Theory of streamer discharge in semiconductors

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Order-of-magnitude values of the main parameters of a streamer in a semiconductor are obtained from qualitative considerations. These are: the propagation velocity, the radius and conductivity of the channel, the fields ahead of the front and inside the channel, and the maximum streamer length.

1. INTRODUCTION

The streamer mechanism of electric discharge in gases was proposed about 50 years ago by Loeb, Meek, and Raether (for a survey of these and later studies see Refs. 1–5). A streamer is a thin highly conducting plasma filament that grows at high speed through ionization in the strong electric field near its head. Streamer discharges in gases, including lightning, were the subject of many investigations. Streamers are observed also in solids—dielectrics and semiconductors.[6–9]

Although the physical processes connected with streamer propagation are well known, there exists at present neither a quantitative nor even a satisfactory qualitative theory of this phenomenon. Inadequately founded models and various semi-empirical equations are customarily used.

The present paper is devoted to a streamer in a semiconductor. Using qualitative considerations, we obtain order-of-magnitude values of the most significant parameters of a streamer. Although the main ideas of our calculations are applicable also to gases (note that some of these ideas were advanced already by Cravath and Loeb,[10] we shall take explicitly into account the specific features of semiconductors, viz., the relatively small difference between the properties of the positive and negative carriers, and the saturation of the drift velocity in a strong field.

A streamer discharge in a dielectric or in a high-resistivity semiconductor is usually produced in the following manner. A metal tip to which a voltage is applied is pressed against the surface of the crystal. When the voltage is raised rapidly enough, an electric discharge is observed in the form of thin glowing filaments that grow into the interior of the crystal at a velocity $v \sim 10^7$–10^9 cm/s. This velocity is as a rule much higher than the saturated carrier drift velocity v_s 10^7 cm/s in a strong field.

The most substantial process responsible for the discharge propagation is generation of electron-hole pairs in the strong electric field at the head of the streamer. We assume that the carriers are generated by impact ionization at a frequency $\beta(E)$ that increases rapidly (exponentially) with increase of the field E and saturates on a level β_0 at $E \sim E_0$. The expression usually employed is

$$\beta(E) = \exp(-E_0/E)\beta_0.$$

Another possible mechanism of carrier generation may be interband tunneling. The dependence of this process on the electric field is qualitatively of the same nature as for impact ionization. The qualitative considerations that follow can therefore be applied, with slight modification, also to the tunneling mechanism of generation.

Carrier generation ahead of the streamer front continues until the field is crowded out as a result of the increased conductivity. The charge of the head then moves to the boundary of the high-conductivity region. Thus, the conducting region moves forward, and its newly produced sections are charged by the current flowing in the streamer channel from the metallic tip. What remains behind the front is a charged conducting filament whose evolution is determined by carrier recombination, by the diffuse broadening of the plasma, and also by the radial scatter of the charges through their electrostatic repulsion.

Streamer propagation is described by a system of equations consisting of the Poisson equation and the continuity equations for the electrons and holes. It is impossible to obtain an analytic solution of these equations in view of their nonlinearity and the non-one-dimensional character of the problem. Using qualitative considerations, however, it is possible to express, accurate to numerical constants, the main characteristics of a streamer in terms of the potential of the tip and the parameters of the material.

We assume hereafter that the drift velocities and the impact-ionization coefficients of the electrons and holes are approximately equal. We neglect the anisotropy of these quantities, which leads to an experimentally observable[6–9] propagation of the streamers in definite crystallographic directions. The streamer will be represented as a conducting filament with a radius r_0 of the order of the rounded end of the head (this representation wil be made more specific in Sec. 11). It is assumed next that the density n_0 of the free or weakly bound carriers in the crystal is high enough to enable the volume $\sim r_0^3$ ahead of the streamer front to contain carriers capable of becoming multiplied by impact ionization ($n_0 r_0^3 \gg 1$). These carriers can also be produced through ionization by the streamer radiation. If the photoionization is substantial, the analysis that follows will be valid under the assumption that the radiation-absorption length is larger than or of the order of r_0.

