Macroscopic theory of superconductors with small coherence length

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One of the main distinguishing features of the now known class of high-temperature superconductors (HTS) is that the superconducting coherence length ξ_0 extrapolated to zero temperature is small (in contrast to ordinary superconductors, for which it is comparable to the average interatomic or interelectronic distance d). This circumstance foreordains the importance of taking into account, for HTS, the fluctuations of the order parameter, viz., the macroscopic wave function $\Psi = \eta \exp(i\varphi)$. A second important feature of the investigated HTS is the quite strong anisotropy of their principal superconducting characteristics. The properties of anisotropic superconductors with small ratio ξ_0/d are discussed here from three standpoints: 1) from the standpoint of the usual macroscopic Ginzburg-Landau or the Ψ theory of superconductivity, in which thermal fluctuations of Ψ are not taken into account; 2) in the framework the same theory, with account taken of small fluctuation corrections; 3) on the basis of the generalized macroscopic Ψ theory of superconductivity, intended for use in the critical region near T_c and in the analog of the generalized Ψ theory of superfluidity of helium II near the λ point. Considered in addition is the case of very strongly anisotropic (layered) superconductors for the description of the superconducting properties of which by the macroscopic approach it is necessary to change from local differential equations to differential-difference equations. In all cases, basic equations are given for the most important observed characteristics of superconductors in terms of a small number of phenomenological parameters of the theory. The theoretical results are compared with available experimental data for $YBa_2Cu_2O_{7-x}$ single crystals.

1. INTRODUCTION

The ordinary superconductors known prior to 1986, with critical temperature $T_c < 25$ K, have a coherence length $\xi_0 \equiv \xi(0)$, extrapolated to zero temperature, that exceeds the characteristic interatomic or interelectronic distance $d \sim 10^{-8}$–10^{-7} cm. The critical region near T_c, in which the fluctuations are appreciable, is therefore small. For this reason, it is practically always possible to use near T_c the mean-field theory and, specifically, the Ginzburg-Landau (GL) macrosopic theory or Ψ theory of superconductivity.[1] On the contrary, according to the available data, the high-temperature superconductors (HTS) observed in 1986–1987 have a small length ξ_0, so that the ratio ξ_0/d cannot be regarded as large. It is therefore relevant, quite independently of the present fundamental problem of the theory, that of elucidating the nature and mechanism of the HTS, to develop a macroscopic superconductivity theory that is suitable also in the critical region.

It is known that the ratio ξ_0/d is not large (specifically, close to unity) also near the λ point of liquid helium, so that in the theory of superfluidity of helium II near the λ point it is necessary to go outside the limits of the Landau theory of phase transitions or of the mean-field theory. A corresponding superfluidity theory that takes into account the most substantial fluctuations has been developed in a number of papers (see Refs. 2 and 3 and the literature cited therein). The theory which we have in mind here is based on the use of modified temperature dependences of the coefficients in the expression for the incomplete thermodynamic potential (the effective Hamiltonian) of the system as a function of the order parameter—the macroscopic wave function $\Psi = \eta \exp(i\varphi)$. As shown in Ref. 3, such a theory of superfluidity near the λ point agrees with the known experimental data.

We consider in the present paper an analogous generalization of the usual Ψ theory of superconductivity. Note that the need for such a generalization has already been indicated in Refs. 4 and 5. In Ref. 4 was considered a hypothetical class of isotropic superconductors containing the so-called local electron pairs that exist, unlike the Cooper pairs, not only below but also above T_c. One cannot exclude the possibility that in any superconductor (including HTS) the superconducting current is transported precisely by such local pairs. As already stated, however, the need for taking fluctuation effects into account in macroscopic superconductivity theory does not depend directly on this circumstance, and is determined only by the value of the ratio ξ_0/d.

An important distinguishing feature of the known class of HTS is also the rather large anisotropy of the critical magnetic fields and of other parameters of the superconducting state. Therefore in the following discussion of fluctuation effects we shall take into account from the very outset the possible crystalline anisotropy of the superconductor. In the local approximation corresponding to the Ψ theory this means introduction of the effective-mass tensor m^*_{ik} of the superconducting electron pairs[5,6] ($m^*_{ik} = m^*\delta_{ik}$ in the isotropic case[1]). Of course, the anisotropy of the properties of a superconductor can set in also as a result of electron pairing in states with nonzero orbital angular momentum l. We, however, confine ourselves to the simplest s-type pairing, i.e., we put $l = 0$, and therefore regard the effective Ψ function as a complex scalar.

In Sec. 2 below we present the main results of the usual Ψ theory for anisotropic superconductors. In Sec. 3 we calculate on the basis of this theory the first fluctuation corrections for various thermodynamic and kinetic quantities at temperatures T higher and lower than T_c. We determine on this basis the temperature widths t_G of the critical (fluctuation) region for anisotropic superconductors. Section 4 is devoted to the development of a generalized Ψ theory of superconductivity, intended for use in the region of large fluctuations, as well as the solution of a number of simplest problems. In Sec. 5 are discussed the changes that must be introduced into the theory in the case of very strongly anisotropic (layered) superconductors. The concluding Sec. 6 contains a discussion of certain presently available experimental data for new superconductors and a list of tasks requiring further research. Taking into account the tremendous interest in HTS, we have deemed it useful to expound the material in sufficient detail, to serve the needs of a large circle of readers.

2. MAIN RELATIONS OF THE USUAL Ψ THEORY OF SUPERCONDUCTIVITY FOR ANISOTROPIC SUPERCONDUCTORS

In the usual Ψ theory of superconductivity,[1,6] with no account taken of thermal fluctuations, the total free energy of an anisotropic superconducting body, corresponding to a certain macroscopic wave function Ψ of the condensate of superconducting pairs, is expressed in the form

$$F=F_{n0}+\int\left\{\frac{\mathbf{B}^2}{8\pi}+a|\Psi|^2+\frac{b}{2}|\Psi|^4+\frac{1}{4m_l^*}\left|\left(-i\hbar\nabla_l-\frac{2e}{c}A_l\right)\Psi\right|^2\right\}dV. \quad (2.1)$$

Here $\mathbf{B}=\operatorname{curl}\mathbf{A}$ is the magnetic-induction vector, F_{n0} the equilibrium free energy of the normal state of the superconductor (in the absence of a magnetic field), $a=\alpha t$, $t=(T-T_c)/T_c$ is the relative distance to the superconducting transition point, and $2m_l^*=\{2m_x^*, 2m_y^*, 2m_z^*\}$ the principal values of the effective-mass tensor of the superconducting electron pairs (with charge 2e). Obviously, at $m_x^*=m_y^*=m_z^*=m^*$ we are dealing with the Ψ theory for isotropic superconductors.[1] Furthermore, α and b in (2.1) are certain positive constants and c is the speed of light. Finally, we assume in (2.1) and everywhere that the coordinate axes are the principal symmetry axes of the crystal, and summation over the repeated subscript $l=\{x,y,z\}$ is implied.

The equilibrium (most probable) value $\Psi=\Psi_m$ corresponds to a minimum of F and is obtained by solving the equations

$$\frac{1}{4m_l^*}\left(-i\hbar\nabla_l-\frac{2e}{c}A_l\right)^2\Psi+a\Psi+b|\Psi|^2\Psi=0, \quad (2.2)$$

$$\operatorname{rot}\mathbf{B}=\frac{4\pi}{c}\mathbf{j}, \quad (2.3)$$

$$j_l=-\frac{ie\hbar}{2m_l^*}(\Psi^*\nabla_l\Psi-\Psi\nabla_l\Psi^*)-\frac{2e^2}{m_l^*c}|\Psi|^2A_l. \quad (2.4)$$

Here $\mathbf{j}$ is the density of the superconducting current (we assume the density of the normal current to be zero). The boundary conditions for Eqs. (2.2)–(2.4) reduce to the continuity of all the components of the induction vector $\mathbf{B}$ on the boundary of the superconductor and to a certain boundary condition for the function Ψ. The character of the latter can be determined in the general case by adding to the bulk functional (2.1) a functional of the surface free energy[2,7,8]:

$$F_S=F_{S,n}+\int[\gamma|\Psi_S|^2+\ldots]dS, \quad (2.5)$$

which depends on the value Ψ_S of the function Ψ on the surface of the superconductor. The coefficient γ in (2.5) can be expressed in terms of the difference between the values of the coefficient a on the surface and in the bulk of the superconductor or, equivalently, in terms of the difference $T_c-T_{c,S}$ of the local values of the temperature of the superconducting transition in the bulk of the superconductor and in a surface layer having a thickness of the order of the lattice constant t:

$$\gamma=\frac{\alpha d(T_c-T_{c,S})}{T_c}. \quad (2.6)$$

This means that in a layer of thickness d the coefficient a in (2.1) takes the form $a^*=\alpha t+\alpha(T_c-T_{c,S})/T_c$.

