Kolmogorov weak turbulence spectra of an inhomogeneous magnetized plasma

A. B. Mikhaĭlovskiĭ, S. V. Novakovskiĭ, V. P. Lakhin, S. V. Makurin, E. A. Novakovskaya, and O. G. Onishchenko

Space Research Institute, Academy of Sciences of the USSR
(Submitted 25 December 1987)
Zh. Eksp. Teor. Fiz. **94**, 159–171 (July 1988)

We study the feasibility of weakly turbulent Kolmogorov spectra in the case of drift-type waves in an inhomogeneous magnetized plasma. We consider the general problem of three-dimensional spectra produced by three-wave interactions between weakly dispersive waves and having a dispersive power-law correction in the equation for the frequency, and the similar problem for strongly dispersive waves. We show that two kinds of Kolmogorov spectra can arise in such problems, one of which is connected with the wave energy flux and the other with the enstrophy flux. The general formalism developed in this paper is applied to the problem of short-wave and long-wave drift waves described by a Hasegawa-Mima kind of equation. We show that in the case of such waves the Kolmogorov spectra connected with the energy flux are local and those connected with the enstrophy flux are nonlocal.

1. INTRODUCTION

The idea of Kolmogorov (power-law) weak-turbulence spectra[1] has been applied before mainly to the case of waves in isotropic and anisotropic media (see the literature cited in Ref. 1). However, drift-type waves in an inhomogeneous magnetized plasma[2] which are of interest from the point of view of anomalous transport in such a plasma[3,4] do not fall into that category. The papers on Rossby waves[5–7] in a rotating fluid are important in that connection. It was shown in those papers that under simplifying assumptions about the smallness of the parameter k_y/k_x (k_y, k_x are characteristic wave numbers in the meridional and zonal directions: we use a notation differing from that in Refs. 5–7) and about the fine-scale nature of the waves compared to the Rossby radius r_R, $k_x r_R \gg 1$, the dispersion law for the waves and the matrix elements for the interactions between the waves have scale invariance.[1] As a result, according to Refs. 5–7, two-dimensional stationary power-law turbulence spectra, simplified in the way indicated, can be realized in the Rossby problem, and one can use the factorization method[1,8] to find them and interpret them in terms of the appropriate fluxes.

As Rossby waves are analogous to some variants of drift-type waves in a plasma,[9,10] one can use Refs. 5–7 as a starting point to develop an analytical theory of weak-turbulence Kolmogorov spectra in an inhomogeneous plasma. The development of such a theory is the aim of the present paper. In the first half (Secs. 2–4) we expound the general formalism of this theory and in the second half (Secs. 5–7) we apply that formalism to the simplest case of drift waves, described by a Hasegawa-Mima type equation.[11] We discuss the results of the paper in Sec. 8.

1. KOLMOGOROV SPECTRUM THEORY METHOD FOR DRIFT-WAVE TURBULENCE

2. Statement of the problem and basic equations

One of the objects of our analysis are weakly dispersive waves with a dispersion law of the form

$$\omega_{\mathbf{k}} \propto k_y + \mu \Omega_{\mathbf{k}}, \tag{2.1}$$

where μ is a small parameter and $\Omega_{\mathbf{k}}$ a power-law function of the wave numbers, which we write in the form

$$\Omega_{\mathbf{k}} \propto |k_y|^a |k_x|^b |k_z|^c \operatorname{sign} k_y. \tag{2.2}$$

Here a, b, c are some numbers which are determined by the actual type of the wave. Together with (2.1) we shall also consider strongly dispersive waves with $\omega_{\mathbf{k}}$ of the form

$$\omega_{\mathbf{k}} \propto \Omega_{\mathbf{k}}. \tag{2.3}$$

The procedure given by us can also be applied through a simple change in the wave number indexes, to the case of waves with $\omega_{\mathbf{k}} \propto k_z + \mu\Omega_{\mathbf{k}}$ and $\omega_{\mathbf{k}} \propto \Omega_{\mathbf{k}}$, where $\Omega_{\mathbf{k}} \propto |k_z|^a |k_x|^b |k_y|^c \operatorname{sign} k_z$, which we shall discuss in more detail below.

We assume that the turbulence is described by kinetic equations for the waves of the form[12]

$$\frac{\partial N_{\mathbf{k}}}{\partial t} \propto \int U(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2)\,[N_{\mathbf{k}_1} N_{\mathbf{k}_2} - N_{\mathbf{k}} N_{\mathbf{k}_1} \operatorname{sign}(\omega_{\mathbf{k}} \omega_{\mathbf{k}_2}) - N_{\mathbf{k}} N_{\mathbf{k}_2} \operatorname{sign}(\omega_{\mathbf{k}} \omega_{\mathbf{k}_1})]\,\delta(\omega_{\mathbf{k}} - \omega_{\mathbf{k}_1} - \omega_{\mathbf{k}_2})\,\delta(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2)\,d\mathbf{k}_1\,d\mathbf{k}_2, \tag{2.4}$$

where $N_{\mathbf{k}}$ is the "number of quanta,"

$$U(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) = |V(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2)|^2, \tag{2.5}$$

and $V(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)$ are the matrix elements of the interaction which satisfy the symmetry properties indicated in Ref. 12. We assume that the matrix elements also possess scale invariance properties[1] so that

$$V(\varepsilon_y k_y, \varepsilon_x k_x, \varepsilon_z k_z;\ \varepsilon_y k_{1y}, \varepsilon_x k_{1x}, \varepsilon_z k_{1z};\ \varepsilon_y k_{2y}, \varepsilon_x k_{2x}, \varepsilon_z k_{2z}) = \varepsilon_y^u \varepsilon_x^v \varepsilon_z^w V(k_y, k_x, k_z;\ k_{1y}, k_{1x}, k_{1z};\ k_{2y}, k_{2x}, k_{2z}), \tag{2.6}$$

where u, v, w are the invariance exponents determined by the properties of the actual type of waves. We write the quantities $N_{\mathbf{k}}$ in the form

$$N_{\mathbf{k}} \propto |k_y|^\alpha |k_x|^\beta |k_z|^\gamma, \tag{2.7}$$

where α, β, γ are the required exponents of the spectrum.

To use (2.4) we need also have a way of determining the number of quanta $N_{\mathbf{k}}$ and the matrix elements $V(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)$. We find the quantity $N_{\mathbf{k}}$ from the condition $N_{\mathbf{k}} \propto W_{\mathbf{k}}/|\omega_{\mathbf{k}}|$, where $W_{\mathbf{k}}$ is the energy density of the oscillations in $\mathbf{k}$-space.

 0038-5646/88/071386-07$04.00

To find the matrix elements we shall introduce the "normalized potential" $C_{\mathbf{k}}$ defined by the relation $|C_{\mathbf{k}}|^2 \propto N_{\mathbf{k}}$. The reduction of the dynamic equations for the corresponding types of wave to canonical form[12]

$$idC_{\mathbf{k}}/dt = \sum_{\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}} V(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2) C_{\mathbf{k}_1} C_{\mathbf{k}_2} \exp[-i(\omega_{\mathbf{k}_1}+\omega_{\mathbf{k}_2}-\omega_{\mathbf{k}})t] \quad (2.8)$$

is at the same time the procedure to calculate the matrix elements.

