Anomalous magnetoresistance associated with the interaction of electrons in cesium films deposited on cold substrates

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An investigation was made of the anomalous magnetoresistance of cesium films deposited at liquid helium temperature. The method employed ensured that the impurity concentration was low and the degree of homogeneity of the films was high. The experimental curves did not obey the theoretical relation predicted usually for the localization of noninteracting electrons in metal films. Moreover, the spin-orbit interaction was weak and it was manifested by a negative sign of the magnetoresistance, which was unexpected for a metal with a large atomic number. The results of the measurements could be explained on the assumption that the weak spin and spin-orbit scattering in cesium films made it possible to observe the anomalous magnetoresistance associated with the electron-electron interaction.

1. INTRODUCTION

One of the results of the successful development of a theory of the properties of disordered two-dimensional systems has been the prediction[1–4] and experimental detection[5–9] of a number of effects associated with the existence of quantum corrections to the conductivity. Suppression of these corrections by a magnetic field (anomalous magnetoresistance) has now been observed for inversion layers in semiconductors, for semiconductor,[6] semimetal,[7] and metal[8,9] films, as well as for other quasitwo-dimensional systems, and all these experiments have been found to be in agreement with the existing theory.

Until recently the only parameter measured in practice for films has been the resistance, i.e., the momentum relaxation time. The anomalous magnetoresistance with its relatively nonstringent demands on the quality of the samples, temperature, and magnetic fields is now becoming a working tool for the investigation of thin films, since the simple procedure needed in the magnetoresistance measurements makes it possible to study the processes of relaxation of electron energy and spin, and thus detect the presence of magnetic and heavy-atom impurities in the samples.

Measurements of the anomalous magnetoresistance can be used as a sufficiently accurate method for the investigation of films if the magnetoresistance itself is sufficiently large, i.e., it is necessary to use films with a high resistance in strong magnetic fields. However, the existing theory describes only low-resistance films in weak fields. One could hope to extend the range of validity of the theory of weak localization in the direction of higher resistances by investigating more homogeneous films. In the case of cold-deposited films, in which the inhomogeneity is at the level of the statistical scatter, the influence of the percolation effects is suppressed even in films of kilo-ohm resistance. Moreover, a high homogeneity means that, for a given resistance, a film has a lower thickness and a longer mean free path l, which is important because the validity of the theory of weak localization requires that the condition $l \gg a$ (a is the interatomic space) be satisfied.

Our investigations were carried out on cesium for which a method has been developed[10] for the preparation of films in ultrahigh vacuum at liquid helium temperatures. As shown in Ref. 10, the properties of such films are independent of the substrate material and of the cooling rate (within the range 0.5–0.005 Å/s). A conductance of the order of $\sim 10^{-10}\ \Omega^{-1}$ appears for an average thickness of ~ 0.7 of a monatomic layer. In the case of films of thickness amounting to one monatomic layer with a resistance of $\sim 80\ \mathrm{k}\Omega$ a transition takes place to a weakly localized metallic conduction. Therefore, such films are the thinnest metal films known at present and this is evidence of a high homogeneity of the films prepared by this method.

Homogeneity of films of such a small thickness formed by atoms randomly incident on a substrate should be understood in the sense that even high-resistivity submonatomic films are continuous from the classical (percolation) point of view and represent not individual islands but a continuous network homogeneous over distances used in the theory. The absence of metallic conduction in the case of these systems is of quantum origin and it is simply related to the fact that the classical resistance of a network exceeds $2\pi^2\hbar/e^2$.

Cesium films have another important advantage as the object of an investigation compared with the films of metals with higher boiling points: it is quite easy to remove impurities from these films. In any case, cold-deposited cesium films should not contain magnetic impurities which would be characterized by a negligible vapor pressure at the working temperature (from -20 to $+20$ °C) of the cesium source.

2. THEORETICAL EXPRESSIONS FOR THE MAGNETORESISTANCE IN THE ABSENCE OF ELECTRON INTERACTION

When a perpendicular magnetic field destroys localization of noninteracting electrons, the dependence of the total conductance on a sufficiently weak field obeying $H \ll kT/g\mu_B$ and $H \ll mc/e\tau$ (μ_B is the Bohr magneton and τ is the elastic-scattering time) is[2]

$$\delta G(H) \equiv \frac{2\pi^2\hbar}{e^2 \delta R(H)} = \ln \frac{HH_3^{1/2}}{H_2^{1/2}} + \frac{3}{2}\psi\left(\frac{1}{2}+\frac{H_2}{H}\right) - \frac{1}{2}\psi\left(\frac{1}{2}+\frac{H_3}{H}\right), \quad (1)$$

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where $\psi(x)$ is the logarithmic derivative of the gamma function (known as the digamma function) and the characteristic values of the magnetic fields

$$H_2=H_i+{}^4/_3H_{so}+{}^2/_3H_s, \quad H_3=H_i+2H_s \tag{2}$$

are related to the corresponding scattering times τ_i, τ_{so}, and τ_s by

$$H=\hbar c/4eD\tau. \tag{3}$$

Here, D is the diffusion coefficient of electrons and the indices refer to inelastic scattering (i), relaxation of the spin because of the spin-orbit interaction (so), and relaxation of the spin due to interaction with magnetic impurities (s).

