Photon emission by a neutron in a plane-wave field

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Photon emission by a chargeless Dirac particle with an anomalous magnetic moment is considered for the cases of the particle interacting with the following fields: a plane wave field with both linear and circular polarization, and a constant crossed field. Closed-form invariant expressions for the emission probability and intensity are found. In the approximation linear in the wave-energy density, some of the results are shown to coincide with earlier ones obtained by cumbersome and noninvariant methods.

The relativistically invariant methodology for the calculations of elastic and inelastic interactions between electrons and a plane electromagnetic wave field has been developed mostly by V. I. Ritus and A. I. Nikishov (see the review in Ref. 1). Their approach is known to be sufficiently general and applicable to processes taking place in arbitrary fields at ultrarelativistic particle energies. One of the most important applications was the study of photon emission by an electron in the field of a plane electromagnetic wave with various polarizations and in a crossed field. That study was a substantial contribution to the solution of the classical problem of the emission of a charge in an external field.

Of equal importance is the problem of photon emission by a neutral Dirac particle with an anomalous magnetic moment, such as a neutron, in a plane-wave field. There have been some earlier attempts to approach this problem[2,3]; however, it was seemingly impossible to obtain closed-form results by using noninvariant solutions of the generalized Dirac equations for a neutron in a plane-wave field; for that reason, the authors of Ref. 3 have limited their consideration to special cases.

In our paper,[4] where the process $n\to n(\nu\bar{\nu})$ in a plane-wave field was studied, we obtained a compact and invariant expression for the wave function; a computational technique was developed using as an example the above inelastic process. In the present paper we consider a purely electrodynamic interaction between a neutron and a plane-wave field, with emission of a photon. Using an invariant technique, we obtain general and closed-form expressions for the emission probability and intensity in a linearly polarized wave and in a crossed field. The general formulas yield the corresponding results of Ref. 3 as a particular case.

1. The relativistically invariant form of the solution of the generalized Dirac equation for a chargeless particle with an anomalous magnetic moment μ in a plane-wave field with a potential $A = af(\varphi)$, $\varphi = (kx)$, can be written as[4]

$$\Psi = \left(\cos z + \frac{B}{z}\sin z\right)\frac{u}{(2p_0)^{1/2}}e^{-i(px)}, \qquad (1)$$

$$B = \frac{\mu}{2(kp)}(\hat{k}\hat{A}\hat{p} + \hat{p}\hat{k}\hat{A}), \quad z = (-B^2)^{1/2} = \mu(-A^2)^{1/2},$$

where $u(p)$ is a solution of the Dirac equation in the absence of a field, with a density matrix

$$u_i\bar{u}_k = {}^1/_2(\hat{p}+m)_{ik}$$

for non-polarized particles (here we do not concern ourselves with the effects of the polarization of the neutrons and of the emitted photon; the appropriate generalization is obvious).

The structure of the $(nn\gamma)$ vertex is defined by the expression:

$$L = -i\mu(\bar{\Psi}\sigma^{\mu\nu}\Psi)\frac{\partial A_\mu^{(\gamma)}}{\partial x^\nu}, \qquad (2)$$

$$\sigma^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu),$$

and using the transformation formulas from Ref. 4, one can readily obtain the following matrix element, which corresponds to the process $n\to n\gamma$ in the field of a monochromatic linearly polarized plane wave $A = a\sin\varphi$:

$$\langle f|S|i\rangle = iC\sum_{s=1}^{\infty} J_s^2\delta(q_s - p' - \varkappa)\bar{u}(p')$$

$$\times\left\{\frac{1+(-1)^s}{2}[\sigma^{\mu\nu} - \Gamma^{\mu\nu}(p',p)] + \frac{1-(-1)^s}{2i}\Gamma^{\mu\nu}(p',p)\right\}u(p)e_\mu\varkappa_\nu,$$

