Processes induced by a charged particle in an electric field, and the Unruh heat-bath concept

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It is shown that in various quantum processes induced by acceleration of a charged particle by a constant electric field the spectra of the final particles deviate from the Unruh "thermodynamic" distribution. It is emphasized that a substantial role is played by the quantum character of the motion of all the participating particles over distances of the order of the reciprocal acceleration, and by the forces that transform virtual particles into real ones. The deviation of the results from those predicted by the Unruh picture is attributed, in particular, to the fact that the Rindler states used in this picture have singularities on the horizons $z = \pm t$, where the composite Rindler-Milne coordinate grids have discontinuities in the metric. These discontinuities correspond physically to particle sources, so that the Unruh picture describes a different physical system.

1. INTRODUCTION

A charged particle in a constant electromagnetic field moves with constant proper acceleration. This means that in the instantaneously comoving Lorentz system, where the particle is at rest in a given instant of time, the acceleration is independent of this instant. If the field is a constant electric one, and the particle momentum transverse to the field $p_\perp = 0$, the particle's proper acceleration has also a constant direction. This is the so-called hyperbolic or equal-acceleration motion. Emission of a photon by an electron in such a motion was considered in Refs. 1 and 2.

A quantum analysis[2] shows that recoil causes an electron to go over from the state $p_\perp = 0$ into an excited state with $p_\perp' \neq 0$, and the ratio of the differential probabilities is the same in the forward ($p_\perp \to p_\perp'$) and inverse ($p_\perp' \to p_\perp$) transitions is equal to

$$\exp\left[-\frac{2\pi m}{|e\varepsilon|}\left(\frac{p_\perp'^2}{2m}-\frac{p_\perp^2}{2m}\right)\right], \tag{1}$$

i.e., it has a Boltzmann form with excitation temperature $|e\varepsilon|/2\pi m$ if, relativism notwithstanding, the excitation energy is taken to be the difference between the nonrelativistic transverse energies.

Starting with Unruh's paper,[3] it has been customary to assume that a particle moving hyperbolically in a vacuum with an acceleration a in an accelerated (Rindler) coordinate frame, where it is at rest in the classical approximation, actually behaves as in a heat bath with temperature $T = a/2\pi$ located in a uniform gravitational field characterized by the same acceleration a. Such a bath is filled with Rindler photons whose states are characterized by the conserved quantum numbers of the solution of the free wave equation written in Rindler coordinates.

It is stated that, from the standpoint of a Rindler observer moving together with a classical electron, the quantum transition $p_\perp = 0 \to p_\perp' \neq 0$ of an electron can be described as absorption of one of the Rindler photons by which the Minkowski vacuum is represented. Such an electron can thus be regarded as a relativistic Unruh counter.

The Unruh picture is used to simulate, in Minkowski space, particle production by a black hole. The equivalence principle is thereby extended to quantum processes.[4–6] Such processes, however, are formed in a finite space-time region, and the use of the local equivalence principle requires that the field of the accelerations be constant and uniform in this region.

Analysis shows that space-time regions of the order of the inverse acceleration are significant for such processes. Yet in Minkowski space it is impossible to produce a uniform gravitational field with acceleration a in a 4-space region with dimensions of order a^{-1}. Thus, in the Rindler system (which is rigid) the variation of the acceleration is inversely proportional to the spatial coordinate. A system with identical spatial points with proper acceleration (e.g., a system of electrons that move hyperbolically in a constant uniform electric field) is rigid, viz., the distances between its observers vary with time and go through a minimum. Arguments are advanced in Ref. 4 in favor of the premise that the gravitational field of a spherically symmetric body can be made uniform over the process-formation length, by a body of sufficient mass. These arguments, however, are not valid outside the gravitational radius.

All this shows that, besides the Rindler sector $R(z>|t|)$, the remaining sectors $F(t>|z|)$, $P(-t>|z|)$ and $L(-z>|t|)$ should also be subtantial in the Unruh picture.

It is known that a true gravitational field differs significantly from the field to which the accelerated coordinate system is equivalent. There is no doubt that the tidal forces due to the nonzero curvature of space can transform a virtual decay into a real one, in analogy with the ability of an electric field to transfer energy to virtual particles in pair production by an electromagnetic field. Covering the Minkowski space by Rindler and Milne coordinate grids[7]

$$\begin{aligned} &ds^2=dz'^2-(az')^2dt'^2,\ z'=\pm(z^2-t^2)^{1/2},\ t'=a^{-1}\,\mathrm{Arth}(t/z),\\ &ds^2=(at'')^2dz''^2-dt''^2,\ t''=\pm(t^2-z^2)^{1/2},\ z''=a^{-1}\,\mathrm{Arth}(z/t), \end{aligned} \tag{2}$$

leads, without changing the geometry of the space inside the above sectors, to jumps of the metric on the boundaries $z = \pm t$ between them, and to the appearance of forces capable of performing work on the virtual particles and convert them into real ones.

Let us corroborate the foregoing considerations by some estimates and equations. We estimate first the length of

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excitation of radiation from a neutral particle of mass μ by a charged particle of mass $m \gg \mu$ that moves hyperbolically in a constant electric field. If we represent it as consisting of a charged and neutral virtual particle with masses $m_1 = m - \mu$ and μ, acceleration of the entire system by the electric field produces an additional force $f = \mu a = \mu e\varepsilon/m$. The work of this creative disrupting force f over a length l, called the formation length, replenishes the mass defect and transforms the virtual particles into real ones. Consequently $fl \sim \mu$, i.e. $l \sim a^{-1}$. This agrees with the estimate of l obtained in Ref. 1, directly from the expression for the probability, for photon emission by an electron with constant acceleration. The estimate $l \sim a^{-1}$ remains valid also for more complicated processes, in which the accelerations of virtual particles differ little from the acceleration of the initial particle, see Sec. 6.

Let us see now whether a gravitational field described by the Schwarzschild solution

$$g(R) = x^2/(1-x)^{1/2}2R_g, \quad x = R_g/R, \quad R_g = 2GM/c^2,$$

can be regarded as uniform over a length $\sim g^{-1}$. To this end it is necessary that the relative change of g over this length

$$\frac{\Delta g}{g} \approx \frac{g'(R)g^{-1}(R)}{g(R)} = -\frac{4-3x}{x(1-x)^{1/2}}$$

be small. It can be seen that this is impossible outside the gravitational radius $(0<x<1)$. This reasoning shows also why it is impossible to draw from the equivalence principle any conclusion whatever concerning the emission of an electron that falls freely in a gravitational field, e.g., an electron on a satellite. The emission and deceleration of such an electron are determined (formed) by the curvature of space, i.e., by distances much greater than the dimensions of the space-time region in which the equivalence principle is applicable.

Note also that an electron resting on the surface of a cold massive star cannot emit a photon as a result of a decrease of the gravitational field and the insufficient work of the creative force over the formation length of the process, even though in a freely falling system the electron moves with an acceleration due to the electric forces of the surface on which it is located.

