of a semiconductor also gives rise to concentric ring structures representing an alternation of crystalline and amorphous phases.⁸ Some results support a nonlinear mechanism of transfer of the laser energy to the lattice at high pump intensities. For example, a reduction in the absorption length by an order of magnitude compared with γ_0^{-1} is reported in Ref. 12. It is shown in Ref. 13 that high-power femtosecond pulses melt the surface of a semiconductor during a pulse, i.e., in a time much shorter than τ_{rel} .

These experimental observations can be interpreted in terms of the formation of laser-induced instabilities on the surface of a semiconductor, leading to a periodic change in the resultant state of the lattice along the semiconductor surface and a strong increase in the effective optical absorption coefficient.

We shall consider two new laser-induced instabilities on the surface of a semiconductor: an electron-deformation instability (EDI) and a deformation-thermal instability (DTI), the theory of which can account for the formation of the ordered structures mentioned above. These two instabilities can occur simultaneously, giving rise to an electrondeformation-thermal instability (EDTI).

The physical mechanism of the EDTI is as follows. The surface deformation $\xi = \text{divu}$ (**u** is the displacement vector of the investigated medium), as well as the density of non-equilibrium carriers *n* and temperature *T*, all of which modulate spatially the width of the band gap:

$$E_g = E_{g0} + \theta \xi - \beta_n n - \beta_T T, \qquad (1)$$

where E_{g0} is the equilibrium value of the band gap: $\partial E_g / \partial \xi = \theta$; $\theta = \theta_{cc} - \theta_{vv}$; θ_{cc} and θ_{vv} are the deformation potentials of the conduction and valence bands, respectively. The phenomenologically introduced coefficient $\beta_n = |\partial E_g / \partial n| > 0$ describes the reduction in E_g because of the breaking of covalent bonds when electrons are transferred from bonding states in the valence band to antibonding states in the

conduction band,⁹ while the coefficient $\beta_T = |\partial E_g / \partial T| > 0$ allows for the reduction in E_g as a result of heating.¹⁰ Modulation of E_g results in modulation of the interband optical absorption coefficient, causing accordingly an additional modulation of *n* and *T*. The resultant forces $\mathbf{F}_n = \theta \operatorname{grad} n$, and $\mathbf{F}_T = -K\alpha \operatorname{grad} T$ (*K* is the bulk modulus and α is the thermal expansion coefficient) maintain initial displacements of the lattice, giving rise to an instability of the amplitude of the surface strain ξ and also of the carrier density *n* and the temperature *T* when a certain critical intensity of the laser pump radiation is exceeded. The mechanism of the DTI in the bulk has been considered earlier^{14,15} for the case of transparent insulators, whereas the EDI mechanism has been discussed for the interior of the semiconductor.¹⁶

The present paper reports the solution of the boundaryvalue problem of the development of the EDTI on the surface of a semiconductor under strong optical absorption conditions. A general dispersion equation is obtained for the EDTI and its solution determines the rate of the exponential growth (with time) depends on the Fourier amplitudes of the coupled fields of ξ , n, and T on the wave vector $[\lambda = \lambda(q) = \lambda' + i\lambda'']$. It follows from this equation that three qualitatively different instabilities can appear under the EDTI conditions. The first is the instability of surface acoustic waves $(\lambda " \neq 0, \lambda' > 0)$ which is initiated by thermal surface acoustic waves. The second is an instability (softening) of the frequencies of acoustic vibrations $(\lambda " \rightarrow 0, \lambda ' < 0)$. Finally, the third instability applies to surface static deformations $(\lambda'' = 0, \lambda' > 0)$. In the last case the instability begins from initial fluctuations of the temperature or carrier density. The present paper represents a detailed study of the last instability.

We shall show that the EDTI can create complex ordered configurations of the coupled surface fields of ξ , n, and T (in the form of gratings, rings, rays, "suns," and radialring cells formed by the intersection of rings and rays). The periods of these structures are determined as a function of the parameters of the investigated material and of the intensity and duration of the laser pulses, and of how far they penetrate from the surface into the material. A nonlinear steady-state EDTI regime is considered, which becomes stabilized because of nonlinear Auger recombination of carriers (and also because of an optoacoustic nonlinearity). The steady-state values of the Fourier amplitudes of the strain, carrier density, and temperature are determined as a function of the wave vector q and of the pump radiation intensity. It is shown that the modulation amplitude of the band gap width along the surface, in accordance with the mechanism described by Eq. (1) may in principle reach values at which E_g vanishes in a spatially periodic manner, i.e., a laser induced semiconductor-metal transition accompanied by the formation of ordered surface structures is induced.

An internal insulator-metal phase transition under the influence of light causing E_g to vanish and due to the terms β_n and β_T in Eq. (1) has been considered previously.^{9,10} Allowance for the deformations (strains) and for the influence of the surface in the present paper is a fundamentally novel feature of the theory of a laser-induced insulator-metal phase transition, which can explain the formation of coplex periodic structures on the surface. Numerical estimates of the parameters of these structures showed that the EDTI mechanism may be responsible for their formation.

2. CLOSED SYSTEM OF EQUATIONS DESCRIBING MODULATION OF THE NONEQUILIBRIUM CARRIER DENSITY, THE TEMPERATURE, AND THE DISPLACEMENT VECTOR IN A MEDIUM

We shall consider a two-band semiconductor filling the half-space z > 0 and assume that a laser wave is incident normally on the z = 0 surface:

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E} \exp(-i\omega t + ikz) + \text{c.c.}$$
(2)

Here $\mathbf{E} = \mathbf{E}(\mathbf{r})f(t)$; f(t) = 1 when $0 \le t \le \tau_p$; f(t) = 0 when $t < 0, t > \tau_p$; τ_p is the duration of a laser pulse; and $\mathbf{r} = \{x, y\}$.

In the case under discussion $(\varepsilon' \ge 1, \varepsilon' \ge \varepsilon'')$ the equation for the temperature of the medium is (see Ref. 4)

$$\partial T / \partial t - \chi \Delta T = 2\omega \varepsilon^{\prime \prime} |\mathbf{E}|^2 e^{-\gamma z} / \pi c_r \varepsilon^{\prime}, \qquad (3a)$$

where χ is the thermal diffusivity; c_v is the specific heat per unit volume; ε' is the real part of the permittivity of the medium; $\gamma = \omega \varepsilon'' / c(\varepsilon')^{1/2}$ is the optical absorption coefficient; and c is the velocity of light in vacuum.

The equation for the density of nonequilibrium carriers can be written in the form

$$\partial n/\partial t - D\Delta n + n/\tau + \gamma_{\text{Aug}} n^3 = 2\varepsilon^{\prime\prime} |\mathbf{E}|^2 e^{-\gamma z} / \pi \hbar \varepsilon^{\prime},$$
 (3b)

where D is the carrier diffusion coefficient, τ is the linear recombination time, and γ_{Aug} is the nonlinear Auger recombination constant.

