New type of the skin effect in a compensated metal subjected to a parallel magnetic field

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It is shown theoretically that the distribution of the field of an electromagnetic wave in a pure compensated metal in an external magnetic field $\mathbf{H}_0$ applied parallel to the surface of a sample is of the two-scale nature. In addition to the usual anomalous skin component of the wave in a metal, there is also a long-wavelength component penetrating the metal to a greater depth, of the order of the mean free path of electrons l. These components compete. It is remarkable that such a skin effect occurs when the conduction electrons are strongly magnetized, i.e., when their Larmor radius is much less than the mean free path l. The two-component nature of the wave results in an unusual behavior of the surface impedance of the metal considered as a function of the external field H_0, of the wave frequency ω, and of temperature. In the high-frequency limit the penetrating component of the field is manifested by a weakly damped electromagnetic wave with a linear dispersion law. The results are in agreement with the experimental data reported by Voloshin and Gaĭdukov [Sov. Phys. **40**, 166 (1975)].

I. INTRODUCTION

It is well known tnat the skin effect governs all the electromagnetic properties of a metal. Experimental and theoretical investigation of the skin effect have been going on for many years (see, for example, Refs. [1–15] and the literature cited there). At low temperatures a pure single crystal can exhibit the anomalous skin effect[1] when

$$\delta_n \equiv (c^2/4\pi\omega\sigma_0)^{1/2} \ll l. \tag{1.1}$$

Here, c is the velocity of light and σ_0 is the static conductivity of a bulky sample. In the case described by Eq. (1.1) an electromagnetic wave of frequency ω penetrates a metal to a depth

$$\delta_a \approx (\delta_n^2 l)^{1/3}, \tag{1.2}$$

which is much less than the electron mean free path l and greater than the classical skin depth δ_n:

$$\delta_n \ll \delta_a \ll l. \tag{1.3}$$

In an external magnetic field $\mathbf{H}_0$ applied parallel to the surface of a sample the nature of the skin effect depends on the value of H_0. In the case of an uncompensated metal an increase in H_0 reduces monotonically the depth of the skin layer $\delta(H_0)$ from δ_a to δ_n (Ref. 2). In weak magnetic fields, when the Larmor radius R of an electron orbit is greater than the mean free path $l(\delta_n \ll l \ll R)$, the influence of a magnetic field on the thickness $\delta(H_0)$ is small and we have $\delta(H_0) \approx \delta_a$. An increase in H_0 enhances the importance of cyclotron returns of electrons to the skin layer and in the range

$$\delta_n \ll R \ll l \tag{1.4}$$

the depth of penetration varies in accordance with the law[2,3] $\delta(H_0) \approx (\delta_n^2 R)^{1/3}$. Finally, in the limit of strong magnetic fields $(R \ll \delta_n \ll l)$ the spatial dispersion is weak and the Hall effect begins to play the main role. In view of the electrical neutrality condition the effective conductivity of an uncompensated metal ceases to depend on H_0 and assumes the value σ_0. Consequently, the skin effect becomes normal and we have $\delta(H_0) \approx \delta_n$. It should be mentioned that the asymptotes of $\delta(H_0)$ in the limits of weak, strong [Eq. (1.4)], and extremely strong magnetic fields transform smoothly from one to the other at points $R = l$ and $R = \delta_n$.

In a compensated metal when the spatial dispersion is strong, i.e., in weak $(\delta_n \ll l \ll R)$ and strong [Eq. (1.4)] magnetic fields H_0 the wave attenuation depth $\delta_s(H_0)$ is the same as for an uncompensated metal: an increase in H_0 changes from δ_a to δ_n in accordance with the law

$$\delta_s(H_0) \approx (\delta_n^2 R)^{1/3}. \tag{1.5}$$

However, in extremely strong fields $(R \ll \delta_n \ll l)$, when there are no spatial dispersion effects, the skin layer depth $\delta_l(H_0) \approx \delta_n l/R$ is much greater than the normal skin depth δ_n. The former depth increases in direct proportion to H_0 beginning from $\delta_l \approx l$ at the point $R = \delta_n$. This behavior of $\delta_l(H_0)$ is due to the fact that the Hall effect in a compensated metal is not the dominant phenomenon and in the limit of very strong magnetic fields the effective conductivity of a metal is identical with the magnetoconductivity $\sigma_0 R^2/l^2$ (see, for example, Refs. 4–6)