Knowing the metal from which a film has been prepared and the mean free path in thin cold-deposited films, we can estimate also the characteristic magnetic fields in Eqs. (1)–(3). In the absence of magnetic fields the value of H_s should vanish. The frequency of inelastic collisions is in most cases in good agreement with the estimate[4]

$$\hbar\tau_i^{-1}=\pi kT\frac{e^2R}{2\pi^2\hbar}\ln\frac{\pi\hbar}{e^2R}, \tag{4}$$

so that for a film with $R\sim 3.5$ kΩ at $T=4.2$ K, we have

$$H_i=\frac{\pi}{2}enct\frac{kT}{\varepsilon_F}\frac{R^2e^2}{2\pi^2\hbar}\ln\frac{\pi\hbar}{e^2R}\approx 0.25 \text{ T}, \tag{5}$$

where n is the electron density and t is the film thickness. The probability of the spin-orbit scattering is governed by a quantity $(\alpha Z)^4$, where α is the fine-structure constant and Z is the nuclear charge,[4] so that for $R\sim 3.5$ kΩ we have

$$H_{so}=\frac{\hbar c(\alpha Z)^4}{4eD\tau}\approx 2.5 \text{ T}. \tag{6}$$

We shall show later that this estimate is far from the experimental value for cesium films.

3. EXPERIMENTAL METHOD AND ANALYSIS OF RESULTS

The main problem which has to be solved in the preparation of continuous films of extremely small thicknesses is the suppression of the mobility of metal atoms after they strike the substrate surface. It is therefore necessary to condense a metal cooled to the lowest possible temperature on a well-cooled substrate with a high thermal conductivity. A method largely satisfying these requirements had been developed[11] for mercury films and then extended[10] to alkali metals. We used one of the variants of the equipment described in Ref. 10. It consisted of a glass ampoule, which was outgassed for several hours in 10^{-7} Torr vacuum at 400 °C, filled with a small amount of high-purity cesium, and then sealed. All the cesium inside the ampoule was sublimated on an evaporation source in the form of a platinum disk 16 mm in diameter located at 5 mm from a polished glass substrate 9 mm in diameter. This geometry ensured homogeneity of the film throughout the substrate surface. During the preparation of a film and in the course of the measurements the ampoule was immersed in liquid helium which ensured that vacuum was 10^{-12} Torr. When the rate of condensation was ~1 Å/min, the cesium atoms did not interact with one another and an ordered film consisting of randomly deposited atoms formed on the surface of a substrate characterized by surface irregularities with an rms size of ~100 Å. Four platinum current leads were soldered across the substrate and covered by evaporated platinum patches 0.7 mm in diameter located at the corners of a square of 3 ± 0.2 mm size, so that the same substrate could be used to measure the resistance and the Hall coefficient by switching between current and potential contacts. In all our experiments the voltage across a film did not exceed 20 mV and was measured with an F30 digital voltmeter. A magnetic field up to 5 T was created by a superconducting solenoid.

The precision of the experimental results was improved and the subsequent analysis and approximation of the results with the theoretical dependence (1) was made convenient by linking the apparatus to a computer which both controlled the experiments and recorded the results. Each experimental point was measured ten times and the results were averaged. The $R(T)$ dependences were obtained by storing about 1000 values of the resistance and temperature in the computer memory. The Hall coefficient was determined by recording the temperature dependences of the Hall emf for opposite directions of a magnetic field ($+3$ and -3 T). Values equidistant on the temperature scale were obtained by interpolating the resultant curves with spline functions and at each point we took the half-difference (in the case of the Hall coefficient) or the half-sum (in the case of the resistance measurements) of the interpolated results.

The magnetoresistance and the Hall emf were measured at a fixed temperature as follows. A magnetic field up to 3 T was applied in accordance with a computer-controlled program at a rate of 5×10^{-3} T/s and the temperature of a helium bath was measured using a carbon resistance thermometer. The error in the temperature measurements, found allowing for the magnetoresistance curve of the thermometer, did not exceed 5×10^{-3} K and the change in the temperature of helium during an experiment could be corrected subsequently. In a standard experiment we determined at each temperature 100 (tenfold averaged) values when increasing the field, the same number of values when decreasing the field. A total of ~80 magnetoresistance curves was obtained for films of different thicknesses at different temperatures.

The experimental results were approximated by Eq. (1) and again the computer was used. In most of the experimental investigations reported so far a comparison with the theory was made employing asymptotic expressions for the limit of weak fields when the quantum correction was $\delta G(H)\propto H^2$ or in the limit of high fields characterized by $\delta G(H)\propto \ln H$, and then the coefficients of proportionality were used to find the required values of H_i and H_{so}. The computer processing of the experimental results increased considerably their reliability because a comparison with the theoretical curve was made simultaneously at all the experimental points. The nonlinearity of the minimization problem did not make it possible to find the exact solution, so that in the program of finding the minimum of the rms deviation we used not only the Gauss-Newton method, but also the modified Newton method. In most cases the calculation time did not exceed several minutes.[1)]

4. EXPERIMENTAL RESULTS

Figures 1a and 1b show typical results of determination of the magnetoresistance of films at different temperatures.

