Interaction of a fluxon with a localized inhomogeneity in a long Josephson junction

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(Submitted 7 July 1987)
Zh. Eksp. Teor. Fiz. **94**, 356–365 (April 1988)

The adiabatic and radiative effects which arise when a fluxon interacts with a localized inhomogeneity consisting of a combination of a microshort or microresistance with a dissipative inhomogeneity are analyzed by perturbation theory. An inhomogeneity of this sort could be produced in a long Josephson junction by, for example, a focused laser beam or a localized short circuit of a normal metal. The threshold values of the static external field which permit the capture of a fluxon by such an inhomogeneity are calculated numerically and analytically for various cases. The energy of the plasma waves (Svihart waves) radiated by a fluxon as it is scattered by a composite inhomogeneity is also calculated. The influence of these effects on the current-voltage characteristic of a long junction is discussed.

1. INTRODUCTION AND STATEMENT OF THE PROBLEM

There is considerable physical interest in the dynamics of nonlinear excitations in long Josephson junctions.[1–3] Various inhomogeneities are usually built into a junction in order to impart given properties to it (Ref. 2, for example). Several theoretical papers have been devoted to the dynamics of fluxons (magnetic flux quanta or Josephson vortices) in inhomogeneous junctions.[4–10] It has been suggested that the radiation of quasilinear plasma waves (Svihart waves) by a fluxon moving in a long annular junction with a periodic sequence of microshorts be utilized[1)] to develop a Josephson microwave source.[4]

A static external current applied to a junction plays the role of an external force acting on a fluxon. If the current is sufficiently low, the interaction of the fluxon with the microshort may result in capture (or pinning) of the fluxon.[4] This effect also may find a variety of applications.[1–3]

In the present paper we analyze the interaction of a fluxon with a localized inhomogeneity in a long Josephson junction in a situation with an external current and with a dissipative loss. We use a model described by the perturbed sine-Gordon equation

$$\varphi_{tt}-\varphi_{xx}+\sin\varphi=-f-\gamma\varphi_t-\beta\delta(x)\varphi_t+\varepsilon\delta(x)\sin\varphi. \qquad (1)$$

Here $\varphi(x,t)=\Phi(x,t)/\Phi_0$ is the magnetic flux, normalized to the corresponding quantum, $\Phi_0=\hbar c/2e$; $f=J/J_{\max}$ is the dimensionless density of the external current, normalized to the maximum Josephson current density $J_{\max}$; and the dimensionless dissipation coefficient γ stems from the tunneling of normal electrons across the junction. The coordinate x, along the junction, and the time t are expressed in units of the Josephson penetration depth λ_J and the reciprocal of the Josephson plasma frequency ω_J, respectively.[2] In the case $\beta=0$, Eq. (1) describes the well-known model of a long Josephson junction with a locally inhomogeneous maximum Josephson current density.[4] In case $\varepsilon<0$ corresponds to a microshort, and $\varepsilon>0$ to a microresistance. A local inhomogeneity of a more general type alters not only the maximum Josephson current density, as described by the term $\varepsilon\delta(x)\sin\varphi$, but also the local value of the dissipation coefficient, which corresponds to the additional term $-\beta\delta(x)\varphi_t$ ($\beta>0$). A local inhomogeneity of this type arises, for example, as the result of the application of a focused laser beam to a junction.[12] Equation (1) was proposed as a model to describe such an inhomogeneity by Chang.[13] The same mode can be used to describe a long Josephson junction with a localized short-circuit made from a normal metal. The interaction of a fluxon with a short circuit of this type was studied experimentally by Akoh *et al.*[14] In the absence of perturbations, i.e., with $f=\gamma=\beta=\varepsilon=0$, the fluxon is described by the well-known solution of the sine-Gordon equation in the form of a so-called kink:

$$\varphi_k(x,t;v)=4\,\mathrm{arctg}\left[\exp\left(\sigma\frac{x-\xi(t)}{(1-v^2)^{1/2}}\right)\right], \qquad (2)$$

where $\xi(t)=vt$ is the coordinate of the center of mass of the fluxon, v is its velocity ($v^2<1$), and $\sigma=\pm1$ is its polarity. In a homogeneous long Josephson junction, with a static external current and a dissipation (i.e., with $\beta=\varepsilon=0$; f, $\gamma\ll1$), the fluxon is described by the following approximate solution of Eq. (1):