As to Rindler states, notwithstanding an opinion most succinctly set forth in Ref. 8, we assume that they cannot be produced in a Minkowski space without sources (of infinite power) on the event horizons. The Rindler solution

$$K_{i\varkappa}(\zeta)\exp(-i\varkappa v + i\mathbf{p}_\perp \mathbf{x}_\perp), \quad v = at' = \operatorname{Arth}\frac{t}{z}, \quad \zeta = m_\perp z' = m_\perp (z^2 - t^2)^{1/2}, \tag{3}$$

which depends on the Rindler coordinate z' and on the Rindler time t', and is characterized by a transverse momentum $\mathbf{p}_\perp$, $m_\perp = (m^2 + p_\perp^2)^{1/2}$ and by a Rindler frequency $\omega = \varkappa a$, is an analytic function of the variables $x_+ = t + z$, $x_- = t - z$ with branch points $x_\pm = 0$. It can therefore be extended from the sector R to all of Minkowski space by analytic continuation from the semiaxes $x_- < 0$, $x_+ > 0$ to the semiaxes $x_- > 0$, $x_+ < 0$ through the lower or upper complex $x_\pm$ planes. Depending on whether the continuation is down or up, positive- or negative-frequency solutions are produced, which coincide in sector R and differ in the remaining sectors. They can be represented by integrals of positive-frequency or negative-frequency plane waves over the rapidity θ:

$$K_{i\varkappa}(\zeta)\exp(-i\varkappa v) = \frac{1}{2}\exp\left(\mp\frac{\pi\varkappa}{2}\right)\int_{-\infty}^{\infty} d\theta \exp[i(p_3 z - p_0 t) \mp i\varkappa\theta], \quad p_0 = \pm m_\perp \operatorname{ch}\theta, \quad p_3 = m_\perp \operatorname{sh}\theta. \tag{4}$$

These integrals demonstrate clearly the analyticity in $x_\pm$, if it is recognized that $p_3 z - p_0 t = -(p_+ x_- + p_- x_+)/2$ and the signs of $p_\pm = p_0 \pm p_3$ coincide with the sign of p_0.

Corresponding to a positive (negative)-frequency solution is a positive (negative) total charge, concentrated at $\varkappa > 0$ mainly in the sector $R(L)$, where the charge density is positive (negative). As the $z = \pm t$ is approached, the charge density executes oscillations of infinitely increasing frequency and amplitude, which acquire alternating signs on passing over to the right (left)-hand halves of the sectors F and P. In the left (right) half of the sectors R and P the charge density again becomes positive (negative) and on approaching the planes $z = \pm t$ it executes oscillations of infinitely increasing frequency and amplitude. On going over to the sector $L(R)$ the density becomes negative (positive) and becomes smaller by a factor $\exp(-2\pi\varkappa)$ than at the points of sector $R(L)$ with the same $|z|$. The reversal of the sign of $\varkappa$ is equivalent to the reflection $z \to -z$ and preserves the described picture apart from an $R \rightleftarrows L$ permutation and a right $\rightleftarrows$ left reversal.

Note that a Rindler solution itself is bounded near the horizons $z = \pm t$ and consists of a continuous and rapidly oscillating term. The oscillation frequency of the latter increases without limit as the horizon is approached, and the amplitude has a discontinuity on passing through the horizon and acquires a factor $\exp(-\pi\varkappa)$. This factor is indicative of particle-pair production near the horizon.

A similar property is possessed by the solutions of the Klein-Gordon equation for a particle in a constant electric field ε, which are characterized by a conserved quantum number p_-, see Ref. 9. In fact, pairs with fixed p_- are created near the event horizon $x_- = p_-/e\varepsilon$, and the amplitude of their creation is given by the same factor if the Rindler energy ω and the acceleration a are taken to mean respectively the quantities $m_\perp^2/2m$ and $|e\varepsilon|/m$ in an electric field.

The described properties of the positive- and negative-frequency solutions give grounds for assuming that these solutions are the limit of the solutions of the Beltrami-Laplace equation

$$(-g)^{-1/2}\partial_\alpha((-g)^{1/2}g^{\alpha\beta}\partial_\beta\Phi) = 0 \tag{5}$$

with a smooth metric $g^{\alpha\beta}$ that differs from the Rindler metric in the sectors R and L and from the Milne metric in the sectors F and P only in the narrow transition region, whose width tends to zero, near the planes $z = \pm t$. The nonzero curvature of the space in the transition leads to particle creation that takes place also in the limit of zero width of the transition region.

If the metric (2) is smoothened in the region $|z^2 - t^2| \lesssim \beta^{-2} \ll a^{-2}$ with the aid of the continuous function $f(x)$, $x = \beta|z^2 - t^2|^{1/2}$, which tends to 0 and 1 as $x \to 0$ and

$x \to \infty$, respectively, by choosing $g^R_{33} = f(\beta z')$ and $g^F_{00} = -f(\beta t')$ in lieu of $g^R_{33} = -g^F_{00} = 1$, we obtain for the curvature-tensor component in the sectors R and F

$$R_{3030} = \mp {}^1/_2 a^2 x \frac{\partial}{\partial x} \ln f(x),$$

which yields, e.g. for $f(x) = \exp(-x^{-2})$, the value $R_{3030} = a^2/\beta^2(t^2 - z^2)$. The curvature in the region $x \lesssim 1$ is $\sim a^2$ and on the planes $z = \pm t$ it reverses sign jumpwise.

Additional evidence favoring particle sources, for example on the $z = t$ plane, is the inequality of the fluxes $\int j_\alpha dS^\alpha$ through the hypersurfaces S_R and S_F that encompass the singled-out part of the hyperplane $z = t$ in the sectors R and F, respectively. An essential role is played thus in the Unruh picture by the particle sources.

We consider in the present paper the emission and decay of a charged particle in a constant electromagnetic field (electric or crossed), and explain the onset of forces whose work over the formation length makes the products of the virtual decay real. The probability of the process in a weak field, when the sum of the masses of the final particles exceeds the mass of the initial particles, is described by an exponential function but is not, as a rule, with the Unruh exponent. The extent to which the Unruh-temperature concept is general and useful is therefore unclear to us.

2. CONVERSION OF A CHARGED PARTICLE IN AN ELECTRIC FIELD INTO A CHARGED AND A NEUTRAL PARTICLE PAIR

Assuming the electric field ε to be directed along the 3 axis and using the same procedure as in Refs. 2 and 10, we write down the matrix element of the conversion of a scalar particle having a charge e, a mass m, and a momentum[1)] $\mathbf{p}$ into a scalar particle with charge e, mass m', and momentum $\mathbf{p}'$, and a scalar neutral particle with mass μ, momentum $\mathbf{k}$, and energy k_0

$$M = f \int d^4x\, {}_+\Phi_{p'}^*(x) \frac{e^{-ikx}}{(2k_0)^{1/2}}\, {}^+\Phi_p(x) N_{p'} N_p. \tag{6}$$

Here

$$\begin{gathered} {}^+\Phi_p(x) = B_p \exp(i\mathbf{px}) D_{-i\nu-1/2}(\exp(i\pi/4)t), \\ t = (2|e\varepsilon|)^{1/2}(x_0 + p_3/e\varepsilon), \\ B_p = (2|e\varepsilon|)^{-1/4}\exp(-\pi\nu/4), \\ N_p = -(2\pi)^{-1/2}\exp(-i\pi/4 + \pi\nu/2)\Gamma(1/2 + i\nu) \end{gathered} \tag{7}$$

is the wave function of a charged particle having a positive frequency at $+\infty$ for the time x_0, $D_\lambda(z)$ is a parabolic-cylinder function, and $p \equiv p_1, p_2, p_3$. The function $+{}^\Phi p(x)$, with positive frequency as $x_0 \to -\infty$, differs from (7) by the complex conjugation of the D-function and by the reversal of the sign of its argument. We write down also the parameters

$$\nu = \frac{m^2 + p_\perp^2}{2|e\varepsilon|}, \quad \nu' = \frac{m'^2 + p_\perp'^2}{2|e\varepsilon|}, \quad \rho = \frac{\mu^2 + k_\perp^2}{2|e\varepsilon|}, \tag{8}$$

which determine both the matrix element and the differential probability in terms of the conserved components, perpendicular to the field, of the kinetic momenta of the particles.