2. FIELD AHEAD OF STREAMER FRONT

Let us show that stable propagation of the streamer requires that the maximum field strength E_m on the front be of the order of the field E_0 at which the frequency $\beta(E)$ of impact ionization reaches saturation. In fact, we assume that $E_m \gg E_0$ at a certain instant. Since the field of the charged filament falls off with distance like $1/r$, it is obvious that the size of region in which $\beta(E) \sim \beta_0$ is of the order of

$$(E_m/E_0) r_0 \gg r_0$$

(see Fig. 1a). Carriers will be generated in this region, so

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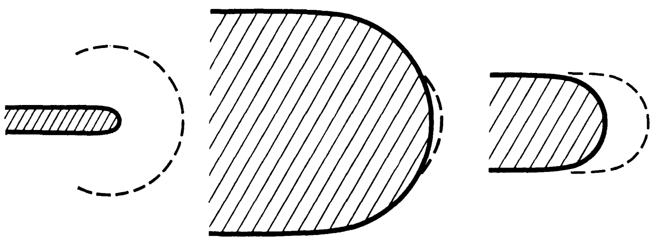

a b c

FIG. 1. Diagram explaining the relation $E_m \sim E_0$. The shaded region is the streamer head. The impact-ionization region is bounded by the dashed line: a—$E_m \gg E_0$, b—$E_m \ll E_0$, c—$E_m \sim E_0$.

that the radius of the head increases rapidly, and the field on the front decreases.

If, however, $E_m \ll E_0$ then, owing to the strong $\beta(E)$ dependence, the size of that region near the front in which $\beta(E) \sim \beta(E_m)$ and noticeable generation takes place will be

$$(E_m/E_0)r_0 \ll r_0$$

(Fig. 1b). The radius of the head is then decreased so that the field on the front increases.

The radius of the head must thus be such that the field on the front be of the order of E_0 (Fig. 1c):

$$E_m \sim E_0. \tag{1}$$

Such a field exists in a region with dimension of order r_0 ahead of the front, and the frequency of impact ionization in this region is of the order of β_0. Similar arguments were contained in the earlier paper by Cravath and Loeb.[10] They were used, however, neither by them (see Ref. 1) nor by others, and the field on the front was estimated from experimental data.

The result (1) can be generalized to include the case when the rapid growth of $\beta(E)$ in a weak field is replaced by a smoother variation in a strong field. Arguments similar to the foregoing lead to the conclusion that the field ahead of the front should be determined from the condition $d\beta/dE \sim \beta/E$. In this case E_m is of the order of field at the point of inflection of the $\beta(E)$ plot.

3. CONNECTION BETWEEN THE PROPAGATION VELOCITY AND THE HEAD RADIUS

The streamer velocity v can be expressed in the form $v \sim r_0/\tau$, where τ is the time during which the front is displaced a distance on the order of the heat radius r_0. This displacement proceeds as follows. Owing to the strong $\beta(E)$ dependence, substantial carrier generation is initiated at a certain point when its distance to the front decreases to a value on the order of r_0. The density increases subsequently at a rate

$$n(t) = n_0 \exp(\beta_0 t),$$

where n_0 is the carrier density far from the front. This increase continues until the field at the considered point becomes noticeably weaker than E_0. The quantity τ is the characteristic time required to crowd the field out of a region of order r_0.

We consider for simplicity first a case when the drift velocity is not saturated up to the field E_0. In this case the density stops increasing when the Maxwell time becomes of the order of the time constant (β_0^{-1}) of this growth, i.e.,

$$\sigma = e\mu n_0 \exp(\beta_0 \tau) \sim \beta_0, \tag{2}$$

where μ is the mobility.

Determining from this the time τ, we obtain the connection between the propagation velocity and the radius of the head:

$$v \sim \beta_0 r_0/\Lambda_1, \quad \Lambda_1 = \ln(N/n_0), \tag{3}$$

where $N = \beta_0/(e\mu)$. In the logarithm, the ratio of the carrier density behind the front (N) to the density ahead of it (n_0) amounts to several orders of magnitude; the logarithm is therefore large.