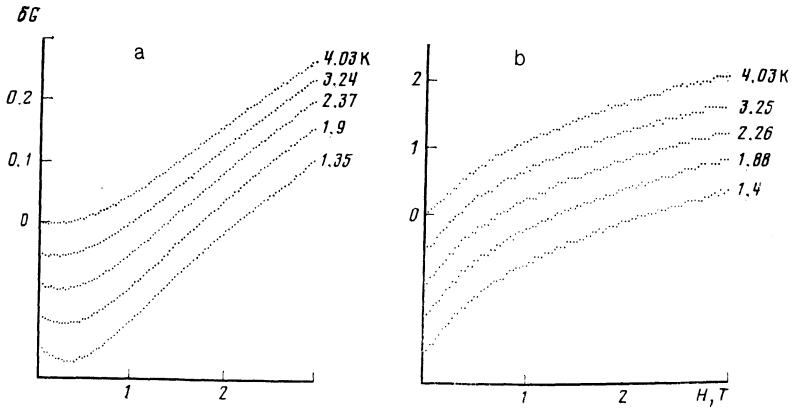

FIG. 1. Dependences of the conductance in units of $e^2/2\pi\hbar$ on the magnetic field obtained at various temperatures for the following films: a) film 6.4 Å thick with a resistance of 3487 Ω; b) film 31.5 Å thick with a resistance 67.8 Ω. Here and in Figs. 3 and 4 the curves are displaced for the sake of clarity along the ordinate; all the curves begin with $\delta G = 0$.

At first sight it seems that the behavior of the resistance in a magnetic field obeys Eq. (1): an anomalous negative magnetoresistance of the correct order of magnitude is observed for thin films (Fig. 1a) and in weak fields there is a region of positive magnetoresistance and the dimensions of this region increase as a result of cooling (increase in τ_i). However, it follows directly from these curves that the spin-orbit interaction is much less than the calculated value [Eq. (6)], which at these temperatures should have resulted in a positive magnetoresistance throughout the range of fields $H \leqslant 3$ T.

The results of the computer calculations indicate that the discrepancies between Eq. (1) and the experimental curves are large for all the films and clearly exceed the experimental error. In the case of thick low-resistivity films the disagreement with the theory manifested by the fact that the values of the fitting parameters H_i and particularly H_{so} are far from the preliminary limits. The best agreement with the experimental results is obtained for the inelastic scattering case characterized by H_i, which is practically independent of temperature, and in total absence of spin-orbit interaction. Naturally, an analysis of monatomic curves without a minimum represents a fairly rough approximation, but the zero value of H_{so} for all the thick films cannot be accidental.

More accurate results can be obtained by selecting those magnetoresistance curves which have positive and negative magnetoresistance regions. It is known[14] that in this case there is practically no indeterminacy in the selection of the fitting values H_i and H_{so}. Therefore, in an analysis of our results we concentrated attention on the alternating-sign dependences similar to those shown in Fig. 1a. This behavior was exhibited by cesium films with resistances close to 100 Ω. It was found that in the case of the alternating-sign magnetoresistance curves it was not possible to approximate the results accurately with the dependence (1) for any values of H_i, H_{so}, and H_s. Curves of the kind described by Eq. (1), ensuring the smallest rms deviation for all the experimental points, did not have any positive magnetoresistance region at all, i.e., they did not even qualitatively follow the alternating-sign experimental curves. The deviations themselves were large and the best approximation was very difficult to identify. Moreover, the values of H_i obtained as a result of such an analysis depended weakly on temperature, approximately to the same extent as the values of H_{so}, which according to Eq. (6) should be constant.

It therefore follows that an attempt to describe the magnetoresistance of cesium films by just delocalization of noninteracting electrons in a homogeneous film of a normal metal has not been successful and a search had to be made for other mechanisms of the loss of localization, which would make it possible to introduce an additional fitting parameter and describe the observed curves.

One possible explanation of the failure to satisfy Eq. (1) in the case of cesium films may be the scattering of electrons by superconducting fluctuations. Although according to Ref. 10 cesium films are not superconducting in the investigated range of temperatures, we cannot *a priori* exclude the possibility of a transition to the superconducting state at lower temperatures. If the experimental curves could be made to fit the theoretical dependence of Eq. (1) after allowance for the Maki-Thompson corrections which describe the scattering of electrons by superconducting fluctuations[3] and to obtain then the correct temperature dependences of H_i and H_s and of the parameter β representing the influence of the superconducting fluctuations[15]

$$\beta(T) - \frac{\pi^2}{6}\left(\ln\frac{T}{T_c}\right)^{-2}, \tag{7}$$

this would have made it possible not only to predict the existence of superconductivity in cesium films at lower temperatures, but also to determine the superconducting transition temperature from the change in the value of $\beta(T)$. Such an analysis was made by us for all the films characterized by $R > 100$ Ω. Three fitting parameters were fully sufficient to achieve agreement within the limits of the experimental error, but the values of H_i, H_{so}, and β obtained in this way and their temperature dependences demonstrated that this procedure was inconsistent.