Calculation of the probability leads to the expression

$$\int \frac{|M|^2}{V} \frac{d^3p'\, d^3k}{(2\pi)^6} = \tau \int_0^{2\pi}\int_0^\infty \frac{d\varphi\, dk_\perp k_\perp}{(2\pi)^3} \frac{\pi f^2 e^{\pi\nu - 2\pi\nu'} |\Psi|^2}{4m|e\varepsilon|(1+e^{-2\pi\nu})(1+e^{-2\pi\nu'})}, \tag{9}$$

where $\Psi \equiv \Psi(\frac{1}{2} + i\nu, 1 + i(\nu - \nu'); -i\rho)$ is a confluent hypergeometric function, ϑ is the angle between the vectors $\mathbf{k}_\perp$ and $\mathbf{p}_\perp$, and $\mathbf{p}'_\perp = \mathbf{p}_\perp - \mathbf{k}_\perp$. The probability is proportional to the proper time τ of motion of the initial particle in the electric field, so that the integral with respect to ϑ and $k_\perp$ in (9) and its integrand are respectively the total and differential probabilities of the process per unit proper time of the initial particle. We denote them respectively by W and w. The total probability is determined by four independent invariants:

$$\beta = \frac{|e\varepsilon|}{m^2}, \quad \nu = \frac{m^2 + p_\perp^2}{2|e\varepsilon|}, \quad \frac{\mu}{m}, \quad \frac{m'}{m}. \tag{10}$$

Using the known relation

$$\Psi(a, c; x) = x^{1-c}\Psi(a - c + 1, 2 - c; x) \tag{11}$$

(see Eq. 6.5(6) in Ref. 11), we can relate the differential probabilities of the direct and inverse processes (which differ by permutation of the charged particles), by means of the equation

$$\frac{w_{p\to p'+k}}{w_{p'\to p-k}} = \frac{m'}{m}\exp\left(\pi\frac{m^2 - m'^2 + p_\perp^2 - p_\perp'^2}{|e\varepsilon|}\right), \tag{12}$$

which generalizes the result obtained by one of us[2,10] to include the case $m = m'$. Introducing the mass defect $\Delta = m' - m$ and the average mass $m = \frac{1}{2}(m + m')$ of the charged particles, the right-hand side of (12) takes the Boltzmann form

$$\frac{m'}{m}\exp\left[-\left(\frac{|e\varepsilon|}{2\pi\overline{m}}\right)^{-1}\left(\Delta + \frac{p_\perp'^2}{2\overline{m}} - \frac{p_\perp^2}{2\overline{m}}\right)\right] \tag{13}$$

with the parameter $|e\varepsilon|/2\pi\overline{m}$ serving as the temperature. In our opinion, this circumstance must not be taken to have a profound physical meaning, all the more since the agreement with the Boltzmann exponent is inexact: relativism notwithstanding, the excitation energy contains the difference of the transverse kinetic equation in a nonrelativistic form, and furthermore with an average mass.

Note that the existence of both the direct and inverse processes is due to the possibility of pair production by the field. If the inverse process can take place also in the absence of a field ($m' > m + \mu$), its probability is hardly changed by a weak field. Relation (12) shows then that in essence the entire dependence of the direct process on the field is determined by the exponential function above.

Note that the differential probability of a process that proceeds also in the absence of a field can vanish in the presence of a field at certain values of the quantum numbers of the final particles, i.e., it is an oscillating function of the quantum numbers and of the field. In a weak field, the probability averaged over the fast oscillations equals the probability when the field is turned off.

In the general case, the differential distribution in (9) has nothing in common with the temperature distribution. We shall therefore consider it below for a positive binding energy $I = m' + \mu - m > 0$ and a weak electric field, when $\nu, \nu' \gg 1$. In this case the process is driven by energy absorption from the field, and its probability is qualitatively determined by an exponential function with a large negative exponent, in which we shall in fact be interested.

2.1. Equal masses of charged particles

Putting $m = m'$ in (9), we put also $p_\perp = 0$, which corresponds to uniformly accelerated notion of the initial particle. We have then $\mathbf{p}'_\perp = -\mathbf{k}_\perp$ and $\nu - \nu' = -k_\perp^2|e\varepsilon|$. In the effective region, we have $k_\perp^2$ of the order of $2|e\varepsilon|$ or less, i.e., a difference $\nu' - \nu \lesssim 1$, whereas $\nu, \nu' \gg 1$ and the effective value of ρ depends on the relation between the parameters $\beta \equiv |e\varepsilon|m^{-2}$ and μ/m, which determine the total probability.

We consider first the classical case, when

$$\beta \equiv |e\varepsilon|/m^2 \ll 1, \quad \mu/m \ll 1, \quad (\mu^2+k_\perp^2)^{1/2} - \mu \sim \beta m \ll m, \quad (14)$$

i.e., when the energy lost by the charge in emission of a neutral particle is small compared with its characteristic energy and momentum change in the region where the radiation is formed (cf. Eqs. (2) and (3) of Ref. 12). Then

$$\nu' - \nu = k_\perp^2/2|e\varepsilon| \sim \mu/m + {}^1/_2\beta \ll 1. \quad (15)$$

The conditions (14) and (15) allow us to use the equation

$$\Psi\left(a, b; \frac{x}{a}\right) \approx \frac{2x^{(1-b)/2}}{\Gamma(1+a-b)} K_{b-1}(2x^{1/2}), \quad a \to \infty, \quad (16)$$

from Ref. 11 and replace Ψ by a Macdonald function (modified Bessel function of the second kind):

$$\Psi\left(\frac{1}{2} + i\nu,\ 1+i(\nu-\nu');\ -i\rho\right) \approx \frac{2K_0(z)}{\Gamma({}^1/_2+i\nu')},$$
$$z = \frac{m(\mu^2+k_\perp^2)^{1/2}}{|e\varepsilon|}. \quad (17)$$

Then, putting $x = m\mu/|e\varepsilon|$, we get from (9)

$$W = \frac{f^2\beta}{8\pi^2 m}\int_x^\infty dz\, zK_0^2(z) = \frac{f^2\beta}{16\pi^2 m} x^2[K_1^2(x) - K_0^2(x)]. \quad (18)$$

This equation agrees with the result of Ref. 12, where the probabilities of the emission of scalar and vector particles by a uniformly accelerated charge were obtained.

The differential and total probabilities are exponential only if $x \gg 1$:

$$W = \frac{f^2\beta}{16\pi m}\int_x^\infty dz\, e^{-2z} = \frac{f^2\beta}{32\pi m} e^{-2x}. \quad (19)$$

Although they have a Boltzmann form with respective excitation energies $(\mu^2 + k_\perp^2)^{1/2}$, and μ, the effective temperature is π times larger than the Davies-Unruh temperature $|e\varepsilon|2\pi m$. Note also that the values $z \sim \max(1, x)$ are significant in the integrals of (18) and (19), i.e., the recoil energy does indeed meet the condition (14), and the motion of the charge is classical.