In semiconductors, as a rule, the drift velocity reaches the saturation value v_0 in a characteristic field E_s substantially weaker than E_0. In this case, the field is crowded out of a region of size on the order of r_0 ahead of the front in a somewhat different manner than in Maxwell relaxation. The time τ to crowd out the field can be determined from the condition that the total charge flowing during that time be equal to the charge on the front. The surface density of the charge on the front is obviously of the order of E_0, and the current density in the region where the carriers are generated is $ev_s n(t)$, where $n(t) = n_0 \exp(\beta_0 t)$. Thus,

$$\int_0^\tau ev_s n(t)\,dt \sim E_0. \tag{4}$$

From this we get

$$\tau \sim \beta_0^{-1} \ln(\beta_0 E_0/ev_s n_0). \tag{5}$$

With the aid of (5) we obtain again the relation (3) between the velocity and the radius, but the density N behind the front is now given by

$$N \sim \beta_0 E_0/ev_s. \tag{6}$$

Note that the expressions for the velocity differ in the two considered cases only by a factor under the logarithm sign.

Relation (3) is very important for the understanding of the development of a streamer discharge. It is valid independently of the character of the variation of the head radius r_0 with time (provided the time of the change is long compared with τ).

An equation of type (3) was first obtained by Loeb,[1] but contained in place the head radius the front width, which remained unknown. In addition, the carrier density N in the channel was not obtained, so that the value of N/n_0 under the logarithm sign was not determined. Equation (3) without the logarithmic factor was used in Ref. 7.

4. MINIMUM VALUE OF STREAMER VELOCITY

Owing to the electrostatic repulsion, charges on the front of the streamer should scatter at a velocity v. Obviously, for a streamer to develop it is necessary that the front velocity exceed the rate of this scatter. Otherwise the charge would become detached from the conducting region and the growth of the latter would cease. Thus, the streamer velocity is bounded from below by a certain threshold on the order of

v_s:

$$v > v_s. \quad (7)$$

It should be borne in mind that this inequality must contain a numerical factor, which we shall not determine. In view of relation (3) between v and r_0, the inequality (7) means also the existence of a minimum radius of the streamer head.

The fact that the streamer velocity in the experiments is always larger than the saturated velocity of the carrier drift was noted in Ref. 8.

5. CONDUCTIVITY OF STREAMER CHANNEL

Directly behind the front, the conductivity is determined by the electron density $N = n(\tau)$. In the first of the cases analyzed here, when no saturation of the drift velocity takes place, it can be seen from (2) that $\sigma \sim \beta_0$. If, however, the drift velocity saturates in a field $E_s \ll E_0$, we obtain, after substituting N from (6) in the expression $\sigma \sim e\mu N$ for the conductivity,

$$\sigma \sim \beta_0 E_0 / E_s. \quad (8)$$

We have used here the relation $v_s \sim \mu E_s$.

It will be shown below that the field in the streamer channel is weak compared with E_s, so that the conductivity is ohmic.

If carrier recombination and diffusion are neglected, Eq. (8) determines the conductivity over the entire length of the channel. The role of the conductivity will be considered in Sec. 10. As to the diffusion spreading of the channel, it leads, of course, to a decrease of the conductivity, but the quantity essential for streamer development, the resistance per unit channel length, obviously remains unchanged. We note that in crystals, as a rule, no noticeable diffusive spreading of the channel takes place during the propagation time t_0. Using for r_0 and t_0 the estimates given below, it can be shown that the parameter $(Dt_0)^{1/2}/r_0$ that describes the diffusive spreading is smaller than or of the order of unity (D is the diffusion coefficient).

6. FRONT WIDTH

Since the characteristic time of density growth in the strong-field region is β_0^{-1}, it is obvious that the front width in the streamer head is $\delta \sim v/\beta_0$. Using Eq. (3) for v, we get

$$\delta \sim r_0/\Lambda_1 \ll r_0. \quad (9)$$

Thus, the characteristic distance δ over which the carrier density is decreased by several times is substantially shorter than the heat radius r_0 (the density decreases over the distance r_0 to the value n_0, i.e., by several orders). Consequently, the conducting region has a very abrupt boundary. The space charge on the front is concentrated in a region whose width is also of the order of δ. In fact, if the drift velocity is not saturated, the charge in the conducting region decreases with time like $\exp(-4\pi\sigma t)$, where $\sigma \sim \beta$ according to (2). For a point at a distance z behind the front we have $t = z/v$. It is from this that we get the statement above. It can be shown that it is valid also for saturation of the drift velocity.

