B^0–$\overline{B}_0$ oscillations and parameters of the standard model

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The production of same-sign dileptons in semileptonic decays of the $(B_d^0\overline{B}_d^0)$ system is observed in the experiments of the ARGUS group, indicating the presence of large B^0–$\overline{B}^0$ oscillations. The next problem is to search for CP-odd effects in the decays of B-mesons. We calculate the CP-odd asymmetry in the yields of same-sign dileptons within the framework of the standard model. We use the experimental value of $x = \Delta M_{B\overline{B}}/\Gamma_B$ and of ε, the CP-violation parameter in the $K^0-\overline{K}^0$ system, to calculate the mass m_t of the t-quark and the CP-odd phase α of the Kobayashi-Maskawa matrix as a function of the ratio $R = \Gamma(b\to ue\nu)/\Gamma(b\to ce\nu)$ in the standard model. If $R<0.1$, then $m_t > 80$ GeV.

1. GENERAL FORMALISM

The two-quark system of neutral pseudoscalar mesons B^0–$\overline{B}^0$ is described by an effective nonhermitian (due to the decay of these mesons) Hamiltonian H, which can be decomposed into the hermitian mass matrix M and width matrix Γ (see, e.g., ref. 1):

$$H=M-i\Gamma/2. \quad (1)$$

As a consequence of CPT invariance one has $H_{BB}=H_{\overline{B}\overline{B}}$. if CP violation is ignored then $H_{B\overline{B}}=H_{\overline{B}B}$. Diagonalizing the Hamiltonian we find the masses and lifetimes:

$$M_\pm-i\Gamma_\pm/2=M_{BB}-i\Gamma_{BB}/2\pm[(M_{B\overline{B}}-i\Gamma_{B\overline{B}}/2)\times(M_{\overline{B}B}-i\Gamma_{\overline{B}B}/2)]^{1/2}, \quad (2)$$

and wave functions of its eigenstates:

$$B_\pm=[(1+\bar\varepsilon)B^0\pm(1-\bar\varepsilon)\overline{B}^0][2(1+|\bar\varepsilon|^2)]^{-1/2},$$
$$(1+\bar\varepsilon)/(1-\bar\varepsilon)=[(M_{B\overline{B}}-i\Gamma_{B\overline{B}}/2)/(M_{\overline{B}B}-i\Gamma_{\overline{B}B}/2)]^{1/2}. \quad (3)$$

A nonzero value of $\bar\varepsilon$ is characteristic of CP violation. A B^0 ($\overline{B}^0$) meson produced at the instant of time $t=0$ will then evolve as a function of time as follows:

$$B^0(t)=\alpha(t)B^0+\beta(t)\frac{1-\bar\varepsilon}{1+\bar\varepsilon}\overline{B}^0,$$
$$\overline{B}^0(t)=\alpha(t)\overline{B}^0+\beta(t)\frac{1+\bar\varepsilon}{1-\bar\varepsilon}B^0,$$
$$\alpha(t)={}^1/_2[\exp(-iM_+t-\Gamma_+t/2)+\exp(-iM_-t-\Gamma_-t/2)],$$
$$\beta(t)={}^1/_2[\exp(-iM_+t-\Gamma_+t/2)-\exp(-iM_-t-\Gamma_-t/2)]. \quad (4)$$

The Pais-Treiman parameters r and $\bar r$ determine the yield of the wrong-sign leptons in the B^0- and $\overline{B}^0$-meson semileptonic decays. We recall that the B^0-meson contains the $\bar b$ quark, whose semileptonic decay results in a positively charged lepton l^+. The B^0–$\overline{B}^0$ oscillations, described by Eq. (4), give rise to the production of negatively charged leptons l^-. Making use of Eq. (4) we obtain

$$r=\frac{N(B^0\to l^-)}{N(B^0\to l^+)}=\frac{(\Delta M)^2+(\Delta\Gamma/2)^2}{2\Gamma^2+(\Delta M)^2-(\Delta\Gamma/2)^2}\left|\frac{1-\bar\varepsilon}{1+\bar\varepsilon}\right|^2,$$
$$\bar r=\frac{N(\overline{B}^0\to l^+)}{N(\overline{B}^0\to l^-)}=\frac{(\Delta M)^2+(\Delta\Gamma/2)^2}{2\Gamma^2+(\Delta M)^2-(\Delta\Gamma/2)^2}\left|\frac{1+\bar\varepsilon}{1-\bar\varepsilon}\right|^2, \quad (5)$$

where

$$\Delta M=M_+-M_-,\quad \Delta\Gamma=\Gamma_+-\Gamma_-,\quad \Gamma=(\Gamma_++\Gamma_-)/2.$$