3. Transformation of the kinetic equation for the waves and calculation of the exponents of stationary spectra

Without loss of generality we assume that in (2.4) $k_y > 0$. Changing in (2.4) to integration over positive k_{1y}, k_{2y}, and using (2.1)–(2.3) we get (cf. Ref. 7)

$$\partial N_{\mathbf{k}}/\partial t \propto I_1+I_2+I_3, \quad (3.1)$$

where

$$I_j = \int_0^\infty dk_{1y}\, dk_{2y} \int_{-\infty}^{\infty} dk_{1x}\, dk_{2x}\, dk_{1z}\, dk_{2z} G_j, \qquad j=1,2,3; \quad (3.2)$$

$$G_{1,2,3} = U_{1,2,3}(N_1N_2 \mp NN_1 \mp NN_2) \times \delta(\mathbf{k} \mp \mathbf{k}_1 \mp \mathbf{k}_2)\delta(\Omega \mp \Omega_1 \mp \Omega_2). \quad (3.3)$$

To simplify the notation we have introduced here the notation: $U_1 = U(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)$, $U_2 = U(\mathbf{k}_2,\mathbf{k}_1,\mathbf{k})$, $U_3 = (\mathbf{k}_1,\mathbf{k},\mathbf{k}_2)$, and the quantities N, N_1, N_2; $\Omega, \Omega_1, \Omega_2$ denote, respectively, $N_{\mathbf{k}}, N_{\mathbf{k}_1}, N_{\mathbf{k}_2}$; $\Omega_{\mathbf{k}}, \Omega_{\mathbf{k}_1}, \Omega_{\mathbf{k}_2}$.

The quantities Ω, Ω_1, Ω_2 in (3.2) [see [3.3)] have the same sign, which we can assume to be the positive one without loss of generality. In other words, by changing to positive k_y, k_{1y}, k_{2y} we have at the same time made the transition to positive $\Omega, \Omega_1, \Omega_2$. This fact is a consequence of the assumed k_y-dependence of $\Omega_{\mathbf{k}}$ [see (2.1)]–(2.3)].

We now introduce dimensionless variables $p_i = k_{iy}/k_y$, $h_i = k_{ix}/k_x$, $q_i = k_{iz}/k_z$, $i = 1, 2$ and change to integration over positive h_i, q_i. We have then

$$\mathbf{k}_i = (k_y p_i,\ k_x \sigma_{h_i} h_i,\ k_z \sigma_{q_i} q_i), \quad i=1,2, \quad (3.4)$$

where $\sigma = \pm 1$. We change similarly to dimensionless frequencies Ω_i and numbers of quanta N_i, by introducing $\nu_i = \Omega_i/\Omega$ and $n_i = N_i/N$; $i = 1, 2$. Moreover, using (2.6) we get dimensionless matrix elements. As a result, we write, for instance, the integral I_1 in the form

$$I_1 = \frac{N_{\mathbf{k}}^2}{|\Omega_{\mathbf{k}}|} |k_y|^{2u+1} |k_x|^{2v+1} |k_z|^{2w+1} \sum_\sigma J_1(\sigma_{h_1}, \sigma_{q_1}; \sigma_{h_2}, \sigma_{q_2}). \quad (3.5)$$

Here

$$J_1(\sigma) = \int dp_1\, dp_2\, dh_1\, dh_2\, dq_1\, dq_2 \hat{U}_{1,\sigma}(n_1n_2-n_1-n_2)\delta(1-p_1-p_2) \times \delta(1-\nu_1-\nu_2)\delta(1-\sigma_{h_1}h_1-\sigma_{h_2}h_2)\delta(1-\sigma_{q_1}q_1-\sigma_{q_2}q_2), \quad (3.6)$$

the quantity $\hat{U}_{1,\sigma}$ denotes

$$\hat{U}_{1,\sigma} = U(\mathbf{1};\ p_1, \sigma_{h_1}h_1, \sigma_{q_1}q_1;\ p_2, \sigma_{h_2}h_2, \sigma_{q_2}q_2), \quad (3.7)$$

and $\mathbf{1}$ is the vector with unit components, $\mathbf{1} = (1,1,1)$.

We write the integrals I_2 and I_3 in similar form. In our problem there appear then the integrals $J_2(\sigma)$, $J_3(\sigma)$ defined by relations such as (3.6), and the quantities $\hat{U}_{2,\sigma}$, $\hat{U}_{3,\sigma}$ defined by analogy with (3.7). It is clear that each of the quantities J_i has in the case of three-dimensional waves the form of a sum of nine terms such as (3.6). In the case of two-dimensional waves we have instead a sum of three terms such as (3.6).

As in the case of Ref. 7, we transform integrals such as $J_2(\sigma)$, $J_3(\sigma)$ to integrals of the type $J_1(\sigma)$ with the corresponding modification of the integrands. The procedure for such a transformation is explained in the Appendix. As a result Eq. (3.1) is reduced to the form

$$\partial N_{\mathbf{k}}/\partial t \propto (N_{\mathbf{k}}^2/|\Omega_{\mathbf{k}}|)|k_y|^{2u+1}|k_x|^{2v+1}|k_z|^{2w+1} I. \quad (3.8)$$

Here

$$I = \int d\mathbf{p}_1\, d\mathbf{p}_2\, n_1 n_2 K R Q_1 \delta(1-\nu_1-\nu_2)\delta(1-p_1-p_2), \quad (3.9)$$

$$d\mathbf{p}_1 = dp_1\, dh_1\, dq_1, \qquad d\mathbf{p}_2 = dp_2\, dh_2\, dq_2, \quad (3.10)$$

$$K = 1-\lambda_1-\lambda_2, \qquad R = 1-n_1^{-1}-n_2^{-1},$$

$$\lambda_i = p_i^{a-2(1+u+\alpha)} h_i^{b-2(1+v+\beta)} q_i^{c-2(1+w+\gamma)}, \quad i=1,2, \quad (3.11)$$

$$Q_1 = \sum_\sigma \hat{U}_{1,\sigma}\delta(1-\sigma_{h_1}h_1-\sigma_{h_2}h_2)\delta(1-\sigma_{q_1}q_1-\sigma_{q_2}q_2). \quad (3.12)$$

Equation (3.8) is the basis of our analysis which follows.

In the framework of our formalism, the exponents of the stationary Kolmogorov spectra are found from the condition that, with allowance for the δ-functions in (3.9), $K = 0$. As a result we get two sets of exponents α, β, γ for stationary spectra, which we shall denote, respectively by $\alpha_0^{(1)}, \beta_0^{(1)}, \gamma_0^{(1)}$ and $\alpha_0^{(2)}, \beta_0^{(2)}, \gamma_0^{(2)}$. These exponents turn out to be (cf. Ref. 1).

$$\alpha_0^{(1)} = -(1+u),\ \beta_0^{(1)} = -(1+v),\ \gamma_0^{(1)} = -(1+w), \quad (3.13)$$

$$\alpha_0^{(2)} = a/2-(3/2+u),\ \beta_0^{(2)} = b/2-(1+v),\ \gamma_0^{(2)} = c/2-(1+w). \quad (3.14)$$

To be specific, we call the spectra corresponding to (3.13) spectra of the first kind, and those corresponding to (3.14) spectra of the second type. It is clear that in the case of two-dimensional (k_y, k_x)-turbulence we must omit in Eqs. (3.13), (3.14) the expressions for $\gamma_0^{(1)}, \gamma_0^{(2)}$, and in the case of (k_y, k_z)-turbulence the expressions for $\beta_0^{(1)}, \beta_0^{(2)}$.