The statements made in this section pertain to the streamer head. The charge distribution and density in the channel will be considered in Sec. 11.

7. STREAMER PROPAGATION

The streamer parameters and the very possibility of its propagation are determined by the potential of the metallic tip and by the character of variation of this potential with time. The velocity v and the radius r_0 of the head are connected by relation (3) and the radius, as indicated in Sec. 2, should be such that the field ahead of the front be of the order of E_0. The problem reduces thus to establishing a connection between the tip potential U, the head radius r_0, and the field ahead of the front. This connection depends generally speaking on the form of the function $U(t)$ and on the character of the prior development of the streamer. We confine ourselves to the simplest case, when the potential drop over the length of the streamer is small, so that the head potential practically coincides with the potential U of the metallic tip. This condition is met provided that the streamer length is not too long and that, furthermore, the potential varies sufficiently slowly with time. The appropriate criteria will be given below.

In this case, the charge distribution over the streamer channel is determined by solving the electrostatic problem of a thin metallic filament with a given potential U. It is known that the charge is distributed along the filament practically uniformly, with a linear density

$$\rho_l \sim U/\ln(l/r_0), \quad (10)$$

where l is the filament length ($l \gg r_0$). The field ahead of the front is obviously of the order of ρ_l/r_0. Equating this quantity to E_0, using (3), and putting $\Lambda_2 = \ln(lE_0/U)$, we get

$$r_0 \sim U/E_0\Lambda_2, \quad v \sim \beta_0 U/E_0\Lambda_1\Lambda_2. \quad (11)$$

The streamer velocity and the head radius are thus proportional during the initial stage to the tip potential U.

8. FIELD IN CHANNEL AND MAXIMUM STREAMER LENGTH

As the streamer grows at a rate v, its newly produced sections acquire charge with a linear density $\rho_l \sim E_0 r_0$. Consequently, a current $I = \rho_l v$ should flow through the conducting channel of the streamer. Maintenance of such a current in the channel requires a longitudinal field E_z determined from the condition $I \sim \sigma E_z r_0^2$. Using Eqs. (3) and (8) we get

$$E_z \sim E_s/\Lambda_1. \quad (12)$$

Owing to the large value of the logarithm Λ_1, the field E_z is substantially weaker than E_s, so that the drift velocity in the channel is not saturated.

We can now indicate the condition, used in the derivation of (11), that the change of potential over the length of the streamer be small:

$$l < l_0, \quad l_0 \sim U/E_z \sim U\Lambda_1/E_s. \quad (13)$$

For the corresponding time $t \sim l/v$ we get with the aid of (11) and (13) the restriction $t < t_0$, where

$$t_0 \sim E_0\Lambda_1^2\Lambda_2/\beta_0 E_s. \quad (14)$$

Note that the time t_0 does not depend on the potential U.

As the streamer propagates, the head potential decreases and, in accordance with (11), its velocity and radius should decrease. When the streamer velocity becomes of the order of v_s, its development should cease (see Sec. 4). The total length of the streamer at the instant of its stopping is apparently of the order of l_0. This conclusion can be drawn from the following considerations. The spreading of the charge along a conducting filament is diffusive,[11,12] with $\sigma r_0^2 \Lambda_2$ acting as the diffusion coefficient. Therefore the characteristic spreading time of the charge (and of the potential) over a length l is $l^2(\sigma r_0^2 \Lambda_2)^{-1}$. For $l \sim l_0$ this quantity is just of the order of the propagation time $t_0 \sim l_0/v$. At $l > l_0$ the head potential decreases strongly compared with the tip parameter, so that the radius r_0 is decreased. The "diffusion coefficient" is then also decreased and the spreading of the charge slows down even more. Calculation shows that at $l > l_0$ the velocity decreases exponentially, with a characteristic time t_0.

These arguments, however, do not take into account the recovery of the head potential after the stopping of the streamer, and the possible broadening of the channel by impact ionization in the region $l > l_0$. The question of streamer development at $t \gtrsim t_0$ is therefore not fully understood and calls for further research.