For B-mesons $\Delta\Gamma\ll\Gamma$, hence if we ignore CP violation then

$$r\approx\bar r\approx x^2/(2+x^2),\quad x=\Delta M/\Gamma. \quad (5')$$

Finally, everything is ready to obtain final formulas for the fraction R and asymmetry a of same-sign dileptons. If the $B^0\overline{B}^0$ pair is produced in a C-odd state then

$$R\equiv\frac{N(l^+l^+)+N(l^-l^-)}{N(l^+l^-)}=\frac{r+\bar r}{2}. \quad (6)$$

For the asymmetry we have

$$a\equiv\frac{N(l^-l^-)-N(l^+l^+)}{N(l^-l^-)+N(l^+l^+)}=\frac{r-\bar r}{r+\bar r}\approx-\mathrm{Im}\frac{\Gamma_{B\overline{B}}}{M_{B\overline{B}}}, \quad (7)$$

where in obtaining the last equality we made use of the inequality $\Gamma_{B\overline{B}}\ll M_{B\overline{B}}$.

2. EVALUATION OF x AND a IN THE STANDARD MODEL

Making use of Eqs. (2), (5′), and (7) we obtain

$$x=[M_{B\overline{B}}M_{B\overline{B}}^*]^{1/2}/\Gamma_BM_B,\quad a=-\mathrm{Im}\,(2\Gamma_{B\overline{B}}M_B/M_{B\overline{B}}), \quad (8)$$

where $M_{B\overline{B}}$ denotes the matrix element of the square of the mass operator. An expression for it can be obtained in the standard model (without assuming $m_t^2\ll M_w^2$) from that given in Ref. 2 for the K^0–$\overline{K}^0$ transition. The amplitude for the B–$\overline{B}$ transition is determined by box diagrams with u-, c- and t-quark exchanges. The main contribution comes from the diagrams with t-quark exchange. We state the result for $M_{B\overline{B}}$, that follows from Ref. 2:

$$M_{B\overline{B}}=-\frac{G_F^2}{6\pi^2}m_t^2f_B^2M_B^2V_{bt}^2V_{td}^2I(\xi),$$
$$I(\xi)=(4-11\xi+\xi^2)/4(1-\xi^2)-3\xi^2\ln\xi/2(1-\xi)^3, \quad (9)$$

where f_B is a constant analogous to f_π, V_{bt} and V_{td} are elements of the Kobayashi-Maskawa matrix, and $\xi=m_t^2/m_w^2$. In evaluating the matrix element of the effective Hamiltonian with $\Delta B=2$ we made use of the vacuum insertion. Expression (9) describes the B_d–$\overline{B}_d$ transition; for the B_s–$\overline{B}_s$ transition one must replace V_{td} by V_{ts}. To obtain the

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absorptive part of the amplitude we make use of Ref. 3. The main part of $\Gamma_{B\bar{B}}$, independent of the internal quark masses, does not contribute to a. It is therefore necessary to isolate the term proportional to m_c^2, which is due to diagrams involving the exchange of two c quarks and c and u quarks:

$$\Gamma_{B\bar{B}}=G_F^2 f_B^2 m_B m_c^2 V_{bc} V_{cd} V_{td} V_{bt}/3\pi. \quad (10)$$

From Eqs. (8)–(10) we obtain

$$x_{B_d}=\frac{G_F^2}{6\pi^2} f_B^2 M_B m_t^2 |V_{tb}^* V_{td}|^2 I(\xi)\tau_B, \quad (11)$$

$$a_{B_d}=4\pi\left(\frac{m_c}{m_t}\right)^2 \operatorname{Im}\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\frac{1}{I(\xi)}, \quad (12)$$

where τ_B is the B-meson lifetime.