The sought boundary condition for the function Ψ, obtained by varying the total functional $F+F_S$ over $\Psi(\mathbf{r})$ and Ψ_S, is of the form

$$n_l\frac{\hbar}{4m_l^*}\left[\frac{\partial\Psi}{\partial x_l}\bigg|_S-i\frac{2e}{\hbar c}A_l\Psi_S\right]=-\gamma\Psi_S, \quad (2.7)$$

where n_l are the components of the unit vector normal to the surface. Using the notation

$$\Lambda_l=\frac{\hbar^2}{4m_l^*\gamma}=\frac{\hbar^2T_c}{4m_l^*\alpha d(T_c-T_{c,S})}=\frac{\xi_l^2(0)}{d}\frac{T_c}{T_c-T_{c,S}}, \quad (2.8)$$

we can write (2.7) also in the form

$$n_l\Lambda_l\left[\frac{\partial\Psi}{\partial x_l}-i\frac{2e}{\hbar c}A_l\Psi\right]\bigg|_S=-\Psi_S. \quad (2.9)$$

The quantities Λ_l, which have the dimension of length, are the phenomenological characteristics of the boundary and are usually called the extrapolation lengths. Their numerical values can be positive as well as negative, depending on whether the boundary hinders or helps the onset of superconductivity. The former case is typical of an interface between a superconductor and a normal metal,[7,9] whereas the latter is realized, for example, near twin boundaries in tin[10,11] and, possibly, also near twin boundaries in certain investigated HTS.[12]

The lengths Λ_l must be compared with the coherence lengths

$$\xi_l(T)=\left(\frac{\hbar^2}{4m_l^*a}\right)^{1/2}=\xi_l(0)|t|^{-1/2},\quad \xi_l(0)=\left(\frac{\hbar^2}{4m_l^*\alpha}\right)^{1/2}, \quad (2.10)$$

which govern the rate of decrease of the perturbation of the function in the bulk of the superconductor. If $\Lambda_l\gg\xi_l(T)$, condition (2.9) is transformed into the free boundary condition

$$n_l\Lambda_l\left[\frac{\partial\Psi}{\partial x}-i\frac{2e}{\hbar c}A_l\Psi\right]\bigg|_S=0, \quad (2.11)$$

customarily used for the interface between a superconductor and a vacuum (insulator).[1] If $\Lambda_l\ll\xi_l(T)$, on the contrary, the condition (2.9) approaches at $\Lambda>0$ the condition

$$\Psi|_S=0, \quad (2.12)$$

which is typical of a He II—solid interface.[2,3]

Note that the condition (2.11) can certainly be used for an interface with vacuum in the case of ordinary (low-temperature) superconductors by virtue of the large ratio $\Lambda/\xi(0) \sim \xi(0)/d \gg 1$ (see Eq. (2.8)] with $T_{c,S} \ll T_c$, and the direct microscopic estimates of Λ in Ref. 7). In recently obtained high-temperature superconductors, however, where the coherence lengths $\xi_l(0)$ are comparable with the lattice constant, it is necessary to use the mixed condition (29) (which goes over into (2.12) for $\Lambda > 0$ near T_c) even for a superconductor-vacuum interface.

It is easily seen that conditions (2.7)–(2.9) lead to the condition that the normal component of the superconducting current be zero at the boundary, i.e., to the condition $(\mathbf{j}\cdot\mathbf{n})|_S = 0$. This condition is perfectly natural and necessary if the medium in which the superconductor is placed is not superconducting. For a junction of two superconductors, however, it is necessary to add to the surface-energy functional (2.5) two new terms, so that

$$F_S=F_{S,n}+\int[\gamma_1|\Psi_1|^2+\gamma_2|\Psi_2|^2+\gamma_{12}|\Psi_1-\Psi_2|^2]dS, \quad (2.13)$$

where the subscripts 1 and 2 refer respectively to the first and second superconductors. As a result we obtain in place of one mixed boundary condition a system of two mixed boundary conditions[7]:

$$n_l\frac{\hbar^2}{4m^*_{l,1}}\left[\frac{\partial\Psi_1}{\partial x_l}-i\frac{2e}{\hbar c}A_{l,1}\Psi_1\right]\Big|_S=-\gamma_1\Psi_1-\gamma_{12}(\Psi_1-\Psi_2),$$
$$-n_l\frac{\hbar^2}{4m^*_{l,2}}\left[\frac{\partial\Psi_2}{\partial x_l}-i\frac{2e}{\hbar c}A_{l,2}\Psi_2\right]\Big|_S=-\gamma_2\Psi_2-\gamma_{12}(\Psi_2-\Psi_1), \quad (2.14)$$

where the normal vector $\mathbf{n}$ is directed from medium 1 to medium 2. Multiplying the first (second) equality of (2.14) by Ψ_1^* (or respectively by Ψ_2^*) and subtracting from the complex conjugate, we obtain for the normal component of the tunnel (Josephson) current flowing through the junction from medium 1 to medium 2 (Ref. 13)

$$\mathbf{jn}=-\frac{2ie}{\hbar}\gamma_{12}(\Psi_1^*\Psi_2-\Psi_1\Psi_2^*)$$
$$=\frac{4e\gamma_{12}}{\hbar}|\Psi_1||\Psi_2|\sin(\varphi_2-\varphi_1), \quad (2.15)$$

where $\varphi_2-\varphi_1$ is the phase difference of Ψ_2 and Ψ_1 at the junction.

The sign of the real coefficient γ_{12} in (2.13) is not determined beforehand and depends on the specific microscopic nature of the boundary. A situation of particular interest (realizable in the case of metallic or dielectric interlayers containing magnetic impurities[14] or of ferromagnetic metallic interlayers[15]), occurs if $\gamma_{12}<0$. In this case, as shown in Refs. 14 and 16, the ground state of the junction corresponds to a phase difference $\varphi_2-\varphi_1=\pi$, so that connection of such a junction into a closed superconducting loop should give rise to a weak spontaneous current that induces in the loop a magnetic flux smaller than or equal to half the flux quantum $\Phi_0=\pi c\hbar/|e|=2\times10^{-7}\,\mathrm{G\cdot cm^2}$.

We dwell now briefly on some basic properties of bulky anisotropic superconductors.

In the absence of a magnetic field and of the current $\mathbf{j}$, the equilibrium order parameter normalized to the density n_p of the superconducting pairs (i.e., to half the density $n_s = 2n_p$ of the superconducting electrons, is

$$|\Psi_e|^2\equiv\eta_e^2=n_{p,e}=\frac{n_{s,e}}{2}=-\frac{a}{b}=\frac{\alpha}{b}|t|,$$
$$t=\frac{T-T_c}{T_c}<0. \quad (2.16)$$

The heat capacity of the superconductor (per unit volume) undergoes at $T=T_c$ a finite discontinuity

$$\Delta C=\frac{\alpha^2}{bT_c}. \quad (2.17)$$

If $T<T_c$, the homogeneous superconducting state becomes thermodynamically less favored than the normal state at magnetic field strengths H exceeding the critical value

$$H_c=\left(\frac{4\pi a^2}{b}\right)^{1/2}=\left(\frac{4\pi\alpha^2}{b}\right)^{1/2}|t|,\quad t<0. \quad (2.18)$$

A uniform magnetic field parallel to the boundary of a superconductor and perpendicular to the principal symmetry axis l (the field-induced current is directed along l) penetrates into the superconductor to a depth

$$\delta_l=\left(\frac{m_l^*c^2b}{8\pi e^2|a|}\right)^{1/2}=\left(\frac{m_l^*c^2b}{8\pi e^2\alpha}\right)^{1/2}|t|^{-1/2}\equiv\delta_l(0)|t|^{-1/2}. \quad (2.19)$$

The depth δ_l can be expressed also in terms of the characteristic plasma frequency of the superconducting electrons oscillating along the l axis:

$$\delta_l=\frac{c}{\omega_{pl,l}},\quad \omega_{pl,l}=\left(\frac{4\pi e^2n_{se}}{m_l^*}\right)^{1/2}. \quad (2.20)$$

Important parameters of a superconductor are the ratios

$$\varkappa_l=\frac{\delta_l}{\xi_l}=\frac{m_l^*cb^{1/2}}{(2\pi)^{1/2}|e|\hbar}. \quad (2.21)$$

If all the ratios $\varkappa_l$ are less than $1/2^{1/2}$ (we confine ourselves for simplicity to this case), placement of the superconductor in an external magnetic field $H=H_c$ destroys the superconductivity (we disregard the possibility of superheating or supercooling) via a first-order phase transition (type-I superconductors).

If, nevertheless, $\varkappa_l>1/2^{1/2}$, quantized vortex filaments appear first in the superconductor in fields $H=H^l_{c1}<H_c$ and only when $H=H^l_{c2}>H_c$ is reached does the superconductivity vanish via a second-order phase transition (type-II superconductors).