The fact that the streamer can stop growing when the potential over its entire length is the total potential applied to the tip was noted earlier.[8,13] The field in the channel and the maximum length l_0, however, have not been theoretically estimated.

9. ROLE OF CONDUCTIVITY OF THE SURROUNDING MEDIUM

It was established by experiment[7–9] that streamers are observed only in samples of sufficiently high resistivity. With increase of the crystal conductivity σ_0 the streamers become shorter and eventually vanish.

The finite conductivity of the medium surrounding the streamer leads to the appearance of a radial current whose density per unit length is of the order of $\sigma_0 E_r r \sim \sigma_0 \rho_l$, where $E_r \sim \rho_l / r$ is the radial component of the electric field. Maintenance of radial currents in the streamer channel requires an additional (on top of the one calculated in Sec. 8) longitudinal current $\tilde{I}$ such that

$$-d\tilde{I}/dz \sim \sigma_0 \rho_l.$$

This current is connected with an additional longitudinal field $\tilde{E}_z \sim \tilde{I}/\sigma r_0^2$. Thus, the presence of a finite sample conductivity leads to an increase of the field in the channel and to a faster decrease of the potential along the streamer.

We denote by $\tilde{l}_0$ the characteristic length over which the potential drop due to the radial current takes place. The longitudinal current $\tilde{I}$ and the field $\tilde{E}$ fall off, obviously, over the same length. The total radial current on this length is of the order of $\sigma_0 \rho_l \tilde{l}_0$, which coincides with the average value of the longitudinal current $\tilde{I}$. Therefore the characteristic value of the longitudinal field is given by the expression

$$\tilde{E}_z \sim \sigma_0 U \tilde{l}_0 / \sigma r_0^2 \Lambda_2. \tag{15}$$

We have used here Eq. (10) for ρ_l. We can now determine the length $\tilde{l}_0$ from the condition $\tilde{E}_z \tilde{l}_0 \sim U$. Hence

$$\tilde{l}_0 \sim r_0 (\sigma \Lambda_2 / \sigma_0)^{1/2}, \tag{16}$$

where the channel conductivity σ and the radius r_0 are given by Eqs. (8) and (11). Using also expressions (13) and (14), we obtain the relation between $\tilde{l}_0$ and l_0:

$$\tilde{l}_0 / l_0 \sim (\sigma_0 t_0)^{-1/2}. \tag{17}$$

Thus, the role of the conductivity of the medium is determined by the parameter $\sigma_0 t_0$, where t_0 is given by Eq. (14). If $\sigma_0 t_0 \ll 1$, the conductivity of the medium can be neglected and the streamer length is of the order of l_0 [see Eq. (13)]. In the opposite limiting case $\sigma_0 t_0 \gg 1$, the streamer length is of the order of $\tilde{l}_0$ and it decreases, with increase of the conductivity of the medium, in proportion to $\sigma_0^{-1/2}$. In this case the characteristic propagation time is of the order of $(t_0/\sigma_0)^{1/2}$. Note that at $\sigma_0 t_0 \gg 1$ the streamer propagation time t_0 is substantially longer than the Maxwell time σ_0^{-1}. This is due to the presence of the current that maintains the streamer charge.

10. INFLUENCE OF CARRIER RECOMBINATION

At a certain distance behind the streamer front, carrier recombination becomes substantial and leads to a lowering of their density and to a corresponding decrease of the channel conductivity. As a result, the longitudinal electric field needed to maintain the current in the channel is increased. This leads in turn to a fast decrease of the potential along the streamer, and its propagation distance decreases.

Carrier recombination in the channel is described by the usual equation

$$\partial n / \partial t = -\gamma n^2. \tag{18}$$

For a point located at a distance z behind the front, the time elapsed from the instant of generation is z/v. Solving Eq. (18) with the initial condition $n(0) = N$, we obtain for $\sigma(z) = e\mu n(z)$ the expression

$$\sigma(z) = \sigma (1 + z/v t_R)^{-1}, \tag{19}$$

where $\sigma = e\mu N$ is the value of the conductivity directly behind the front [Eq. (8)], and $t_R = (\gamma N)^{-1}$ is the characteristic recombination time.