4. Dynamic properties of the Kolmogorov spectra

4.1. Conservation laws. We now assume that the turbulence is almost stationary, so that

$$\alpha = \alpha_0+\delta_\alpha, \quad \beta = \beta_0+\delta_\beta, \quad \gamma = \gamma_0+\delta_\gamma, \quad (4.1)$$

where the quantities α_0, β_0, γ_0 correspond to stationary spectra [see (3.13), (3.14)], and $\delta_\alpha, \delta_\beta, \delta_\gamma$ are some small corrections. We expand the integrand of (3.9) in a series of these small corrections and we find that in the case considered, of almost stationary turbulence, the kinetic Eq. (3.8) for the waves takes the form

$$\frac{\partial N_{\mathbf{k}}}{\partial t} \propto \delta\begin{pmatrix} |\Omega_{\mathbf{k}}|^{-1} \mathbf{J}^{K_1} \\ |k_y|^{-1} \mathbf{J}^{K_2} \end{pmatrix} |k_y|^{-1+2\delta_\alpha} |k_x|^{-1+2\delta_\beta} |k_z|^{-1+2\delta_\gamma}. \quad (4.2)$$

Here

$$J^{K_i}=\int d\mathbf{p}_1\, d\mathbf{p}_2\, Q_1\delta(1-\nu_1-\nu_2)\,\delta(1-\mathbf{p}_1-\mathbf{p}_2)\,\Lambda^{K_i}(n_1n_2R)_{\alpha=\alpha_0^{(i)}}, \quad (4.3)$$

$$\Lambda^{K_i}=(\partial K/\partial\alpha)_{\alpha=\alpha_0^{(i)}},\quad i=1,2, \quad (4.4)$$

and the vectors $\boldsymbol{\delta}$ and $\boldsymbol{\alpha}$ stand for $\boldsymbol{\delta}=(\delta_\alpha,\delta_\beta,\delta_\gamma)$, $\boldsymbol{\alpha}=(\alpha,\beta,\gamma)$. The integrals $\mathbf{J}^{K_i}$ are assumed to be finite (convergent). The convergence of these integrals must be verified in each actual case of turbulent spectra.

Using (3.10) and (3.11) we find that the vectors Λ^{K_i} are

$$\Lambda^{K_1}=\{\nu_1\ln p_1+\nu_2\ln p_2,\ \nu_1\ln h_1+\nu_2\ln h_2,\ \nu_1\ln q_1+\nu_2\ln q_2\}, \quad (4.5)$$

$$\Lambda^{K_2}=\{p_1\ln p_1+p_2\ln p_2,\ p_1\ln h_1+p_2\ln h_2,\ p_1\ln q_1+p_2\ln q_2\}. \quad (4.6)$$

In the case of two-dimensional (k_y, k_x)-turbulence we must omit from Eqs. (4.2) the factors with k_z and the terms with δ_γ. Similar modifications must be made in Eqs. (4.2) in the case of two-dimensional (k_y, k_z)-turbulence.

Introducing the functions $D_{\mathbf{k}}^{(1)}\equiv|\Omega_{\mathbf{k}}|N_{\mathbf{k}}$, $D_{\mathbf{k}}^{(2)}\equiv|k_y|N_{\mathbf{k}}$, using the relations $\delta x^{-1+\delta}=\partial x^\delta/\partial x$, and afterwards taking the limit as $\delta\to 0$ (cf. Refs. 13, 14) we reduce Eqs. (4.2) to the form

$$\partial D_{\mathbf{k}}^{(i)}/\partial t+\mathrm{div}_{\mathbf{k}}\mathbf{P}^{(i)}(\mathbf{k})=0,\quad i=1,2, \quad (4.7)$$

where

$$\mathbf{P}^{(i)}(\mathbf{k})\propto-(J_1^{K_i}/|k_xk_z|,\ J_2^{K_i}/|k_yk_z|,\ J_3^{K_i}/|k_yk_x|),\quad i=1,2. \quad (4.8)$$

In the case of weakly dispersive waves the quantity $D_{\mathbf{k}}^{(1)}$ has the meaning of the enstrophy (or the "dispersive part" of the wave energy), while $D_{\mathbf{k}}^{(2)}$ is the main part of the wave energy, denoted by us by $W_{\mathbf{k}}$ and called simply the wave energy (see Sec. 2). In that case. Eq. (4.7) with $i=1$ is the enstrophy conservation law, and with $i=2$ the energy conservation law. For such waves $\mathbf{P}^{(1)}(\mathbf{k})$ corresponds to the enstrophy (or generalized enstrophy) flux, and $\mathbf{P}^{(2)}(\mathbf{k})$ to the energy flux.

In the case of strongly dispersive waves the physical meaning of Eqs. (4.7) and the quantities occurring in it turns out to be the opposite. The quantities $D_{\mathbf{k}}^{(1)}$ and $\mathbf{P}^{(1)}(\mathbf{k})$ correspond in that case to the wave energy and the wave energy flux, and $D_{\mathbf{k}}^{(2)}$ and $\mathbf{P}^{(2)}(\mathbf{k})$ to the enstrophy and the enstrophy flux. For such waves, correspondingly, Eq. (4.7) with $i=1$ is the energy conservation law and with $i=2$ the enstrophy conservation law.

The physical meaning of Eqs. (4.7) will in what follows be illustrated also by actual examples.

The quantum-mechanical meaning of Eqs. (4.7) is clear: in the case $i=1$ it is the quasiparticle energy conservation law, and in the case $i=2$ the conservation law for the y-component of the quasiparticle momentum.

In the case of two-dimensional (k_y, k_x)-turbulence we must omit from Eqs. (4.8) for the $\mathbf{P}^{(i)}(\mathbf{k})$ the factor k_z^{-1}, and these factors themselves must be understood to be two-dimensional. Similar remarks hold for two-dimensional (k_y, k_z)-turbulence.

We can also use instead of (4.7) conservation laws in (k_y,Ω,k_z)- or (k_y,k_x,Ω)-space. We consider the case of the (k_y,Ω,k_z)-space. Instead of $N_{\mathbf{k}}$ we introduce $N(k_y,\Omega,k_z)$ normalized such that

$$\int N_{\mathbf{k}}\,dk_y\,dk_x\,dk_z=\int N(k_y,\Omega,k_z)\,dk_y\,d\Omega\,dk_z. \quad (4.9)$$