The equation for the displacement vector of the medium is of the form

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = c_i^2 \Delta \mathbf{u} + (c_i^2 - c_i^2) \operatorname{grad} \operatorname{div} \mathbf{u} + \sum_{j=n,T} f_j \operatorname{grad} Y_j, \quad (4)$$

where c_i and c_i are the longitudinal and transverse velocities of sound, $f_n = \theta / \rho$, $f_T = -K\alpha / \rho$, and ρ is the density of the medium. It follows from Eq. (1) that the optical constants ε'' and ε' depend on n, T, and ξ . In the range of values of n, T, and ξ of interest to us the dependence of ε'' can be represented in the following form if we use the expression for the interband permittivity of a semiconductor $\varepsilon(\omega) = \varepsilon' + i\varepsilon''$ (Ref. 17) and allow for Eq. (1):

$$\varepsilon'' = \varepsilon_0'' + \frac{\partial \varepsilon''}{\partial \omega} \frac{1}{\hbar} (\beta_n n + \beta_T T - \theta \xi) = \varepsilon_0'' + \varepsilon_1''.$$
 (5)

A similar dependence can be written down also for ε' . However, since in the range of frequencies of interest to us we have $\varepsilon'_0 \ge \varepsilon''_0$ ($\varepsilon'_0 \sim 10$, $\varepsilon''_0 \sim 10^{-1}$) and $\partial \varepsilon' / \partial \omega \sim \partial \varepsilon'' / \partial \omega$, we shall ignore the change in ε' and assume later that $\varepsilon' = \text{const.}$ Therefore, we have $\gamma = \gamma_0 + \gamma_1$, where $\gamma_1 \propto \varepsilon''_0$.

We can see from Eq. (3) that, in the presence of a laser field, Eq. (5) should give rise to a relationship between the diffusing fields of n and T and the deformation (strain) of the medium.

We shall represent the required solutions in the form

$$n=n_0+n_1, T=T_0+T_1, \xi=\xi_0+\xi_1, u=u_0+u_1,$$

where n_0 , T_0 , ξ_0 , and \mathbf{u}_0 are the solutions of Eqs. (3) and (4) subject to Eq. (5), obtained in the zeroth approximation with respect to n, T, and ξ under the appropriate boundary conditions at z = 0 (see below). The actual form of these solutions is unimportant for the purpose of the present treatment; the only significant factor is that these solutions are constant along the surface over distances $r \ll r_0$ of interest to us (r_0 is the radius of the laser beam) and they vary sufficiently slowly with time over intervals on the order of the

instability development time (see below). We shall now consider the stability of the solutions, n_0 , T_0 , and ξ_0 . Linearizing Eq. (3) and applying Eq. (5), we yield equations for $n_1 \equiv Y_{n_1}$ and $T_1 \equiv Y_{T_1}$, which can be written in a unified manner

$$\partial Y_{ji}/\partial t + \Delta_j(t) Y_{ji} = (\varepsilon_{ji}\xi_i + \varepsilon_{jr}T_i + \varepsilon_{jn}n_i) e^{-\tau_0 z} (1 - \gamma_0 z), \qquad (6)$$

where j = n, T; $\Delta_j(t) = -\chi_j \Delta + \tau_j^{-1}(t)$; $\chi_T = \chi$; $\chi_n = D$; $\tau_T^{-1} = 0$, $\tau_n^{-1} = \tau^{-1} + 3\gamma_{Aug}n_0^2(t)$. The coupling coefficients are described by the following expressions

$$\epsilon_{ji} = 2 \frac{\partial \epsilon_0''}{\partial \omega} |\mathbf{E}|^2 \sigma_{ji} / \epsilon' \pi \hbar, \quad i = n, T, \xi,$$

$$\sigma_{n\xi} = -\theta / \hbar, \ \sigma_{n\tau} = \beta_T / \hbar, \ \sigma_{nn} = \beta_n / \hbar, \ \sigma_{\tau\xi} = -\omega \theta / c_v,$$

$$\sigma_{\tau\tau} = \omega \beta_T / c_v, \ \sigma_{\tau n} = \omega \beta_n / c_v. \tag{6a}$$

The boundary conditions for Y_{j1} can be written in the form

$$\left. \frac{\partial Y_{j_1}}{\partial z} \right|_{z=0} = 0, \quad Y_{j_1}|_{z=\infty} = 0.$$
(7)

It should be noted that, in general, the boundary condition at z = 0 on a surface with relief is of the form

$$\left(\left.\frac{\partial Y_{j_1}}{\partial z}\right)\right|_{z=0} + \left.\frac{\partial^2 Y_{j_0}}{\partial z^2}\right|_{z=0} u_{1z}(0) = 0,$$

where $u_{1z}(0)$ is the displacement of the surface points z = 0along z, which describes the surface relief. However, under the conditions of interest to us the additional term is small (representing $10^{-1}-10^{-4}$ of the first term, see Sec. 6), so that we shall ignore it and we use the boundary conditions specified by Eq. (7). The nature of the boundary conditions for \mathbf{u}_1 depends on the symmetry of the laser field and on the required type of solution (see Sec. 3).

The system of equations (4) (where $\mathbf{u} = \mathbf{u}_1$, $Y_j = Y_{j1}$) and (6) can be solved analytically in two limiting cases: a) weak optical absorption (bulk case); b) strong optical absorption, when γ_0^{-1} is less than the depths of penetration of the surface material excitations, n_1 , T_1 , ξ_1 into the medium (surface case). We shall consider case b because it is of greater practical interest. We can then write Eq. (6) in a simpler form:

$$\partial Y_{j1}/\partial t + \Delta_{j0}(t) Y_{j1} = (\varepsilon_{j\xi} \xi_1 + \varepsilon_{jT} T_1 + \varepsilon_{jn} n_1) |_{z=0} e^{-\gamma_0 z}.$$
(8)

Here,
$$\Delta_{n0} = -D\Delta + \tau_{n0}^{-1}, \tau_{n0}^{-1} = \tau^{-1} + 3\gamma_{Aug}n_0^2(t)|_{z=0},$$

 $\Delta_{T0} = -\chi\Delta.$

3. GENERAL DISPERSION EQUATION FOR THE ELECTRON-DEFORMATION-THERMAL INSTABILITY; FORMATION OF ONE-DIMENSIONAL SURFACE GRATINGS

We assume that as a result of breakdown of the symmetry of the field or medium there is some preferred direction xon the $\{x, y\}$ surface. Then, the boundary conditions on the z = 0 surface for the vector \mathbf{u}_1 can be written in the form (we write $\mathbf{u}_1 \equiv \mathbf{u}$)

$$\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} = 0, \quad \sum_{j=n,T} \frac{f_j Y_{ji}}{c_l^2} + \frac{\partial u_z}{\partial z} + (1-2\beta) \frac{\partial u_x}{\partial x} = 0,$$
(9)

where $\beta = c_l^2/c_l^2$. We look for the simultaneous solution of

the system of equations (4)-(6) assuming $\mathbf{E}(\mathbf{r}) = \text{const}$ in Eq. (2).