It follows from (19) that with increase of the distance from the front the longitudinal field in the channel increases linearly from the value (12). The characteristic length l_R over which the potential drop takes place can be easily calculated in analogy with the procedure used in Sec. 9. It is determined by the relation

$$l_R / l_0 + l_R^2 t_0 / l_0^2 t_R \sim 1, \tag{20}$$

where l_0 and t_0 are given by Eqs. (13) and (14). The ratio l_R / l_0 depends on the parameter t_0 / t_R, if $t_0 \ll t_R$, the recombination is insignificant and $l_R \sim l_0$. For $t_R \ll t_0$ we obtain from (20)

$$l_R \sim l_0 (t_R / t_0)^{1/2}, \tag{21}$$

i.e., the distance over which the streamer propagates is substantially decreased by the recombination in comparison with l_0. Note that the propagation time $l_R / v \sim (t_0 t_R)^{1/2}$ exceeds in this case noticeably the recombination time.

The results of this and of the preceding two items show that the distance negotiated by the streamer is limited either by an increase of the channel resistance or by leakage of the charge into the surrounding medium, or else by carrier recombination. We have accordingly three lengths, l_0, $\tilde{l}_0$, and l_R which are proportional to the tip potential U [Eqs. (13), (17), and (21)]. The shortest of these lengths is the one which determines the maximum streamer length.

Note that these three lengths can also be obtained by equating the characteristic charge flow time $l^2(\sigma r_0^2\Lambda_2)^{-1}$ of the charge along the conducting filament either to the Maxwell relaxation time $1/\sigma_0$ in the medium, or to the recombination time t_R.

11. DISTRIBUTION OF CHARGE AND OF CARRIER DENSITY IN STREAMER CHANNEL

So far, we have regarded the streamer channel as a thin conducting uniformly charged filament having a radius approximately equal to the radius r_0 of the rounded head. This representation calls for some elaboration. Actually, the radial field near the surface of such a filament is of the same order as the field E_0 at the head. The conducting region should therefore broaden by impact ionization.[1)] The charge would go to the newly produced conducting region, and this would decrease the radial field.

The actual distribution of the charge and density in the streamer channel can qualitatively be determined from the following considerations. The radial field at a distance $r > r_0$ from the channel axis is

$$E_r \sim E_0 r_0 / r$$

(if the distance to the charge is less than r). The impact-ionization frequency in such a field is

$$\beta(E_r) = \beta_0 \exp(-r/r_0).$$

For a point with coordinates r and z, the impact ionization continues for a time z/v. The conductivity at the considered point is therefore

$$\sigma \sim e\mu n_0 \exp(\beta(E_r) z/v).$$

This expression is valid for points outside the surface on which the charge is located (see Fig. 2a). Inside this surface, there is practically no field and no impact ionization produced. For a given r the density and conductivity increase thus up to a certain value of z (section AB in Fig. 2a), and remain unchanged beyond (until the recombination becomes substantial). Let us determine the form of the charged surface for the simplest case when the drift velocity is not saturated. Reasoning as in Sec. 3, we arrive at the conclusion that the charge should be located at a point where $\sigma \sim \beta(E_r)$. Consequently,

$$e\mu n_0 \exp(v^{-1}\beta_0 z \exp(-r/r_0)) \sim \beta_0 \exp(-r/r_0). \qquad (22)$$

From this, using Eq. (3), we obtain with logarithmic accuracy the form of the charged surface:

$$r \sim r_0 \ln(z/r_0), \qquad (23)$$

where $z \gtrsim r_0$. It can be shown that the result (23) remains valid, with the same accuracy, also when the drift velocity is saturated.

Thus, the radius of the charged surface surrounding the streamer channel increases slowly with increasing distance from its head. As to the radial distribution of the conductivity, its characteristic scale coincides with the head radius r_0. Indeed, on the charged surface we have

$$\sigma \sim \beta(E_r) \sim \beta_0 \exp(-r/r_0)$$

(or

$$\sigma \sim (\beta_0 E_0/E_s) \exp(-r/r_0),$$

if the drift velocity saturates), and inside the surface the conductivity at constant r is unchanged.