Because of the very method used in their preparation, the investigated cesium films should be quite homogeneous in the investigated range of thicknesses. Nevertheless, we could not exclude the possibility of percolation effects, particularly in the case of high resistance films. The absence of a theory describing the two-dimensional case prevented us from a direct comparison, but it was possible to carry out a qualitative analysis by multiplying the whole quantum correction in Eq. (1) by a coefficient γ reducing the resistance to the square geometry and by varying this coefficient. It was reported in Ref. 9 that variation of this coefficient made it possible to match the observed curves with the theoretical dependences up to resistances of hundreds of kilo-ohms. The coefficient γ then increased from $\gamma \sim 1$ for $R \sim 10$ kΩ to $\gamma \sim 15$ for $R \sim 300$ kΩ, which physically meant an increase in the path along which the current was flowing. An analysis using fitting parameters H_i, H_{so}, and γ also failed to clear up

the situation. The major changes in H_{so} and γ with temperature indicated that this fitting procedure was incorrect. It was interesting to note that γ was found to be less than unity and, therefore, the quantum correction was greater than that expected. In this analysis the coefficient γ was found to be dependent on the film thickness. It varied monotonically from $\gamma \sim 0.5$ for thin films with $R \sim 5$ kΩ to $\gamma \sim 1$ for films with $R \sim 50$ Ω or less.

Some of the films were annealed at different temperatures right up to room value. The magnetoresistance curves then changed greatly and the films annealed at temperatures up to 200–250 K exhibited a positive magnetoresistance throughout the full range of fields $H \leqslant 3$ T. Again all the curves disagreed with Eq. (1).

In addition to the magnetoresistance, we determined the temperature dependences of the Hall coefficient of the majority of the investigated films. The error in the Hall coefficient was considerably greater, because in determining this coefficient it was necessary to record a much lower voltage than in determination of the resistance for the same current through a sample. Moreover, because of the nonideal geometry of the substrate, these components of the voltage were superimposed, which gave rise to an additional error. Consequently, the temperature-induced change in the Hall coefficient was only several times larger than the experimental error and the scatter of the data. Within the limits of the experimental error, we found that the relative change in the Hall coefficient with temperature for films with $R \gtrsim 50$ Ω was approximately twice as large as the relative change in the resistance. The value of the Hall coefficient for films with $R > 100$ Ω was 2–3 times less than the calculated value. The Hall emf varied linearly with the field within the limits of the experimental error (0.01%).

5. PROBLEM OF MAGNETIC IMPURITIES

The field dependences of the magnetoresistance observed for cesium films are in conflict with the formula applicable to the case of localization of noninteracting electrons [Eq. (1)] which has been checked on tens of previous occasions and confirmed in various experimental investigations, including those carried out in the same ranges of temperatures and fields. The question arises: why are cesium films special? The first possible answer is that these films contain magnetic impurities. In the case of thick films it has been suggested that magnetic impurities can account for a weak temperature dependence of H_i and zero values of H_{so}. It follows from Eq. (2) that introduction of H_s independent of temperature postulates the same value of H_{so}. In the case of thin films introduction of H_s as an additional parameter is insufficient to fit the experimental results to Eq. (1). However, if we assume that in fields of the order of $kT/g\mu_B$ the value of H_s itself begins to vary with the external magnetic field, then such an auxiliary function $H_s(H)$ could describe the experimental dependences.

As mentioned already, one of the reasons why cesium was selected for our experiments is the practically absolute absence of magnetic impurities in films. Moreover, the presence of impurities on the surface of a glass substrate is highly unlikely because of the careful washing and outgassing during the preparatory stages. It would also be useful to form, in a similar ampoule, and investigate by the same method a film of some other thoroughly investigated metal with a high atomic number Z and a strong spin-orbit scattering field H_{so}. We selected gold, investigated thoroughly by Bergmann[14] and later by other authors. The procedure used in the preparation of the films was the same as in Ref. 14. At 4.2 K we deposited a film with $R \sim 150$ Ω, but a subsequent annealing to several tens of kelvin resulted in an irreversible reduction of the resistance to 100 Ω. The substrates were the same in the case of cesium films and the only difference was an evaporation source which was now a drop of high-purity gold[2)] supported by a previously etched tungsten wire.