Writing N in the form $N\propto|k_y|^r|\Omega|^s|k_z|^f$, where r,s,f have a meaning similar to α,β,γ, and using (2.2), (2.7), we find a relation between r,s,f and α,β,γ:

$$r=\alpha-a\beta/b,\quad s=\beta/b,\quad f=\gamma-c\beta/b. \quad (4.10)$$

Correspondingly, one can introduce stationary values of r,s,f, denoted by $r_0^{(i)}, s_0^{(i)}, f_0^{(i)}$ and connected with $\alpha_0^{(i)},\beta_0^{(i)},\gamma_0^{(i)}$ through Eqs. (4.10), and small corrections $\delta_r,\delta_s,\delta_f$ to them, which characterize almost stationary spectra. Recognizing also that according to (4.9)

$$N_{\mathbf{k}}\propto|\partial\Omega_{\mathbf{k}}/\partial k_x|N(k_y,\Omega,k_z)\propto|\Omega_{\mathbf{k}}/k_x|N(k_y,\Omega,k_z), \quad (4.11)$$

we get instead of (4.1) an equation of the form

$$\frac{\partial N(k_y,\Omega,k_z)}{\partial t}\propto\boldsymbol{\delta}\begin{pmatrix}|\Omega_{\mathbf{k}}|^{-1}\mathbf{H}^{K_1}\\|k_y|^{-1}\mathbf{H}^{K_2}\end{pmatrix}\times|k_y|^{-1+2\delta_r}|\Omega|^{-1+2\delta_s}|k_z|^{-1+2\delta_f}, \quad (4.12)$$

where now $\boldsymbol{\delta}=(\delta_r,\delta_s,\delta_f)$ while the vectors $\mathbf{H}^{K_1}$ and $\mathbf{H}^{K_2}$ are defined by equations similar to (4.3):

$$\mathbf{H}^{K_i}=\int d\mathbf{p}_1\,d\mathbf{p}_2\,Q_1\delta(1-\nu_1-\nu_2)\,\delta(1-p_1-p_2)\,\mathbf{M}^{K_i}(n_1n_2R)^{(i)}. \quad (4.13)$$

Here

$$\mathbf{M}^{K_i}=(\partial K/\partial\mathbf{r})_{\mathbf{r}=\mathbf{r}_0^{(i)}}, \quad (4.14)$$

where $\mathbf{r}\equiv(r,s,f)$. Explicitly, the vectors $\mathbf{M}^{K_i}$ are [cf. (4.5), (4.6)]

$$\mathbf{M}^{K_1}=\{\nu_1\ln p_1+\nu_2\ln p_2,\ \nu_1\ln\nu_1+\nu_2\ln\nu_2,\ \nu_1\ln q_1+\nu_2\ln q_2\}, \quad (4.15)$$

$$\mathbf{M}^{K_2}=\{p_1\ln p_1+p_2\ln p_2,\ p_1\ln\nu_1+p_2\ln\nu_2,\ p_1\ln q_1+p_2\ln q_2\}. \quad (4.16)$$

From a comparison of (4.15), (4.16) with (4.5), (4.6) it is clear that

$$M_1^{K_i}=\Lambda_1^{K_i},\quad M_3^{K_i}=\Lambda_3^{K_i}. \quad (4.17)$$

Moreover, in accordance with (4.10)

$$M_2^{K_i}=a\Lambda_1^{K_i}+b\Lambda_2^{K_i}+c\Lambda_3^{K_i}. \quad (4.18)$$

The integrals $\mathbf{J}^{K_i}$ and $\mathbf{H}^{K_i}$ are correspondingly connected with one another through the relations

$$H_{k_y}^{K_i}=J_{k_y}^{K_i},\ H_{k_z}^{K_i}=J_{k_z}^{K_i},\quad H_\Omega^{K_i}=aJ_{k_y}^{K_i}+bJ_{k_x}^{K_i}+cJ_{k_z}^{K_i}. \quad (4.19)$$

For spectra of the first and second kind we find hence from (4.12) the conservation laws [cf. (4.7)]:

$$\frac{\partial D^{(i)}(k_y,\Omega,k_z)}{\partial t}+\frac{\partial}{\partial k_y}P_{k_y}^{(i)}+\frac{\partial}{\partial\Omega}P_\Omega^{(i)}+\frac{\partial}{\partial k_3}P_{k_z}^{(i)}=0. \quad (4.20)$$

The functions $D^{(i)}(k_y,\Omega,k_z)$ can be expressed in terms of $N(k_y,\Omega,k_z)$ in the same way as the $D_{\mathbf{k}}^{(i)}$ were in terms of $N_{\mathbf{k}}$. The fluxes $P_{k_y}^{(i)}, P_\Omega^{(i)}, P_{k_z}^{(i)}$ are [cf. (4.8)]

$$(P_{k_y}^{(i)},P_\Omega^{(i)},P_{k_z}^{(i)})\propto-(H_1^{K_i}/|\Omega k_z|,\ H_2^{K_i}/|k_yk_z|,\ H_3^{K_i}/|k_y\Omega|). \quad (4.21)$$

When one uses k_y, k_x, Ω as independent variables one gets the corresponding conservation laws and formulae for the fluxes by an obvious change of notation in the equations given above.

4.2. The problem of determining the signs of the fluxes and the locality problem. According to (4.3), (4.13), (4.21) the signs of the fluxes are determined by the signs of the vectors $\mathbf{\Lambda}^{K_i}$, $\mathbf{M}^{K_i}$ and the signs of $R^{(i)} \equiv (R)^{(i)}_{\alpha=\alpha_0}$. We consider the problem of determining the signs of the fluxes in the case of two-dimensional (k_y, k_x)-turbulence. We shall work in the (k_y, Ω)-space. We are then dealing with the quantities $M^{K_i}_{k_y}$, $M^{(i)}_{\Omega}$, $i = 1, 2$. Starting from (4.15), (4.16) and using the fact that $(p_1, p_2, \nu_1, \nu_2) \leqslant 1$ we conclude that all these quantities are negative:

$$\operatorname{sign} \mathbf{M}^{K_i} = -1, \quad i=1, 2. \tag{4.22}$$

In the case of functions $R^{(i)}$ with a fixed sign we find, using (4.21), (4.19), that

$$\operatorname{sign}(P^{(i)}_{k_y}, P^{(i)}_{\Omega}) = \operatorname{sign} R^{(i)} \qquad i=1\ 2. \tag{4.23}$$

On the other hand, according to (3.10), (3.13), (3.14),

$$R^{(1)} = 1 - p_1^{1+u-a(1+v)/b} \nu_1^{(1+v)/b} - p_2^{1+u-a(1+v)/b} \nu_2^{(1+v)/b}, \tag{4.24}$$

$$R^{(2)} = 1 - p_1^{3/2+u-a(1+v)/b} \nu_1^{-1/2+(1+v)/b} - p_2^{3/2+u-a(1+v)/b} \nu_2^{-1/2+(1+v)/b}. \tag{4.25}$$

In the case of $R^{(i)}$ with alternate sign it is necessary to turn to (4.22) to find the sign of the fluxes.

In agreement with Ref. 1 the locality condition of the Kolmogorov spectra is equivalent to the condition that the spectral fluxes are finite, i.e., the condition that the integrals in $\mathbf{J}^{K_i}$ or $\mathbf{H}^{K_i}$ converge [see (4.3), (4.13)].

We also note that in the case of divergent integrals $\mathbf{J}^{K_i}$ Eqs. (4.2), (4.8), (4.12), and (4.20) are invalid.