We specify the strain ξ_1 and the diffusion field Y_{j1} at z = 0:

$$\xi_{i} = A(t) \exp\left(iqx + \int_{0}^{t} \lambda dt\right), \quad Y_{ji} = A_{j}(t) \exp\left(iqx + \int_{0}^{t} \lambda dt\right),$$
(10)

where A(t) = A(q,t) is a function proportional to the amplitude of the initial strain and $A_j(t)$ is a slow function of time. This solution describes the surface field of the strain, carrier density, and temperature in the form of one-dimensional gratings (Fig. 1b). The solution of Eq. (8) subject to Eq. (10) can be found in the form

$$Y_{j1} = [B_j(t) e^{-\tau_0 z} + C_j(t) e^{-\delta_j z}] \exp\left(iqx + \int_0^{\infty} \lambda \, dt\right). \tag{11}$$

Substituting Eq. (11) into Eq. (8), allowing for the boundary conditions of Eq. (7), and applying the condition that Eqs. (11) and (10) be self-consistent, we find that if $\gamma_0 \gg \delta_j$ (i.e., if $C_j \gg B_j$), then

$$Y_{j1} = A \varepsilon_{j1} \exp\left(iqx + \int_{0}^{t} \lambda \, dt - \delta_{j}z\right) \left[\chi_{j}\gamma_{0}\delta_{j}\left(1 - \sum_{j=n,T} \frac{\varepsilon_{jj}}{\chi_{j}\gamma_{0}\delta_{j}}\right)\right]^{-1},$$
(12)

where

$$\delta_{j} = \delta_{j}(t) = \{ q^{2} + [\lambda + \tau_{j0}^{-1}(t)]/\chi_{j} \}^{\frac{1}{2}}.$$
(13)

The functions $f(t) = A_j(t)$, $\delta_j(t)$, $\lambda(t)$ obey the condition for slow change with time $f^{-1}(t)\partial f(t)/\partial t \ll \lambda$, which reduces to the condition that the functions n_0 , T_0 , and ξ_0 vary little on the scale of λ^{-1} (adiabatic approximation, see Ref. 18). We now solve Eqs. (4) and (9). We represent the vector **u** in the form $\mathbf{u} = \mathbf{u}_l + \mathbf{u}_t$, where

$$\operatorname{div} \mathbf{u}_t = 0, \operatorname{curl} \mathbf{u}_t = 0. \tag{14}$$

Equation (4) then yields equations for the vectors \mathbf{u}_i and \mathbf{u}_i :



FIG. 1. Spatially periodic patterns of coupled deformation, nonequilibrium carrier density, and temperature fields resulting from the EDTI: a) "sun" structure (obtained for an axial symmetry of the laser pump field and an isotropic surface); b) "grating" (the direction \mathbf{q} is selected by the one-dimensional geometry of the pump field or by the crystallographic axis).

$$\partial^2 \mathbf{u}_{\alpha} / \partial t^2 = c_{\alpha}^2 \Delta \mathbf{u}_{\alpha} + \delta_{\alpha,i} \operatorname{grad} \sum_{j=n,T} f_j Y_{ji},$$
 (15)

where $\alpha = l, t; \delta_{\alpha,l} = 1$ if $\alpha = l; \delta_{\alpha,l} = 0$ if $\alpha = t; Y_{j1}$ is given by Eq. (12). The solution of Eq. (15) is assumed to be in the form

$$\mathbf{u}_{\alpha} = \left(\mathbf{B}_{\alpha} e^{-\varkappa_{\alpha} t} + \delta_{\alpha, t} \sum_{j=n, T} \mathbf{D}_{j} e^{-\delta_{j} t} \right) \exp\left(i q x + \int_{0}^{t} \lambda \, dt \right). \quad (16)$$

Substituting this solution in the form of Eq. (16) into Eq. (15) and allowing for the condition (14), we express the components B_{lx} and B_{lz} in terms of some constant *m*, and the components B_{tx} and B_{tz} in terms of a constant *N*. Then, using the conditions for self-consistency of the solutions described by Eq. (16) and of the expression for $\xi_1|_{z=0}$ given by Eq. (10), we can express *A* in terms of *M*:

$$A = \frac{\lambda^{2}M}{c_{l}^{2}\Phi},$$

$$\Phi = 1 - \sum_{j=n,T} \frac{R_{j}c_{l}^{2}(q^{2}-\delta_{j}^{2})}{\chi_{i}\gamma_{0}\delta_{j}[\lambda^{2}+c_{l}^{2}(q^{2}-\delta_{j}^{2})]} \left[1 - \sum_{j=n,T} \frac{\varepsilon_{jj}}{\chi_{j}\gamma_{0}\delta_{j}} \right]^{-1},$$

$$R_{n} = 2\theta^{2} \frac{\partial \varepsilon''}{\partial \omega} |\mathbf{E}|^{2}/\varepsilon' \pi \hbar^{2} \rho c_{l}^{2},$$

$$R_{T} = -2K\alpha \frac{\partial \varepsilon''}{\partial \omega} \theta \omega |\mathbf{E}|^{2}/\varepsilon' \pi \rho \hbar c_{\mathbf{v}} c_{l}^{2}.$$
(17)

The solution with the vector \mathbf{u} can be expressed in terms of M and N, and is of the form

$$u_{x} = \left[\varkappa_{i} N e^{-\varkappa_{i} z} + iq M e^{-\varkappa_{i} z} - \lambda^{2} M iq \sum_{j=n,T} (\Phi_{j}/\delta_{j}) e^{-\delta_{j} z} \right]$$

$$\times \exp\left(iq x + \int_{0}^{t} \lambda dt \right),$$

$$u_{z} = \left(iq N e^{-\varkappa_{i} z} - \varkappa_{i} M e^{-\varkappa_{i} z} + \lambda^{2} M \sum_{j=n,T} \Phi_{j} e^{-\delta_{j} z} \right)$$

$$\times \exp\left(iq x + \int_{0}^{t} \lambda dt \right).$$
(18)

Here,

$$\kappa_{l,t} = (q^2 + \lambda^2 / c_{l,t}^2)^{\frac{1}{4}}, \tag{19}$$

$$\Phi_{j} = R_{j} \{ \chi_{j} \gamma_{0} [\lambda^{2} + c_{i}^{2} (q^{2} - \delta_{j}^{2})] \}^{-1} \Big[1 - \sum_{j=n,r} \{ \frac{\varepsilon_{jj}}{\chi_{j} \gamma_{0} \delta_{j}} - \frac{R_{j} c_{i}^{2} (q^{2} - \delta_{j}^{2})}{\chi_{j} \gamma_{0} \delta_{j} [\lambda^{2} + c_{i}^{2} (q^{2} - \delta_{j}^{2})]} \Big\} \Big]^{-1} .$$
(20)

Substituting Eq. (18) into Eq. (9), we obtain two homogeneous equations for M and N:

$$N(\varkappa_{i}^{2}+q^{2})+2iqM\left(\varkappa_{i}-\lambda^{2}\sum_{j=n,T}\Phi_{j}\right)=0,$$

$$-2i\varkappa_{i}qN+(\varkappa_{i}^{2}+q^{2})M\left(1-\lambda^{2}\sum_{j=n,T}\Phi_{j}/\delta_{j}\right)=0.$$
(21)

Equating the determinant of the system (21) to zero, we obtain a general dispersion equation for the EDTI (which is valid also in the case of radial-ring structures, see Sec. 5):