Our measurements were carried out in the temperature range 1.2–70 K. Above 4.2 K the experimental curves obeyed the theoretical relation (1) within the limits of the experimental error and the value of $H_{so} = 1.7$ T was reasonable (Bergmann[14] reported $H_{so} = 1.5$ T); a strong temperature dependence of the inelastic scattering field H_i was observed. The value of τ_i deduced from an analysis of the data on gold was tens of times less than the estimate given by Eq. (5) and was close to the results of Bergmann (Fig. 2). The continuous curve in this figure represents a straight line $\tau_i = 3.72 \times 10^{-11} T^{-1.14}$ characterized by the smallest rms deviation of the points in the range $T > 4.2$ K.

At temperatures below 4.2 K there were deviations from the dependence (1). A formal analysis carried out in this region gave rise to an error in the approximation, which increased at lower temperatures, and a constant (or even increasing) strength of inelastic scattering at low temperatures. The deviations from Eq. (1) were small (amounting to a few percent of the total magnetoresistance) and could be due to the fact that the magnetic field was no longer low ($g\mu_B H \gtrsim kT$). The constant component of H_i could be due to a small ($\sim 2 \times 10^{-5}$) amount of a magnetic impurity.

The fact that gold films had few magnetic impurities did not necessarily mean that there were none in cesium films prepared on the same substrates, but it did indicate that the number of magnetic impurities in cesium could not be sufficiently high to account for all the observed deviations.

6. INTERACTION OF ELECTRONS AND ITS CONTRIBUTION TO THE MAGNETORESISTANCE OF CESIUM FILMS

The conclusion that there were no magnetic impurities in the investigated films could be drawn already as a result of a qualitative analysis of the behavior of films of different

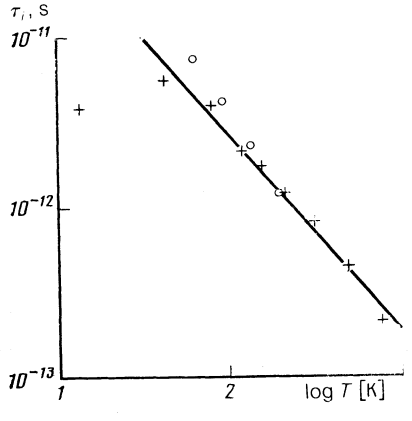

FIG. 2. Temperature dependence of the inelastic relaxation time for a film of gold with a resistance 100.8 Ω (crosses) deposited on the same substrate as cesium films. The circles represent the results of Bergmann[14] and the continuous curve is a plot of $\tau_i = 3.72 \times 10^{-11} T^{-1.14}$.

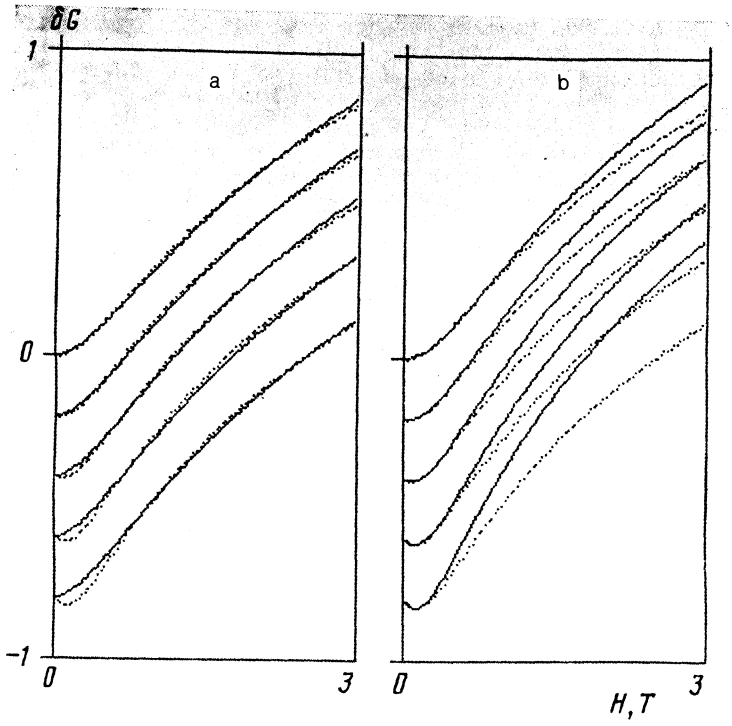

FIG. 3. Dependences of the conductance on the magnetic field (points) and its approximation based on Eq. (1) (continuous curves): a) throughout the full range of fields $H \leqslant 3$ T; b) only in weak fields $H \leqslant 0.2kT/g\mu_B$. Film thickness 11.5 Å, $R = 508$ Ω, temperature from 4.1 to 1.4 K.

thicknesses at different temperatures. It is clear from Figs. 1a and 1b that whereas at high fields $H > 1$ T the curves obtained at different temperatures practically coincided, in the lower fields there were large qualitative changes in the magnetoresistance which would have been impossible in the presence of magnetic impurities. In fact, the presence of such impurities gives rise to a characteristic temperature below which τ_i^{-1} becomes smaller than τ_s^{-1} and the curves cease to vary with temperature.