If $\beta \ll 1$, and μ/m is not small, the motion of the charge is not classical. In this case the values $k_\perp^2 \sim |e\varepsilon|$ are effective. The parameters in the function Ψ are of the following order of magnitude:

$$\nu \sim \beta^{-1} \gg 1, \quad -\nu+\nu' \sim 1, \quad \rho \sim \nu \gg 1. \quad (20)$$

An asymptotic expression for the confluent hypergeometric function in this region is given in Sec. 8 of the book by Buchholz.[13] Assuming

$$\rho = 2(\nu+\nu')\cos^2\alpha \quad \text{and} \quad \rho = 2(\nu+\nu')\operatorname{ch}^2\alpha \quad (21)$$

corresponding to the cases $(\mu/2m) < 1$ and $(\mu/2m) > 1$, we obtain for the first of them

$$\exp(\pi\nu - 2\pi\nu')\,|\Psi({}^1/_2+i\nu,\ 1+i(\nu-\nu');\ -i\rho)|^2$$
$$\approx \exp[-2\pi\nu' + (\nu+\nu')\delta]/\rho\operatorname{tg}\alpha, \quad (22)$$

where $\delta = 2\alpha - \sin 2\alpha$. Expanding the argument of the exponential function in (22) in powers of $k_\perp^2$ we get

$$W = \frac{f^2}{16\pi m\mu^2\operatorname{tg}\alpha_0}\int_0^\infty dk_\perp^2$$
$$\times\exp\left[-\frac{2m^2}{|e\varepsilon|}[\arcsin\xi + \xi(1-\xi^2)^{1/2}] - \left(\pi - \alpha_0 + \frac{1}{2}\operatorname{tg}\alpha_0\right)\frac{k_\perp^2}{|e\varepsilon|}\right]$$
$$= \frac{f_m^2\beta m\exp\{-(2m^2/|e\varepsilon|)[\arcsin\xi + \xi(1-\xi^2)^{1/2}]\}}{8\pi\mu^2\operatorname{tg}\alpha_0(2\pi - 2\alpha_0 + \operatorname{tg}\alpha_0)},$$
$$\xi = \cos\alpha_0 = \frac{\mu}{2m}. \quad (23)$$

If, however, $(\mu/2m) > 1$, we get in place of (23)

$$W = \frac{f^2}{16\pi m\mu^2\operatorname{th}\alpha_0}\int_0^\infty dk_\perp^2\exp\left(-\pi\frac{m^2+k_\perp^2}{|e\varepsilon|}\right)$$
$$= \frac{f^2 m\beta}{16\pi^2\mu^2\operatorname{th}\alpha_0}\exp\left(-\frac{\pi m^2}{|e\varepsilon|}\right), \quad \operatorname{ch}\alpha_0 = \frac{\mu}{2m}. \quad (24)$$

For $\mu/2m$ very close to unity, it is necessary to make in the "dangerous" places of the pre-exponential factors of Eqs. (23) and (24) the substitution

$$\operatorname{tg}\alpha_0, \quad \operatorname{th}\alpha_0 \to \pi\Gamma^{-2}({}^1/_3)(3\beta)^{1/3}, \quad (25)$$

and put in the remaining ones $\alpha_0 = 0$. As a result, these equations are continuously transformed into one another when the parameter $\mu/2m$ is changed. As seen from (23) and (24), the values $k_\perp^2 \sim |e\varepsilon|$ are significant, as assumed above.

Note also that if $\beta \ll \mu/2m \ll 1$, then $\cos\alpha$ becomes a small parameter. In this case $\delta \approx \pi - 4\cos\alpha$ and we obtain Eq. (19) for the probability. The classical approximation (14), (17) and the approximation (20), (22) are different particular cases of the semiclassical approximation, and are contiguous in the region

$$\beta \ll \mu/2m \ll 1.$$

Thus, the argument of the exponential function in both the total and the differential probabilities depends on the mass μ of the neutral particle only in the region $0 < \mu < 2m$ and ceases to depend on μ at $\mu > 2m$, where it coincides with the exponent of the differential probability of pair production by an electric field. The emission of a neutral particle with mass $\mu > 2m$ proceeds in two stages: the field creates first a pair with probability $\sim\exp(-\pi m^2/|e\varepsilon|)$, and next the initial charged particle annihilates with one of the pair particles into a neutral particle with a nonexponentially low probability, so that the two annihilating particles are real and can be accelerated by the field towards each other to an energy sufficient to overcome the neutral-particle creation threshold.

On the other hand, emission of a neutral particle with mass $\mu < 2m$ proceeds via annihilation of an initial particle by an oppositely charged particle of virtual pair that has acquired from the field a kinetic energy $+\mu/2$ (if it is regarded as a hole, it is located on a level $-\mu/2$). The energy $m+\mu/2$ released thereby goes over into the energy $m-\mu/2$ needed for conversion of a virtual particle having the same charge (and located on a level $+\mu/2$) into a real particle, and into the energy μ needed for production of a neutral particle:

$$m+\mu/2=(m-\mu/2)+\mu.$$

The probability of this annihilation is $\sim f^2m^{-2}$, so that the total probability is determined by multiplying f^2m^{-2} by the probability of production, by the field, of a virtual pair of particles with kinetic energies $\mu/2$ (particle and hole on levels $\pm\mu/2$). The latter probability can be calculated semiclassically, as was done in § 129 of Ref. 14, the only difference being that the integration over the coordinate z must be carried out in the interval (z_1, z_2), where according to the single-particle theory the kinetic energy $\pm(m^2+p_3^2(z))^{1/2}=p_0+e\varepsilon z$ changes from a value $-\mu/2$ to a value $+\mu/2$. We have then for the exponent of the sought probability

$$-2\int_{z_1}^{z_2} dz|p_3(z)|=-2\int_{z_1}^{z_2} dz[m^2-(p_0+e\varepsilon z)^2]^{1/2}$$

$$=-\frac{2m^2}{|e\varepsilon|}[\arcsin\xi+\xi(1-\xi^2)^{1/2}],\quad \xi=\frac{\mu}{2m},\tag{25}$$

i.e., the same expression as contained in the probability (23).

2.2. Mass difference of charged particles small compared with their masses

Let now $0<\Delta\equiv m'-m\ll m, m'$. Assuming a weak field, we put

$$\nu,\ \nu'\gg\nu'-\nu\sim m\Delta/|e\varepsilon|\gg1.\tag{27}$$

In this case we can again use Eq. (16), but the index of the Macdonald function is no longer small. We get thus for the probability

$$W=\int_0^{2\pi}\int_0^{\infty}\frac{d\varphi\, dk_\perp k_\perp}{(2\pi)^3}\frac{f^2e^{\pi(\nu-\nu')}}{2m|e\varepsilon|}K^2_{i(\nu-\nu')}(z),\tag{28}$$

$$z=m_\perp\mu_\perp/|e\varepsilon|,\quad m_\perp=(m^2+p_\perp^2)^{1/2},\quad \mu_\perp=(\mu^2+k_\perp^2)^{1/2}.$$

We assume next for simplicity that $p_\perp=0$, so that the differential probability does not depend on ϑ, but depends only on $k_\perp$ ($\mathbf{p}'_\perp=-\mathbf{k}_\perp$) via the quantities

$$\nu-\nu'=-\left(\frac{m\Delta}{|e\varepsilon|}+\frac{\Delta^2+k_\perp^2}{2|e\varepsilon|}\right)\equiv -s,\quad z=\frac{m(\mu^2+k_\perp^2)^{1/2}}{|e\varepsilon|}.\tag{29}$$

The Langer asymptotic representation[15] of a Macdonald function with large index and argument

$$K_{-is}(z)\approx e^{-\pi s/2}\left(\frac{4u}{z^2-s^2}\right)^{1/4}\Phi(u),\quad s\sim z\gg1,\tag{30}$$

expresses it in terms of the Airy function

$$\Phi(u)=\int_0^\infty dt\cos\left(ut+\frac{1}{3}t^3\right)\tag{31}$$

with a real argument

$$u=\left(\frac{s}{2}\right)^{2/3}\frac{w^2}{k(w)},\quad w^2=\frac{z^2}{s^2}-1,$$

$$k(w)=\left[\frac{w^3}{3(w-\operatorname{arctg} w)}\right]^{2/3},\tag{32}$$

the sign of which coincides with the sign of ω^2, $-1\leqslant\omega^2<\infty$.