 $4q^{2}\varkappa_{\iota}\varkappa_{\iota} - (\varkappa_{\iota}^{2} + q^{2})^{2}$ $= \frac{\lambda^{2}}{c_{\iota}^{2}} \sum_{j=n,T} R_{j} [4q^{2}\varkappa_{\iota}\delta_{j} - (\varkappa_{\iota}^{2} + q^{2})^{2}] / \left\{ \chi_{j}\gamma_{0}\delta_{j} \left(\frac{\lambda^{2}}{c_{\iota}^{2}} + q^{2} - \delta_{j}^{2} \right) - \sum_{j=n,T} \left[\frac{\lambda^{2}}{c_{\iota}^{2}} \varepsilon_{jj} + (R_{j} + \varepsilon_{jj}) (q^{2} - \delta_{j}^{2}) \right] \right\}.$ (22)

The dispersion equation (22) describes three qualitatively different types of the EDTI. We shall consider each of them separately and limit our treatment of Eq. (22) either to an allowance for the EDI for $R_n/D\delta_n \gg R_T/\chi\delta_T$ or to the DTI in the opposite limit.

a) Laser generation of surface acoustic waves: $\lambda = \lambda' + i\lambda''$, $\lambda' > 0, \lambda'' \gg \lambda'$

The dispersion equation (22) has been derived ignoring the viscosity of the medium. If we allow for the viscosity in Eq. (22) and also in the corresponding expressions for Φ_i , $\varkappa_{l,t}$, and R_j , we have to replace $c_{l,t}^2$ with $c_{l,t}^2 (1 + \eta_{l,t} \lambda / \rho c_{l,t}^2)$, where $\eta_t = \eta$ and $\eta_l = 4\eta/3 + \zeta$ (η and ζ are the first and second viscosities). Then, separating in Eq. (22) the real and imaginary parts subject to the conditions $\lambda' + \tau_{i0}^{-1}/\gamma_{i}$ $2\lambda'\chi_j/c_t^2 \ll 1,$ $\ll \lambda "^2/c_{t,l},q^2,$ $R_{j}, \quad \varepsilon_{jj} \ll \chi_{j} q \gamma_{0},$ and $\lambda''^2 \eta \chi_i / \rho c_i^4 \ll 1$, we find the usual expression for the frequency of surface acoustic waves: $\lambda'' = \sigma c_1 q$; 0.87 < σ < 0.95 (Ref. 19). Using the imaginary part of Eq. (22) and applying the conditions $\lambda''/2Dq^2 \ll 1$, we obtain the following expression for the growth rate of surface acoustic waves in the EDI case:

$$\lambda' = -\frac{q^2 \sigma^2 \eta}{2\rho} - \frac{4(1-\sigma^2)c_i^2}{D^2 q \sigma^2 \gamma_0} R_n.$$
(23)

The expression for the growth rate of a surface acoustic wave in the DTI case is found subject to the condition $\lambda''/2\chi q^2 \gg 1$:

$$\lambda' = -\frac{\sigma^2 q^2 \eta}{2\rho} + \frac{2q (1-\sigma^2)^{\frac{\gamma_2}{\gamma_0}}}{\gamma_0} R_T.$$
 (24)

It is clear from Eqs. (18), (23), and (24) that the generation of surface acoustic waves $(\lambda' > 0)$ due to the EDI is possible only if $\partial \varepsilon'' / \partial \omega < 0$, whereas in the case of the DTI surface acoustic waves can be excited if $\partial \partial \varepsilon'' / \partial \omega < 0$. The critical value of the pump radiation is obtained from Eqs. (23) and (24) subject to the condition $\lambda' = 0$.

Equations (23) and (24) were derived ignoring the term in the sum in the denominator of the right-hand side of Eq. (22) on the assumption that ε_{jj} , $R_j \ll \chi_j q \gamma_0$. It therefore follows that changes in the band gap due to heating (ε_{TT}) and breaking of covalent bonds (ε_{nn}) play no significant role in the process of excitation of surface acoustic waves [but they may be significant in the generation of static structures (see below)].

b) Softening of acoustic frequencies: $\lambda' < 0, \lambda'' \rightarrow 0$

Separating Eq. (22) into real and imaginary parts subject to the conditions λ' , $\lambda'' \ll qc_{t,l}$, $\lambda'/c_{t,l}$, $\lambda''/c_{t,l} \ll q$, $\left[(\lambda' + \tau_{j0}^{-1}) / \chi_j \right]^{1/2}$, $\lambda', \lambda'' \ll q^2 \chi_j$, we find that the pumprenormalized acoustic frequencies are described by the expression

$$\lambda^{\prime\prime\prime 2} = \Omega_q^2 \left(1 - \frac{2\beta}{1-\beta} \frac{qR_j}{(q+(q^2+\tau_{j0}^{-1}/\chi_j)^{\gamma_i})(\chi_j\gamma_0(q^2+\tau_{j0}^{-1}/\chi_j)^{\gamma_j}-R_j-\varepsilon_{jj})}}{\Omega_q = 2(1-\beta)^{\gamma_i} qc_i/(3-2\beta)^{\gamma_j}.$$

We can see from Eq. (25) and (17) that softening of the acoustic frequencies occurs in the EDI case if $\partial \varepsilon'' / \partial \omega > 0$, whereas in the DTI case this occurs if $\partial \partial \varepsilon'' / \partial \omega < 0$. The critical intensity is found from the condition $\lambda'' = 0$. In fact, the acoustic frequencies do not go to zero but to $\lambda' \sim \sigma^2 q^2 \eta / 2\rho \ll \Omega_q$.

c) Laser generation of static surface structures

Subsituting $\lambda'' = 0$ in Eq. (22) and using the notation $\lambda' \equiv \lambda$, we find the following equation subject to the conditions $\lambda^2/c_{i,l}^2 < q^2$, $(\lambda' + \tau_{i0}^{-1})/\chi_i$

$$(q+\delta_j)(\delta_j-q_{R\varepsilon}) = \frac{2\beta}{1-\beta}q_{R}, \qquad (26)$$

where $q_R = R_j / \chi_j \gamma_0$, $q_{R\varepsilon} = (R_j + \varepsilon_{jj}) / \chi_j \gamma_0$. The solution of Eq. (26) is

$$\lambda = \chi_{j} \left\{ \left[\frac{(q+q_{Re})^{2}}{4} + q q_{R} \frac{2\beta}{1-\beta} \right]^{\frac{1}{2}} + \frac{q_{Re}-q}{2} \right\}^{2} - \chi_{j}q^{2} - \tau_{j_{0}}^{-1}.$$
(27)

The growth rate reaches its maximum value λ_{\max} at a pont q_{\max} , where

$$q_{max} = q_{Re} [(5a^{2} + 6a + 1)^{\frac{1}{2}} - (2a + 1)], \quad a = \frac{2\beta}{1 - \beta} (q_{R}/q_{Re}),$$
(28a)

$$\lambda_{max} = \chi_{j} q_{Re}^{2} \frac{1}{2} \left[(5a+1) (5a^{2}+6a+1)^{\frac{1}{2}} - 11a^{2} - 8a+1 \right].$$
(28b)