The general picture of the skin effect in a compensated metal described above is not self-consistent, because the asymptotes $\delta_s(H_0)$ and $\delta_l(H_0)$ do not match at the point $R = \delta_n$. This conflict can be avoided in a natural manner by assuming that in the range of strong magnetic fields of Eq. (1.4) the scale of variation of the electromagnetic field $\delta_s(H_0)$ coexists with another scale $\delta_l(H_0)$, which at the point $R = \delta_n$ becomes $\delta_l(H_0) \approx \delta_n l/R$.

We shall show that in the range defined by Eq. (1.4) the distribution of an electromagnetic field in a compensated metal is of two-component nature. In addition to a short-wavelength (anomalous skin) component with an attenuation depth $\delta_s(H_0)$ of Eq. (1.5), there is also a long-wavelength (normal skin) component which penetrates the metal to a depth of the order of the mean free path of electrons (Fig. 1):

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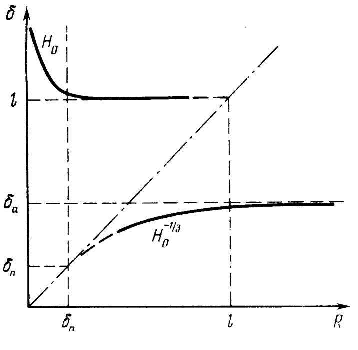

FIG. 1. Schematic dependence of the long-wavelength δ_l (upper curve) and short-wavelength δ_s (lower curve) depths of attenuation of an electromagnetic field on the reciprocal of an external magnetic field $H_0(R \propto H_0^{-1})$. The notation is explained in the text.

$$\delta_l(H_0) \approx l. \tag{1.6}$$

It should be pointed out that neither the origin of this long-wavelength component of the field nor the role it plays in the skin effect are analogous to the Sondheimer component.[1] It is well known that the Sondheimer component, penetrating in the absence of a magnetic field to a depth of the order of l, is due to the ballistic mechanism of field transfer by electrons. However, our long-wavelength component occurs in strong magnetic fields defined by Eq. (1.4), when electrons are strongly magnetized ($R \ll l$) and the ballistic mechanisms cannot ensure penetration of the field to a depth l equal to the mean free path. It should be stressed that both scales δ_s and δ_l are very different in all external magnetic fields H_0:

$$\delta_s(H_0) \leqslant \delta_a \ll l \leqslant \delta_l(H_0). \tag{1.7}$$

The two-component nature of the electric magnetic field with such very different "wavelengths" means that in strong fields H_0 the skin effect is neither normal nor anomalous. This is reflected in the magnitude of the surface impendance Z, which is governed not by the local characteristics $\delta_s(H_0)$ and $\delta_l(H_0)$ but by the integral complex skin layer depth:

$$\delta_{eff} = \int_0^\infty dx\, H(x)/H(0) = ic^2 Z/4\pi\omega. \tag{1.8}$$

Here, $H(x)$ is the projection of an alternating magnetic field collinear with the vector $\mathbf{H}_0$ and the x axis is directed along the inner normal to the surface of a metal at $x = 0$. Under the conditions of such an unusual "mixed" skin effect the contributions of the long- and short-wavelength components $H(x)$ to $\delta_{\rm eff}$ of Eq. (1.8) are in competition with one another. This competition gives rise to a specific relationship between the real and imaginary parts of the impedance, and to new dependences of Z on the value of H_0, wave frequency ω, mean free path l, and parameters of the surface scattering of quasiparticles. It is interesting to note that at high frequencies $\omega \gg \nu$ (ν is the relaxation frequency) the long-wavelength component represents a weakly damped wave with a linear dispersion law and a phase velocity of the order of the Fermi velocity v_F. In the limit of very strong magnetic fields it reduces to a magnetoplasma wave (see also Refs. 4–6).