It is known that the Airy function becomes exponentially small only for $u>1$. It follows therefore from (28) and (30) that the differential probability is $\sim\exp(-2\pi s)$ at $u\lesssim1$ and $\sim\exp[-2s(\pi+\omega-\arctan\omega)]$ at $u\gg1$. If $\Delta^2/|e\varepsilon|$ is assumed small and the parameter $\omega_0^2=(\mu^2-\Delta^2)/\Delta^2$ is introduced, the exponent of the total probability is equal to

$$-\frac{2\pi m\Delta}{|e\varepsilon|}\quad\text{for}\quad -1\leqslant w_0^2\lesssim\left(\frac{|e\varepsilon|}{m\Delta}\right)^{2/3}\ll1,\tag{33}$$

$$-\frac{2\pi m\Delta}{|e\varepsilon|}-\frac{2m\Delta}{|e\varepsilon|}(w_0-\operatorname{arctg} w_0)\quad\text{for}\quad\left(\frac{|e\varepsilon|}{m\Delta}\right)^{2/3}\ll w_0^2.\tag{34}$$

It can be seen that the exponent agrees with the Davies-Unruh exponent $-2\pi m(\Delta+\mu)/|e\varepsilon|$ only if $\mu\ll\Delta$. If $\mu\sim\Delta$ its dependence on μ and Δ is essentially nonlinear, and although it becomes linear in Δ and μ for $\mu\gg\Delta$,

$$-\pi m\Delta/|e\varepsilon|-2m\mu/|e\varepsilon|,\tag{35}$$

the "temperatures" are not equal to the Davies-Unruh temperature; see also Ref. 16.

3. SPLITTING OF A CHARGED PARTICLE IN AN ELECTRIC FIELD INTO TWO PARTICLES OF LIKE CHARGE

We denote the charge and mass of the initial particle by e and m, and of the final particles by e', m' and e'', m''. According to Refs. 2 and 10, the matrix element can be expressed in the form

$$M=f\int d^4x\,N_{p''}N_{p'}N_{p+}\Phi_{p'}{}^*(x)_+\Phi^*_{p''}(x)^+\Phi_p(x).\tag{36}$$

In this matrix element, which is more complicated than (6), it is more convenient to use the eigenfunctions of the conserved operators Π_1, Π_2, $P_-=\Pi_-+e\varepsilon x_-$ with eigenvalues p_1, p_2, p_- designated in (36) by the single symbol p. Such symbols have a semiclassical form and are treated in detail in Ref. 9.

Calculation of the $e\to e'+e''$ splitting probability per unit proper time of the initial particle leads to the following result:

$$W=\frac{f^2}{8(2\pi)^3m}\times\int\frac{dp_1'\,dp_2'\,|Q|^2}{|e'\varepsilon|[1+\exp(-2\pi\nu')][1+\exp(-2\pi\nu'')]},$$

$$Q=AF\left(\frac{1}{2}+i\nu',\ \frac{1}{2}+i(\nu'+\nu''-\nu);\ 1+i(\nu'+\nu'');\ \frac{e}{e'}+i\delta\right), \tag{37}$$

$$|A|^2=\frac{2\pi[1-\exp(-2\pi(\nu'+\nu''))]}{(\nu'+\nu'')[1+\exp(-2\pi(\nu'+\nu''-\nu))][1+\exp(-2\pi\nu)]}$$

if the integration with respect to p'_-, which led to the proper time of the initial particle, was carried out in the interval

$$p_-'<(e'/e)p_-, \tag{38}$$

and to

$$W=\frac{f^2}{8(2\pi)^3m}\times\int\frac{dp_1'\,dp_2'\,|\tilde{Q}|^2}{|e''\varepsilon|[1+\exp(-2\pi\nu')][1+\exp(-2\pi\nu'')]}$$

$$\tilde{Q}=AF\left(\frac{1}{2}+i\nu'',\ \frac{1}{2}+i(\nu'+\nu''-\nu);\ 1+i(\nu'+\nu'');\ \frac{e}{e''}+i\delta\right), \tag{39}$$

if the integration with respect to p'_- was carried out in the interval

$$(e'/e)p_-<p_-'. \tag{40}$$

The conservation laws $\mathbf{p}_\perp=\mathbf{p}_\perp'+\mathbf{p}_\perp''$ and $p_-=p'_-+p''_-$ hold. In (37) and (39), $F(a,b;c;z)$ is a hypergeometric function; its argument z takes on values e/e' and e/e'' larger than unity on the upper edge of the cut $1\leqslant z<\infty$, $\delta\to+0$. The parameters ν' and ν'' differ from the value of ν defined by Eq. (8) in that e, m, $p_\perp$ are replaced be e', m' $p_\perp{}'$ or by e'', m'', $p_\perp{}''$.

Thus, the distribution of the final particles over the perpendicular momenta and the decay rate depend on which of the two intervals (38) and (49) the number $p_-{}'$ is located, i.e., there are two modes of the reaction $e\to e'+e''$. The distributions corresponding to them differ by permutation of the final particles: $e'\rightleftarrows e''$, $\nu'\rightleftarrows\nu''$.

The two-mode character of the splitting of particles into similarly charged particles will be the subject of a separate paper. Here we are interested in the form of the dependence of the differential or total probability on the rather weak field ε. To this end we must investigate the asymptotic behavior, at $\nu,\nu',\nu''\gg1$, of the integral

$$Q=\int_0^\infty d\xi\frac{e^{if(\xi)}}{[\xi(\xi+1)(\xi-e''/e')]^{1/2}},$$

$$f(\xi)=\nu\ln\xi-\nu'\ln\left(\xi-\frac{e''}{e'}\right)-\nu''\ln(\xi+1) \tag{41}$$

and of the integral $\tilde{Q}$ which differs from Q by the permutations $e'\rightleftarrows e''$, $\nu'\rightleftarrows\nu''$.

3.1. Weakly differing charge accelerations

Consider the particular case when

$$e'/m'=e''/m''=e/m(1+\delta),\quad \delta\ll1, \tag{42}$$

i.e., the charge accelerations differ little and the binding energy $I=m'+m''-m=m\delta\ll m$ is low. Using for the calculation of the integral Q the saddle-point method and the smallness of certain linear combinations of the parameters ν, ν', and ν'' compared with the parameters themselves:

$$\nu'+\nu''-\nu\sim\nu-e\nu'/e'\sim\nu-e\nu''/e''\sim\delta\nu,\quad\delta\ll1, \tag{43}$$

it can be shown that the real part of the function if (ξ) at the saddle point $\xi=\xi_1$ is equal to

$$\mathrm{Re}\,if(\xi_1)=-\pi(\nu'+\nu''-\nu)/2+\dots\,. \tag{44}$$

The three dots denote terms $\sim\delta^{\frac{1}{2}}$ compared to those written out. The argument of the exponential function that determines the differential probability is then

$$-\pi(\nu'+\nu''-\nu)=-\frac{\pi m}{|e\varepsilon|}\left[I+\frac{p_\perp'^2}{2m'}+\frac{p_\perp''^2}{2m''}-\frac{p_\perp^2}{2m}+\delta\left(\frac{1}{2}I+\frac{p_\perp'^2}{2m'}+\frac{p_\perp''^2}{2m''}\right)\right], \tag{45}$$

where the term $\sim\delta$ can be neglected and the exponent of the total probability is $-\pi mI/|e\varepsilon|$. Consequently, under the chosen conditions (42), which make the reaction $e\to e'+e''$ an ideal detector of uniformly accelerated motion, the effective excitation temperature turns out to be double the Davies-Unruh temperature. Obviously, the exponent of the second mode, defined by the integral $\tilde{Q}$, coincides in this approximation with (45).