The upper critical field of a bulky superconductor whose principal crystallographic axis (say, the z axis) is directed along the magnetic field) is given by

$$H_{c2}^z=(2\varkappa_x\varkappa_y)^{1/2}H_c=\frac{\Phi_0}{2\pi\xi_x(0)\xi_y(0)}|t|, \quad (2.22)$$

where $\Phi_0=\pi c\hbar/|e|=ch/2|e|$ is the flux quantum. The lower critical field, on the other hand, is in this case (if $\varkappa_l\gg1$)

$$H_{c1}^z\approx\frac{\ln(\varkappa_x\varkappa_y)^{1/2}}{(2\varkappa_x\varkappa_y)^{1/2}}H_c=\frac{\Phi_0|t|}{4\pi\delta_x(0)\delta_y(0)}\ln(\varkappa_x\varkappa_y)^{1/2}. \quad (2.23)$$

Numerous problems solved on the basis of Eqs. (2.2)–

(2.4) and their corresponding boundary conditions (2.11) and (2.14) can be found, for example in Refs. 1, 7, and 17.

3. FLUCTUATION EFFECTS AND TEMPERATURE WIDTH OF THE CRITICAL REGION OF AN ANISOTROPIC SUPERCONDUCTOR (CALCULATED BY PERTURBATION THEORY)

In studies of fluctuation effects (see, e.g., Ref. 18 and the references cited in this review), the free energy (2.1) is regarded as a certain Hamiltonian that determines the probability

$$w \propto \exp\left\{-\frac{1}{k_B T}\left(F[\Psi(\mathbf{r})]-F[\Psi_m(\mathbf{r})]\right)\right\} \tag{3.1}$$

of finding the system in a state with a specified function $\Psi(\mathbf{r})$ different from the most probable function $\Psi_m(\mathbf{r})$ which corresponds to the minimum of the free energy (2.1) and is the solution of Eqs. (2.2)–(2.4).

The total equilibrium free energy F is expressed in this case in terms of the logarithm of the partition function:

$$F=-k_B T \ln \int w[\Psi(\mathbf{r})] D\Psi(\mathbf{r}), \tag{3.2}$$

taken over all possible configurations (functions) $\Psi(\mathbf{r})$ that vary little over distances smaller than or of the order of the lengths $\xi_l(0)$, i.e., that contain the Fourier transforms

$$\Psi_{\mathbf{k}}=\frac{1}{V}\int \Psi(\mathbf{r}) e^{i\mathbf{kr}} dV$$

of the function $\Psi(\mathbf{r})$ with wave numbers $k_l \lesssim \xi_l^{-1}(0)$ (it is known that the shorter-wavelength components $\Psi_{\mathbf{k}}$ are assumed to be included in the expression (2.1) for the free energy and are precisely the ones responsible for the temperature dependence of the coefficient a).

The configuration integral in (3.2) cannot be evaluated exactly in three and two dimensions. If the fluctuations of Ψ are small, however, the integral in (3.2) can be calculated approximately by expanding the functional (2.1) in powers of the difference $\delta\Psi = \Psi - \Psi_m$ and retaining only the first (quadratic) terms in this expansion.

In this quadratic (Gaussian) approximation we obtain for the fluctuation contribution to the heat capacity of a bulky superconductor (per unit volume) in a zero magnetic field

$$C'=C_{fl}^{+}=\frac{k_B}{8\pi\xi_x(0)\xi_y(0)\xi_z(0)}|t|^{-1/2}, \quad t=(T-T_c)/T_c>0, \tag{3.3}$$

$$C'=C_{fl}^{-}=\frac{2^{1/2}k_B}{8\pi\xi_x(0)\xi_y(0)\xi_z(0)}|t|^{-1/2}, \quad t<0. \tag{3.4}$$

If $\xi_x(0)=\xi_y(0)=\xi_z(0)$, these expressions are transformed, as they should be, into the well-known expression for the fluctuation contribution to the heat capacity of a bulky isotropic superconductor.[19–21]

Note that the increase of C_{fl}^{-} by a factor $2^{1/2}$ compared with C_{fl}^{+} is due, firstly, to the $2^{1/2}$-fold decrease, below T_c, of the superconducting coherence lengths for the modulus of Ψ (indeed, below T_c we have

$$\xi_l^{(-)}(t)=\frac{\hbar}{[4m_l^*(a+3b\eta_e^2(t))]^{1/2}} =\frac{\hbar}{[4m_l^*(-2a)]^{1/2}}=\xi_l^{+}(t)\left(\frac{1}{2^{1/2}}\right),$$

and secondly, to the fact that for $T<T_c$ the phase fluctuations of the order parameter are independent of $(T-T_c)$ and make therefore no contribution to the heat capacity.[2,3,22]

Above T_c, fluctuations of Ψ lead likewise to a small diamagnetic contribution to the magnetic susceptibility (we assume the z axis to be directed along the field)

$$\chi_{zz}'=-\frac{\pi}{6}\frac{k_B T}{\Phi_0^2}\frac{\xi_x(t)\xi_y(t)\xi_z(t)}{\xi_z^2(t)} \propto t^{-1/2} \tag{3.5}$$

and to a positive contribution to the conductivity tensor relative to the direct current

$$\sigma_{ik}'=\frac{1}{16\pi}\frac{\hbar e^2 k_B T\tilde{\gamma}}{m_{ik}^* a^2(t)\xi_x(t)\xi_y(t)\xi_z(t)} \propto t^{-1/2}. \tag{3.6}$$

In the last expression, $\tilde{\gamma}$ is the coefficient in the temporal relaxation equation for the function Ψ at $T>T_c$ (Ref. 18):

$$-\tilde{\gamma}\hbar\dot{\Psi}=a\Psi+b|\Psi|^2\Psi-\frac{\hbar^2}{4m_l^*}\nabla_l^2\Psi, \tag{3.7}$$

where the dot denotes differentiation with respect to time. Note that in the case of superconductors described by the BCS theory we have

$$\tilde{\gamma}=\frac{\alpha}{k_B T_c}=\frac{\alpha}{k_B T t}. \tag{3.8}$$

This relation does not hold, however, in the general case.

Fluctuation effects in bulky (three-dimensional) superconductors at temperatures lower than T_c have hardly been discussed in the literature. In superconductors having a low coherence length, such as the present-day HTS, fluctuations may nevertheless be quite noticeable even below T_c. The pertinent effects can be analyzed by using the results of Ref. 21, where a procedure was developed for a systematic calculation of the fluctuation corrections to the equilibrium thermodynamic quantities and to the order-parameter correlation function $K_{\mathbf{k}}\langle\Psi_{\mathbf{k}}\Psi^*_{\mathbf{k}}\rangle$ both above and below T_c, by expanding them in series in terms of the parameter

$$u\equiv(t_G/|t|)^{1/2}=\frac{1}{4\pi\sqrt{2}}\frac{k_B T_c b}{\alpha^2\xi_x(0)\xi_y(0)\xi_z(0)}|t|^{-1/2}.$$

To be sure, only a one-component order parameter was considered in Ref. 21. Below T_c, however, this circumstance is apparently immaterial since, as already noted, the phase fluctuations of the order parameter are independent of the difference $|T-T_c|$ if $T<T_c$, and should make no contribution, at least in lowest-order perturbation theory, to the anomalies of the physical quantities. Assuming that this is really so, and using the results of Ref. 21, we obtain for the temperature dependences of the coefficients of the thermodynamic-potential (21) at $T<T_c$ the following expressions which take into account the fluctuation corrections:

$$a=-\alpha|t|\left(1+\frac{3}{4}\left(\frac{t_G}{|t|}\right)^{1/2}\right), \tag{3.9}$$

$$b=b_0\left(1-\frac{9}{4}\left(\frac{t_G}{|t|}\right)^{1/2}\right), \qquad (3.10)$$

$$\frac{\hbar^2}{4m_l^*}=\frac{\hbar^2}{4m^*_{l,0}}\left(1+\frac{3}{16}\left(\frac{t_G}{|t|}\right)^{1/2}\right). \qquad (3.11)$$

The parameter $u=(t_G/|t|)^{1/2}$ in these expressions has the meaning of the ratio of the anomalous (i.e., dependent on $|t|$) part of the total mean square of the fluctuations of Ψ to the squared modulus of the equilibrium Ψ at $t<0$:

$$u=\left(\frac{t_G}{|t|}\right)^{1/2}=\frac{\langle|\delta\Psi|^2\rangle_T}{|\Psi_e|^2}=\frac{1}{\pi 2^{5/2}}\frac{k_BT_cb}{a^2\xi_x(t)\xi_y(t)\xi_z(t)}. \qquad (3.12)$$

Obviously, the condition for the validity of all the foregoing expressions is that u be small compared with unity or, equivalently, that the following inequality be satisfied[23]:

$$|t|\gg t_G=\frac{1}{32\pi^2}\frac{(k_BT_cb)^2}{\alpha^4\xi_x^2(0)\xi_y^2(0)\xi_z^2(0)}. \qquad (3.13)$$