In the derivation of (23) it was assumed that the change of the charge density with time is due to the conduction current flowing in the conducting quasineutral region. With increasing distance from the channel axis, however, the carrier density decreases exponentially, whereas the charge density decreases like $1/r$. At a certain distance from the axis the quasineutrality condition is therefore violated and the current is determined not by the conduction current but by the electrostatic repulsion of the charges. The radial velocity of the charged surface is then equal to v_s. This circumstance becomes significant when v_s turns out to exceed the radial velocity $v\,dr/dz$ determined from (23), i.e., at $z > z_0$, where $z_0 \sim r_0 v/v_s$. At $z > z_0$ Eq. (23) no longer holds and the charged surface takes the form of a cone $r \sim z v_s/v$. When the field on this surface decreases below the saturation field E_s, the drift velocity decreases and the broadening slows down.

The charge and carrier-density distributions in the streamer channel are shown schematically in Fig. 2. The conducting region that contains the bulk of the carriers is a cylinder of radius on the order of the head radius r_0. With increasing distance from the axis, all the way to the charged surface, the density decreases like $\exp(-r/r_0)$.[2)] At the same time, everywhere with the exception of the streamer head, the charge is located outside the high-conductivity region, and the radius of the charged surface increases with increase of the distance from the head, as described above.

The charge distribution is essential for the solution of the electrostatic problem of the connection between the tip potential and the field ahead of the front. For a thin conducting filament of constant radius r_0, this connection is given by

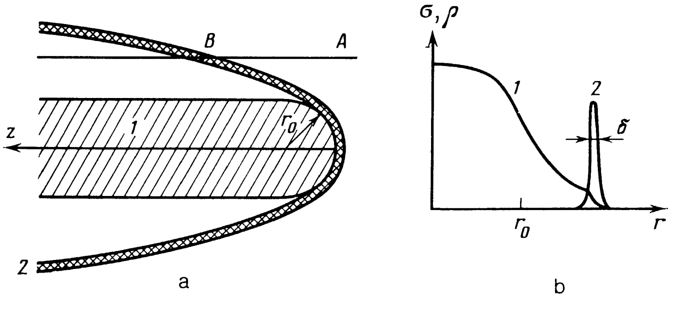

FIG. 2. Distribution of the conductivity σ and of the charge density ρ in a streamer channel at $v \gg v_s$: a—schematic diagram of the conducting region (1) and of the region where the charge is located (2). The carrier density on an arbitrary straight line parallel to the axis increases on section AB and remains unchanged to the left of the point B; b—radial dependences of the conductivity (1) and of the charge density (2) in the streamer channel at $r_0 < z < z_0$.

the first of Eqs. (11). Obviously, at $v \gg v_s$ the charged surface is strongly elongated in the streamer propagation direction and the aperture angle dr/dz is small everywhere. In this case the difference between the streamer and a charged filament can be neglected. If $v \sim v_s$, however, this difference becomes quite substantial, and this is indeed why a streamer with $v < v_s$ cannot propagate (see Sec. 4).

12. THRESHOLD CONDITIONS FOR STREAMER INITIATION

It has been established by experiment[7-9] that for a streamer to start the potential of the tip must increase with time rapidly enough, i.e., a threshold $(dU/dt)_{th}$ exists. If, however, a rectangular voltage pulse is applied to the tip [the slope of the leading front is much larger than $(dU/dt)_{th}$], the streamer is produced only at a sufficient pulse amplitude $U > U_{th}$. Let us obtain estimates of the threshold values of U_{th} and $(dU/dt)_{th}$.

We consider first the case when the tip potential increases jumpwise to a certain value U. We assume that the field $E \sim U/R$ near the tip is such that $E_s \ll E \ll E_0$ (R is the radius of the rounded part of the tip). Carriers will then be generated mainly in a region having a width on the order of RE/E_0 near the vertex of the tip (see Sec. 2 and Fig. 1b, in which the shaded region must now be taken to be the picture of the tip). The time needed to crowd the field out of this region is $\tau \sim \beta^{-1}(E)\Lambda_1$ (see Sec. 3), and the rate of displacement of the boundary of the conducting region is[3)]