3.2. Noticeably different charge accelerations

We consider now a second particular case, when $e'=e''=e/2$, $m''=\alpha m'$, $m=(1+\alpha-\delta)m'$. Since

$$\frac{e'}{m'}=\alpha\frac{e''}{m''}=\frac{1+\alpha-\delta}{2}\frac{e}{m}, \tag{46}$$

the charge accelerations differ substantially when α differs noticeably from unity and $\delta\ll1$, and the binding energy is low: $I\equiv m'+m''-m=\delta m'\ll m',\ m'',m$. Without loss of generality, it can be assumed that $\alpha>1$, so that the largest acceleration is that of the charge e'. Since the field is assumed weak, it follows that ν, ν', $\nu''\gg1$ and that the integral Q, in which now

$$f(\xi)=\nu\ln\xi-\nu'\ln(\xi-1)-\nu''\ln(\xi+1), \tag{47}$$

can again be evaluated by the saddle-point method.

The value of the function 2 Re if (ξ) at the saddle point $\xi=\xi_1$, which is the exponent of the differential probability, is in the lowest approximation in δ:

$$2\,\mathrm{Re}\,if(\xi_1)=-\frac{4[2\alpha(\alpha+1)]^{1/2}}{3(\alpha-1)\beta'}\Delta^{3/2}+\dots,\quad \Delta=\delta+\frac{1+\alpha}{2\alpha}\frac{p_\perp'^2}{m'^2}+\dots,\quad \beta'=\frac{|e'\varepsilon|}{m'^2}. \tag{48}$$

We have put here for simplicity $\mathbf{p}_\perp=0$, so that $\mathbf{p}_\perp'=-\mathbf{p}_\perp''$. The exponent of the total probability is obtained from (48) by putting $p_\perp'=0$. At $\alpha\gg1$ it coincides with the atom-ionization exponent:

$$-2(2\delta)^{1/2}/3\beta' = -2\varepsilon_0/3|\varepsilon|, \qquad (49)$$

where $\varepsilon_0 = (2m'I)^{3/2}/|e'|m'$ is the characteristic atomic field (see §77 in Ref. 17). This is as it should be, for in this case m, $m'' \gg m'$. It is difficult to compare the probability exponents with temperature ones, since they are nonlinear in the excitation energy $\Delta m'$. If the comparison is nevertheless made, we get for the temperature

$$\frac{3(\alpha-1)}{8}\left(\frac{\alpha+1}{2\alpha\delta}\right)^{1/2}\frac{|e\varepsilon|}{m},$$

which is much higher than the Davies-Unruh temperature since δ is small.

Note that the second mode defined by the integral Q is much less probable and has an exponent equal to

$$2\,\mathrm{Re}\, if(\xi_1) = 2\pi(\nu-\nu'-\nu'') + \dots$$
$$= -\frac{\pi}{2\beta'}\left[(\alpha-1)^2 + 4\frac{p_\perp'^2}{m'^2}\right] + \dots . \qquad (50)$$

Assuming $p_\perp' = 0$ we obtain the exponent $-\pi(\alpha-1)^2/2\beta'$ of the total probability of the second splitting mode.

4. EMISSION OF A NEUTRINO PAIR BY AN ELECTRON IN AN ELECTRIC FIELD

Since the lightest charged particle is the electron, the most realistic examples of the reactions considered in Sec. 2 are $e \to e + \gamma$ and $e \to e + \nu + \tilde{\nu}$. The first was already accounted for in Ref. 2. We shall therefore dwell here on the second, which is of interest also because the neutrino pair $\nu + \tilde{\nu}$ can be regarded in it as a neutral particle with mass $\mu = [-(k' + k'')^2]^{1/2}$ (k', k'' are the 4-momenta of ν and $\tilde{\nu}$), having a continuous spectrum of values $0 < \mu < \infty$ and playing the role of excitation energy. The probability is then an integral with respect to μ^2 of an excitation spectrum in the form of the right hand side of Eq. (9), with corresponding complications due to the electron and neutrino spins and to the nonscalar character of their interaction.

In fact, starting with the matrix element

$$M = \frac{G}{2^{1/2}}\int d^4x\left[\left(2\xi + \frac{1}{2}\right)\bar{e}'\gamma_\alpha e + \frac{1}{2}\bar{e}'\gamma_\alpha\gamma_5 e\right]\bar{\nu}_1\gamma_\alpha(1+\gamma_5)\nu_2, \qquad (51)$$

in which ν_1, ν_2, e, and e' are the wave functions of the neutrino, antineutrino, and initial and final electrons in the field, and $\xi = \sin^2\theta_W$ is the Weinberg parameter (see pp. 130 and 183 of Okun's book[18]), we obtain for the probability of emission of a neutrino pair by an electron per unit of its proper time

$$W = \int_0^\infty d\mu^2 \int \frac{dp_1'\,dp_2'\,G^2 e^{\pi\nu - 2\pi\nu'}}{6(2\pi)^4 m\beta\nu\nu'(1-e^{-2\pi\nu})(1-e^{-2\pi\nu'})}$$
$$\left\{\left(\xi^2 + \frac{1}{2}\xi + \frac{1}{8}\right)\mu^2 X\right.$$
$$\left. + \left(\xi + \frac{1}{4}\right)\mu^2 Y + \frac{1}{4}Z\right\}, \qquad (52)$$

$$X = 2\frac{p_\perp^2 + p_\perp'^2}{m^2}\nu|\Psi|^2$$
$$-4\frac{(\mathbf{p}_\perp\mathbf{p}_\perp' + m^2)(\nu-\nu') + p_\perp^2\rho}{m^2}\,\mathrm{Re}\,\Psi\Psi'^*$$
$$+4\frac{\mathbf{p}_\perp\mathbf{p}_\perp' + m^2 + |e\varepsilon|\rho}{m^2}\rho|\Psi'|^2, \qquad (53)$$

$$Y = 2\frac{e\varepsilon[\mathbf{p}'\mathbf{p}]}{|e\varepsilon|m^2}(\nu-\nu')\,\mathrm{Im}\,\Psi\Psi'^*, \qquad (54)$$

$$Z = (2\mu^2 + k_\perp^2)\nu|\Psi|^2$$
$$+[(\mu^2 - k_\perp^2)(\nu-\nu') - (2\mu^2 + k_\perp^2)\rho]\,\mathrm{Re}\,\Psi\Psi'^*$$
$$+(k_\perp^2 - \mu^2)\rho|\Psi'|^2. \qquad (55)$$

Here $\Psi = \Psi(i\nu, 1 + i(\nu - \nu'); -i\rho)$ is a confluent hypergeometric function, Ψ' is its derivative with respect to $-i\rho$, and $\mathbf{k} = \mathbf{p} - \mathbf{p}'$ is the momentum of the neutrino pair. On the basis of Eq. (11) we can prove again relation (12) between the differential probabilities of processes that differ by permutations of the states of the initial and final electrons.