II. DRIFT WAVE TURBULENCE DESCRIBED BY THE HASEGAWA-MIMA EQUATION

5. Initial canonical equations

We consider two-dimensional (k_y, k_x) waves with a dispersion relation of the form

$$\omega_{\mathbf{k}} = k_y V_* / (1 + k_\perp^2 \rho_0^2), \tag{5.1}$$

where V_* is a scale velocity and ρ_0 some scale length. The best known representatives of waves of the kind (5.1) in a plasma are the electron drift waves in a plasma with cold ions. In that case $V_* = V_{ne}$, where $V_{ne} = -cT_e \varkappa_n / eB_0$ is the electron drift velocity along the density gradient, $\varkappa_n = \partial n_0 / \partial x$, n_0 is the equilibrium plasma density, T_e the electron temperature $\rho_0^2 = T_e / m_i \omega_{Bi}^2$ the square of the ion Larmor radius with respect to the electron temperature, $\omega_{Bi} = eB_0 / m_i c$ the ion cyclotron frequency, e and m_i the ion charge and mass, and c the light velocity. In the case of Rossby waves V_* and ρ_0 are, respectively, the Rossby speed and the Rossby radius (see, e.g., Refs. 9, 10 for the definition of these quantities).

We assume that the dynamic equation describing the interaction of these waves with one another has in the Fourier representation the form (cf. Ref. 4).

$$\frac{\partial \varphi_{\mathbf{k}}}{\partial t} \propto \sum_{\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}} [\mathbf{k}_1 \mathbf{k}_2]_z \frac{\mathbf{k}_{2\perp}^2 - \mathbf{k}_{1\perp}^2}{1+\mathbf{k}_\perp^2 \rho_0^2} \varphi_{\mathbf{k}_1} \varphi_{\mathbf{k}_2} \exp[-i(\omega_{\mathbf{k}_1} + \omega_{\mathbf{k}_2} - \omega_{\mathbf{k}}) t], \tag{5.2}$$

where φ is the potential of the field of the waves considered. Equations (5.1), (5.2) are the consequence of the well known equations for Rossby waves and also of the Hasegawa-Mima equation.[11] We use also the fact that the wave energy, apart from a constant, equals (see, e.g., Ref. 4)

$$W_{\mathbf{k}} \propto (1+\mathbf{k}_\perp^2 \rho_0^2) |\varphi_{\mathbf{k}}|^2, \tag{5.3}$$

so that

$$N_{\mathbf{k}} \propto (1+\mathbf{k}_\perp^2 \rho_0^2)^2 \varphi_{\mathbf{k}}^2 |k_y|^{-1/2}. \tag{5.4}$$

We can therefore use for the normalized potential $C_{\mathbf{k}}$ the quantity

$$C_{\mathbf{k}} \propto (1+\mathbf{k}_\perp^2 \rho_0^2) \varphi_{\mathbf{k}} |k_y|^{-1/2}. \tag{5.5}$$

Taking (5.5) into account and also the fact that the "phase mismatch" $\omega_{\mathbf{k}_1} + \omega_{\mathbf{k}_2} - \omega_{\mathbf{k}} \approx 0$ is small we reduce (5.2) to the form (2.8) with matrix elements of the form (cf. Refs. 5–7)

$$V(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) \propto |k_y k_{1y} k_{2y}|^{1/2} \times \left(\frac{k_{1x}}{1+\mathbf{k}_{1\perp}^2 \rho_0^2} + \frac{k_{2x}}{1+\mathbf{k}_{2\perp}^2 \rho_0^2} - \frac{k_x}{1+\mathbf{k}_\perp^2 \rho_0^2} \right). \tag{5.6}$$

Thereby we have all that is necessary to use the kinetic equation for waves of the form (2.4).

6. Short-wavelength turbulence

Let $k_\perp^2 \rho_0^2 \gg 1$, $k_x \gg k_y$. It follows then from (5.1), (5.6) that

$$\omega_{\mathbf{k}} \propto k_y k_x^{-2}, \tag{6.1}$$

$$V(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) \propto |k_y k_{1y} k_{2y}|^{1/2} \left(\frac{1}{k_{1x}} + \frac{1}{k_{2x}} - \frac{1}{k_x} \right). \tag{6.2}$$

According to (2.2), (2.6) this corresponds to the case

$$a=1, \quad b=-2, \quad u=3/2, \quad v=-1. \tag{6.3}$$

6.1. Stationary Kolmogorov spectra. It follows from (6.3) and (3.13), (3.14) that the exponents of the stationary Kolmogorov spectra of short-wavelength turbulence are equal to

$$\alpha_0^{(1)} = -5/2, \quad \beta_0^{(1)} = 0 \tag{6.4}$$

or to

$$\alpha_0^{(2)} = -5/2, \quad \beta_0^{(2)} = -1. \tag{6.5}$$

These exponents correspond to the energy spectra

$$W_{\mathbf{k}}^{(1)} \propto k_y^{-3/2} k_x^{-2} \tag{6.6}$$

or

$$W_{\mathbf{k}}^{(2)} \propto k_y^{-3/2} k_x^{-3}. \tag{6.7}$$

The spectra (6.6), (6.7) are the same as two of the three found in Refs. 5–7 for the case of short-wavelength Rossby waves. In those papers is given yet another spectrum

$W_{\mathbf{k}} \propto k_y^{-1} k_x^{-7/2}$, corresponding to the exponents $\alpha = -2$ and $\beta = -3/2$. However, in finding such indexes one must neglect $\delta(1-h_1+h_2)\delta(1+h_1-h_2)$ and retain only $\delta(1-h_1-h_2)$ in Eq. (3.12) for Q_1. By a direct substitution into these δ-functions of the quantities h_1, h_2, expressed in terms of the p_i, ν_i ($i = 1, 2$), one can check that this nelgect is inadmissible. The "additional" spectrum is thus not realized.

According to Refs. 4 and 9, numerical experiments about short-wavelength turbulence described by the Hasegawa-Mima equation give forms of spectra close to $W_{\mathbf{k}} \propto k_\perp^{-4}$. At the limits of applicability of our analysis, i.e., when $k_y \approx k_x \approx k_\perp$, it follows from (6.6), (6.7) that $W_{\mathbf{k}} \propto (k_\perp^{-7/2}, k_\perp^{-9/2})$, so that the numerical value of $W_{\mathbf{k}}$ lies between the two Kolmogorov values.

6.2. Dynamical properties of short-wavelength Kolmogorov turbulence. Using (4.24), (4.25) we find that in the case considered

$$R^{(1)}=1-p_1^{5/2}-p_2^{5/2}, \quad R^{(2)}=1-p_1^3\nu_1^{-1/2}-p_2^3\nu_2^{-1/2}. \tag{6.8}$$

As $0 \leqslant (p_j, \nu_j) \leqslant 1$, it follows from (6.8) that $R^{(1)}$ is a function with a fixed sign and that

$$\operatorname{sign} R^{(1)}=1. \tag{6.9}$$

Hence, according to (4.23),

$$\operatorname{sign}(P_{k_y}^{(1)}, P_{\Omega}^{(1)})=1. \tag{6.10}$$

This means that in the (k_y, Ω)-space the spectrum (6.6) corresponds to an energy flux towards larger k_y and Ω. Using (4.19) we note that in the (k_y, k_x) space the energy flux is directed towards larger k_y and smaller k_x. At the limit of applicability of our analysis when $k_y \approx k_x$ and $\omega_{\mathbf{k}} \propto 1/k_\perp$, these results mean qualitatively that a spectrum of the type (6.6) corresponds to an energy flux in the direction of larger frequencies and smaller wave numbers. A similar behavior is observed also in the numerical calculations of Ref. 9.