The region $|t|\gg t_G$, where all the fluctuation corrections are small, can be called the classical fluctuation region. When expressions (3.3)–(3.6) and (3.9)–(3.11) are used, it must be borne in mind that they were derived without allowance for the influence of the magnetic field (more accurately, the field in the case (3.5) was of course taken into account, but in an approximation linear in the field). In expressions (2.10), (2.19), and (2.21) for ξ_l, δ_l, and $\varkappa_l$, the fluctuation corrections are obtained by substituting in them expressions (3.9)–(3.11):

$$\xi_l(t)=\xi_l(0)|t|^{-1/2}\left(1-\frac{15}{32}\left(\frac{t_G}{|t|}\right)^{1/2}\right),$$

$$\delta_l(t)=\delta_l(0)|t|^{-1/2}\left(1-\frac{45}{32}\left(\frac{t_G}{|t|}\right)^{1/2}\right), \qquad (3.14)$$

$$\varkappa_l(t)=\varkappa_l(0)\left(1-\frac{15}{16}\left(\frac{t_G}{|t|}\right)^{1/2}\right), \qquad \varkappa_l(0)=\frac{\delta_l(0)}{\xi_l(0)}.$$

The values of $\varkappa_l$ thus become dependent on $|t|$ when the fluctuations are taken into account. As to the corrections to the fields H_{c1}, H_{c2}, and H_c, however, it would be necessary, strictly speaking, to repeat all the calculations with account taken of the influence of the magnetic field on the equilibrium value $\Psi_e(\mathbf{r})$ and on the fluctuations $\delta\Psi(\mathbf{r})$. No such calculation was performed so far. We are of the opinion, however, that the fluctuation corrections to the critical fields can be obtained at least qualitatively by substituting expressions (3.9)–(3.11) and (3.14) in (2.18), (2.22), and (2.23). With this stipulation, we have

$$H_c=H_c(0)|t|\left(1+\frac{15}{8}\left(\frac{t_G}{|t|}\right)^{1/2}\right), \qquad H_c(0)=\left(\frac{4\pi\alpha^2}{b}\right)^{1/2}, \qquad (3.15)$$

$$H_{c1}^x=H_{c1}^x(0)|t|\left(1+\frac{45}{16}\left(\frac{t_G}{|t|}\right)^{1/2}\right),$$

$$H_{c1}^x(0)=\frac{\Phi_0\ln(\varkappa_y\varkappa_z)}{8\pi\delta_y(0)\delta_z(0)}, \qquad (3.16)$$

$$H_{c2}^x=H_{c2}^x(0)|t|\left(1+\frac{15}{16}\left(\frac{t_G}{|t|}\right)^{1/2}\right),$$

$$H_{c2}^x(0)=\frac{\Phi_0}{2\pi\xi_z(0)\xi_y(0)}. \qquad (3.17)$$

Note that in the condition (3.13) we can express t_G in the form

$$t_G=\frac{1}{32\pi^2}\left(\frac{k_B}{\Delta Cd^3}\right)^2\left(\frac{d}{\bar\xi_0}\right)^6, \qquad \bar\xi_0=[\xi_x(0)\xi_y(0)\xi_z(0)]^{1/3}, \qquad (3.18)$$

i.e., this quantity is determined primarily by the ratio d/ξ_0. However, t_G contains also the ratio $(k_B/\Delta Cd^3)^2$ and the small numerical factor $1/32\pi^2\approx3\times10^{-3}$. The latter explains, in particular why the width of the critical-fluctuations region for the λ transition in helium, where $d\sim\xi_0$ and $(k_B/\Delta Cd^3)\sim1$, is in fact no longer as large ($t_G\sim10^{-3}$ at the saturated-vapor pressure, i.e., the width of the critical region is $\Delta T=|T-T_\lambda|\approx2\times10^{-3}$ K).

4. MACROSCOPIC THEORY OF SUPERCONDUCTIVITY IN THE CRITICAL REGION

As T approaches T_c, or when the measurement accuracy is increased, the first-order fluctuation corrections can obviously no longer be used in expressions (3.9)–(3.11) to obtain a more accurate temperature dependence of the coefficients of the thermodynamic potential (2.1). Furthermore, starting with a certain value of $u=(t_G/|t|)^{1/2}$, perturbation theory in terms of this parameter becomes in itself meaningless. In the intermediate range of $|t|$, when $u\sim1$, no definite theoretical predictions whatever can be made with respect to the coefficients a, b, and m_l^* in (2.1). The situation, however, is again improved in a certain closer vicinity of T_c, which is in fact the one usually called critical. To determine the temperature dependences of the coefficients in the expression for the density of a renormalized thermodynamic potential of the type (2.1) [averaged over Ψ fluctuations with scales smaller than or of the order of $\xi_l(t)$], use can be made of the principles of universality and scale invariance of the critical phenomena, as well as of the results of calculation of these dependences by the renormalization-group-theory method. By virtue of the universality principle we can resort, in our description of superconductors with the aid of the complex scalar function Ψ, also to experimental data on the λ transition in liquid ^{4}He (recall that in Refs. 2 and 3 we describe the λ transition by an analogous function).

All these considerations lead to the conclusion[4,5] that for superconductors in the critical temperature region near T_c one can use in lieu of (2.1) the following expression for the thermodynamic potential of nonequilibrium states:

$$F=F_{n,0}+\frac{C_0T_c}{2}t^2\ln|t|+\int\Big[a_0t|t|^{1/3}|\Psi|^2+\frac{b_0}{2}|t|^{2/3}|\Psi|^4+\frac{g_0}{3}|\Psi|^6+\frac{\hbar^2}{4m_l^*}\left|\left(\nabla_l-\frac{2ie}{c\hbar}A_l\right)\Psi\right|^2\Big]dV. \qquad (4.1)$$

In this expression we have neglected the possible small deviation from zero of the critical exponents $\hat\alpha$ and $\hat\sigma$ that are indicative of the divergence, as $|t|\to0$, of the heat capacity $C_p\propto|t|^{-\hat\alpha}$ and of the coefficients $\hbar^2/4m_l^*\propto|t|^{-\hat\sigma}$ preceding the terms with spatial derivatives in (4.1). As evidenced by renormalization-group calculations and by experimental data on the λ transition in helium (see Refs. 2 and 3 and the literature cited therein), the corresponding differences are very small ($|\alpha| \sim \sigma \lesssim 0.02$) and are immaterial for all practical purposes.[1)] Note that expressions similar to (4.1), with the various simplifications, have already been used in Refs. 4 and 24–26.

In accordance with (4.1), the equilibrium equation for the function Ψ takes now the form

$$-\frac{\hbar^2}{4m_l^*}\left(\nabla_l-\frac{2ie}{c\hbar}A_l\right)^2\Psi$$
$$+(a_0 t|t|^{1/3}+b_0|t|^{2/3}|\Psi|^2+g_0|\Psi|^4)\Psi=0. \qquad (4.2)$$

It differs from Eq. (2.2) of the usual Ψ theory of superconductivity not only by a different temperature dependence of the coefficients, but also by the presence of Ψ raised to the fifth power. Two other basic equations of the theory, Eq. (2.3) for the vector potential **A** and Eq. (2.4) for the superconductor-current density, as well as the boundary conditions (2.9) and (2.14) for Eqs. (4.1), (2.3), and (2.40) retain their form intact also in the critical region.

From (4.1) and (4.2) we find that the temperature dependences of the equilibrium value of $|\Psi_e|$ at $t<0$ and of the anomalous part C^* of the heat capacity of a bulky superconductor are determined in the absence of a magnetic field by the expressions

$$\Psi_e=\Psi_{e0}|t|^{1/3},\qquad \Psi_{e0}^2=\frac{1}{2g_0}(-b_0+(b_0^2+4a_0g_0)^{1/2}),\qquad t<0, \qquad (4.3)$$

$$C=\begin{cases} C_0\ln|t|^{-1}, & t>0\\ C_0\ln|t|^{-1}+\Delta C, & t<0\end{cases}, \qquad (4.4)$$
$$\Delta C=\frac{a_0\Psi_{e0}^2}{2T_c}\left(1+\frac{1}{3}\frac{g_0\Psi_{e0}^4}{a_0}\right).$$

To describe the effects occurring near the λ transition in helium and due to the deviation from zero of the coefficient g_0 of the term with Ψ raised to the sixth power in Eq. (4.1), a dimensionless parameter

$$M=g_0\Psi_{e0}^4/a_0, \qquad (4.5)$$

with an experimental value close to 1/2, was introduced in Refs. 2 and 3. One can expect, on the basis of the universality and the scaling invariance of the critical phenomena, this parameter to have the same numerical value for the superconducting-transition case of interest to us. Moreover, according to the universality and scaling-invariance hypotheses (see Ref. 3 for the pertinent literature) all dimensionless combination of the coefficients ("amplitudes") of the critical temperature dependences of the physical quantities should be the same for all three-dimensional systems described by the complex scalar function Ψ. From this standpoint it seems probable that the ratio $C_0/\Delta C$ of superconductors in the critical region is, just as in superfluid helium, approximately equal to 1/4.