$$\varphi_f(x,t)=f+\varphi_k(x,t;v_0), \qquad (3)$$

where the dimensionless fluxon velocity v_0 is determined unambiguously by the balance between the external force and the frictional force,[4]

$$v_0=\sigma[1+(4\gamma/\pi f)^2]^{-1/2}. \qquad (4)$$

In the present paper, except in Sec. 3, we will be dealing with the "nonrelativistic" case $v_0^2\ll1$, i.e.,

$$v_0\approx\pi\sigma f/4\gamma, \qquad (5)$$

$$f\ll\gamma \qquad (6)$$

(for definiteness, we are assuming $f>0$).

When there is an inhomogeneity, the motion of a fluxon can be described by perturbation theory.[4] It is easy to show that in the adiabatic approximation (in which the radiative loss and the distortions of the shape of the fluxon are ignored) the equation of motion for the coordinates ξ of a "nonrelativistic" fluxon is the same as the equation for a classical particle of mass $m=8$ which is moving in a potential

$$U(\xi)=-2\pi\sigma f\xi-2\varepsilon\,\mathrm{sech}^2\,\xi\equiv-2\pi\sigma f\xi+U_0(\xi) \qquad (7)$$

with a friction force

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$$F = -8\gamma \frac{d\xi}{dt} - 4\beta \frac{d\xi}{dt} \operatorname{sech}^2 \xi = F_1 + F_2. \quad (8)$$

The capture of a fluxon by a microshort ($\beta = 0$, $\varepsilon < 0$) was examined in the well-known study by McLaughlin and Scott[4] (a microshort repels a fluxon, regardless of its polarity). Under the condition

$$|\varepsilon| \gg \gamma^2 \quad (9)$$

the maximum (threshold) value (f_{thr}) of the external current which permits the capture of a fluxon by an inhomogeneity of this type was found by equating the kinetic energy of a fluxon moving far from the inhomogeneity, $E_{kin} \approx 4v_0^2$, to the height of the potential barrier, $2|\varepsilon|$ [see (7)]:

$$f_{thr} = (2\gamma/\pi)(2|\varepsilon|)^{1/2}. \quad (10)$$

A microresistance differs from a microshort in that it attracts a fluxon. In Sec. 2 we analytically and numerically calculate the threshold value of the external current, f_{thr}, which allows the capture of a fluxon by an inhomogeneity of this type. This threshold turns out to be substantially lower than (10). We also calculate the correction to expression (10) for a microshort, and through a comparison with the numerical results we show that this correction is important. In the same section we calculate the threshold value f_{thr} for a local inhomogeneity of a general type, i.e., with $\beta \neq 0$, for either sign of ε.

These results were derived in the adiabatic approximation. At the same time, it is worthwhile to examine the radiative effects which accompany the interaction of a fluxon with an inhomogeneity. In Sec. 3 we calculate the total energy radiated by a fluxon as it is scattered by a purely dissipative inhomogeneity ($\varepsilon = 0$), and we show that this energy is substantially greater than the prediction of Refs. 4, 5, and 15 of the energy radiated by a fluxon as it is scattered by an ordinary (nondissipative) inhomogeneity.

In the final section, Sec. 4, we briefly discuss the influence of these effects on the current-voltage characteristic of a long junction with a composite inhomogeneity.

2. CAPTURE OF A FLUXON BY AN INHOMOGENEITY

2.1 Microresistance ($\beta = 0$, $\varepsilon > 0$). In the case $\varepsilon > 0$, the capture of a fluxon is possible under the necessary condition $f < f_c \equiv (4\sqrt{3}/9\pi)$ in which case the corresponding potential, (7), has two equilibrium positions, $dU/d\xi = 0$, which are determined by the equation

$$\operatorname{th} \xi (1 - \operatorname{th}^2 \xi) = \pi f / 2\varepsilon \quad (11a)$$

(here and below we are assuming $\sigma = +1$ for definiteness). It is easy to see that the smaller root, ξ_1, corresponds to a stable equilibrium (a minimum of the potential), while the larger, ξ_2, corresponds to an unstable equilibrium (a maximum). In particular, under the condition $f/\varepsilon \ll 1$ we have

$$\xi_1 \approx \pi f / 2\varepsilon, \quad \xi_2 \approx {}^1/_2 \ln(\varepsilon / f). \quad (11b)$$