The dependence of λ on q is plotted for the EDI case in Fig. 2a and for the DTI case in Fig. 2b. If $R_i \ge \varepsilon_{ii}$ (deformation limit of the EDTI), the maximum of $\lambda(q)$ selects a dominant structure with $q = q_{\max} \neq 0$, whereas for $R_j \ll \varepsilon_{jj}$ (diffusion limit of the EDTI) the maximum in the spectrum of $\lambda = \lambda(q)$ is reached at q = 0. However, even in the diffusion limit of the EDTI a dominant structure with $q \neq 0$ is selected. We can demonstrate this by considering first in greater detail the nature of the initial fluctuations in the EDTI case. At a moment t = 0 the boundary condition of Eq. (9) contains $Y_n(t=0) = n_1(t=0)$ and $Y_{T1}(t=0) = T_1(t=0)$, which are the Fourier amplitudes of the initial fluctuations of the population and temperature on the surface (when the simultaneous correlation function is $\langle Y_{i1} (t=0) Y_{i1} (t=0) \rangle = \text{const and is independent of } q$). (18) relationship Using Eq. and of the $N = -2iq \varkappa_1 M / (\varkappa_1^2 + q^2)$ that follows from Eq. (21), on the assumption that R_j , $\varepsilon_{jj} = 0$ at t = 0, and also taking from Eq. (27) the expression $\tilde{\lambda} = \lambda(t=0) = -\chi_j q^2 - \tau_{j0}^{-1}$, we obtain from Eq. (9)

$$M = -f_j Y_{ji}(t=0) / (1-\beta) (\chi_j q^2 + \tau_{j0}^{-1})^2.$$
(29)

Equations (18) and (29) together with the relationship between M and N, give the final solution for the vector **u** in the EDTI case when static structures are formed. Then, in the

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diffusion limit $(\varepsilon_{jj} \ge R_j)$ the EDTI growth rate is given by the following expression which is deduced from Eq. (27):

$$\lambda = \chi_j (q_{R\epsilon}^2 - q^2 - \tau_{j0}^{-1} / \chi_j).$$
 (30)

Allowing for Eq. (29), we find that Eq. (17) yields an expression for the amplitude of the deformation (strain) on the surface in Eq. (10) at time t:

$$A = A(q, t) = -\frac{\lambda^2}{\chi_j q^2 + \tau_{j0}^{-1}} \frac{f_j Y_{j1}(t=0)}{\Phi c_l^2 (1-\beta)},$$

where λ is given by Eq. (30) [if $\varepsilon_{jj} \ge R_j$, the function Φ of the system (17) is independent of q]; allowing for this question in Eq. (10), we find that the resultant strain on the surface at the end of a laser pulse $t = \tau_p$ is

$$\xi(x,\tau_p) = \sum_{\mathbf{q}} A(q,\tau_p) \exp\left(iqx + \int_{\mathbf{0}}^{\mathbf{r}} \lambda \, dt\right).$$

The strain correlation function is

$$\langle \xi^{2}(x) \rangle = \sum_{\mathbf{q}} \langle A^{2}(\mathbf{q}, \tau_{p}) \rangle \exp\left(2\int_{0}^{\infty} \lambda \, dt\right)$$
$$\sim \int_{0}^{\infty} dq \langle A^{2}(\mathbf{q}, \tau_{p}) \rangle \exp(2\lambda\tau_{p}) \sim \int_{0}^{\infty} \xi_{q}^{2} \, dq.$$

where the spectral strain function is

$$\xi_q^2 \sim [(q_{Re}^2 - q^2 - \tau_{j0}^{-1}/\chi_j)/(q^2 + \tau_{j0}^{-1}/\chi_j)]^4 \exp(2\lambda\tau_p),$$

and λ is given by Eq. (30). The value $q = q_{\text{max}}$ at which the maximum of the spectral function is attained at time $t = \tau_p$ determines, under linear conditions, the period of a grating in the case when $\varepsilon_{ij} \gg R_j$. For example, in the DTI case $(\tau_{i0}^{-1} = 0)$, we have

$$q_{max} = q_{R\varepsilon} \text{, for } q_{R\varepsilon} > (4\tau_{P})^{-4}, \qquad (31a)$$

$$q_{max} = (4\tau_p \chi)^{-\gamma_b}$$
 for $q_{R^p} < (4\tau_p \chi)^{-\gamma_b}$. (31b)

If we calculate the critical intensity necessary for the EDTI (found from the condition $\lambda = 0$) by expanding Eq.



FIG. 2. Growth rates λ of static structures vs the absolute value of the wave vector q: a) electron-deformation instability (EDI); b) deformation-thermal instability (DTI). Curves labeled I correspond to the condition $R_j \gg \varepsilon_{ij}$ and are calculated for typical values of semiconductor parameters listed in Sec. 6; curves labeled II correspond to the $R_j \ll \varepsilon_{ij}$ case (schematic representation).

(22) in terms of the parameters $\lambda/c_{i,l}q$, where $(\lambda + \tau_{j0}^{-1})/\chi_j q^2 \ll 1$. We then obtain a softer relaxation mode:

$$\lambda = -(2\chi_j q^2 + \tau_{j_0}^{-1}) + \frac{2q}{\gamma_0} \left(\frac{R_j}{1 - \beta} + \varepsilon_{jj}\right), \qquad (32)$$

where R_i is given by Eq. (17) and ε_{ii} by Eq. (5).

In the case of the EDI we obtain from Eq. (32) a soft carrier-density mode

$$\lambda = -(2Dq^{2} + \tau^{-1}) + \frac{4q}{\gamma_{o}} \frac{|\mathbf{E}|^{2}}{\pi\hbar^{2}\varepsilon'} \frac{\partial\varepsilon''}{\partial\omega} \left[\left| \frac{\partial E_{g}}{\partial n} \right| + \frac{\theta^{2}}{(1-\beta)\rho c_{i}^{2}} \right].$$
(33a)

We can therefore see that the EDI appears only when the conditions $\partial \varepsilon'' / \partial \omega > 0$ is satisfied. This condition is obeyed by various semiconductors in the range of frequencies of generally available lasers. In the case of the DTI, we can deduce from Eq. (32) a soft temperature mode:

$$\lambda = -2\chi q^{2} + \frac{4q}{\gamma_{0}} \frac{|\mathbf{E}|^{2} \omega}{\pi \hbar \varepsilon' c_{v}} \frac{\partial \varepsilon''}{\partial \omega} \left[\left| \frac{\partial E_{s}}{\partial T} \right| - \frac{\theta K \alpha}{(1-\beta)\rho c_{l}^{2}} \right].$$
(33b)

We can see that the DTI can be achieved for different signs of $\partial \varepsilon'' / \partial \omega$, depending on the sign of θ and on the relative magnitudes of the first and second terms in the brackets of Eq. (33b). the condition $\lambda = 0$ in Eqs. (33a) and (33b) yields the critical intensity E_{cr}^2 for the appearance of the ETI and DTI, respectively.

It therefore follows from this section that if $|\mathbf{E}|^2 > E_{cr}^2$, then specific Fourier amplitudes of coupled fields of static deformations [Eq. (18)], of the nonequilibrium carrier density, and of the temperature [Eq. (12)] appear on the surface exponentially with time and the process is characterized by a growth rate λ_{max} [Eq. (28b)]. Consequently, in the linear EDTI case we can expect structures in the form of one-dimensional gratings with a period

$$d=2\pi/q_{max},\tag{34}$$

where in the case $R_j \ge \varepsilon_{jj}$ the value of q_{\max} is given by Eq. (28a), while for $R_j \ll \varepsilon_{ij}$, it is given by Eq. (31).