Taking (4.5) into account, we can rewrite expressions (4.3) and (4.4) Ψ_{e0} for and ΔC in the form

$$\Psi_{e0}^2=\frac{a_0}{b_0}(1-M),\qquad \Delta C=\frac{a_0^2}{b_0T_c}(1-M)\left(1+\frac{M}{3}\right), \qquad (4.6)$$

which is close to expressions (2.16) and (2.17) of the usual Ψ theory of superconductivity.

Disregarding certain differences, due to the need for taking into account in (4.1) the term with Ψ raised to the sixth power, between the numerical values of the coefficients, all the results of the usual Ψ theory of superconductivity can be made automatically valid in the critical region by replacing in them $a=\alpha t$ by $a_0 t|t|^{1/3}$ and $b=\text{const}$ by $b_0|t|^{2/3}$. In particular, for the temperature dependences of the principal values of the tensors of the superconducting coherence lengths and of the magnetic-field penetration depths we now have [cf. Eqs. (2.10) and (2.19)]

$$\xi_l=\frac{\hbar}{(4m_l^*a_0|t|^{1/3})^{1/2}}\infty|t|^{-2/3}, \qquad (4.7)$$

$$\delta_l=(m_l^*c^2/8\pi e^2\Psi_e^2)^{1/2}=(m_l^*c^2b_0/8\pi e^2a_0(1-M)|t|^{4/3})^{1/2}\infty|t|^{-1/3}. \qquad (4.8)$$

We see that in the critical region the temperature dependences of the superconducting coherence lengths become stronger, and the penetration depths weaker, compared with their linear dependences on $|t|$ far from T_c. In the critical region, as a result, the ratios of these lengths are no longer constant but decrease as $|t|\to 0$ like

$$\varkappa_l=\frac{\delta_l}{\xi_l}=\frac{m_l^*cb^{1/2}}{(2\pi)^{1/2}|e|\hbar(1-M)^{1/2}}\infty|t|^{1/3}, \qquad (4.9)$$

so that a superconductor that is of type-II far from T_c becomes of type-I close enough to T_c. In the case of an isotropic superconductor, this transformation should take place at $\varkappa(t)$ equal to

$$\varkappa_c=\left[\frac{(1-M)(3+M)}{6}\right]^{1/2}\approx 0.54, \qquad (4.10)$$

whereas in the case of an anisotropic superconductor an analogous critical value will be possessed by definite $\varkappa_l$ combinations that depend on the magnetic-field orientation relative to the principal axes of the crystal see, e.g., (2.22)].

Changes of the temperature dependences of the lengths ξ_l and δ_l give rise to changes of the temperature dependences of the upper and lower critical magnetic fields. thus, for a field directed along the crystallographic x axis we have

$$H_{c2}^x=\frac{\Phi_0}{2\pi\xi_y(t)\xi_z(t)}\infty|t|^{4/3} \qquad (4.11)$$

and (in the case $\ln(\varkappa_y\varkappa_z)\gg 1$)

$$H_{c1}^x=\frac{\Phi_0\ln(\varkappa_y\varkappa_z)}{8\pi\delta_y(t)\delta_z(t)}\infty|t|^{2/3}\ln|t|^{-1}. \qquad (4.12)$$

At the same time, the temperature dependence of the thermodynamic critical field remains linear in the critical region:

$$H_c=\left[\frac{4\pi a_0^2}{b_0}(1-M)\left(1+\frac{M}{3}\right)\right]^{1/2}|t|. \qquad (4.13)$$

When account is taken of the nonzero critical exponent $\hat{\alpha}$ in the expression $C_p=\propto|t|^{-\hat{\alpha}}$ for the heat capacity, we have

$$H_c\infty|t|^{1-\hat{\alpha}/2}, \qquad (4.14)$$

where, remember, $|\hat{\alpha}|\lesssim 0.02$.

We point out (see also Refs. 4 and 26) that according to (4.11) the curvature (d^2H_{c2}/dT^2) of the plot of H_{c2} vs $|t|$ is positive in the critical region. At the same time, this curvature is negative in classical-fluctuation region [see (3.17)].

The $H_{c2}(t)$ curves should thus have an inflection point at the boundary between the regions of the classical and critical fluctuations.[2)]

Attainment of a critical region above T_c can be attested to by changes in the character of the temperature dependences of the fluctuation contribution to the susceptibility

$$\chi_{ik}^* = -\frac{\pi}{6}\frac{k_BT}{\Phi_0}\cdot\frac{\bar{\xi}^3(t)}{\xi_i(t)\xi_k(t)} \propto |t|^{-2/3}, \quad \bar{\xi}=(\xi_x\xi_y\xi_z)^{1/3} \tag{4.15}$$

and to the conductivity tensor

$$\sigma_{ik}^* = \frac{1}{16\pi}\frac{\hbar e^2\tilde{\gamma}k_BT}{m_{ik}^* a^2(t)\bar{\xi}^3(t)} \propto |t|^{-1/3}. \tag{4.16}$$

The first of these equations is obtained simply by substituting in Eqs. (3.5) and (2.10) the modified temperature dependence of the coefficient a, while the second is obtained by substituting in Eq. (3.6) the same $a(t)$ dependence and the modified temperature dependence of the relaxation coefficient [2,3]

$$\tilde{\gamma}=\tilde{\gamma}_0|t|^{1/3}$$

(which is contained in the temporal Equation (3.7) for the function Ψ).

Special attention must be paid, in the case of superconductors having a small coherence length, to the peculiarities of the solutions of various kinds of surface problems and of problems connected with the action of a magnetic field on thin superconducting plates, films, small particles, etc. The point is that in the solutions of problems of this type the amplitude distribution of the function Ψ in a direction transverse to the film, or inside the small particle, is usually regarded as uniform; this is justified (see Sec. 2), inasmuch as in ordinary superconductors the extrapolation length Λ for a superconductor-vacuum interface is of the order of $\xi_0^2/d\sim 1$ cm and greatly exceeds the length $\xi(T)$. The latter is the reason why a boundary condition (2.11) of the type $d\Psi/dz=0$ is used. For superconductors with $\xi_0\sim d$, however, as already emphasized in Sec. 2, the inequality $\Lambda\gg(T)$ is not strong even far from T_c, while near T_c, in a region that may be accessible to experimentation, this inequality is reversed, so that on the interface with the vacuum the derivative $d\Psi/dz$ must now not be regarded as zero, and the more general boundary condition (2.9) must be used. This difficulty is even more aggravated in the critical region, for in the calculation of the distribution of the function $\Psi(z)$ near the boundary it is necessary to take into account the term with Ψ raised to the sixth power in Eq. (4.1) for the density of the thermodynamic potential.

In view of the foregoing, we have not attempted to use in the case of the critical region the solutions of many known problems dealing, for example with oscillations of the critical temperature of a superconducting cylindrical film as a function of the external magnetic field (the Little-Parks effect), calculations of the thermodynamic critical field for a small superconducting sphere,[27] and so forth.

5. LAYERED SUPERCONDUCTORS (USE OF DIFFERENTIAL-DIFFERENCE EQUATIONS)

The analysis in the preceding section, which involves introduction of the effective-mass tensor,[6,28,29] is suitable for layered compounds only so long as the coherence length $\xi_\perp(T)\equiv\xi_z(T)$, which is perpendicular to the layers, exceeds substantially the distance d between the layers. If this condition is not met, differential-difference equations can be used.[30,31]

For simplicity, we assume hereafter that the electron motion inside the layers is isotropic, and use the tight-binding approximation to describe the electron transitions between layers. In this approach, the electron spectrum is given by

$$\varepsilon(\mathbf{k})=\frac{\hbar^2}{2m_\parallel}(k_x^2+k_y^2)+2I(1-\cos k_zd), \tag{5.1}$$

where $\mathbf{k}$ represents the electron quasimomenta, x and y are the coordinate axes in the layer plane, d is the distance between layers, $m_\parallel$ is the electron mass for motion inside the layer, and I is the resonance integral for electron motion between layers. The spectrum (5.1) leads to anisotropy of the electron velocity on the Fermi surface. Within the framework of the BCS theory we obtain for (2.1) in a pure superconductor the following values of the reciprocal effective mass tensor components along the principal axes[28,29]:

$$1/m_i^*\propto\langle v_{F,i}v_{F,i}\rangle,\quad 1/m_x^*=1/m_y^*=1/m_\parallel,\quad 1/m_z^*=Id^2/\hbar^2.$$

Corresponding to these values are the principal values of the superconducting coherence length tensor at $T=0$

$$\xi_x(0)=\xi_y(0)=\hbar v_{F\parallel}/1.76\pi k_BT_c$$

and $\xi_z(0)=\xi_x(0)(m_\parallel/m_z)^{1/2}$, which determine the dimensions of the Cooper pair in the directions of the axes x, y, and z, respectively.

So long as $\xi_z(T)\gg d$, the layered structure of the crystal along the direction z is immaterial and the differential equations (2.2)–(2.4) are assumed valid. For $\xi_z(T)\lesssim d$, however, this approximation, as already stated, is not suitable and account must be taken of the inhomogeneity of the electron density in the direction transverse to the layers. The distribution of the order parameter Ψ along z must accordingly be regarded in this case as strongly nonuniform.