To determine the complete conditions for the capture of a nonrelativistic fluxon, we write a balance equation for its kinetic energy $E_{kin} \equiv 4(d\xi/dt)^2$. From (7) and (8) we have

$$\frac{d}{dt} E_{kin} = \left(-8\gamma \frac{d\xi}{dt} + 2\pi f - \frac{dU}{d\xi} \right) \frac{d\xi}{dt}. \quad (12)$$

The threshold value of the external current which allows capture, f_{thr}, is determined by the condition that the velocity $d\xi/dt$ vanishes at the point $\xi = \xi_2$ (in the limit $t \to \infty$). In the opposite limit $t \to -\infty$, i.e., in the limit $\xi \to -\infty$, $d\xi/dt$ takes on the value (5). Accordingly, by integrating Eq. (12) we find an equation which determines f_{thr}:

$$4v_0^2 = \int_{-\infty}^{\xi_2} \left(8\gamma \frac{d\xi}{dt} - 2\pi f_{thr} \right) d\xi + U_0(\xi_2). \quad (13)$$

Let us assume $f_{thr} \ll \varepsilon$. The right side of (13) will then be dominated by the integral over the region $|\xi| \lesssim 1$, where the law of motion takes the approximate form

$$d\xi/dt = (\varepsilon/2)^{1/2} \operatorname{sech} \xi. \quad (14)$$

The next contribution comes from the region $1 \ll \xi \lesssim \xi_2$, where, by virtue of our assumption $f_{thr} \ll \varepsilon$, the value of ξ_2 is determined by (11b) with $f = f_{thr}$. As a result, we find the final expression

$$G = (2\pi\Gamma^3)^{1/2} + {}^9/_4 \Gamma^2 \ln \Gamma, \quad (15)$$

$$G \equiv \pi f_{thr} / 2\varepsilon, \quad \Gamma \equiv \gamma (2/\varepsilon)^{1/2}. \quad (16)$$

If follows from (15) and (16) that the incorporation of the term $U_0(\xi_2)$ in (13) leads to a correction $\sim \Gamma^2$ to (15). With regard to our original assumption $f_{thr} \ll \varepsilon$, i.e., $G \ll 1$, we note that it reduces to the condition $\Gamma \ll 1$ [condition (9)].

To determine the functional dependence $G(\Gamma)$ throughout the region $\Gamma \lesssim 1$, we have numerically integrated the complete equation of motion of a nonrelativistic fluxon:

$$8 \frac{d^2\xi}{dt^2} = F - \frac{dU}{d\xi}, \quad (17)$$

where U and F are defined in (7) and (8). The results of this calculation are shown in Fig. 1. At $\Gamma = \Gamma_{max} \approx 1.14$, the value $f_{thr} = f_c \equiv (4\sqrt{3}/9\pi)\varepsilon$, is reached; i.e., we reach the value $G = G_{max} = 2\sqrt{3}/9$, at which bound states disappear. As $\Gamma \to \Gamma_{max}$ we have $dG/d\Gamma \to 0$. This result could be found by analyzing the phase paths of an effective particle at $\Gamma \approx \Gamma_{max}$. This result is confirmed by the numerical calculations. In the region $\Gamma > \Gamma_{max}$, a capture occurs at all values $G < 2\sqrt{3}/9$ (Fig. 1). Incorporating the correction term in

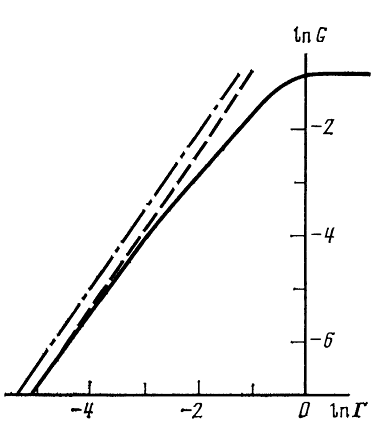

FIG. 1. The threshold external current f_{thr} versus the parameters ε and γ for a dissipationless microresistance [the quantities G and Γ are defined in (16)]. Solid line—Result of a numerical solution of Eq. (17); dashed line—complete analytic functional dependence (15); dot-dashed line—dependence (15) without the second term.