Using (6.1), (6.2), (3.12) we find that the function Q_1 in the integrals (4.3), (4.13) are in the case considered

$$Q_1 = p_1p_2\Big[\Big(1-\frac{1}{h_1}-\frac{1}{h_2}\Big)^2 \delta(1-h_1-h_2)+\Big(1+\frac{1}{h_1}-\frac{1}{h_2}\Big)^2 \times\delta(1+h_1-h_2)+\Big(1-\frac{1}{h_1}+\frac{1}{h_2}\Big)^2 \delta(1-h_1+h_2)\Big]. \tag{6.11}$$

Moreover, according to (6.1)

$$h_j=(p_j/\nu_j)^{1/2}, \quad j=1,\ 2. \tag{6.12}$$

Using (6.12) we change from the variables h_j to the variables ν_j. After that we use the fact that h_j is of the form (6.12) and that $0 \leqslant (p_j, \nu_j) \leqslant 1$, $\delta(1-h_1-h_2) \equiv 0$, so that the first term in the square brackets of the right-hand side of Eq. (6.11) drops out. The integral of the two remaining terms in Q_1 reduce one to the other through a change of variables. Moreover, we recognize that

$$(n_1n_2)^{(1)}=(p_1p_2)^{-5/2}, \quad (n_1n_2)^{(2)}=(p_1p_2)^{-3}(\nu_1\nu_2)^{1/2}. \tag{6.13}$$

As a result, the integrals $\mathbf{H}^{K_i}$ reduce, for instance, to the form

$$\mathbf{H}^{K_i} \infty \int_0^1 dp_1\, dp_2\, d\nu_1\, d\nu_2 \Big[1+\Big(\frac{\nu_1}{p_1}\Big)^{1/2}-\Big(\frac{\nu_2}{p_2}\Big)^{1/2}\Big]^2 \mathbf{F}^{K_i} \times\delta\Big[1+\Big(\frac{p_1}{\nu_1}\Big)^{1/2}-\Big(\frac{p_2}{\nu_2}\Big)^{1/2}\Big]\delta(1-\nu_1-\nu_2)\delta(1-p_1-p_2), \tag{6.14}$$

where

$$\mathbf{F}^{K_i}=\{(p_1p_2)^{-1}(\nu_1\nu_2)^{-1/2}R^{(1)}\mathbf{M}^{K_1}, \quad (p_1p_2)^{-1/2}(\nu_1\nu_2)^{-1}R^{(2)}\mathbf{M}^{K_2}\}. \tag{6.15}$$

The quantities $R^{(1)}$, $R^{(2)}$ are given by Eqs. (6.8) and the vectors $\mathbf{M}^{K_1}$, $\mathbf{M}^{K_2}$ by Eqs. (4.15), (4.16).

Our problem is, firstly, to study the problem of whether the integrals (6.14) are finite, which means as we noted in Sec. 4.2 clarifying the locality problem, and secondly, to determine the sign of the integrals $\mathbf{H}^{K_i}$ in cases when these integrals are finite, which is necessary, according to what has been said earlier, to elucidate the direction of the fluxes.

"Dangerous" regions of integration occurs as $(p_1, \nu_1) \to 0$ and $(p_2, \nu_2) \to 0$. In the case $(p_1, \nu_1) \to 0$ the δ-functional connection between p and ν of (6.14) gives $p_1 = \nu_1^{3/4}/4$ and the contribution from the corresponding region to the integrals $\mathbf{H}^{K_i}$ is given by the expression

$$(\mathbf{H}^{K_i})_1 \infty \int \mathbf{F}^{K_i}\, d\nu_1. \tag{6.16}$$

Taking into account that for the indicated p_1 and ν_1 we have $R^{(1)} \propto \nu_1^3$ and $M_{k_y}^{K_1} \propto M_{\Omega}^{K_1} \propto \nu_1 \ln \nu_1$, we find, using (6.15), that in that region

$$\mathbf{F}^{K_1} \infty \nu_1^{-1/2} \ln \nu_1, \tag{6.17}$$

so that $(\mathbf{H}^{K_1})_1 \to 0$. On the other hand, in the case $i = 2$ we have $R^{(2)} = -\nu_1$, $M_{k_y}^{K_2} \propto \nu_1^3 \ln \nu_1$, $M_{\Omega}^{K_2} \propto -\nu_1$. In that case

$$(F_{k_y}^{K_2}, F_{\Omega}^{K_2}) \infty -\nu_1^{-7/2}(\nu_1^2 \ln \nu_1, -1), \tag{6.18}$$

so that $(\mathbf{H}^{K_2})_1 \to \infty$.

In the region $(p_2, \nu_2) \to 0$ we have $p_2 = 4\nu_2$ and the contribution from that region to the integrals $\mathbf{H}^{K_i}$ can be written in the form

$$(\mathbf{H}^{K_i})_2 \infty \int \nu_2 \mathbf{F}^{K_i}\, d\nu_2. \tag{6.19}$$

Moreover, now $R^{(i)} \propto \nu_2$, $M_{k_y}^{K_i} \propto M_{\Omega}^{k_i} \propto \nu_2 \ln \nu_2$, $i = 1, 2$, and in that case [cf. (6.17)]

$$\mathbf{F}^{K_i} \infty \nu_2^{-1/2} \ln \nu_2. \tag{6.20}$$

Hence $(\mathbf{H}^{K_i})_2 \to 0$.

The integrals $\mathbf{H}^{K_1}$ thus turn out to converge. Hence, the spectrum (6.6) connected with the energy flux is local. The integral $\mathbf{H}^{K_2}$ turns out to diverge as $(\nu_1, p_1) \to 0$. In that sense, the spectrum (6.7), connected with the enstrophy flux, is nonlocal. The nonlocality is caused by the long-wavelength part of the spectrum with $k_y \propto k_x^3$. This part of the spectrum corresponds to "zonal flows."[9] It is clear from physical considerations that the ideas expounded here about the turbulence are no longer applicable to such waves.

7. Long-wavelength turbulence

We now consider the case of long waves, $k_\perp^2 \rho_0^2 \ll 1$, again assuming that $k_x \gg k_y$. The frequency of the oscillations can in this case be written in the form (2.1) with

$$\Omega_{\mathbf{k}} \infty k_y k_x^2, \tag{7.1}$$

so that

$$a=1, \quad b=2. \tag{7.2}$$

The matrix elements (5.6) under the given assumptions take the form

$$V(\mathbf{k},\ \mathbf{k}_1,\ \mathbf{k}_2) \propto |k_y k_{1y} k_{2y}|^{1/2} (k_{1x}^3 + k_{2x}^3 - k_x^3). \tag{7.3}$$

Hence it follows that

$$u = 3/2, \quad v = 3. \tag{7.4}$$

We then get, similarly to (6.4), (6.5), the following two pairs of exponents α, β:

$$\alpha_0^{(1)} = -5/2, \quad \beta_0^{(1)} = -4 \tag{7.5}$$

or

$$\alpha_0^{(2)} = -5/2, \quad \beta_0^{(2)} = -3. \tag{7.6}$$

As in the case considered $\omega_\mathbf{k} \sim k_y$, we have

$$W_\mathbf{k} \propto |k_y| N_\mathbf{k} \sim k_y^{1+\alpha} k_x^\beta. \tag{7.7}$$