A model with Josephson interaction between the layers has been proposed in Ref. 30 for the description of layered systems with $\xi_z(T)\gg d$. Each layer is described by a discrete variable (the number n of the layer), and a discrete order parameter $\Psi_n(x,y)$ that depends on the layer number n and on the continuous coordinates x and y inside the layer is introduced. The equations for the functions $\Psi_n(x,y)$ are now difference equations in the coordinate n and differential equations in the coordinate $\boldsymbol{\rho}=(x,y)$. The latter can be obtained from the BCS scheme by introducing a localized Wannier representation for the electron wave functions [the functions $w_n(\boldsymbol{\rho},z)$] along the coordinate z and by expanding the initial microscopic order parameter $\Delta(\boldsymbol{\rho},z)$ in terms of these functions:

$$\Delta_{n_1n_2}(\boldsymbol{\rho})=\int_{-\infty}^{+\infty}\Delta(\boldsymbol{\rho},z)\,w_{n_1}(z)\,w_{n_2}(z)\,dz, \tag{5.2}$$

and discarding the terms with $n_1\neq n_2$. The result for the parameters $\Delta_n(\boldsymbol{\rho})\equiv\Delta_{nn}(\boldsymbol{\rho})$ is an integral equation in the variable $\boldsymbol{\rho}$ and a difference equation in the discrete variable n. For a pure superconductor with an electron mean free path $\lambda\gg\xi_\parallel(0)$ and $\lambda\gg\hbar v_{F\parallel}/I$, for $I\ll k_BT_c$, the interaction between layer n and layer $n+m$ is proportional to $(I/k_B T_c)^{2m}$. Confining ourselves to terms of lowest order in $(I/k_B T_c)^2$, we obtain the sought differential-difference equations for the functions $\Psi_n(x,y)$ (Refs. 30–33):

$$\left[\frac{\hbar^2}{4m_\parallel}\left(-i\frac{\partial}{\partial\rho}-\frac{2e}{\hbar c}A_\rho\right)^2+\alpha t+b|\Psi_n|^2\right]\Psi_n$$
$$+\alpha r(2\Psi_n-\Psi_{n+1}e^{-i\chi_n}-\Psi_{n-1}e^{i\chi_n})=0,$$
$$t=(T-T_c)/T_c,\quad \chi_n=\frac{2e}{\hbar c}\int_{nd}^{(n+1)d}A_z\,dz, \tag{5.3}$$

$$\alpha=(2\pi k_BT_c)^2/7\zeta(3)\varepsilon_F,\quad r=7\zeta(3)I^2/(8\pi k_BT_c)^2,$$
$$\zeta(3)=1.204.$$

The condition $\lambda\gg\hbar v_{F\parallel}/I$ means that the characteristic time $\tau=\lambda/v_{F\parallel}$ of the electron mean free path in the layer is much longer than the electron transit time $\hbar/I$ between layers. Near T_c, where

$$\xi_z(T)=\xi_z(0)|t|^{-1/2}\gg d,$$

Eqs. (5.3) go over into the ordinary Ψ-theory differential equations (2.2) with a correlation length $\xi_z(T)=d(r/|t|)^{-1/2}$. Such a transition is always possible if $r\gg1$, but if $r\ll1$ and $|t|\gg r$ the description of the superconductivity must be based on the difference equations (5.3), and not on an equation that is differential in the variable z. Thus, Josephson interaction between the layers is realized for $r\ll1$, i.e., for pure superconductors at $I\ll k_BT_c$.

In dirty superconductors with $k_BT_c\tau/\hbar\ll1$ the coefficients $1/\alpha$ and r must be multiplied by the factor $k_B T_c\tau/\hbar$ (Ref. 33). The parameter r is then decreased and Josephson interaction between the layers is possible at larger values of I/k_BT_c. In the considered case of dirty superconductors, the interaction of layer n with layer $n+m$ is also proportional to the factor r^m, and Eqs. (5.3) are applicable as before only so long as $r\ll1$ and $|t|\gg r$.

Equations (5.3) can be regarded also as phenomenological equations that are valid irrespective of the particular superconductivity model. It must be emphasized, however, that it is sensible to use them only if $r\ll1$, for if $r\gtrsim1$ it is no longer possible to confine oneself to interaction between nearest-neighbor layers.

Transition to Josephson interaction of the layers leads to the appearance of qualitatively new superconducting properties. Thus, a change takes place in the temperature dependence of the upper critical magnetic field $H_{c2,\parallel}$ parallel to the layers. In the immediate vicinity of T_c, but not yet in the critical region, the field $H_{c2,\parallel}$ in layered superconductors, just as in ordinary three-dimensional systems, is proportional to $|t|$ [see (2.22)]. However, on approaching the point $|t|=t^*=2r$, in which $\xi_z(t^*)=d/2^{1/2}$, the orbital pair-breaking effect of the magnetic field weakens, and at $|t|>t^*$ it vanishes completely.[31–33] This vanishing of the orbital effect at $|t|>t^*$, which takes place only in the self-consistent-field approximation,[34] can be attributed to the fact that for $\xi_z<2^{-1/2}d$ the normal core of the vortex is in a space between layers and the superconducting order parameter inside the layers changes little under the influence of the field. Thus, the field $H_{c2,\parallel}$ increases steeply as the temperature approaches the point $|t|=t^*=2r$ from above. Below this temperature, the field $H_{c2,\parallel}$ is bounded only by the paramagnetic effect and by the order-parameter fluctuations that play a noticeable role in view of their quasi-two-dimensional character.[34]

Since the upper critical field increases near T_c linearly with decrease of temperature, the parameter r can be expressed in terms of the derivatives of $H_{c2,\parallel}$ and $H_{c2,\perp}$ with respect to T at the point T_c. Calculating these derivatives with the aid of Eqs. (5.3), we obtain for the parameter r, in both pure and in dirty superconductors,

$$r=\Phi_0H_z'/2\pi d^2T_cH_x'H_y',\quad \Phi_0=\pi\hbar c/|e|, \tag{5.4}$$

where $H_l'\equiv-(dH_{c2,l}/dT)_{T=T_c}$, $l=x,y,z$. Equation (5.4) permits r to be calculated from measurements of the $H_{c2,l}(T)$ dependences near T_c.

Josephson interaction of the layers is also manifested by the singularities of the behavior of layered systems at $|t|\gtrsim2r$ in an electric field perpendicular to the layers. Highly pronounced effects, similar to the nonstationary Josephson effect, should be observed in this case. Unfortunately, there are at present no detailed investigations whatever of such Josephson effects in layered systems.

We emphasize, finally, that the screening (penetration depth) of the magnetic field in the regime of Josephson interaction between layers should be strongly anisotropic. Indeed, it follows from Eqs. (5.3) that the ratio of the penetration depth δ_z for a field directed along the z axis (i.e., perpendicular to the layers) to the penetration depth $\delta_\parallel$ for a field perpendicular to z (i.e., along the layers) is

$$\frac{\delta_\parallel}{(\delta_\parallel\delta_\perp)^{1/2}}=\left(\frac{4\alpha rd^2m_\parallel^*}{\hbar^2}\right)^{1/2}=r^{1/2}\frac{d}{\xi_x(0)}. \tag{5.5}$$

The lower critical magnetic field $H_{c1,\parallel}$, which determines the start of penetration of the vortices along the layers, is therefore likewise weak. For the ratio of the fields $H_{c1,\parallel}$ and $H_{c1,\perp}$ we have ($\xi_\parallel=\xi_x=\xi_y$)

$$\frac{H_{c1,\parallel}}{H_{c1,\perp}}=\frac{4\alpha rm_\parallel^*d^2}{\hbar^2}\left(\ln\frac{\delta_\parallel}{d}\Big/\ln\frac{\delta_\perp}{\xi_\perp}\right)\approx\frac{rd^2}{\xi_x^2(0)}. \tag{5.6}$$

We have replaced $\delta_\parallel/\xi_\parallel$ by $\delta_\parallel/d$ under the logarithm sign in the expression for $H_{c1,\parallel}$. The reason is that in a field parallel to the layers the vortex axes penetrate into the space between the layers, meaning that it makes no sense to speak of a normal core for them (this is why the cutoff parameter $\xi_\parallel$ is replaced in (5.6) by d).

We have used everywhere above the self-consistent-field approximation. This approximation is justified for $T<T_c$ if long-range superconducting order exists in the system. In a strictly two-dimensional system, however, there is no such order in a certain temperature interval between the initial critical temperature T_c (calculated in the self-consistent-field approximation) and the temperature T_{BKT} of the Berezinskii-Kosterlitz-Thouless approximation. In this temperature interval $T_{BKT}<T<T_c$ the correlation function $\langle\Psi(\mathbf{r})\Psi(0)\rangle$ decreases with distance exponentially, owing to spontaneous vortex production.[35] Below T_{BKT}, individual point vortices are bound into pairs and the exponential decrease of the correlation function gives way to a power-law decrease. This suffices for a superconducting behavior of the system, i.e., for the presence of infinite conductivity and of the Meissner effect, although formally there is no long-range

superconducting order at $T < T_{BKT}$, as before, owing to the divergence of the long-wave phase fluctuations.