$$\Gamma^{\mu\nu}(p',p) = \frac{M'\sigma^{\mu\nu}M}{(kp')(kp)a^2},$$

$$\Gamma^{\mu\nu}(p',p) = \frac{1}{(-a^2)^{1/2}}\left(\frac{M'\sigma^{\mu\nu}}{kp'} - \frac{\sigma^{\mu\nu}M}{kp}\right), \qquad (3)$$

$$C = {}^1/_2\mu(4\pi)^{1/2}(2\pi)^4V^{-3/2}(2p_0 2p_0' 2\varkappa_0)^{-1/2},$$

$$M = {}^1/_2(\hat{k}\hat{a}\hat{p} + \hat{p}\hat{k}\hat{a}), \quad M' = M(p\to p'),$$

$$\arg J_s = 2\mu(-a^2)^{1/2}, \quad q_s = p + sk,$$

where $\varkappa$ and e are the photon momentum and polarization vector, $\mathbf{p}$ and $\mathbf{p}'$ are the momenta of the initial and final neutrons (for simplicity, we will omit the subscript "s" in q_s in the remainder of this section). The probability of the process per unit time, for unpolarized particles, can be written as:

$$W = \frac{w_0}{\pi m^4}\sum_{s=1}^{\infty} J_s^2\mathcal{F}_s\left\{\left[\frac{1+(-1)^s}{2}T^{(even)}_{\mu\nu\alpha\beta} + \frac{1-(-1)^s}{2}T^{(odd)}_{\mu\nu\alpha\beta}\right]g^{\mu\alpha}\varkappa^\nu\varkappa^\beta\right\},$$

$$T^{(even)}_{\mu\nu\alpha\beta} = {}^1/_4\,\mathrm{Sp}\{(\hat{p}'+m)[\sigma_{\mu\nu} - \Gamma_{\mu\nu}(p',p)]\times(\hat{p}+m)[\sigma_{\alpha\beta} - \Gamma_{\alpha\beta}(p,p')]\},$$

$$T^{(odd)}_{\mu\nu\alpha\beta} = {}^1/_4\,\mathrm{Sp}\{(\hat{p}'+m)\Gamma_{\mu\nu}(p',p)(\hat{p}+m)\Gamma_{\alpha\beta}(p,p')\},$$

$$w_0 = \mu^2m^4/4p_0, \qquad (4)$$

where each term of the series corresponds to capture of s photons from the wave, and the operator $\mathcal{F}_s$ is defined by

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the expression:

$$\mathcal{F}_s[G]=\int\frac{d^3p'}{2p_0'}\int\frac{d^3\varkappa}{2\varkappa_0}\delta(q-p'-\varkappa)G. \tag{5}$$

Performing straightforward calculations, one can easily verify that

$$T^{(even)}_{\mu\nu\alpha\beta}=T^{(odd)}_{\mu\nu\alpha\beta}=T_{\mu\nu\alpha\beta}, \tag{6a}$$

$$T\equiv g^{\mu\alpha}\varkappa^\nu\varkappa^\beta T_{\mu\nu\alpha\beta}=8\Big[(\varkappa p)(\varkappa p')+(kp)(kp')\frac{m^2}{a^2}\Big(\frac{ap}{kp}-\frac{ap'}{kp'}\Big)^2 +m^2(k\varkappa)\Big(\frac{\varkappa p}{kp}+\frac{\varkappa p'}{kp'}\Big)-\frac{m^2(pp')}{(kp)(kp')}(k\varkappa)^2\Big]. \tag{6b}$$