In the weak-field region, when ν and $\nu' \gg 1$ and Ψ and Ψ' assume exponential forms, the argument of the exponential function that determines the differential probability does not differ from the one obtained in Sec. 2.1. We confine ourselves therefore to a region that is classical with respect to the electron motion: $\nu \gg 1$, $\nu - \nu' \sim 1$, $\nu\rho \sim 1$. Using (16), we obtain

$$W = \int_0^\infty d\mu^2 \int \frac{dp_1'\,dp_2'\,G^2 e^{\pi(\nu-\nu')}}{3\pi(2\pi)^4 m\beta}$$
$$\times\left\{4\left(\xi^2 + \frac{1}{2}\xi + \frac{1}{8}\right)\mu^2\left[\frac{p_\perp^2}{m^2}K^2\right.\right.$$
$$\left. + \frac{p_\perp^2 + m^2}{m^2}\left(K'^2 - \left(\frac{\nu-\nu'}{z}\right)^2 K^2\right)\right] + \frac{1}{4}\left[(k_\perp^2 + 2\mu^2)K^2\right.$$
$$\left.\left. + (k_\perp^2 - \mu^2)\left(K'^2 - \left(\frac{\nu-\nu'}{z}\right)^2 K^2\right)\right]\right\}. \qquad (56)$$

Here $K \equiv K_{i(\nu-\nu')}(z)$ is a Macdonald function, K' its derivative, $z = 2(\nu\rho)^{1/2}$, $\nu - \nu' \approx \mathbf{p}_\perp\mathbf{k}_\perp/|e\varepsilon|$ and in contrast to (18) we have preserved $p_\perp \neq 0$. It can be seen that in the excitation region where $z \gg 1$ the integrand becomes exponential and the exponents of the differential probability and of the distribution in μ are, respectively,

$$-2m_\perp(\mu^2 + k_\perp^2)^{1/2}/|e\varepsilon| \text{ и } -2m_\perp\mu/|e\varepsilon|, \quad m_\perp = (m^2 + p_\perp^2)^{1/2}, \qquad (57)$$

i.e., the effective temperature $|e\varepsilon|/2m_\perp$ differs from the Davies-Unruh temperature even if $p_\perp = 0$.

Note that the probabilities (52) and (56) can be transformed into the probabilities of photon emission $e \to e + \gamma$ by retaining in them only the terms proportional to ξ^2, omitting the integration with respect to μ^2, making the substitutions $G^2 \to 2e^2$, $\mu^2/3\pi \to 2\pi$, $\xi^2 \to \frac{1}{4}$ and putting $\mu = 0$ in the parameter ρ.

5. SPLITTING OF A CHARGED PARTICLE IN A CROSSED FIELD

We have seen that the exponents of the probabilities of various processes in an electric field depend substantially on the relation between the proper accelerations of the charges. We shall show now that they are quite sensitive to a change of acceleration in the region where the reaction is formed. To this end, we consider the charged-particle splitting

$e \to e' + e''$ in a crossed field ($\mathbf{E} \perp \mathbf{H}$, $E = H$), where the proper acceleration $[(eF_{\mu\nu}p_\nu)^2]^{1/2}m^{-2}$ is constant and the direction changes with the proper time.

Starting with the matrix element

$$M = f\int d^4x\, \Phi^*_{p''}(x)\Phi_{p'}^*(x)\Phi_p(x), \tag{58}$$

in which

$$\Phi_p(x) = (2p_0)^{-1/2}\exp\left[ipx + i\frac{eap}{2kp}(kx)^2 - i\frac{e^2a^2}{6kp}(kx)^3\right], \tag{59}$$

etc., are the wave functions of scalar particles with charges e, e' and e'' and momenta p_α, p'_α, and p''_α in a crossed field $F_{\alpha\beta} = k_\alpha a_\beta - k_\beta a_\alpha$, described by a potential $A_\alpha = a_\alpha(kx)$, $k^2 = ka = 0$, it is easy to obtain the following expressions for the splitting probability per unit proper time of the incident particle:

$$W = \frac{f^2}{16\pi^2 p_0}\int_0^1 dv\, \Phi_1(z), \tag{60}$$

$$z(v) = \frac{vm''^2/m^2 + (1-v)m'^2/m^2 - v(1-v)}{[\chi v(1-v)|v - e'/e|]^{2/3}}. \tag{61}$$

Here

$$\Phi_1(z) = \int_z^\infty dx\, \Phi(x)$$

is an integral of the Airy function, $\varkappa = [(eF_{\alpha\beta}p_\beta)^2]^{1/2}m^{-3}$, and $v = p'_-/p_-$. Note that p_1, p_2, p_- are the eigenvalues of conserved operators $\Pi_1, \Pi_2, \Pi_- \equiv \Pi_0 - \Pi_3$, if $\mathbf{E}$ and $\mathbf{H}$ are directed along axes 1 and 2. One more component p_α of the 4-vector is set by the condition $p^2 = -m^2$. The following conservation laws hold: $p_1 = p'_1 + p''_1$, $p_2 = p'_2 + p''_2$, $p_- = p'_- + p''_-$.

The function $\Phi_1(z)$ in (60) is thus the distribution of the final particles in p'_- and p''_-. Recall that $\Phi_1(z)$ for real z is positive, falls off exponentially as $z \to +\infty$, and tends to π as $z \to -\infty$ while executing damped oscillations of increasing frequency [19,15]:

$$\Phi_1(z) \approx \begin{cases} \dfrac{\pi^{1/2}}{2z^{3/4}}\exp\left(-\dfrac{2}{3}z^{3/2}\right), & z \to +\infty \quad (62) \\ \pi - \pi^{1/2}(-z)^{-3/4}\cos\left(\dfrac{2}{3}(-z)^{3/2} + \dfrac{\pi}{4}\right), & z \to -\infty \quad (63) \end{cases}.$$

We assume that the binding energy $I = m' + m'' - m > 0$, i.e., there is no reaction in the absence of a field. In that case $z(v) > 0$ in the physical region $0 < v < 1$ and becomes infinite at its end points $v = 0$ and $v = 1$. For splitting into particles of like charge, $z(v)$ becomes infinite also at the point $v = e'/e$, inside the interval $0 < v < 1$. Therefore the distribution in v vanishes at the points $v = 0, e'/e, 1$ and consists thus of two spectral lines with maxima in the ranges $0 < p'_- < p_- e'/e$ and $p - e'/e < p'_- < p_-$. If $\varkappa \ll 1$, then $z(v) \gg 1$ and the spectral lines are described by the exponential function (62). Its maxima are located at the minima of $z(v)$. The positions of the minima of $z(v)$ are given by the roots of the equation $z'(v) = 0$ that reduces to a cubic one. In view of the complexity of the latter, we confine ourselves to particular cases. In the case $e' = e'' = e/2$, $m' = m'' = 1/2m(1+\delta)$, $I = m\delta \ll m$, which is a particular case of (42), the equation $z'(v) = 0$ reduces to a quadratic one. For the positions $v_{1,2}$ of the maxima of the distributions and for the corresponding exponents we obtain

$$v_{1,2} = \frac{1}{2} \mp \frac{1}{2}\delta^{1/2} + \ldots, \qquad -\frac{2}{3}z^{3/2}(v_{1,2}) = -2\cdot 3^{1/2}\frac{\delta}{\chi} + \ldots, \tag{64}$$

where the triple dot denotes terms of order $\sim\delta$ relative to those written down. Comparison with (45) shows that the difference between the probability exponents for splitting in an electric field and in crossed fields is that π in the former is replaced by $2\sqrt{3}$ in the latter (the parameter $\varkappa$ must be identified here with β, since both represent the electric field strength in the proper system of the incident particles in units of m^2/e or the proper acceleration of the charge in units of m).