3. NUMERICAL ESTIMATES

By making use of QCD sum rules the quantity f_B is estimated in Ref. 4. to be 130 MeV $\pm$ 20%. A somewhat smaller value was obtained in Ref. 5. However, these values overlap within the errors and in what follows we use the results of Ref. 4. The lifetime of the B meson equals 1.26 ps, also accurate to about 20%. The modulus of the matrix element V_{tb} is equal to unity with good accuracy, and for V_{td} we shall make use of the following representation:

$$V_{td}\approx[1+(9.8R)^{1/2}(c_{23}/c_{13})e^{i\alpha}]s_{12}s_{23},$$
$$s_{12}=0.22,\quad s_{23}=0.05(1+R)^{-1/2},$$
$$R\equiv\Gamma(b\to ue\nu)/\Gamma(b\to ce\nu).$$

Introducing the experimental result[6] $x_{B_d}=0.77\pm0.25$ we obtain from Eq. (11)

$$\frac{m_t^2}{1+R}I\left(\frac{m_t^2}{M_W^2}\right)\left|1+\frac{c_{23}}{c_{13}}(9.8R)^{1/2}e^{i\alpha}\right|^2=(1.9\pm0.6)10^4\ \text{GeV}^2. \quad (13)$$

If $R\ll0.1$ then $m_t\approx200$ GeV; if instead $R\approx0.1$, then $m_t\approx80$ GeV. For x_{B_s} we obtain from Eq. (11) the following prediction:

$$x_{B_s}=x_{B_d}|V_{ts}/V_{td}|^2=20.7|1+(c_{23}/c_{13})(9.8R)^{1/2}e^{i\alpha}|^{-2}x_{B_d}. \quad (14)$$

It is apparent that $x_{B_s}\gg x_{B_d}$ and in the $B_s^0-\bar{B}_s^0$ system same-sign and opposite-sign dileptons should be produced at the same rate.

Let us move on to charge asymmetry. The imaginary part, contained in (12) of the combination of elements of the Kobayashi-Maskawa matrix equals Im $[1+(c_{23}/c_{13})(9.8R)^{1/2}e^{i\alpha}]^{-1}$. Using $m_c=1.3$ GeV we obtain for a_{B_d} the following upper bound:

$$|a_{B_d}|<4\cdot10^{-3}, \quad (15)$$

which is too small to be detected experimentally. For the B_s^0 meson the effect is even smaller:

$$|a_{B_s}|<4\cdot10^{-4}. \quad (16)$$

4. THE K^0–$\bar{K}^0$ SYSTEM

The mixing in the K^0–$\bar{K}^0$ system determines two experimentally measured quantities: the K_L- and K_S-meson mass difference Δm_{LS} and the CP-violation-determining parameter ε. We give the relevant formulas. The following was obtained in Ref. 2 for the matrix element of the K–$\bar{K}$ transition

$$M_{K\bar{K}}=-\frac{G_F^2}{6\pi^2}f_K^2M_K^2\left[m_c^2V_{cs}^{*2}V_{cd}^2+2m_c^2\ln\left(\frac{m_t^2}{m_c^2}\right)\right.$$
$$\left.\times V_{cs}^*V_{ts}^*V_{cd}V_{td}+m_t^2I\left(\frac{m_t^2}{M_W^2}\right)V_{ts}^{*2}V_{td}^2\right]. \quad (17)$$

For the mass difference and for ε we have

$$\Delta m_{LS}=-[M_{K\bar{K}}M_{\bar{K}K}]^{1/2}/M_K=3.5\cdot10^{-12}\ \text{MeV}, \quad (18)$$

$$|\varepsilon|=\operatorname{Im}M_{K\bar{K}}/2^{3/2}M_K\Delta m_{LS}=2.3\cdot10^{-3}. \quad (19)$$

The numbers in Eqs. (18) and (19) determine the experimental values of the corresponding quantities.