(15) substantially improves the agreement between the analytic dependence and the numerical dependence.[2)]

2.2 Microshort ($\beta=0$, $\varepsilon<0$). In the case $\varepsilon<0$, the equilibrium positions in effective potential (7) exist under the same condition, $f<f_c\equiv(4\sqrt{3}/9\pi)|\varepsilon|$, as in the case $\varepsilon>0$, and these positions are again given by Eq. (11a). Since ε is negative, both of the real roots of this equation are negative. The root which is larger in magnitude, ξ_1, corresponds to a stable equilibrium, while the smaller in magnitude, ξ_2, corresponds to an unstable equilibrium. In particular, under the condition $f\ll|\varepsilon|$ we have

$$\xi_1\approx-{}^1/_2\ln(|\varepsilon|/f),\quad \xi_2\approx-\pi f/2|\varepsilon| \tag{18}$$

[cf. (11b)].

As was mentioned above, the value given in (10) for f_{thr} for the case $\varepsilon<0$ was found under condition (9) in Ref. 4. In the notation

$$G=\pi f_{thr}/2|\varepsilon|,\quad \Gamma=\gamma(2/|\varepsilon|)^{1/2}$$

[cf. (16)], relation (10) take the form $G=\Gamma$. As we will see below, from a comparison with the numerical results, it is important to find the correction $\sim\Gamma^2$ to (10). As in the case $\varepsilon>0$, the energy balance equation is written in the form (12), and the threshold-capture condition—the condition that $d\xi/dt$ must vanish at $\xi=\xi_2$—leads to Eq. (13) for f_{thr}. In contrast with the preceding case, the basic approximation which leads to (10) is determined not by the integral term on the right-hand side of (13) but by the term $U(\xi_2)$. To find the first correction to (10) it is sufficient to substitute into the integral the approximate law of motion of a fluxon at $|\xi|\lesssim1$ which follows from the general equations (17), (7), (8), where f_{thr} is taken in the form in (10):

$$d\xi/dt=(|\varepsilon|/2)^{1/2}\,\mathrm{th}\,\xi \tag{19}$$

[cf. (14)]. As a result we find the following refined expression for the thershold value of the external current:

$$G=\Gamma-\Gamma^2\ln 2. \tag{20}$$

Expression (20), like (15), was derived under the assumption $\Gamma\ll1$. Nevertheless, the results of a numerical calculation of the pinning threshold show that this expression, in contrast with (10), gives a fairly good approximation to the function $G(\Gamma)$ even at nonzero values of Γ (Fig. 2). According to (20), the maximum value $G=1/4\ln2\approx0.360$ is reached at $\Gamma=1/2\ln2\approx0.721$. Each of these quantities differs by less than 8.6% from the values found through the numerical calculations: $G_{max}=0.385$ and $\Gamma_{max}=0.788$. In the limit $\Gamma\to\Gamma_{max}$, as in the case of an attractive microscopic inhomogeneity, we have $dG/d\Gamma\to0$. In the region $\Gamma>\Gamma_{max}$, capture occurs for any $G<G_{max}$.

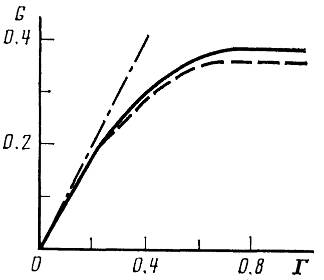

FIG. 2. Threshold external current f_{thr} versus the parameters ε and γ for a dissipationless microshort (the parameters G and Γ are defined in the text proper). Solid line—Result of numerical calculations; dashed line—dependence (20); dot-dashed line—dependence (10).

Comparison of Figs. 1 and 2 leads to the conclusion that the threshold value of the external current in the case of an attractive microscopic inhomogeneity in the region $\Gamma<1.14$ is lower (considerably lower if Γ is small) than for a repulsive inhomogeneity.