In a layered system with a large enough number N of layers, spontaneous formation of vortices perpendicular to the layers becomes impossible, since the energy E of formation of such a vortex increases in proportion to the vortex length, i.e., to the number N of layers. It is therefore meaningless to speak of a Berezinskii-Kosterlitz-Thouless transition in a layered system. In addition, the Josephson interaction between the layers eliminates the divergence of the long-wave phase fluctuations below T_c. By the same token, the use of the self-consistent-field approximation for layered systems can be regarded as justified both above and below T_c, with the obvious exclusion of the region of critical fluctuations. The width t_G of this region must now be calculated on the basis of (5.3), as will be done below.

Within the framework of the differential-difference equations (5.3), the expressions for the Gaussian corrections to the temperature dependences of various physical quantities differ substantially for $|t| \ll r$ and $|t| \gg r$. In the $|t| \ll r$ region, where the differential-difference equations (5.3) are close to the differential equations (2.2), the corresponding equations, as expected, are transformed into those of Sec. 3 by the substitution $\xi_z(0) = dr^{1/2}$. The region $|t| \ll r$ can be therefore called the region of ordinary three-dimensional fluctuations. In the region $|t| \gg r$, on the contrary, one can disregard in first approximation the correlations between the Ψ fluctuations in neighboring layers, and use for the fluctuation corrections the same expressions as for a system of thin (two-dimensional) films of thickness d. In this quasi-two-dimensional case we obtain, above T_c, for the Gaussian fluctuation contributions to the heat capacity C^*_{2D}, to the conductivity $\sigma^*_{2D,\parallel}$ along the layers, and to the susceptibility $\chi^*_{2D,\perp}$ in a field perpendicular to the layers[18,32]

$$C_{2D}^{\cdot} = \frac{k_B T_c}{2td\Phi_0} H_z' = C_{3D}^{\cdot} \left(\frac{4r}{t}\right)^{1/2}, \tag{5.7}$$

$$\dot{\sigma}_{2D,\parallel} = \frac{k_B T_c \tilde{\gamma}}{\alpha} \frac{e^2}{16\pi\hbar\, dt} = \dot{\sigma}_{3D,\parallel} \left(\frac{4r}{t}\right)^{1/2}, \tag{5.8}$$

$$\dot{\chi}_{2D,\perp} = \frac{H_z'}{6d\Phi_0 t} = \dot{\chi}_{3D,\perp} \left(\frac{4r}{t}\right)^{1/2}, \tag{5.9}$$

where C^*_{3D}, $\sigma^*_{3D,\parallel}$ and $\chi^*_{3D,\perp}$ are determined by Eqs. (3.4)–(3.6) with $\xi_z(t) = d(r/t)^{1/2}$.

Equation (5.7) with t replaced by $|t|$ remains in force also below T_c, with no change of the numerical value of the coefficient. As to the fluctuation corrections to the coefficients a, b, and $m^*_\parallel$ for $T < T_c$, they have not been calculated for the two-dimensional case. We note also that account is taken in Eqs. (3.6) and (5.8) of only that part of the fluctuation contribution to the conductivity which can be calculated without going outside the framework of the temporal equation (3.7). More general expressions for $\sigma^*_{ik}(t)$ are given in Refs. 18 and 32.

The width of the critical region in the regime of two-dimensional fluctuations can be estimated from the condition $C^*_{2D}(t) = \Delta C$, where $\Delta C = \alpha^2/bT_c$ is the heat-capacity discontinuity at the transition point. Using this condition, we get

$$t_{G,2D} = \frac{k_B T_c H'_{c2,z}}{2d\Phi_0 \Delta C} = (2rt_{G,3D})^{1/2}, \tag{5.10}$$

where $t_{G,3D} \equiv t_G$ is given by Eq. (3.13).

Within the framework of the microscopic BCS theory, we have for $t_{G,2D}$ and $t_{G,3D}$ in pure superconductors ($k_B T_c \tau/\hbar \ll 1$) the estimate

$$t_{G,2D} \sim k_B T_c/\varepsilon_F k_F^3 d \xi_{0,x}^2, \quad t_{G,3D} \sim (k_B T_c/\varepsilon_F)^4, \tag{5.11}$$

and in the case of dirty superconductors ($k_B T_c \tau/\hbar \gg 1$),

$$t_{G,2D} \sim 1/dlk_F^2, \quad t_{G,3D} \sim k_B T_c/\varepsilon_F (k_F l)^3, \tag{5.12}$$

where $l = v_F \tau$ is the mean free path, ε_F is the Fermi energy, and $\hbar k_F$ is the Fermi momentum.

It is appropriate to use the estimate (5.10) only if $r \ll t_{G,2D}$, for if $r > t_{G,2D}$ the transition from the region of Gaussian two-dimensional fluctuations into the region of the three-dimensional ones occurs before the critical region is reached. Interest attaches to the character of the temperature dependences of the coefficients in Eqs. (5.3) in the region of two-dimensional critical fluctuations (i.e., in the interval $r \lesssim |t| \lesssim t_{G,2D}$). To our knowledge, however, this question has not been investigated theoretically at all.

6. DISCUSSION OF RESULTS AS APPLIED TO HIGH-TEMPERATURE SUPERCONDUCTORS

We examine now the available experimental data on the high-temperature superconductor $YBa_2Cu_3O_{7-x}$ and attempt to determine for it the parameters r and t_G.

Measurements, near T_c, of the upper magnetic field H_{c2} of a single crystal yield according to Ref. 36 $H'_{c2,x} = H'_{c2,y} = 2.3 \times 10^4$ Oe/K and $H'_{c2,z} = 0.46 \times 10^4$ Oe/K, and according to Ref. 37 $H'_{c2,x} = H'_{c2,y} = 3.9 \times 10^4$ Oe/K and $H'_{c2,z} = 1.1 \times 10^4$ Oe/K. Substituting these values in (5.4) we obtain $r \approx 2$ for a distance $d \approx 4$ Å between the conducting Cu–O layers. It can therefore be concluded that no Josephson interaction between layer is reached in the $YBa_2Cu_3O_{7-x}$ superconductor and that even far from T_c the situation is more readily intermediate between a three-dimensional anisotropic system and a Josephson system. It must be noted, to be sure, that the quantities $H'_{c2,l} = (-dH_{c2,l}/dT)_{T=T_c}$, used above call for some caution. The point is that the temperature dependence of the resistance $R(T)$ (from which H_{c2} is in fact determined) is not so much shifted to the left by the magnetic field as it is broadened without a shift of the start of the resistance falloff. The H_{c2} determined from the half-width of the normal resistance, or from its 10% or 90% level, is not strictly speaking the quantity usually referred to as H_{c2}. It is therefore difficult as yet to determine the true value of r. Thus if the cited values of $H'_{c2,l}$ are assumed, we get $r \approx 2$ and the fluctuations must be regarded as three-dimensional near T_c. We can then determine t_G from the equation (obtained by substituting in (3.13) the values $\xi_l^2(0) = 2\pi T_c H'_{c2,l}/\Phi_0$)

$$t = \frac{1}{32\pi^2} \left(\frac{2\pi T_c}{\Phi_0}\right)^3 \frac{H'_{c2,x} H'_{c2,y} H'_{c2,z}}{(\Delta C)^2}, \tag{6.1}$$

where ΔC is the experimentally measured heat-capacity discontinuity. Reference 38 cites for this discontinuity a value $4 \cdot 10^5$ erg/cm$^3 \cdot$K, while Ref. 39 gives somewhat higher values (up to $6 \cdot 10^5$ erg/cm$^3 \cdot$K). For all the above values, we find from (6.1) that $t_G = (0.2 - 2) \cdot 10^{-4}$. This estimate shows that in the presently available HTS the range of critical fluctuations is relatively narrow (on the order of 10^{-2} K) and difficult to obtain in experiment. The smallness of t_G, however, still does not prevent observation of quite noticeable Gaussian fluctuation effects at $|t| \gg t_G$. In fact, the scale of these effects is determined by the ratio $(t_G/|t|)^{1/2}$, equal to about 1% when the distance from T_c is of the order of T_c, but reaching already 10% for $|t| = |T - T_c|/T_c \sim 10^{-2}$.