If we confine ourselves to the contribution of the c quark to (17), we obtain for Δm_{LS}

$$\Delta m_{LS}^c=(G_F^2/6\pi^2)f_K^2M_Km_c^2\sin^2\theta_c=2{,}4\cdot10^{-12}\ \text{MeV}, \quad (20)$$

where we used $f_K=165$ MeV and $m_c=1.3$ GeV. In Ref. 2, gluon corrections to the amplitude (17) were calculated in the leading-logarithm approximation; they are responsible numerically for the appearance in (17) and (20) of the factor $\eta=0.6$. Let us note that the main contribution to η comes from the evolution of α_s from 0.2 to 1. Obviously, the use of the Gell-Mann–Low formula for $\alpha_s\approx1$ is not legitimate, if instead we confine ourselves to the value $\alpha_s(u)=0.3$, we obtain for the renormalization factor $\eta=0.8$. We see that the c-quark exchange provides for only half of the value of Δm_{LS} and it is natural to attempt to describe the second half by contributions of the t quark. Exchange of one t and one c quark, described by the second term in the square brackets in Eq. (17), can not compete with the first term; the relative size of the third term, describing the exchange of two t-quarks, equals

$$\frac{tt}{cc}\approx\left(\frac{m_t}{m_c}\right)^2I\left(\frac{m_t^2}{M_W^2}\right)\frac{\operatorname{Re}V_{ts}^{*2}V_{td}^2}{\sin^2\theta_c}\approx10^{-5}\left(\frac{m_t}{m_c}\right)^2I\left(\frac{m_t^2}{M_W^2}\right), \quad (21)$$

and it reaches the value needed for saturation of Δm_{LS} when $m_t\approx600$ GeV. So heavy a t quark is unacceptable in the minimal version of the standard model, as it gives rise, due to radiative corrections, to a Higgs fields potential unbounded from below.[7, 1)] Consequently we are unable to attribute in the standard model the quantity $\Delta m_{K_LK_S}$ to contributions from short distances.

Making use of numerical values for the mixing angles we obtain for $|\varepsilon|$ from Eqs. (17) and (19)

$$|\varepsilon|=\frac{s_{12}s_{13}\sin\alpha}{2^{3/2}s_{12}\cdot3.5/2.4}\left\{2\ln\left(\frac{m_t^2}{m_c^2}\right)-2+2\frac{m_t^2}{m_c^2}I\left(\frac{m_t^2}{M_W^2}\right)\right.$$
$$\left.\times s_{23}^2\left[1+\frac{s_{13}}{s_{12}s_{23}}\frac{c_{23}}{c_{13}}\cos\alpha\right]\right\}$$
$$=2.3\cdot10^{-3}\frac{\sin\alpha(9{,}8R)^{1/2}}{3.8(1+R)}\left\{2\ln\left(\frac{m_t^2}{m_c^2}\right)\right.$$
$$\left.-2+\frac{I(m_t^2/M_W^2)}{200(1+R)}\frac{m_t^2}{m_c^2}\left[1+\frac{c_{23}\cos\alpha}{c_{13}}(9.8R)^{1/2}\right]\right\}. \quad (22)$$

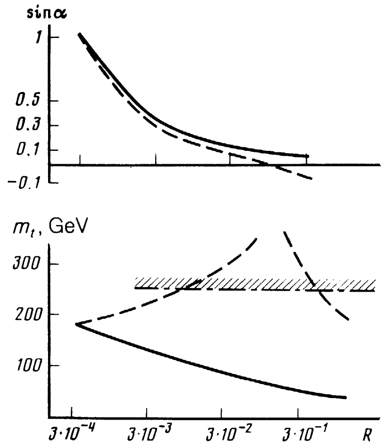

FIG. 1. The dependence of $\sin\alpha$ and m_t on $R=\Gamma(b\to uev)/\Gamma(b\to cev)$. Solid curves correspond to $c_{13}c_{23}\cos\alpha>0$, dashed curves to $c_{13}c_{23}\cos\alpha<0$. The hatched region above the dot-dash line is forbidden by the requirement $m_t<250$ GeV (see text). Values $R>0.5$ are excluded experimentally, $R<3\cdot10^{-4}$ are excluded theoretically.