4. NONLINEAR ELECTRON-DEFORMATION-THERMAL INSTABILITY AND POSSIBILITY OF A SEMICONDUCTOR-METAL PHASE TRANSITION

We consider the nonlinear case for the EDI stabilized by nonlinear Auger recombination. The various functions are described at z = 0 by $n_1(0) = n_q e^{iqx}$, $\xi_1(0) = \xi_q e^{iqx}$, and $T_1(0) = T_q e^{iqx}$ where $E_g(0) = E_{g0} + E_{g1} e^{iqx}$. Under steady-state condition $(\lambda = 0)$ near the threshold $(|\mathbf{E}|^2 \gtrsim E_{cr}^2)$ the substitution $\tau_n^{-1} = \tau^{-1} + \gamma_{Aug} |n_g|^2$ gives

$$n_{1}(0) = |n_{q}| = \frac{1}{\gamma_{Aug}^{1/2}} \left[\frac{2q}{\gamma_{0}} \left(\frac{R_{n}}{1-\beta} + \varepsilon_{nn} \right) - 2Dq^{2} - \tau^{-1} \right]^{\gamma_{1}}.$$
(35)

The threshold for the appearance of steady-state values of n_q corresponds to E_{cr}^2 and it naturally coincides with the EDI threshold of Eq. (33a). Far from the threshold ($|\mathbf{E}|^2 \gg E_{cr}^2$) we similarly obtain from Eq. (27)

$$n_{q} = \frac{1}{\gamma_{Aug}^{1/2}} \left\{ D \left[\left(\frac{(q+q_{Re})^{2}}{4} + q q_{R} \frac{2\beta}{1-\beta} \right)^{\frac{1}{2}} + \frac{q_{Re}-q}{2} \right]^{2} - Dq^{2} - \tau^{-1} \right\}^{\frac{1}{2}}.$$
(36)

Therefore, the dependence of n_q on q repeats the dependence $\lambda = \lambda(q)$. Equations (10) and (12) yield the amplitude of a steady-state deformation wave on the surface

$$\xi_q = n_q (D\gamma_0 \delta_n - \varepsilon_{nn}) / \varepsilon_{n\xi}, \qquad (37)$$

where $\delta_n = [q^2 + (\tau^{-1} + \gamma_{Aug} n_q^2)/D]^{1/2}$, while the amplitude of a steady-state temperature wave is

$$T_q = \frac{\varepsilon_{T_k}}{\varepsilon_{nk}} \frac{D\delta_n}{\chi q} n_q.$$

We now substitute Eq. (37) and the value of T_q in Eq. (1). Then, a wave of the normalized band gap at z = 0 obtained using an expression for ε_{nn} given by Eq. (6a) on the assumption that $|\mathbf{E}|^2 > E_{cr}^2$ is described by

$$E_{g}(0) = E_{g0} - E_{g1}(q) \cos qx,$$

$$E_{q1}(q) = D\delta_{n}n_{q} \left(\epsilon' \pi \hbar^{2} \gamma_{0}/2 \frac{\partial \epsilon''}{\partial \omega} |\mathbf{E}|^{2} + \frac{\hbar \omega}{\chi q c_{v}} \left| \frac{\partial E_{g}}{\partial T} \right| \right).$$
(38)

Note that the term β_n in Eq. (1) is balanced exactly by a part of the second term on the right-hand side of Eq. (37), i.e., that effective renormalization in Eq. (38) occurs only due to the deformation and temperature waves. When the modulation amplitude in Eq. (38) obeys $E_{g1} \ge E_{g0}$, we find that a grating appears on the surface, which represents alternation of the metallic and semiconductor phases. In the $R_j \ge \varepsilon_{jj}$ case the period of this grating is given by the value of q_{\max} of Eq. (28a). In the opposite limit of $R_j \ll \varepsilon_{jj}$ the maximum n_q in Eq. (36) is attained at q = 0. Then, in Eq. (38) we can sum over the surface modes:

$$E_{g}(0) = E_{g_{0}} - \sum_{q} E_{g_{1}}(q) \cos(qx) = E_{g_{0}} - [E_{g_{1}}(x)]_{eff}.$$
 (39)

It follows from Eq. (36) subject to the condition $\varepsilon_{jj} \gg R_j$ that $n_q \propto (q_{R\varepsilon}^2 - q^2)$ and with the aid of Eqs. (39) and (38), where $\gamma_{Aug} n_q^2 \gg q^2 D$ and the second term in parentheses is retained, that

$$\left[E_{g1}(x)\right]_{\text{eff}} \sim \int_{0}^{q_{Re}} dq \cos(qx) \left(q_{Re}^{2} - q^{2}\right) \sim \sin(q_{Re}x).$$
(40)

Therefore, for $\varepsilon_{jj} \gg R_j$, a grating with a period $d = 2\pi/q_{R\varepsilon}$ forms in the nonlinear EDTI case.

In addition to the instability of the Fourier amplitudes with $q \neq 0$ in the EDTI, the amplitudes of the deformation (strain), carrier density, and temperature fields for q = 0also increase exponentially in time, which gives rise to a spatially homogeneous reduction of E_g , i.e., it increases the optical absorption coefficient. This could account for the experimental observation of a strong reduction in the optical absorption length in semiconductors at a high rate of excitation with light characterized by a photon energy $\hbar\omega$ much higher than E_g (Ref. 12).

5. FORMATION OF RADIAL-RING STRUCTURES

Let us assume that the laser field has axial symmetry (relative to the z axis). We seek a simultaneous solution of the system of equations (4), (7), and (8) subject to the boundary condition (9) written down in cylindrical coordinates, ignoring at first the dependence of **E** on *r* in Eq. (2) (the effect of the influence of the dependence of the laser field on the coordinate *r* will be discussed later). The process of solution of the problem in cylindrical coordinates represents repetition of the procedure in Sec. 3, provided that in the expressions of Eq. (10) we make the substitution $e^{iqx} \rightarrow J_m(qr) \cos m\varphi$, where J_m is a Bessel function of the first kind of order *m*, and *m* is an integer. We then find that the components of the displacement vector ($\alpha = r, \varphi, z$) are given by

$$u_{\alpha} = \left(a_{\alpha}Ne^{-\varkappa_{i}z} + b_{\alpha}Me^{-\varkappa_{i}z} + M\sum_{j=n,T}c_{\alpha j}\Phi_{j}e^{-\delta_{j}z}\right)\Psi_{\alpha}\exp\left(\int_{0}^{\cdot}\lambda dt\right),$$
(41)

where

$$a_{z}=q, \ a_{\varphi}=-a_{r}=\varkappa_{i}, \ b_{z}=\varkappa_{i}, \ b_{\varphi}=-b_{r}=q, \ c_{zj}=-\lambda^{2},$$

$$c_{rj}=-c_{qj}=\lambda^{2}q/\delta_{j}.$$

$$\Psi_{z}=J_{m}(qr)\cos m\varphi, \quad \Psi_{r}=\frac{1}{2}\left[J_{m-1}(qr)-J_{m+1}(qr)\right]\cos m\varphi,$$

$$\Psi_{\varphi}=\frac{1}{2}\left[J_{m+1}(qr)+J_{m-1}(qr)\right]\sin m\varphi.$$

The dispersion equation for the EDTI deduced from the boundary conditions for **u** and from Eq. (41) is exactly identical with Eq. (22). Therefore, the expressions for $q_{\rm max}$ and $\lambda_{\rm max}$ [Eq. (28a) and (28b)], which separate the dominant structure in the linear regime, are valid also in the case of the radial-ring structures under consideration.