The energy spectra

$$W_\mathbf{k}^{(1)} \propto k_y^{-3/2} k_x^{-4}, \tag{7.8}$$

$$W_\mathbf{k}^{(2)} \propto k_y^{-3/2} k_x^{-3}, \tag{7.9}$$

follow from (7.5)–(7.7)

By analogy with Sec. 6.2 we consider the dynamic properties of these spectra. We have now according to (7.1) instead of (6.12)

$$h_j = (\nu_j/p_j)^{1/3}, \quad j = 1,\ 2, \tag{7.10}$$

and the functions $R^{(i)}$ are in accordance with (4.24), (4.25)

$$R^{(1)} = 1 - p_1^{1/2}\nu_1^2 - p_2^{1/2}\nu_2^2, \quad R^{(2)} = 1 - p_1\nu_1^{5/3} - p_2\nu_2^{5/3}. \tag{7.11}$$

Similar to (6.11), we find an expression for the function Q_1

$$Q_1 = p_1 p_2 [(1-h_1^3-h_2^3)^2 \delta(1-h_1-h_2)$$
$$+ (1+h_1^3-h_2^3)^2 \delta(1+h_1-h_2)$$
$$+ (1-h_1^3+h_2^3)^2 \delta(1-h_1+h_2)]. \tag{7.12}$$

Using (7.10) we note that, as in the case of short-wavelength turbulence (see Sec. 6), $\delta(1-h_1-h_2) \equiv 0$. Using (4.13), (7.12) we write down, similarly to (6.14), the integrals $\mathbf{H}^{K_i}$. We then get

$$\mathbf{H}^{K_i} \propto \int_0^1 dp_1\, dp_2\, d\nu_1\, d\nu_2 \left[1 + \left(\frac{\nu_1}{p_1}\right)^{1/3} - \left(\frac{\nu^2}{p_2}\right)^{1/3} \right]^2 \mathbf{F}^{K_i}$$
$$\times \delta \left[1 + \left(\frac{\nu_1}{p_1}\right)^{1/3} - \left(\frac{\nu_2}{p_2}\right)^{1/3} \right] \delta(1-\nu_1-\nu_2)\, \delta(1-p_1-p_2), \tag{7.13}$$

where

$$\mathbf{F}^{K_i} = \{(\nu_1\nu_2)^{-5/2} R^{(1)} \mathbf{M}^{K_1}, \quad (p_1p_2)^{-1/2} (\nu_1\nu_2)^{-2} R^{(2)} \mathbf{M}^{K_2}\}. \tag{7.14}$$

As in Sec. 6, the "dangerous" regions of integration correspond to the cases $(p_1, \nu_1) \to 0$ and $(p_2, \nu_2) \to 0$. In the first case $\nu_1 = p_1^3/4$ and the corresponding contribution to $\mathbf{H}^{K_i}$ can be written in the form [cf. (6.16)]

$$(\mathbf{H}^{K_i})_1 \propto \int p_1^4 \mathbf{F}^{K_i}\, dp_1. \tag{7.15}$$

Using (7.11), (4.15), (4.16) we get

$$R^{(1)} \propto R^{(2)} \sim p_1, \quad \mathbf{M}^{K_1} \propto (-p_1,\ p_1^3 \ln p_1), \quad \mathbf{M}^{K_2} \propto p_1 \ln p_1. \tag{7.16}$$

It then follows from (7.14) that

$$\mathbf{F}^{K_1} \propto p_1^{-11/2} (-1,\ p_1^2 \ln p_1). \tag{7.17}$$

It is clear from (7.15), (7.17) that $(H_{k_y}^{K_1})_1 \to \infty$, $(H_\Omega^{K_1})_1 \to 0$. Similarly, we have for $i = 2$

$$\mathbf{F}^{K_2} \propto p_1^{-9/2} \ln p_1. \tag{7.18}$$

Substituting (7.16) into (7.15) we find that $(\mathbf{H}^{K_2})_1 \to 0$. We now consider the region $(p_2, \nu_2) \to 0$. In that case $p_2 = \nu_2/4$, and the corresponding contribution to $\mathbf{H}^{K_i}$ has the form (6.19) while $\mathbf{F}^{K_i}$ is given by Eq. (6.20). Therefore, as in Sec. (6.2), $(\mathbf{H}^{K_i})_2 \to 0$.

Thus, the spectrum $W_\mathbf{k}^{(1)}$ turns out to be nonlocal,and the spectrum $W_\mathbf{k}^{(2)}$ to be local. The region $(\nu_1, p_1) \to 0$ with $\nu_1 \propto p_1^3$, causing the nonlocality of the enstrophy spectrum, corresponds to waves with $k_{1x} \approx k_{1y}$. However, according to the assumption which we have made, $k_x \gg k_y$, such waves must be excluded from our analysis. This indicates that in principle it is possible to regularize the integral $H_{k_y}^{K_1}$ and, correspondingly, that it is possible to realize the spectrum $W_\mathbf{k}^{(1)}$. On the other hand, the insensitivity of the spectrum $W_\mathbf{k}^{(2)}$ to waves with $\nu_1 \propto p_1^3$, i.e., with $k_{1x} \approx k_{1y}$ indicates that our initial assumption $k_x \gg k_y$ is adequate, when applied to such a spectrum. According to Ref. 9 a numerical simulation and experimental observations of Rossby waves with $k_\perp \rho_0 \ll 1$ also indicate that the main part of the energy is contained in waves with $k_x \gg k_y$. In this connection the ideas presented above about the spectrum $W_\mathbf{k}^{(2)}$ are in agreement with the picture following from numerical and real experiments.

According to (7.11) the function $R^{(2)}$ has a fixed sign and sign $R^{(2)} = 1$. We then conclude in accordance with (4.25) that the energy flux in the $W_\mathbf{k}^{(2)}$ spectrum is in the direction of larger frequencies (in the direction of shorter waves).

8. Discussion of the results

We have considered the problem of three-dimensional weakly turbulent power-law spectra which are established when weakly dispersive waves with a dispersion relation such as (2.1) interact, and the similar problem of strongly dispersive waves with a dispersive relation of the form (2.3), assuming that the matrix elements are scale invariant. We have shown that in this kind of problem two kinds of Kolmogorov spectra can be realized with power-law exponents (3.13), (3.14). One of those spectra is connected with the flux of the wave energy and the other with the enstrophy flux.

The formalism expounded above can be used for a wide class of problems of drift-type waves in an inhomogeneous plasma. By applying this formalism to the problem of drift waves described by a Hasegawa-Mima type equation, we have established that short-wavelength drift turbulence is characterized by stationary spectra of the form (6.6), (6.7) and long-wavelength turbulence by spectra of the form (7.8), (7.9). From the analysis of the locality of these spectra it follows that the spectra connected with the energy flux are local and those connected with the enstrophy flux are nonlocal.

The local spectra (6.6), (7.9) can be interpreted as spectra connected with the energy flux. The problem of the interpretation of the nonlocal spectra (6.7), (7.8) requires an additional analysis.

The authors are grateful to S. V. Nazarenko and A. I. Smolyakov for useful discussions.