2.3 Dissipative inhomogeneity ($\beta\neq0$). We turn now to a study of the capture of a fluxon by a dissipative inhomogeneity. This effect has been observed experimentally.[14] We first consider the case $\varepsilon<0$ (a combination of a microshort and a dissipative inhomogeneity). In the limit of small values of Γ and G, we can set

$$U(\xi)\approx U_0(\xi),\quad F\approx F_2,\quad \xi_2=0 \tag{21}$$

in the equation describing the motion of an effective particle [see (7), (8), and (18)]. It can be shown that the path of a particle corresponding to the pinning threshold, i.e., satisfying the condition $\xi\to\xi_2$ as $t\to\infty$, is described by the law of motion

$$d\xi/dt={}^1/_4[\beta+(\beta^2+8|\varepsilon|)^{1/2}]\,\mathrm{th}\,\xi.$$

From the condition $d\xi/dt(-\infty)=v_0$ we finally find

$$G={}^1/_2\Gamma[B+(B^2+4)^{1/2}], \tag{22}$$

where we have introduced

$$B=\beta/(2|\varepsilon|)^{1/2}. \tag{23}$$

At small values of Γ, expression (22) is confirmed to high accuracy by the numerical calculations.

At finite values of G, an analytic result can be found under the condition $B\gg1\gg\Gamma$. Integrating the approximate equation of motion

$$\frac{d^2\xi}{dt^2}=-\frac{1}{2}\beta\frac{d\xi}{dt}\mathrm{sech}^2\,\xi$$

for this case, we find

$$\frac{d\xi}{dt}=v_0-\frac{1}{2}\beta(1+\mathrm{th}\,\xi). \tag{24}$$

At $f=f_{thr}$ the fluxon stopping point is the same as the point at which the potential reaches its maximum, which is given by Eq. (11a). We then find a relation among G, Γ, and B which can be put in the form

$$G/C(G)=\Gamma B,\quad C(G)=1+\mathrm{th}\,\xi_2(G). \tag{25}$$

With increasing ΓB, the value of G increases monotonically from 0 to $G_{max}=2\sqrt{3}/9$; the coefficient C falls off from 1 to $1-\sqrt{3}/3$. The value of the parameter Γ_{max}, above which capture occurs for any $G<G_{max}$, can be found analytically in the case $B\gg1$:

$$\Gamma_{max}=G_{max}/B(1-\sqrt{3}/3).$$

We have calculated Γ_{max} numerically (line 1 in Fig. 3) for finite values of the parameter B. The results satisfy the

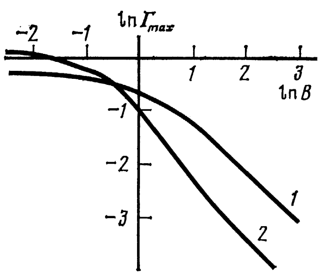

FIG. 3. The parameter Γ_{max} versus the ratio of the intensities of a dissipative inhomogeneity and of an ordinary inhomogeneity for (1) a microshort and (2) a microresistance.

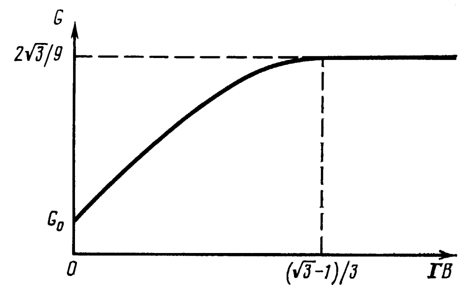

FIG. 4. The threshold external current $f_{thr} \sim G$ [see (16)] as a function of the parameter $\gamma\beta \sim \Gamma B$ [see (16), (23)] at a fixed value of ε. According to (15), the value $B = 0$ corresponds to $G_0 \approx (2\pi\Gamma^3)^{1/2}$.

asymptotic expressions found above for the cases $B \to 0$ and $B \gg 1$.

We turn now to a study of the interaction of a fluxon with an inhomogeneity of a different type, with $\varepsilon > 0$ (a "hybrid" of a microresistance and a dissipative inhomogeneity). Analytic expressions can be found for the fluxon capture threshold in the case $\Gamma \ll 1$ the two limiting cases $B \ll 1$ and $B \gg 1$.