Equations (13) and (22) contain three currently unknown quantities: R, α, and m_t. In Fig. 1 we show the dependence of $\sin\alpha$ and m_t on R. In the numerical estimates we made use of the value $m_c=1.3$ GeV. The two solutions correspond to the two possible signs of the quantity $c_{13}c_{23}\cos\alpha$. The region $R<3\cdot10^{-4}$ is forbidden: ε for such R is too small, and to obtain the experimental value $2.3\cdot10^{-3}$ it is necessary to take $\sin\alpha>1$. For $R=3\cdot10^{-4}$ we obtain $\sin\alpha=1$ and the two solutions coincide. For $(9.8R)^{1/2}\approx1$ the solution corresponding to $c_{13}c_{23}\cos\alpha<0$ has a singularity—the mass of the t quark becomes infinite. At that point V_{td} vanishes and $m_t\to\infty$ is necessary to satisfy Eq. (13). It follows then from Eq. (22) that $\sin\alpha=0$.

To obtain restrictions on the parameters of interest in the standard model one may use, beside the equations for x_{B_d} and ε, the limitation arising from the analysis of the $K_L\to2\mu$ decay.[8]

$$\zeta(m_t^2/M_W^2)m_t^2\,\mathrm{Re}\,V_{ts}^*V_{td}/s_{12}<57\ \mathrm{GeV}^2,$$

$$\zeta(x)=\frac{1}{4}+\frac{3}{4}\frac{1}{1-x}+\frac{3}{4}\frac{x}{(1-x)^2}\ln x, \qquad (23)$$

where the function $\zeta(x)$ was evaluated in Ref. 9.

A stronger restriction results from the required agreement between values of $\sin^2\theta_w$ obtained from the ratio of the W- and Z-boson masses and from νN scattering data: $m_t<250$ GeV.[10] This restriction forbids the solution with too heavy a t quark, obtained for $c_{13}c_{23}\cos\alpha<0$ and $5\cdot10^{-3}<R<0.3$. The existence of a quark with mass in excess of 250 GeV is impossible in the standard model as it gives rise to a Higgs field-quark coupling constant larger than 1.

We conclude that the analysis of $K^0-\bar K^0$ and $B_d^0-\bar B_d^0$ transitions permits the determination of the Kobayashi-Maskawa matrix parameters and the t-quark mass as functions of the ratio $R=\Gamma(b\to uev)/\Gamma(b\to cev)$. An experimental determination of these parameters and of the quantity R would provide one way of clarifying whether or not the standard model is in need of modification at the scale ~100 GeV.

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APPENDIX

To make this presentation self-contained we give the expression, widely used by us, for the quark-mixing-matrix parametrization given in the reviews, Ref. 1.

$$V=\begin{pmatrix}V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{pmatrix}$$

$$=\begin{pmatrix}c_{12}c_{13} & -s_{12} & -c_{12}s_{13}\\ s_{12}c_{13}c_{23}-s_{13}s_{23}e^{i\alpha} & c_{12}c_{23} & -s_{12}s_{13}c_{23}-c_{13}s_{23}e^{i\alpha}\\ s_{12}c_{13}s_{23}+s_{13}c_{23}e^{i\alpha} & c_{12}s_{23} & -s_{12}s_{13}s_{23}+c_{13}c_{23}e^{i\alpha}\end{pmatrix},$$

$$s_{12}=0.221\pm0.02,\qquad s_{23}=(0.05\pm0.007)(1+R)^{-1/2},$$
$$s_{13}=(R/2.1)^{1/2}s_{23},\qquad R=\Gamma(b\to uev)/\Gamma(b\to cev).$$

[1)] According to Ref. 7 the Higgs potential has the form

$$V(\varphi_c)=[6M_W^4(\varphi_c)+3M_Z^4(\varphi_c)+M_H^4(\varphi_c)-4m_t^4(\varphi_c)]\ln(\varphi_c^2)/64\pi^2$$

and the expression in the square brackets should be positive. If we do not wish to deal with a Higgs boson with mass ~1 TeV we cannot allow the existence of a t quark with mass ~600 GeV.

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