The calculation of $\mathcal{F}_s[T]$ can be carried out by utilizing the following relations and some particular cases which stem from them:

$$\mathcal{F}_s[p_\mu'p_\nu']=-\frac{\pi}{24}\frac{(q^2-m^2)^3}{q^4}g_{\mu\nu}+\frac{\pi}{6}\frac{(q^2-m^2)}{q^6}(q^4+m^2q^2+m^4)q_\mu q_\nu, \tag{7a}$$

$$\mathcal{F}_s\Big[\frac{p_\mu'}{kp'}\Big]=\frac{\pi}{2}\frac{q^2-m^2}{q^2(kq)}q_\mu+\frac{\pi}{2(kq)^2}\Big[-q^2+m^2+\frac{1}{2}(q^2+m^2)\ln\frac{q^2}{m^2}\Big]k_\mu, \tag{7b}$$

$$\mathcal{F}_s\Big[\frac{p_\mu'p_\nu'}{kp'}\Big]=\frac{\pi}{4(kq)}\Big[m^2\ln\frac{q^2}{m^2}-\frac{q^4-m^4}{2q^2}\Big]g_{\mu\nu}+\frac{\pi}{4}\frac{q^4-m^4}{q^4(kq)}q_\mu q_\nu+\dots, \tag{7c}$$

note that we have omitted in (7c) two tensor combinations that do not contribute to $\mathcal{F}_s[T]$.

Denoting

$$u_s=2s(kp)/m^2, \tag{8}$$

and performing some transformations with Eqs. (4) and (5) taken into account, we obtain the following expression for the total probability:

$$W=w_0\sum_{s=1}^{\infty}J_s^2g(u_s), \tag{9}$$

$$g(u_s)=\frac{u_s(u_s+2)}{2(1+u_s)^2}(u_s^2+2u_s+2)-2\ln(1+u_s), \tag{9a}$$

$$g(u_s)|_{u_s\ll1}\approx{}^4/_3u_s^3. \tag{9b}$$

A plot of the function $g(u_s)$ is shown in Fig. 1. If the result of

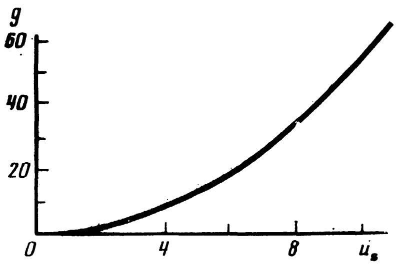

FIG. 1.

Ref. 3 is expressed in a simpler form, it follows from ours in the lowest order in μ.

2. For more general potentials, including circular polarization as well, we shall try to find the solution of the generalized Dirac equation [Eq. (1) in Ref. 4] in the following form:

$$\Psi=\Big(\cos z+\frac{B}{z}\sin z\Big)F\frac{u}{(2p_0)^{1/2}}e^{-i(px)}. \tag{10}$$

For the function F we obtain after some transformations (the derivative is taken with respect to the phase $\varphi=(kx)$).

$$F'=-\Big(\cos z-\frac{B}{z}\sin z\Big)\frac{1}{2}[BB']\times\Big[\frac{\sin z}{z}+\Big(\frac{\sin z}{z^3}-\frac{\cos z}{z^2}\Big)B\Big]F. \tag{11}$$

For potentials $A=af(\varphi)$, the commutator $[BB']=0$ and we return to the solution (1). Applying Eq. (11) and limiting ourselves to the accuracy $\sim B\otimes B\ (\sim\mu^2)$ inclusive, we obtain:

$$F'\approx-\frac{1}{4}[BB']\frac{\sin(2z)}{z}F$$

with the solution:

$$F\approx1-\frac{1}{4}\int[BB']\frac{\sin(2z)}{z}d\varphi,$$

and with the same accuracy:

$$\Psi=\Big\{\cos z+\frac{B}{z}\sin z-\frac{1}{4}\cos z\int[BB']\frac{\sin(2z)}{z}d\varphi\Big\}\frac{u}{(2p_0)^{1/2}}e^{-i(px)}. \tag{12}$$