We first assume $\Gamma \ll B \ll 1$. As we will show below, the relation $G \ll 1$ holds in this case. The threshold value of the external current is calculated by a procedure similar to that which we used in Subsec. 2.1, but in this case we take account of the circumstance that the change in the kinetic energy of the particle is now determined primarily by the dissipative loss associated with friction force F_2 [see (8)]. This loss is equal to $\pi\beta\varepsilon^{1/2}$. From the energy balance equation we find

$$G = \Gamma(\pi B)^{1/2}. \tag{26}$$

In the case $B \gg 1 \gg \Gamma$, the requirement that the fluxon stopping point coincide with the maximum of potential (7) [which is determined by Eq. (11a)] leads us again to relations (25). A straightforward analysis of Eqs. (11a) and (25) shows that we have

$$G = \Gamma B[(1-4\Gamma B)^{1/2}+3]/2 \tag{26a}$$

for $\Gamma B < (\sqrt{3} - 1)/3$. For $\Gamma B = (\sqrt{3} - 1)/3$, expression (26a) reaches the value $G_{max} = 2\sqrt{3}/9$, at which the minimum and the maximum of the potential (7) merge and disappear; i.e., we have $G \equiv G_{max}$ as $\Gamma B > (\sqrt{3} - 1)/3$. Figure 4 shows the complete functional dependence $G(\Gamma B)$ in this case ($\varepsilon > 0$, $B \gg 1 \gg \Gamma$) (cf. Fig. 2). According to (26a), we have $dG/d(\Gamma B) = 0$ at the point $\Gamma B = (\sqrt{3} - 1)/3$; i.e., the functional dependence $G(\Gamma B)$ is "smooth" at this point.

In summary, at a fixed value of B, the quantity G increases with increasing Γ in accordance with (26a), reaching the value G_{max} at $\Gamma = \Gamma_{max} = (\sqrt{3} - 1)/3B$. For $\Gamma > \Gamma_{max}$, the capture of a fluxon by an inhomogeneity of the type under consideration ($\varepsilon > 0$, $\beta \neq 0$) occurs for all $G < G_{max}$. For arbitrary $B \gtrless 1$ and $\Gamma \lesssim 1$, the functional dependence $\Gamma_{max}(B)$ can be found numerically (line 2 in Fig. 3). For $B \gg 1$, the numerical dependence can be approximated well by the expression $\Gamma_{max} = (\sqrt{3} - 1)/3B$ which was found above.

Finally, for $B \sim 1$ and small values of Γ, we can find the following result analytically: $G = \Gamma V(B)$ where $V(B)$ is some function with the asymptotic behavior $V \approx (\pi B)^{1/2}$ as $B \to 0$ [cf. (26)] and $V \approx 2B$ as $B \to \infty$ [cf. the asymptotic behavior of expression (26a) as $\Gamma B \to 0$].

3. RADIATIVE LOSS

The results reported above were derived in the adiabatic approximation. There is also considerable interest in studying the radiative effects which accompany the scattering of a fluxon by an inhomogeneity. In terms of the inverse scattering method,[16] the radiative component of the wave field described by the sine-Gordon equation is characterized by a complex amplitude (the so-called Jost coefficient of the scattering problem) $b(\lambda)$, where the real spectral parameter λ is related to the frequency ω and the wave number k of the linear waves which are excited: $\omega = \lambda + 1/4\lambda$, $k = \lambda - 1/4\lambda$. The evolution of the coefficient $b(\lambda)$ governed by the perturbation is determined by a well-known equation,[17,18] which takes the following form in our case:

$$\frac{dB(\lambda,t)}{dt} = \frac{i\varepsilon}{4} a(\lambda) \int_{-\infty}^{\infty} dx\, P(\varphi) \{[\Psi_2^*(x,t;\lambda)]^2 - [\Psi_1^*(x,t;\lambda)]^2\}. \tag{27}$$

Here $B(\lambda,t) \equiv b(\lambda,t)\exp(i\omega(\lambda)t)$, $\varepsilon P(\varphi)$ specifies the perturbation, i.e., the right side of (1); and $\Psi_{1,2}(x,t;\lambda)$ are the components of the so-called Jost function (the normalized eigenfunction of the auxiliary scattering problem associated with the sine-Gordon equation[16]). A general method for calculating the radiation generated when perturbations act on solitons was formulated in Refs. 15, 19, and 20. Under the assumption that there is no radiation before the interaction, i.e., choosing $B(\lambda, t = -\infty) = 0$, we can find the final value of the radiation amplitude,[20] $B_f(\lambda)$:

$$B_f(\lambda) \equiv B(\lambda, t=+\infty) = \int_{-\infty}^{\infty} \frac{dB(\lambda,t)}{dt} dt. \tag{28}$$