These qualitative conclusions can be confirmed by a quantitative analysis if we compare the results of the processing of the curves at all the experimental points (Fig. 3a) and the results obtained only for the initial parts of the plots (Fig. 3b). An analysis based on Eq. (1) in low fields is largely equivalent to the approximation by a quadratic parabola used by other authors. We carried out such an analysis for the ranges $H \leqslant 0.5$ T and $H \leqslant 1$ T for all the curves beginning from the very first systematic measurements. Moreover, some of the films were studied in detail in fields $H \leqslant 0.3$ T.

In the case of thick films (those in Fig. 1b and thicker) the initial temperature-dependent region was too narrow to obtain reliable values of H_i and H_{so} in view of the error in the experimental results. In the case of thin kilo-ohm films (those in Fig. 1a or thinner) the approximation of the initial part did not result in good agreement with Eq. (1) and did not predict the correct temperature dependences of H_i and H_{so}. However, there was always an intermediate range of thicknesses where the dependence (1) was already satisfied and the experimental error was sufficiently low for reliable determination of H_i and H_{so} (Fig. 3b). An analysis of the data obtained for films of these thicknesses, carried out in the limit of low fields $H \to 0$ (Figs. 3b and 4), gave a constant value of H_{so} and a value of H_i increasing linearly with temperature. The spin-orbit interaction was an order of magnitude less than $(\alpha Z)^4$, as expected from the negative sign of the magnetoresistance, and the frequency of inelastic collisions was close to the temperature $\hbar\tau_i^{-1} \propto kT$.

It is clear from Fig. 3b that the discrepancy between the approximation based on Eq. (1) and the experimental data in fields up to 3 T was of the order of the magnetoresistance itself. A comparison of the discrepancies at different temperatures indicated that they became lower when plotted as a function of the magnetic field reduced to temperature $h = g\mu_B H/kT$ (Fig. 5) and this made it possible to make a comparison with the theoretical dependence of the magnetoresistance allowing for the electron-electron interaction.

The correction to the conductance in a magnetic field due to the interaction of an electron and a hole with a total spin $j = 1$ is[16]

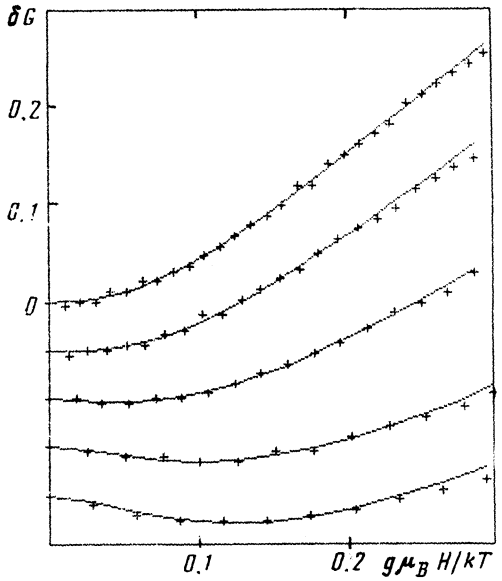

FIG. 4. Initial parts of the curves in Fig. 3b and their approximation by Eq. (1) in fields $H \leqslant 0.2kT/g\mu_B$.

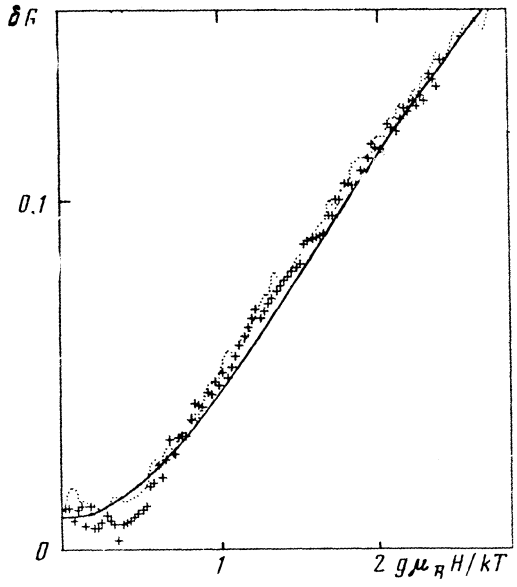

FIG. 5. Conductance not described by the localization of noninteracting electrons (difference between the experimental curve and the continuous curve in Fig. 3b plotted as a function of the reduced magnetic field $g\mu_B H/kT$. The points correspond to $T=1.67$ K and the crosses to $T=1.44$ K; the continuous curve is a plot of (8) with allowance for the interaction of electrons and including the parameter $\lambda=-0.253$.

$$\delta G(H,T)=\frac{\lambda}{2(\hbar D)^{d/2-1}}\int_0^\infty d\omega\,\frac{\partial^2(\omega\,\mathrm{cth}(\omega/2T))}{\partial^2\omega}$$
$$\times\{\varphi_d(\omega+g\mu_B H)+\varphi_d(|\omega-g\mu_B H|)-2\varphi_d(\omega)\}, \qquad (8)$$

where in the two-dimensional case we have $d=2$ and $\varphi_2(\omega)=\ln\omega$; this dependence can be written in the form

$$\delta G(h)=\frac{\lambda}{2}\int_0^\infty \frac{\partial^2(x\,\mathrm{cth}\,x)}{\partial x^2}\ln\left|1-\frac{h^2}{x^2}\right|dx. \qquad (9)$$