$$v(E) \sim \beta(E) R (E/E_0) \Lambda_1^{-1}. \tag{24}$$

As shown in Sec. 2, the radius of the conducting region should decrease, the field on the front should increase, and a streamer should be formed. The competing process is the scatter of the charge with velocity v_s, which leads to a decrease of the field at the front (see Sec. 4). Thus, the threshold field $E_{th} \sim U_{th}/R$ is determined by the condition $v(E_{th}) \sim v_s$ or, with logarithmic accuracy

$$U_{th} \sim E_0 R / \ln(\beta_0 R / v_s). \tag{25}$$

We turn now to an estimate of the threshold value $(dU/dt)_{th}$. Let the tip potential increase linearly with time: $U = At$. The carrier density near the tip increases like

$$N(t) = n_0 \exp\left(\int_0^t \beta(E)\, dt\right), \tag{26}$$

where $E \sim At/R$. When the field is crowded out of the produced conducting region, the density reaches a definite value N. The time needed for this depends on the potential growth rate A. If the field at the tip is at the instant stronger than E_{th}, a streamer is produced. The threshold value A is therefore determined from the condition $E \sim E_{th}$ at the instant when the density N is reached.

Using the steep growth of $\beta(E)$ at $E \ll E_0$, we can calculate the integral in (26):

$$\int_0^t \beta(E)\, dt \sim \beta(E) E^2 R / E_0 A, \tag{27}$$

where the value of E in the right-hand side is taken at the instant t. For $N(t) = N$ we have

$$\beta(E) E^2 R / E_0 A \sim \Lambda_1. \tag{28}$$

Putting $E = E_{th}$ in (28), and using Eq. (24) and the relation $v(E_{th}) \sim v_s$, we obtain the threshold value $A_{th} = v_s E_{th}$. With the aid of (25) we get ultimately

$$(dU/dt)_{th} \sim E_0 v_s / \ln(\beta_0 R / v_s). \tag{29}$$

Note that $(dU/dt)_{th}$, just as E_{th} depends very little on the tip radius.

13. ESTIMATES AND COMPARISON WITH EXPERIMENT

The results above establish the dependence of the streamer parameters on the properties of the material and on the potential of the tip. For streamers in crystals, however, these relations have been little investigated in experiment. It appears that no experiments were performed in which the tip potential remained constant during the streamer propagation, as proposed in our calculations. It is therefore impossible to verify whether the radius of the head, the propagation velocity, and the streamer length are indeed proportional to potential, as follows from Eqs. (11) and (13).

Streamer discharges in semiconductors were investigated in greatest detail in Refs. 7 and 8. Let us compare our estimates of the streamer parameters with the results of these experiments. It must be borne in mind here that the numerical coefficients of our equations are unknown, that β_0 and E_0 are known with large errors, and in addition, under the conditions of Refs. 7 and 8, the tip potential increased substantially during the streamer propagation.

We use the CdS parameters cited in Ref. 7: $\beta_0 = 6 \cdot 10^{12}$ s^{-1}, $E_0 = 10$ V/cm, $v_s = 10^7$ cm/s, and $\mu = 200$ cm^2/V·s. From the equations derived above, at a typical potential value $U = 10$ kV, we obtain then $r_0 \sim 10\,\mu$m, $v \sim 10^9$ cm/s, $l_0 \sim 1$ cm, and $N \sim 10^{18}$ cm^{-3}. The experimental values[7] of these parameters are $r_0 \sim 1\,\mu$m, $v = (1\text{–}5) \cdot 10^8$ cm/s, and $l_0 \lesssim 3$ cm. Equation (29) yields an estimate $(dU/dt)_{th} \sim 10^{12}$ V/s, whereas experiment 8 gives $6 \cdot 10^{11}$ V/s. Our estimates and experiment are thus in reasonable agreement.

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1) The diffusion broadening is negligible (see Sec. 5).

2) It can be shown that at $z < z_0$ this behavior is replaced on the charged surface by a steeper decrease of characteristic scale δ [Eq. (9)]. The bulk charge is also concentrated in a layer of thickness δ.

3) Here, as in (3), $\Lambda_1 = \ln(N/n_0)$, where N can be shown to be obtainable from Eq. (6) by replacing $\beta_0 E_0$ by $\beta(E) E^2/E_0$. Note that the quantity Λ_1 is immaterial in the sequel.

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