The basic physical characteristic of the radiated waves is the spectral density $\mathscr{E}(k)$ of their total energy E_{em} (see, for example, Refs. 11, 15, 19, and 20):

$$\mathscr{E}(k) = \frac{dE_{em}}{dk} = \frac{4}{\pi} \left| B_f\left(\lambda = \frac{1}{2}[k + (1+k^2)^{1/2}]\right)\right|^2. \tag{29}$$

A calculation of this quantity with the help of expressions (27)–(29) leads to the following result for the spectral density of the radiated energy as a function of the wave number k:

$$\mathscr{E}(k)=\frac{\pi[\varepsilon^2(1-v_0^2)^2[(1+k^2)^{1/2}-kv_0]^2+4\beta^2v_0^4]}{4v_0^6\,\mathrm{ch}^2[\pi(1-v_0^2)^{1/2}(1+k^2)^{1/2}/2v_0]},\qquad(30)$$

where v_0 is defined in (4). It can be seen from (30) that the spectral density of the energy radiated when a fluxon is scattered by a "composite" inhomogeneity ($\varepsilon\neq0$, $\beta\neq0$) is equal to the sum of the spectral densities radiated in the cases $\varepsilon=0$ and $\beta=0$ (the latter was calculated previously in Ref. 15). This circumstance—the absence of interference—is a consequence of the fact that the phases of the waves excited by the terms $\sim\varepsilon$ and $\sim\beta$ on the right side of (1) differ by $\pi/2$. We also note tht in the case $\beta=0$, $\varepsilon\neq0$ nearly all of the energy is radiated backward with respect to the direction in which the fluxon is moving [the backward direction corresponds to $v_0k<0$ in (30)], while in the case $\varepsilon=0$, $\beta\neq0$ the spectral density is symmetric (it does not depend on the sign of k).

The total radiated energy can be calculated in two limiting cases: the case $\varepsilon^2, \beta^2\ll v_0^2\ll1$ and the case $1-v_0^2\ll1$. In the first of these cases we find from (30)

$$E_{em}=\int_{-\infty}^{\infty}\mathscr{E}(k)\,dk\approx4\frac{\pi\varepsilon^2}{2^{1/2}}v_0^{-11/2}\exp(-\pi/v_0).\qquad(31)$$

In other words, for small values of v_0 the radiated energy is exponentially small and is determined primarily by the first term in (30). In the nonrelativistic case, the radiative loss is actually negligible in comparison with the dissipative loss $E_{\rm diss}=8\gamma v_0/(1-v_0^2)^{1/2}$. In the second case, $1-v_0^2\ll1$, we have

$$E_{em}\approx2(\varepsilon^2/3)(1-v_0^2)^{1/2}+4\beta^2(1-v_0^2)^{-1/2}.\qquad(32)$$

In other words, the total radiated energy in the "ultrarelativistic" limit is determined by the dissipative properties of the inhomogeneity, while the contribution of the first term in (30) ($\sim\varepsilon^2$) to the radiation tends toward zero. This result means that the dependence of the total radiated energy E_{em} on the fluxon velocity v_0 is definitely not monotonic in the case $\beta\sim\varepsilon$; it goes through a maximum at $v_0\lesssim1$, due to the contribution of the first term in (30). Furthermore, the increase in the radiated energy as $v_0\to1$ for dissipative inhomogeneities means that the use of dissipative inhomogeneities to develop a microwave source using a Josephson junction may be more effective than to use microshorts for similar purposes.

4. CURRENT-VOLTAGE CHARACTERISTIC OF A LONG JUNCTION WITH A COMPOSITE INHOMOGENEITY

Let us briefly discuss the manifestation of the analytic results derived above in a measurement of the current-voltage characteristic of a long junction with a composite inhomogeneity in which there is one fluxon. An isolated fluxon would be seen experimentally either as a "shuttle flux quantum" oscillating between the reflecting ends of a linear junction of finite length[21] $L(L\gg\lambda_J)$ or in uniform circular motion in a uniform annular junction.[22] In either case, the potential difference V across the junction would be $\Phi_0v_0c_0/Lc$, where c is the velocity of light in vacuum, and c_0 is the Svihart velocity [the corrections to V for end effects were found in Ref. 23 for a linear junction of finite length; for an annular junction of finite circumference, there are obviously no such corrections, but there are some small corrections to expression (4) because of the difference between the shape of the kink in a problem with periodic boundary conditions and the shape corresponding to expression (2)].