It is clear from Eq. (22) that surface harmonics with any value of m are characterized by the same growth rate. This degeneracy in m is a consequence of neglect of the dependence of the intensity of the laser field E on r in Eq. (2) and it is also due to diffraction effect (Sec. 6).

We can find the parameters m by solving the problem of the growth of the EDTI assuming a Gaussian distribution of the laser radiation intensity. We shall do this by considering an example of another possible class of surface structures in the form of radial rays.

Let us assume that the pump field has a Gaussian distribution:

$$|\mathbf{E}|^{2} = E_{0}^{2} \exp(-r^{2}/r_{0}^{2}).$$
(42)

Then, Eq. (8) becomes

$$\partial Y_{j_1}/\partial t + \Delta_j Y_{j_1} = (\varepsilon_{j_1} \xi_1 + \varepsilon_{j_1} n_1 + \varepsilon_{j_T} T_1)|_{z=0} \exp(-\gamma_0 z - r^2/r_0^2),$$
(43)

where $\varepsilon_{j\xi}$, ε_{jn} , and ε_{jT} are given by expressions in Eq. (6a) provided we replace $|E|^2$ with E_0^2 . The boundary conditions for Y_{j1} are still the same. We specify ξ_1 and Y_{j1} at z = 0 in the form

$$\xi_{i} = A \left(r/r_{0} \right)^{m} \exp \left(-r^{2}/r_{0}^{2} \right) \cos \left(m\varphi \right) \exp \left(\int_{0}^{\infty} \lambda \, dt \right), \quad (44)$$

$$Y_{ji} = A_{j}(t) \left(r/r_{0} \right)^{m} \exp \left(-r^{2}/r_{0}^{2} \right) \cos \left(m\varphi \right) \exp \left(\int_{0}^{t} \lambda \, dt \right). \quad (45)$$

Here, as in Eqs. (9) and (10), we are using A for the initial amplitudes and A_j for the slow functions of time, where m is an integer; we shall assume that $m \ge 1$. Then a maximum of the function $(r/r_0)^m \exp(-r^2/r_0^2)$ lies at a point $r_{\max} = r_0 (m/2)^{1/2} \ge r_0$. In the range $r < r_0$ we can assume that $(r/r_0)^m \exp(-r^2/r_0^2) \approx (r/r_0)^m$ and Eq. (43) can be written as follows:

$$\partial Y_{j_1}/\partial t + \Delta_j Y_{j_1}$$

$$= (\varepsilon_{j_k}A + \varepsilon_{j_n}A_n + \varepsilon_{j_n}A_n) (r/r_0)^m \exp\left(-r^2/r_0^2 - \gamma_0 z + \int_0^t \lambda dt\right).$$
(46)

The solution of equations of the form (46) subject to the conditions $r/r_0 \ll m$, and $\gamma_0 \gg m^{1/2} r_0$ can be obtained in the form

$$Y_{j1} = \frac{\varepsilon_{jt}A \exp\left(-\tilde{\delta}_{j}z - r^{2}/r_{0}^{2} + \int \lambda \, dt\right) (r/r_{0})^{m} \cos m\varphi}{\chi_{j}\gamma_{0}\tilde{\delta}_{j}\left(1 - \sum_{j=n,T} \varepsilon_{jj}/\chi_{j}\gamma_{0}\tilde{\delta}_{j}\right)},$$
(47)

where $\tilde{\delta}_i$ is given by Eq. (13) when q^2 is replaced with \tilde{q}^2 :

$$\tilde{q}^2 = 4(m+1)/r_0^2. \tag{48}$$

Using the Eqs. (4) and the boundary conditions for the vector **u** in cylindrical coordinates and following the solution procedure similar to that described in Sec. 3, and also allowing for the solutions Y_{j1} described by Eq. (47) and the conditions subject to which the solution of Eq. (46) is obtained, we find that the components of the displacement vector are described by

$$u_{i} = \frac{r^{m-1}}{r_{0}^{m}} \exp\left(-\frac{r^{2}}{r_{0}^{2}} + \int_{0}^{t} \lambda \, dt\right) \Psi_{i} \left(a_{i} M e^{-\varkappa_{i} z} + b_{i} N e^{-\varkappa_{i} z} + M \sum_{j=n,T} c_{ij} \Phi_{j} e^{-\delta_{j} z}\right),$$
(49)

where

$$i=z, \varphi, r; \Psi_r=m \cos m\varphi, \Psi_{\varphi}=-m \sin m\varphi,$$
$$\Psi_z=r \cos m\varphi; a_r=a_{\varphi}=1,$$
$$a_z=-\varkappa_i; b_r=b_{\varphi}=-\varkappa_i, b_z=q; c_{rj}=c_{\varphi j}=-\lambda^2/\delta_j, c_{zj}=\lambda^2.$$

Here, $\varkappa_{l,t}$ and Φ_j are given by Eq. (19) and (20), where q^2 is replaced with \tilde{q}^2 given by Eq. (48). The dispersion equation derived from Eqs. (9) and (49) is identical with the dispersion equation (22) if q^2 is replaced with \tilde{q}^2 , i.e., if we specify $\lambda(m)$. If the expressions for q_{\max} [Eq. (28a)] and λ_{\max} [Eq. (28b)] are modified by replacing q^2 with \tilde{q}^2 , they give the value of m_{\max} at which λ reaches its maximum:

$$m_{max} = (q_{Re}^2 r_0^2 / 4) \left[(5a^2 + 6a + 1)^{\frac{1}{2}} - (2a + 1) \right]^{\frac{1}{2}};$$
(50)

the value of m_{max} given by Eq. (50) determines the number of rays in the dominant structure.