The function described by Eq. (8) (represented by the continuous curve in Fig. 5), like the digamma function of Eq. (1), gives rise to a logarithmic asymptote in the limit $H\to\infty$ and to a quadratic parabola for $H\to 0$. The constant λ is related to the screening factor F:

$$\lambda=\frac{32}{3}\,\frac{1+{}^3/_4F-(1+{}^1/_2F)^{3/2}}{F}. \qquad (10)$$

The factor F can in turn be estimated from the Fermi momentum and the reciprocal of the screening radius of the Coulomb interaction[17]:

$$F=\frac{2}{x^2}\ln(1+x^2),\qquad x=\frac{2p_F}{\hbar}. \qquad (11)$$

Therefore, the magnetoresistance associated with the interaction of electrons can be found simply knowing the interatomic spacing in cesium and assuming one conduction electron per atom. Such an estimate gives $F=0.365$ and $\lambda=-0.18$. The continuous curves in Fig. 3 correspond to the value $\lambda=-0.253$, which agrees within an order of magnitude to the value just found.

It is clear from the above estimates that the anomalous magnetoresistance of the type described by Eq. (9) should be the property of any metal film and it should be of the same order of magnitude for all the metals. However, as pointed out already, in the case of atoms of other metals the magnetoresistance is described well by Eq. (1) for noninteracting electrons and the correction associated with the interaction has not been observed experimentally. This is due to the fact such a correction is readily suppressed by even relatively small amounts of a magnetic impurity in a film or by any significant spin-orbit interaction. The dependence of the type described by Eq. (9) is then observed only in high fields such that $g\mu_B H\gg\frac{4}{3}\hbar(\tau_s^{-1}+\tau_{so}^{-1})$ (Ref. 16). In the case of cesium films the absence of magnetic impurities and the weak spin-orbit interaction make it possible to observe this effect practically throughout the investigated range of fields (Fig. 4).

In the case of thick films (Fig. 1a) and thin films in high fields $H>1$ T the superposition of the dependences for noninteracting [Eq. (1)] and interacting [Eq. (10)] electrons gives a combined curve which hardly varies with temperature, although the two contributions considered separately vary strongly, and which corresponds to the total absence of the spin-orbit interaction $H_{so}=0$. It is possible that it is not due to an accidental identity of the parameters, at least in the range of the logarithmic asymptotes of the dependences (1) and (9).

7. CHARACTERISTIC FEATURES OF COLD-DEPOSITED CESIUM FILMS AND WEAK SPIN-ORBIT INTERACTION

As stressed already, cold-deposited films are very homogeneous in respect of the thickness. The dependence of the resistance on the thickness can be used to show that a cesium film becomes continuous before it begins to conduct and that the classical percolation conduction threshold lies at lower thicknesses than the quantum threshold. In the majority of other films for which investigations of the anomalous magnetoresistance have been carried out the situation is opposite and metallic conduction appears at $R\sim\hbar/e^2$ because of the quantum tunneling between islands in a film which is discontinuous in the classical sense.

Figure 6 shows the dependence of the resistance of the thinnest cesium films on their thickness. The transition to metallic conduction occurs when the resistance is several tens of kilo-ohms and the thickness is about one monatomic layer. Calculations carried out using percolation theory indicate that in the case of accidental arrival of atoms on a substrate the first infinitely conducting clusters appear when the average thickness of the film is only ~ 0.5 monolayer. However, until the amount of the metal is sufficient for classical resistance of a conducting network to become comparable with $\hbar/e^2$, electrons do not have sufficient energy to travel large distances and they remain localized.

Even a qualitative comparison of the behavior of the conductance of a film in the submonatomic range of thicknesses with the hypothesis that the film is continuous and does not conduct only because of localization of electrons, requires an estimate of the classical conductance. It is not yet possible to calculate with any reasonable accuracy the resistance of such a conducting network, since it is necessary then to deal with many arbitrary assumptions about the film structure. Therefore, we calculated the classical resistance by extrapolation of the Fuchs equation from the range of thick (50–200 Å) films. This approximation is represented

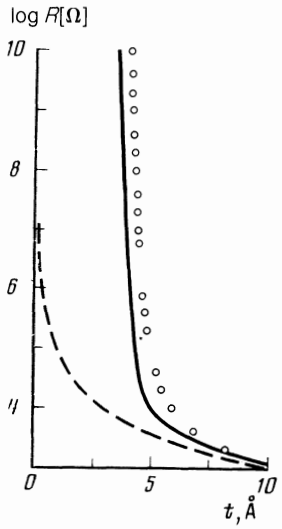

FIG. 6. Dependence of the resistance of a cesium film on its thickness at liquid helium temperature. The dashed curve is the approximation based on the Fuchs expression extrapolated from large thicknesses and the continuous curve is the same approximation corrected for the localization effects.

by a dashed curve in Fig. 6 and it is clearly far too rough because it is assumed that a submonatomic film is absolutely homogeneous in respect of its thickness and is characterized by a constant mean free path for electrons scattered within the film ($l_\infty = 273$ Å follows from the resistance of thick films).