If a long Josephson junction has a single inhomogeneity of the type described above, then over one half-period $T/2=L/v_0$ the fluxon radiates, during the scattering, the energy D_{em} which was found in Sec. 3. In an oscillatory motion, the power of the radiation generated by a fluxon is $P(v_0)=v_0E_{\rm em}(v_0)/2L$. The radiative loss ($\sim\varepsilon^2,\beta^2$) and the dissipative loss ($\sim\gamma$) are balanced by the work performed by the external force (the external current). The energy balance condition leads to the relation

$$2\pi v_0f=8\gamma v_0^2/(1-v_0^2)^{1/2}+P(v_0),\qquad(33)$$

in which we have ignored the change in the fluxon velocity near the inhomogeneity, as we are justified in doing under the conditions $v_0^2\gg\varepsilon,\beta$. Since $f=J/J_{\max}$, and $v_0=LcV/\Phi_0c_0$, relation (33) actually determines the shape of the current-voltage characteristic (the dependence of J on V) of a long Josephson junction with an inhomogeneity in a region in which there is no pinning of a fluxon near an inhomogeneity. Figure 5 shows this current-voltage characteristic. In the case $\varepsilon^2\gg\gamma$, the function $f(v_0)$ is not monotonic, and it contains hysteresis (the presence of hysteresis in the current-voltage characteristic of a junction containing nondissipative inhomogeneities was pointed out in Ref. 6). As can be seen from (32), however, regardless of the relation among the parameters the asymptotic behavior of the function $f(v_0)$ in the limit $v_0^2\to1$ is substantially different from that of the current-voltage characteristic of a homogeneous junction (the dashed line in Fig. 5). As the external current is reduced to $f_{\rm thr}$, the fluxon becomes captured by an inhomogeneity, and the voltage across the junction falls abruptly to zero (Fig. 5). If, on the other hand, the external current is increased again, the fluxon will remain at rest until the current supplied to the junction reaches the critical value of c corresponding to the disappearance of the bound state. At f_c, the fluxon is necessarily torn away from the inhomogene-

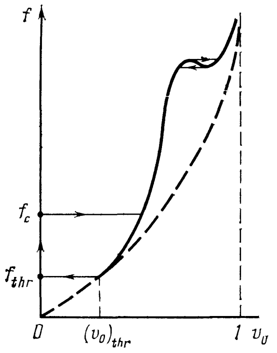

FIG. 5. Current-voltage characteristic of a long junction with a composite inhomogeneity in the case $\gamma\ll\varepsilon^2$. Here $f_c=(4\sqrt{3}/9\pi)|\varepsilon|$ is the current at which the fluxon is "torn away" from the inhomogeneity, and $f_{\rm thr}\equiv f_{\rm thr}(\varepsilon,\beta)$ is the current at which a fluxon is captured by an inhomogeneity. Dashed line—Current-voltage characteristic (4) of a homogeneous junction ($\varepsilon=0,\beta=0$).

ity, and it is accelerated until the external force becomes balanced by the ohmic (dissipative) and radiative losses.

In summary, the threshold value $f_{\rm thr}$ and the critical value f_c of the external current are clearly evident on the current-voltage characteristic of the junction. Since f_c is determined exclusively by the strength of the effective potential of the inhomogeneity (by the parameter ε), while $f_{\rm thr}$ is also determined by the dissipative properties of the inhomogeneity (the parameters ε and β), one can determine the parameters of inhomogeneities in any of the cases discussed above by measuring these quantities.

We are indebted to A. V. Ustinov for calling our attention to Ref. 14. We are also indebted to J. J. Chang for sending us a copy of Ref. 13.

[1)]For these purposes one could also use the radiation generated by a collision of two fluxons of opposite polarities in a homogeneous junction.[11]

[2)]Actually, the dashed line in Fig. 1 corresponds not to expression (15) but to its inverse, $\Gamma = (G^2/2\pi)^{1/2} - (1/2\pi)G \ln G$, which was derived from (15), with the same accuracy. This inverse leads to an agreement with the numerical results better than that achieved with (15).

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Translated by Dave Parsons