6. COMPARISON WITH THE EXPERIMENTS AND NUMERICAL ESTIMATES. CONCLUSIONS

As already mentioned, the symmetry of the structures formed as a result of the EDTI should be governed by the symmetry of the laser beam for an isotropic surface or by the symmetry of the crystal surface. We now obtain numerical estimates of the parameters and compare the theoretical predictions with experimental results. For typical values $c_{\rm m} = 2 \times 10^7$ $erg cm^{-3} K^{-1}$, $K \approx 10^{12}$ erg/cm³. $\alpha \approx 2 \times 10^{-15} \,\mathrm{K}^{-1}, \partial \varepsilon'' / \partial \omega \approx 10^{-15} \,\mathrm{s}^{-1}, \theta \approx 10^{-11} \,\mathrm{erg}, \rho \approx 5$ g/cm^3 , $c_l = 5 \times 10^5 cm/s$, $\beta = 0.4$, $\omega = 4 \times 10^{15} s^{-1}$, $|\partial E_g/$ $\partial T | \approx 4 \times 10^{-4} \,\mathrm{eV/K}, |\partial E_g / \partial n| = 10^{-32} \,\mathrm{erg \cdot cm^3}, \chi = (0.1)^{-32} \,\mathrm{erg \cdot cm^3}$ cm^2/s , $\gamma_0 = 10^5 cm^{-1}$, $\gamma_{Aug} = 4 \times 10^{-31} cm^6/s$, $\hbar\omega \approx 1 eV$, $D \approx 10^2 cm^2/s$, we find from Eq. (17) that $R_T \sim R_n \sim \varepsilon_{nn} \sim \varepsilon_{TT} \sim 10^4 - 10^5 |\mathbf{E}|^2$ cgs esu. We find from Eq. (33b) that the critical intensity for the appearance of the DTI is $I_{\rm cr} \approx 2 \times 10^6 \, {\rm W/cm^2}$ when $q \approx 10^4 \, {\rm cm^{-1}}$. The critical intensity for the EDI is $(D/\chi)\hbar\omega |\partial E_g/\partial T|/(c_v |\partial E_g/\partial n|)$ \approx 10 times larger. It follows from Eq. (28a) that in the EDI case we have $q_{\text{max}} \sim q_{R\varepsilon} \sim 10^{-2} |\mathbf{E}|^2 \text{ cgs esu} \approx 10^4 - 10^5 \text{ cm}^{-1}$ for $I = 6 \times 10^8$ W/cm², i.e., the period d of the structures is of the order of 1–5 μ m. Equation (28b) yields $\lambda_{\text{max}} \sim Dq_{\text{max}}$ $\approx 10^{10}$ -10¹¹ s⁻¹ for the growth rate. The amplitude of the renormalization E_g for the above values of the parameters is $E_{g1} \approx 1 \text{ eV}$, i.e., the EDTI can give rise to an insulator-metal phase transition.

This conclusion and estimates of the structure periods as well as of their growth times are in agreement with the experimental results reported in Ref. 11, where laser pulses of $\tau_p = 5 \times 10^{-11}$ s duration were used. It was found that gratings of alternate metallic and semiconducting phases or structures of the "sun" type formed on an isotropic surface of VO₂, depending on the symmetry of the laser field. It follows from Eq. (50) that for $r_0 \sim 5 \times 10^{-3}$ cm (Ref. 11), the number of rays in the "sun" structure can be m = 50, which agrees with the experimental results; moreover, we have $\lambda_{max} \tau_p \sim 5 > 1$.

These estimates allow us to justify the boundary conditions given in Eq. (7). If we use Eq. (17), we find from Eq. (12) an expression for $(\partial Y_{j1}/\partial z)|_{z=0}$ and from Eq. (18) an expression for $u_z(0)$. We shall assume that in the case of picosecond laser pulses we have $T_0(z,t) \approx T(0,t)e^{-\gamma_0 z}$ and $n_0(z,t) = n_0(0,t)e^{-\gamma_0 z}$, so that in the DTI case we obtain the following estimate

$$\frac{\partial^2 T_0}{\partial z^2} u_z |_{z=0} / \frac{\partial T_1}{\partial z} \Big|_{z=0} \sim (\gamma_0^2 \chi \lambda^{-1}) [K \alpha T_0(0)] / \rho c_1^2 \sim 10^{-4},$$

whereas in the EDI case, the corresponding estimate is

$$\frac{\partial^2 n_0}{\partial z^2} u_z \big|_{z=0} \left/ \frac{\partial n_1}{\partial z} \right|_{z=0} \sim (\gamma_0^2 D \lambda^{-1}) \left[\theta n_0(0) \right] / \rho c_l^2 \sim 10^{-1}$$

for $n_0(0) \sim 10^{21} \text{ cm}^{-3}$, which confirms the validity of the boundary conditions given by Eq. (7) for the case in question.

In the experiments described in Ref. 20 a (111) surface of crystalline silicon was irradiated with laser pulses of wavelength $\lambda = 0.53 \,\mu$ m, creating one-dimensional gratings of the surface relief in which the lines were parallel to one of the [110] axes. The dependence of the grating orientation on the crystallographic axes indicated that a deformation mechanism was responsible for the formation of these structures. The structure period was independent of λ , angle of incidence, and the polarization of the incident radiation, but was governed by the pulse duration. It was found that $d_1 \approx 3 \times 10^{-5}$ cm for $\tau_{p1} = 10^{-11}$ s, whereas $d_2 \approx 6 \times 10^{-6}$ cm for $\tau_{p2} = 5 \times 10^{-12}$ s, i.e., the relationship $(d_1/d_2) \approx (\tau_{p1}/\tau_{p2})^{1/2}$ was obeyed. In the case of Si one should have $\varepsilon_{nn}/R_n \sim 10$, so that the period of the EDTI structures should be given by Eq. (31b), i.e., $d \propto \tau_p^{1/2}$, which was in agreement with the experimental result of Ref. 20.

It should be pointed out that the mechanism for the appearance of a superlattice with alternating metallic and insulating phases, formed as a result of a laser-induced phase transition, was considered in Refs. 9 and 10. The mechanism of the EDTI and of the corresponding laser-induced phase transition, and the class of superstructures formed as a result of the development of the EDTI considered here, are fundamentally different from those discussed in Refs. 9 and 10. It is assumed in Refs. 9 and 10 that a superlattice forms because of interference between an incident optical wave with a wave diffracted by the initial grating of the permittivity, which appears because of a fluctuation Fourier harmonic n_1 and T_1 in Eq. (1) (the influence of deformation is ignored). Such interference superstructures have a period $d \sim \lambda_L$ (where λ_L) is the wavelength of the incident radiation) and their orientation is rigidly linked to the polarization of the incident wave.

Similar interference structures of modulation of the surface relief with $d \propto \lambda_L$ may appear because of the diffraction of the incident wave on the surface relief. We can describe them by including, on the rigth-hand side of Eq. (4), interference sources of the EE_1 type, where $E_1 \sim u_{1z} \mid_{z=0}$ is the amplitude of the diffracted wave. These structures appeared in the solid phase due to an interference instability of the sublimation²¹ or due to generation of coupled diffracted waves and surface acoustic waves²² (see also the review given in Ref. 4). The growth rates of the interference instability as a function of *q* have very narrow resonances (see Ref. 4). the positions of which are generally different from the positions of resonances of the EDTI increments, so that we can regard these instabilities as independent in different ranges of q. Therefore, we ignored the contribution of interference sources in Eq. (4) when considering the EDTI.

In contrast to the interference instability case, the characteristic scale of a structure formed because of the EDTI is not governed directly by the wavelength λ_L and its geometry is not related to the polarization of the incident radiation, so that these two types of structure are easily distinguished experimentally.

We can see that in addition to static structures, the EDTI can induce also dynamic surface acoustic waves (Sec. 3). It should be mentioned in this connection that threshold generation of nonthermal surface acoustic waves as a result of interaction of laser radiation with the surface of GaAs was reported in Ref. 23. An estimate of the threshold for generation of surface acoustic waves obtained on the basis of Eq. (24) gave a critical intensity $I_{\rm cr} \approx 10^4$ W/cm², which was in agreement with the experimental data of Ref. 23.

The EDTI mechanism considered here may be of interest also in the case of ultrafast melting¹³ ($I \gtrsim 10^{10}$ W/cm²), which in the EDTI case can be due to softening of acoustic modes; this mechanism may apply also to the generation of point defects,²⁴ dislocations, and other types of interaction of high-power laser radiation pulses with semiconductor surfaces.

The authors are grateful to S. A. Akhmanov for his encouragement and discussions, and to M. I. Tribel'skiĭ for valuable critical comments.

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Translated by A. Tybulewicz