In the case $m'' = e'' = 0$, $e' = e$, $I = m' - m = m\delta \ll m$ the equation $z'(v) = 0$ becomes linear. For the position of the distribution maximum and of the probability exponent we get

$$v_1 = 1 - \delta + \ldots, \qquad -\frac{2}{3}z^{3/2}(v_1) = -2\cdot 3^{1/2}\frac{\delta}{\chi} + \ldots. \tag{65}$$

Finally, in the case (46), when the proper accelerations of the charges differ noticeably, the position of the principal maximum of the spectrum coincides essentially with the position of the minimum of the denominator of $z(v)$, which varies in this region approximately δ^{-1} times faster than the denominator. An arbitrarily accurate position of the principal maximum can be obtained by perturbation theory in the small parameter δ. As a result we obtain for the position and exponent of the principal spectral maximum

$$v_1 = \frac{1}{\alpha+1} - \frac{\delta}{3(\alpha-1)} + \ldots,$$

$$-\frac{2}{3}z^{3/2}(v_1) = -\frac{4[2\alpha(\alpha+1)]^{1/2}}{3(\alpha-1)\chi'}\delta^{3/2} + \ldots. \tag{66}$$

In the approximation considered, the exponent (66) is equal to the exponent (48) of the dominant mode of splitting in an electric field. The position of the second, lower maximum of the spectrum can be found by reducing the equation $z'(v) = 0$ to a quadratic one with the aid of the just obtained root v_1. We obtain thus for the position and the exponent of the second maximum in the spectrum

$$v_2 = \frac{(\alpha^2-\alpha+1)^{1/2}+\alpha-2}{3(\alpha-1)} + \frac{(\alpha+1)\delta}{6(\alpha-1)(\alpha^2-\alpha+1)^{1/2}} + \ldots,$$

$$-\frac{2}{3}z^{3/2}(v_2) = \begin{cases} -9(\alpha-1)^2/4\chi' & \text{for} \quad \delta \ll \alpha - 1 \ll 1 \\ -8\alpha^2/3\chi' & \text{for} \quad \alpha \gg 1 \end{cases}. \tag{67}$$

This exponent has qualitatively the same dependence on α as the exponent (50) of the second splitting mode in an electric field, but is somewhat higher in value.

6. CREATIVE FORCE AND FORMATION LENGTH

We see thus that at a low binding energy ($\delta \equiv I/m \ll 1$), in the case of noticeably different accelerations of the charged particles participating in the reaction, the exponent turns out to be smallest and proportional to $\delta^{3/2}$, whereas if the accelerations differ little it is proportional to δ. A particle (e, m) can be regarded as consisting of virtual particles (e', m_1) and (e'', m_2) with masses m_1 and m_2 adding up to m and different from the masses m' and m'' of real particles with energies Δ' and Δ'' of the order of the binding energy: $m_1 = m' - \Delta'$, $m_2 = m'' - \Delta''$, $m_1 + m_2 = m$, $\Delta' + \Delta'' = I$. As the particle (e, m) is then accelerated, an additional force is produced between its constituent virtual particles; this force can be found from Newton's equations $m_1 a = e'\varepsilon - f$, $m_2 a = e''\varepsilon + f$:

$$f=m_1\varepsilon\left(\frac{e'}{m_1}-\frac{e}{m}\right)=-m_2\varepsilon\left(\frac{e''}{m_2}-\frac{e}{m}\right). \qquad (68)$$

It can be seen that this force is due to the difference between e/m of the initial particle and e'/m_1 or e''/m_2, i.e., to the difference of the accelerations imparted by the electric field to the virtual and particles and to the incident one. At low binding energy, $f \sim e\varepsilon$ for noticeably differing accelerations of the real charged particles and $f \sim \delta e\varepsilon$ for slightly differing accelerations.

Obviously, the splitting process evolves over a length l over which the force f produces work I:

$$fl \sim I. \qquad (69)$$

We obtain hence, in the case of different accelerations, a formation length $l \sim I/|e\varepsilon|$. The kinetic energy acquired by the particles over this length through the action of the external force is also of the order of I, i.e., the charged particles move nonrelativistically, in the proper system of the initial particle, during the process formation time. The difference between processes in equally strong electric and crossed fields is therefore insignificant, see (48) and (66).

If the accelerations are close, we have $l \sim m/|e\varepsilon|$. The particles therefore become relativistic in the formation region and the actions of the electric and crossed fields on them are noticeably different, cf. (45) and (64).

Returning to the concept of force f, we note that owing to the spread of order I in the values of the virtual masses m_1 and m_2 the force f is always different from zero. The situation is different if the particle is accelerated by a uniform gravitational field. The charges turn then into masses and it is necessary to replace $e\varepsilon$, $e'\varepsilon$, $e''\varepsilon$ in (68) by mg, $m_1 g$, $m_2 g$, and this leads to $f = 0$: a uniform gravitational field accelerates virtual and real particles in the same manner, and the process does not take place. A change to an inertial system that falls together with the particle, where the latter is not acted upon by forces, leads to the same conclusion. A true gravitational field, however, is not uniform, and it can transfer its energy via tidal forces to virtual particles and convert them into real ones.

In sum, it can be stated that the processes due to acceleration of a particle depend substantially on the nature of the accelerating force, and on its ability to alter the interaction with the virtual particles of the vacuum and imparting to them the energy needed to convert them into real ones (to excite the detector). It is just the latter circumstance which leads to the simultaneous existence of the direct and inverse processes and to a relation of type (12).

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[1)]The components of **p** are the eigenvalues of the conserved operators Π_1, Π_2, $p_3 = \Pi_3 - e\varepsilon x_0$, where $\Pi_\alpha = -i\partial_\alpha - eA_\alpha$ is the kinetic-momentum operator.

[1]A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. **56**, 2035 (1969) [Sov. Phys. JETP **29**, 1093 (1969)].
[2]A. I. Nikishov and V. I. Ritus, *ibid.* **59**, 1262 (1970) [**32**, 690 (1971)].
[3]W. G. Unruh, Phys. Rev. **D14**, 870 (1976).
[4]L. P. Grishchuk, Ya. B. Zel'dovich, and L. V. Rozhanskiĭ, Zh. Eksp. Teor. Fiz. **92**, 20 (1987) [Sov. Phys JETP **65**, 11 (1987)].
[5]T. H. Boyer, Phys. Rev. **D29**, 1096 (1984).
[6]V. L. Ginzburg and V. P. Frolov, Usp. Fiz. Nauk **153**, 633 (1987) [Sov. Phys. Usp. **30**, 1073 (1987)].
[7]N. Burrell and P. C. W. Davies, Quantized Fields in Curved Space-Time [Russ. transl., Mir, 1984].
[8]W. Greiner, B. Muller, and J. Rafelski, *Quantum Electrodynamics of Strong Fields*, Springer, 1985.
[9]N. B. Narozhnyĭ and A. I. Nikishov, Trudy FIAN **168**, 175 (1986).
[10]A. I. Nikishov, *ibid.* **111**, 152 (1979).
[11]A. Erdelyi, ed. *Higher Transcendental Functions*, Vol. 1, McGraw, 1953.
[12]V. I. Ritus, Zh. Eksp. Teor. Fiz. **82**, 1375 (1982) [Sov. Phys. JETP **55**, 799 (1982)].
[13]H. Buchholz, *The Confluent Hypergeometric Function with Special Emphasis on Its Applications*, Springer, 1969.
[14]V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, Quantum Electrodynamics [in Russian], Nauka, 1980.
[15]M. Abramowitz and I. A. Stegun, eds., *Handbook of Mathematical Functions*, Dover, 1964.
[16]S. Takagi, Progr. Theor. Phys. Suppl. **88**, 1 (1986).
[17]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics. Nonrelativistic Theory*, Pergamon, 1977.
[18]L. B. Okun', Leptons and Quarks [in Russian], Nauka, 1981.
[19]A. I. Nikishov and A. I. Ritus, Asymptotic Representations for Some Functions and Integrals Connected with the Airy Function, FIAN Preprint No. 253, 1985.

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