The continuous curve in Fig. 6 shows how the classical resistance (dashed curve) increases if we allow for the localization effects. It is surprising that such a simple approximation as the Fuchs formula taken as the starting point predicts a dependence sufficiently close to the experimental one. This means that even in the case of a monatomic film the percolation effects do not yet lead to a strong increase in the current flow path.

In the region of weak localization $R < \hbar/e^2$ the calculated resistance differs from the classical value by a logarithmic correction

$$R^{-1}(t) = R_0^{-1}(t) - \frac{e^2}{2\pi^2\hbar}\ln\left(\frac{L}{t}\right), \qquad (12)$$

where R_0 depends on the thickness d in accordance with the Fuchs equation and the correction is governed by the dephasing length $L \propto (D\tau_i)^{1/2} \propto (D\tau\varepsilon_F/kT)^{1/2}$, which in this case plays in fact the role of the size of the sample. In the strong localization region $R > \hbar/e^2$ the wave functions of electrons decrease exponentially with distance and we have

$$R = \frac{2\pi^2\hbar}{e^2}\exp\left(\frac{L}{L_c}\right), \qquad (13)$$

where L_c is the length in which the film resistance becomes comparable with $2\pi^2\hbar/e^2$ can be found from the condition

$$R_0^{-1}(t) = \frac{e^2}{2\pi^2\hbar}\ln\left(\frac{L_c}{t}\right). \qquad (14)$$

These relationships allow us also to estimate the magnetoresistance in the strong localization region.

The continuous curve in Fig. 6 deduced from Eqs. (12)–(14) is practically universal. If we plot the thickness in atomic layers, we find that this curve describes the resistance of any metal condensed in such a way that there is no mobility of atoms on the substrate surface. The nature and shape of this curve are relatively little affected by a change in the Fuchs parameter l_∞, because in this range of thicknesses the elastic scattering of electrons in cesium films occurs mainly on the surface. However, if we assume that the mean free path is of the order of several atomic spacings, the thickness at the onset of metallic conduction amounts to about two monatomic layers, which corresponds to the experimental data on cold-deposited films of other metals.

In the case of cesium films, because of the large value of l_∞, the scattering by the surface determines the mean free path even in the case of relatively thick films with R = 50–100 Ω. This may be the reason for the observed weak spin-orbit interaction, which should be of the order of $(\alpha Z)^4$ only in the case of scattering by defects inside a given substance. The scattering by the surface gives[18] a spin-orbit contribution of the order of $\frac{1}{60}\alpha^4(\hbar k_F/me^2) \sim (\frac{1}{60})(v_F/c)^4$, which gives rise to negligible values of H_{so}. True, this small value applies only to the case of specular reflection, although in the case of cesium films it reaches tens of percent, but it is still not the dominant factor.

The observation that high-resistance cesium films are described well by Eqs. (18) and (14) ignoring the percolation effect demonstrates that in the case of low-resistance films we cannot expect inhomogeneities with large characteristic lengths $\sim (D\tau_i)^{1/2}$. Therefore, they cannot demonstrate quasi-one-dimensional effects[19] or saturation associated with the Aharonov-Bohm interference.[20] It is equally unlikely that any percolation effects may appear even in kiloohm films.

The good agreement between the theory and experiment (Fig. 5) obtained after correcting for the interaction (9) shows that cesium films exhibit an anomalous magnetoresistance associated with the electron-electron interaction and calculated earlier theoretically,[16] but not observed experimentally before because it has been suppressed by the spin-orbit interaction.

This investigation started during the lifetime of Academician A. I. Shal'nikov on his initiative and can be regarded as a continuation of his ideas on the physics of thin films. The author owes Aleksandr Iosifovich all that he knows and it is pleasant to confirm that these experiments confirm his ideas on the properties and structure of films.

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[1)]A very convenient and fast algorithm for the calculation of the digamma functions was proposed by Luke.[12] Unfortunately, the recurrence expression for the term B_n in Luke's book[13] suffers from a misprint. It should read

$$(2n-3)(n+1)B_n(z) = (2n-1)\{3(n-1)z+7n^2-9n-6\}B_{n-1}(z) - (2n-3)(z-n-1)\{3(n-1)z-7n^2+19n-4\}B_{n-2}(z) + (z-n)(2n-1)(n-3)(z-n-1)(z+n-4)B_{n-3}(z).$$

There is also a misprint in the Russian translation of Luke's book.[13] In the original paper[12] this expression is given correctly.

[2)]The author is grateful to Yu. P. Gaĭdukov for supplying gold samples checked for the absence of magnetic impurities and pure in the sense of the Kondo effect.

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