In a wave with circular polarization

$$A=a_1\cos\varphi+a_2\sin\varphi,$$
$$a_1^2=a_2^2=a^2,\quad(a_1a_2)=0,\quad z=\mu(-a^2)^{1/2} \tag{13}$$

the interference of the first and second terms of (12) in the matrix element describes the one-photon capture. The exact solution of (11), with Eq. (13) taken into account, would also describe multiphoton processes in a circularly polarized wave. We shall limit ourselves to the investigation of the contribution W_1 to the total probability. Using the representation (12), we obtain

$$\langle f|S|i\rangle_1=\frac{i}{2}C\sin(2z)\delta(q_1-p'-\varkappa)\times[\bar u(p')\tilde\Gamma_{\mu\nu}(p',p)u(p)]e^\mu\varkappa^\nu, \tag{14}$$

where in $\tilde\Gamma$ $a\to a_1-ia_2$. The corresponding probability is

$$W_1=w_0\frac{\sin^2(2z)}{4\pi m^4}\mathcal{F}_1\Big\{\frac{1}{4}\mathrm{Sp}[(\hat p'+m)\tilde\Gamma_{\mu\nu}(p',p)(\hat p+m)\times\tilde\Gamma^*_{\alpha\beta}(p,p')]g^{\mu\alpha}\varkappa^\nu\varkappa^\beta\Big\}, \tag{15}$$

where in $\tilde\Gamma^*$ $a\to a_1+ia_2$. The result of calculation of the expression the braces in Eq. (15) is

$$\{(15)\}=16\Big[(\varkappa p)(\varkappa p')-m^2(kp)(kp')\alpha^2+m^2(k\varkappa)\Big(\frac{\varkappa p}{kp}+\frac{\varkappa p'}{kp'}\Big)-\frac{m^2(pp')}{(kp)(kp')}(k\varkappa)^2\Big], \tag{16}$$

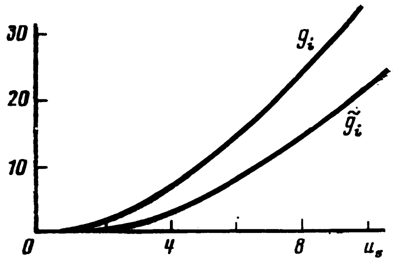

FIG. 2.

where

$$\alpha^2=-\frac{1}{2a^2}\left[\left(\frac{a_1p}{kp}-\frac{a_1p'}{kp'}\right)^2+\left(\frac{a_2p}{kp}-\frac{a_2p'}{kp'}\right)^2\right].$$

It is readily seen (see also Ref. 1), that

$$\alpha^2=-\frac{1}{2}\left(\frac{p}{kp}-\frac{p'}{kp'}\right)^2.$$

The subsequent integration over the phase-space volume in (15), with the Eqs. (7) and (8) taken into account, results in:

$$W_1={}^1/_2w_0\sin^2(2z)g(u_1). \qquad (17)$$

Those terms in Eqs. (9) and (17), which are linear in wave energy density, do not depend on the state of polarization and are, therefore, identical (their seeming discrepancy by the factor of 2 is due to the difference in the relation between a^2 and the average energy density, see Ref. 1). Apart from notation, they describe photon scattering on a magnetic moment.

3. The intensity I_σ of the "photon 4-momentum emission" can be derived from the expressions (4) and (15) by adding the factor $\varkappa_\sigma=q_\sigma-p'_\sigma$ to the braces. The integration over the phase-space volume is carried out utilizing Eq. (7) and an additional expression:

$$\mathcal{F}_\bullet\left[\frac{p_\sigma'p_\alpha'p_\beta'}{kp'}\right]=-\frac{\pi}{24(kq)}\frac{(q^2-m^2)^3}{q^4}\times\left(g_{\sigma\alpha}q_\beta+g_{\sigma\beta}q_\alpha+g_{\alpha\beta}q_\sigma-k_\sigma\frac{q_\alpha q_\beta}{kq}\right)+\frac{\pi}{6(kq)}\frac{q^2-m^2}{q^6}(q^4+m^2q^2+m^4)q_\alpha q_\beta q_\sigma+\frac{\pi}{4(kq)^2}\left[m^2\left(-q^2+m^2+\frac{q^2+m^2}{2}\ln\frac{q^2}{m^2}\right)-\frac{(q^2-m^2)^3}{12q^2}\right]k_\sigma g_{\alpha\beta}+\ldots.$$

We omitted those tensor combinations which do not contribute to I_σ.