One such Gaussian fluctuation effects was investigated recently in Ref. 40, where the fluctuation conductivity σ_x^* of epitaxial oriented $YBa_2Cu_3O_{7-x}$ films was measured in the x direction along the copper-oxygen films. The question of the $H_{c2}(T)$ dependence for these films is still open, for in this case, too, the magnetic field mainly broadens the superconducting transition without shifting its starting point. It was observed in Ref. 40 that near T_c the $\sigma_x^*(t)$ dependence corresponds to the regime of three-dimensional fluctuations ($\sigma_x^* \propto t^{-1/2}$), but with increasing distance from T_c the $\sigma_x^*(t)$ dependence approaches the relation $\sigma_x^* \propto t^{-1}$ corresponding to two-dimensional fluctuations. The crossover temperature $T_0 \approx 1.1\ T_c$ at which the Josephson interaction regime sets in between the layers, corresponds to $r \approx 0.1$, which does not agree with the data of Refs. 36 and 37 for single crystals. In addition, the numerical value of σ_x^* was found to be three to seven times lower than that calculated on the basis of Eqs. (3.6) and (3.8) or (5.8) and (3.8). This could have been due to incomplete filling of the sample by the superconducting phase. However, the interpretation of the data of Ref. 40 and of analogous data [41] for single crystals is ambiguous also for another and more important reason. The point is that all the investigated HTS contain twin domains and, as already noted in Sec. 2, local superconductivity can set in on the boundaries of such domains at a certain temperature T_c' higher than the superconducting transition temperature T_c in the bulk. Indicating this possibility are measurements of the $H_{c2}(T)$ temperature dependence near the point at which superconductivity sets in.[12] It turned out that here $H_{c2} \propto (T_c' - T)^{1/2}$, as it should in the case of local superconductivity.[11] The presence of superconductivity localized near twin boundaries is evidenced also by measurements of the heat capacity of $YBa_2Cu_3O_{7-x}$ single crystals,[38] which we now proceed to discuss.

In Ref. 38 were observed a discontinuity and a peak of the heat capacity at $T_c = 89$ K. They were preceded above T_c, at $T_c' = 93$ K, by a smooth growth of the heat capacity and by an additional small discontinuity amounting to approximately 20% of the main discontinuity. Below T_c, when the temperature was lowered to 87 K, the heat capacity underwent a rapid decrease that slowed down and smoothened subsequently. The anomaly of the heat capacity at the point T_c' and its smooth increase in the interval from T_c' to T_c can be interpreted as a contribution from the superconductivity localized on the twinning planes.[42] This superconductivity can also account for the smooth $\sigma_x^*(t)$ increase observed in Refs. 40 and 41 and attributed there to fluctuations. On the other hand, the maximum of the heat capacity in the range from 87 to 89 K cannot be ascribed to the influence of the twinning boundaries, since they decrease rather than increase the heat-capacity anomaly below T_c.

The experimental data available above T_c can thus not be unambiguously interpreted as yet. Below T_c, however, they attest to the presence of a rather noticeable fluctuation contribution exceeding the negative contribution to the heat capacity from the domain walls[42] and varying near T_c like

$$C_{fl}^- = k_B/2^{5/2}\pi\xi^3(0)\,|t|^{1/2}$$
$$\xi(0) = [\xi_x(0)\,\xi_y(0)\,\xi_z(0)]^{1/3} = (10\pm1)\ \text{Å}.$$

This value of $\xi(0)$ is in satisfactory agreement with the $H_{c2,l}'$ data[36,37] that yield for $\xi(0)$ the values 16.5 and 12 Å, respectively.

TABLE I. Parameters of the superconducting single crystal $YBa_2Cu_3O_{7-x}$.

Experimental parameter values	Calculated parameter values	References
$T_c = 90$ K, $n = 6\cdot10^{21}$ cm^{-3}, $\Delta C = 4\cdot10^5$ erg/cm$^3\cdot$K	$H_c' = -\left(\frac{dH_c}{dT}\right) = 2.4\cdot10^2$ Oe/K; $\alpha = 1.2\cdot10^{-14}$ erg, $b = 4\cdot10^{-36}$ erg$\cdot$cm^3	[38]
$H_{c2,x}' = H_{c2,y}' = 2{,}3\cdot10^4$ Oe/K; $H_{c2,z}' = 0.46\cdot10^4$ Oe/K	$m_z^* = 200\ m_e$, $m_x^* = m_y^* = 8\ m_e$; $\xi_z = 5.6$ Å, $\xi_y(0) = \xi_x(0) = 28$ Å, $\xi(0) = 16.5$ Å; $\delta_z(0) = 19.6\cdot10^2$ Å, $\delta_x(0) = \delta_y(0) = 3.9\cdot10^2$ Å; $\varkappa_z = 350$, $\varkappa_x = \varkappa_y = 14$; $H_{c1,z}' = 43$Oe/K, $H_{c1,x}' = H_{c1,y}' = 10$Oe/K, $t_G = 2\cdot10^{-5}$, $r = 2$	[36]
$H_{c2,x}' = H_{c2,y}' = 3.9\cdot10^4$ Oe/K; $H_{c2,z}' = 1.1\cdot10^4$ Oe/K	$m_z^* = 130\ m_e$, $m_x = m_y = 10\ m_e$; $\xi_z(0) = 5.1$ Å, $\xi_x(0) = \xi_y(0) = 18$ Å, $\xi(0) = 12$ Å; $\delta_z(0) = 2.1\cdot10^3$ Å, $\delta_x(0) = \delta_y(0) = 0.6\cdot10^3$ Å, $\varkappa_z = 410$; $\varkappa_x = \varkappa_y = 33$, $H_{c1,z}' = 18$ Oe/K, $H_{c1,x}' = H_{c1,y}' = 6.9$ Oe/K; $t_G = 1.3\cdot10^{-4}$, $r = 1.7$	[37]

On the basis of the data of Refs. 36–38 we can calculate all the principal parameters of the superconducting state, and also the coefficients α and b in (2.1). The pertinent results are summarized in Table I.

Note that the coefficients α and b and the masses m_x^*, m_y^*, m_z^* were determined assuming isotropy of m_l^* in the planes of the Cu–O layers, and the square $|\Psi_e|^2 = n_s(0)|t|/2$ of the equilibrium value of the order parameter was normalized to the value $n_s(0)/2 = 3\cdot 10^{21}$ cm^{-3}. This accords with certain published estimates of the conduction-electron density ($n = 6\cdot 10^{21}$ cm^{-3}) in HTS crystals at $T > T_c$. This choice of $n_s(0)$ does not influence, however, the values of $\xi_l(0)$, $\delta_l(0)$, $\varkappa_l$, and of other actually measurable physical quantities, since all these quantities are independent of the normalization of Ψ.

We call particular attention to the unusually large values of the ratio $2\Delta C/k_B n$ for the investigated HTS. Recall that according to the BCS theory this ratio is proportional to $k_B T_c/\varepsilon_F$ and is therefore usually small. In Pb, for example, the discontinuity is $\Delta C = 2.3\cdot 10^4$ erg/cm$^3\cdot$K and consequently $\Delta C/k_B n = 1.7\cdot 10^{-2}$ (for $n = 10^{22}$ cm^{-3}). The ratio $C/k_B n$ of the available HTS, however, is close to unity (thus, $2\Delta C/k_B n = 1.0$ for $\Delta C = 4\cdot 10^5$ erg/cm^3 K and $n = 6\times 10^{21}$ cm^{-3}). This circumstance, in addition to the proximity of the ratios $d/\xi_l(0)$ to unity emphasizes all the more the similarity of this class of superconductors to superfluid^4 He, where the $\Delta C/k_B n$ equals 2.6.

Of course, the proximity of $\xi(0)$ to d in no way prevents the use of the macroscopic theory in the region close to T_c, since application of the macroscopic approach requires only that the ratios $\xi(T)/d$ and $\xi(T)/\xi(0)$ be large. Note that the value of $\xi(0)$ used by us is arbitrary (it is obtained by extrapolation from the region near T_c) and need not necessarily agree with the value of the superconducting coherence length at $T = 0$.

The existing HTS fluctuation effects are thus fully observable (amount to several percent) at deviations as large as 10 K from T_c. One can not exclude the possibility of investigating some HTS even in critical fluctuation region itself.

What information can one hope to extract by investigating fluctuation effects? Firstly, such investigations permit assessment of the degree of anisotropy (two-dimensionality) of the superconductor. Second, as already emphasized in Ref. 38, comparison of the fluctuation contributions above and below T_c yields direct information on the number of order-parameter components. On the other hand, in the case of the critical region the quantities dependent on the structure (number of components) of the order parameter are not only the ratios of the coefficients of the power-law dependences of the various physical quantities, but also the critical exponents (indices) themselves. Finally, by investigating fluctuation effects one can ascertain whether a superconducting transition belongs to the same universality class as the λ transition in helium. Within the framework of a four-fermion Hamiltonian of the BCS type, the similarity of the superfluid and superconducting transitions is beyond doubt. There is no proof, however, for the general case. Moreover, according to Ref. 43, in the case of strong electron-phonon interaction the functional (2.1) is not suitable for the description of even the first fluctuation corrections to fluctuation effects in superconductors.

All the foregoing makes further research into fluctuation effects in HTS, as well as other questions touched upon here, of particularly great interest.

[1)]Certain remarks were made in Ref. 44 concerning the possibility of using expression (4.1) in the critical region. We, however, disagree with these remarks (see Refs. 2, 3, and 22).

[2)]In nonideal crystals with an appreciable twinning-plane density, an inflection can appear near T_c on the plot of H_{c2} vs $|t|$ even if no account is taken of fluctuation effects.[11,12]

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