APPENDIX

Derivation of Eq. (3.8)

To begin with, to be specific, we consider the integrals $J_j(\sigma) = 1,2,3$ for $\sigma \equiv (\sigma_{h_i}, \sigma_{q_i}) = 1$, $i = 1,2$. The initial expressions for these integrals with $N_{\mathbf{k}}$ of the form (2.7) are

$$J_{1,2,3,}(\sigma) = \int dp_1\, dp_2\, dh_1\, dh_2\, dq_1\, dq_2 \hat{U}_{1,\sigma;2,\sigma;3,\sigma} R_{1,2,3}\delta \times (1 \pm p_1 \mp p_2)\,\delta(1 \pm \nu_1 \mp \nu_2)\,\delta(1 \pm h_1 \mp h_2)\,\delta(1 \pm q_1 \mp q_2), \qquad (A1)$$

where

$$R_{1,2,3} = (p_1p_2)^\alpha (h_1h_2)^\beta (q_1q_2)^\gamma \mp p_1^\alpha h_1^\beta q_1^\gamma \mp p_2^\alpha h_2^\beta q_2^\gamma. \qquad (A2)$$

We give here the procedure to transform the integral $J_2(\sigma)$. We change the expression for $\hat{U}_{2,\sigma}$ which occurs in that integral for the given values of σ_{h_i}, σ_{q_i} to the form

$$\hat{U}_{2,\sigma} = (1+p_1)^{2u}(1+h_1)^{2v}(1+q_1)^{2w} \times U(1;\ p_1/(1+p_1),\ h_1/(1+h_1),\ q_1/(1+q_1);\ 1/(1+p_1),\ 1/(1+h_1),\ 1/(1+q_1)), \qquad (A3)$$

where we have used (2.5), (2.6) and the appropriate δ functions. Hence, performing an obvious change of variables (Cf. Ref. 15),

$$p_1/(1+p_1) = p_1',\quad h_1/(1+h_1) = h_1',\quad q_1/(1+q_1) = q_1', \qquad (A4)$$

we get

$$\hat{U}_{2,\sigma} = (1-p_1')^{-2u}(1-h_1')^{2v}(1-q_1')^{-2w} \times U(1;\ p_1',\ h_1',\ q_1';\ 1-p_1',\ 1-h_1',\ 1-q_1'). \qquad (A5)$$

The Jacobian of the transformation (A4) equals

$$D(p_1, h_1, q_1)/D(p_1', h_1', q_1') = [(1-p_1')(1-h_1')(1-q_1')]^{-2}. \qquad (A6)$$

Moreover, using (2.2), (A4), we transform the δ function of the frequencies which occurs in $J_2(\sigma)$:

$$\delta(1+\nu_1-\nu_2) = (1-p_1')^a(1-h_1')^b(1-q_1')^c\delta(1-\nu_1'-\nu_2'), \qquad (A7)$$

where ν_1', ν_2' are the same functions of the primed variables as ν_1, ν_2 are of the unprimed ones. Finally, we transform R_2:

$$R_2 = -(1-p_1')^{-2\alpha}(1-h_1')^{-2\beta}(1-q_1')^{-2\gamma}R_1', \qquad (A8)$$

where R_1' is the same function of the primed variables as R_1 is of the unprimed ones. Dropping the primes of the corresponding integration variables we get

$$J_2(\sigma) = -\int dp_1\, dp_2\, dh_1\, dh_2\, dq_1\, dq_2\, U_{1,\sigma} R_1 \lambda_2 \times \delta(1-p_1-p_2)\,\delta(1-\nu_1-\nu_2)\,\delta(1-h_1-h_2)\,\delta(1-q_1-q_2), \qquad (A9)$$

where λ_2 is given by Eq. (3.11).

One can transform $J_3(\sigma)$ also to a form similar to (A9). The only difference with (A9) consists in the substitution $\lambda_2 \to \lambda_1$. As a result we get with the given σ

$$J_1(\sigma) + J_2(\sigma) + J_3(\sigma) = \int dp_1\, dp_2\, dh_1\, dh_2\, dq_1\, dq_2\, U_{1,\sigma} R_1 \times (1-\lambda_1-\lambda_2)\,\delta(1-p_1-p_2)\,\delta(1-\nu_1-\nu_2) \times \delta(1-h_1-h_2)\,\delta(1-q_1-q_2). \qquad (A10)$$

Similarly we transform also the sums of integrals with other values of σ. We then get formulae such as (A10) with appropriate modifications of the values $U_{1,\sigma}$ and the substitutions

$$\delta(1-h_1-h_2)\,\delta(1-q_1-q_2) \to \delta(1-\sigma_{h_1}h_1-\sigma_{h_2}h_2) \times \delta(1-\sigma_{q_1}q_1-\sigma_{q_2}q_2). \qquad (A11)$$

Taking into account what we have said and using Eq. (3.5) and similar relations for I_2, I_3 we bring (3.1) to the form (3.8).

[1]V. E. Zakharov, *Basic Plasma Physics* (Eds., A. A. Galeev and R. N. Sudan), North Holland, Amsterdam, Vol. II, 3 (1984).

[2]A. B. Mikhaĭlovskiĭ, Teoriya plazmennykh neustoĭchivosteĭ (Theory of Plasma Instabilities), Atomizdat, Moscow, Vol. 2, Chs. 5, 6 (1977) [English translation published by Consultants Bureau].

[3]B. B. Kadomtsev, Vopr. teor. Plazmy **4**, 188 (1964) [English translation published by Academic Press as Plasma Turbulence].

[4]W. Horton, *Basic Plasma Physics* (Eds. A. A. Galeev and R. N. Sudan), North Holland, Amsterdam, Vol. II, 383 (1984).

[5]A. G. Sazontov, Tonkaya struktura i sunopticheskaya izmenchivost' moreĭ (Fine Structure and Synoptic Variability of Seas), Tallin (1980), p. 147.

[6]A. G. Sazontov, Preprint No. 30, Inst. Appl. Phys., Gor'kiĭ (1981).

[7]A. S. Monin and L. I. Piterbarg, Dokl. Akad. Nauk SSSR **295**, 86 (1987) [Sov. Phys. Doklady **32**, 560 (1987)].

[8]B. B. Kadomtsev and V. M. Kontorovich, Radiofiz. **32**, 511 (1974) [Radiophys. Qu. Electron. **32**, 386 (1975)].

[9]A. Hasegawa, C. G. Mclennan, and Y. Kodama, Phys. Fluids **22**, 2122 (1979).

[10]A. B. Mikhailovskiĭ, Nonlinear Phenomena in Plasma Physics and Hydrodynamics (Ed. R. Z. Sagdeev) Mir, Moscow, p. 8 (1986).

[11]A. Hasegawa and K. Mima, Phys. Fluids **21**, 87 (1978).

[12]A. A. Galeev and R. Z. Sagdeev, Vopr. teor. Plazmy **7**, 3 (1973) [Rev. Plasma Phys. **7**, 1 (1979)].

[13]V. I. Karas', S. S. Moiseev, and V. E. Novikov, Zh. Eksp. Teor. Fiz. **71**, 1421 (1976) [Sov. Phys. JETP **44**, 744 (1976)].

[14]A. V. Kats, Zh. Eksp. Teor. Fiz. **71**, 2104 (1976) [Sov. Phys. JETP **44**, 1106 (1976)].

[15]E. A. Kuznetsov, Zh. Eksp. Teor. Fiz. **62**, 584 (1972) [Sov. Phys. JETP **35**, 310 (1972)].

Translated by D. ter Haar