For linear polarization we obtain:

$$I_\sigma=w_0\sum_{s=1}^{\infty}J_s^2[g_i(u_s)k_\sigma^{(s)}+\tilde g_i(u_s)p_\sigma], \qquad (18)$$

$$g_i(u_s)=\frac{1}{3(1+u_s)^3}(u_s^5+3u_s^4-3u_s^3-22u_s^2-30u_s-12)+\frac{4}{u_s}\ln(1+u_s), \qquad (18a)$$

$$\tilde g_i(u_s)=\frac{u_s}{6(1+u_s)^3}(u_s^4+7u_s^3+22u_s^2+30u_s+12)-2\ln(1+u_s),$$

$$k_\sigma^{(s)}=sk_\sigma. \qquad (18b)$$

Note that in the nonrelativistic approximation

$$g_i\approx{}^4/_5u_s^4,\quad \tilde g_i\approx{}^2/_3u_s^4. \qquad (18c)$$

Plots of the functions $g_i(u_s)$ and $\tilde g_i(u_s)$ are shown in Fig. 2. In the case of circular polarization, when one photon has been emitted

$$I_\sigma^{(1)}={}^1/_2w_0\sin^2(2z)\,[g_i(u_1)k_\sigma+\tilde g_i(u_1)p_\sigma]. \qquad (19)$$

It follows from Eqs. (18) and (19) that in the approximation linear in the wave energy density the intensity likewise does not depend on the state of polarization. In the same approximation, the asymptotic behavior of functions g_i and $\tilde g_i$ yields the same results as in Ref. 3, which were written in noninvariant notation.

4. The limiting case of a constant crossed field F can be obtained if we remove the summation over s in Eq. (9) and introduce the following substitutions (see Ref. 4[1)]):

$$J_s^2\to{}^1/_2,\quad u_s\to4\chi, \qquad (20a)$$

$$\chi=[\mu^2(pF^2p)]^{1/2}/m^2, \qquad (20b)$$

and make in equation (18) by the additional substitution

$$k_\sigma^{(s)}\to\frac{2\mu^2}{m^2\chi}(pF^2)_\sigma. \qquad (20c)$$

In particular, in the ultrarelativistic case $\chi\gg1$ we have:

$$W=4\chi^2w_0, \qquad (21a)$$

$$I_\sigma=\frac{4}{3}\chi^2w_0\left[\frac{4\mu^2}{m^2\chi}(pF^2)_\sigma+p_\sigma\right]. \qquad (21b)$$

If these formulas are applied to a purely magnetic field, then in the ultrarelativistic case the asymptotic behavior, evaluated in Refs. 5 and Ref. 6, does not coincide with (21a) and (21b). In Ref. 5 and Ref. 6 the authors took into account the first nonvanishing term of the expansion in μ for interaction not only with the radiation field, but also with the external field; this is not justified in the ultrarelativistic asymptotic case, where the true expansion parameter is χ. The aforementioned results of Ref. 5 and Ref. 6, correspond in fact, to the Eqs. (9), (9b), and (18), (18c) for the temporal component of I_σ, with (20a) and (20c) taken into account.

The effect under consideration is quite likely to be observed in the interaction of a high-power directed laser beam with a high-intensity neutron beam; this would allow one to draw some conclusions as to the extent of the method's applicability. The invariant technique developed here could also be used in research into the effects of polarization, by introducing the proper density matrix (see Ref. 7) and, in addition, to calculations of the probability of the crossed channels $\gamma\to n\bar n$, $n\bar n\to\gamma$ in a plane-wave field.

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[1)]The statement $J_s^2\to1/4$ in Ref. 4 is inaccurate.

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