2. FIELD DISTRIBUTION AND SURFACE IMPEDANCE

We shall consider a half-space occupied by a metal and subjected to a static and homogeneous magnetic field $\mathbf{H}_0$ parallel to the metal-vacuum interface. The x axis is directed into the metal at right-angles to the interface $x = 0$ and the z axis is parallel to the vector $\mathbf{H}_0$. An electromagnetic wave of frequency ω incident on such a sample is polarized so that its magnetic component is collinear with the vector $\mathbf{H}_0$ (z axis) and the electric component is directed along the y axis. The direction of propagation of the wave is parallel to the x axis.

The tangential projections of the electric $E_y(x,t) = E(x)\exp(-i\omega t)$ and magnetic $H_z(x,t) = H(x)\exp(-i\omega t)$ fields in the metal are found by solving simultaneously the Maxwell equations and the Boltzmann transport equation for the distribution function of conduction electrons. We shall not repeat cumbersome but conventional procedures (see, for example, Refs. 4 and 7–11), but simply write down the results of calculations in the form

$$\frac{H(x)}{H(0)} = \frac{2}{\pi} \int_0^\infty \frac{k \sin(kx)\, dk}{k^2 - 4\pi i\omega c^{-2}\sigma(k)}, \tag{2.1}$$

$$\frac{E(x)}{H(0)} = -\frac{i\omega}{c} \frac{2}{\pi} \int_0^\infty \frac{\cos(kx)\, dk}{k^2 - 4\pi i\omega c^{-2}\sigma(k)}. \tag{2.2}$$

Here, $\sigma(k)$ is the electrical conductivity renormalized by the quasineutrality condition $j_x = 0$ ($\mathbf{j}$ is the current density):

$$\sigma(k) = \sigma_{yy}(k) + \sigma_{xy}^2(k)/\sigma_{xx}(k), \tag{2.3}$$

where $\sigma_{\alpha\beta}(k)$ is the cosine transformation of the integral kernel of the electrical conductivity of an unbounded sample ($\alpha,\beta = x,y$). We shall simplify the subsequent analysis by ignoring the influence of the surface of the metal on the conductivity, so that the results obtained are not affected qualitatively if we allow for the surface electron scattering. The exception to this rule is the case of specular reflection of electrons from the surface of a metal, which requires a special study. The last section in the present paper will be devoted to the role of the surface scattering processes.

Equation (2.1) for the distribution of the magnetic field of the wave allows for the fact that the effective conductivity $\sigma(k)$ tends to zero in the limit $k \to \infty$. It follows from Eqs. (2.1) and (2.2) that the surface impedance Z of the metal is given by the expression

$$Z \equiv \frac{4\pi}{c} \frac{E(0)}{H(0)} = -\frac{4\pi i\omega}{c^2} \frac{2}{\pi} \int_0^\infty \frac{dk}{k^2 - 4\pi i\omega c^{-2}\sigma(k)}. \tag{2.4}$$

In the case of a compensated metal containing two groups of carriers the conductivity tensor $\sigma_{\alpha\beta}(k)$ is a sum of two terms: electron $\sigma^e_{\alpha\beta}(k)$ and hole $\sigma^h_{\alpha\beta}(k)$. Following Refs. 4 and 7, we shall simplify the treatment by assuming that the dispersion laws of electrons and holes are quadratic and isotropic. It should be pointed out that the equality of the radii p_F of the electron and hole Fermi spheres in a compensated metal does not imply the equality of the mobilities of electrons and holes, i.e., the masses and relaxation frequencies of quasiparticles (and, consequently, the mean free paths) may differ. Therefore, the adopted model may simple but it still retains all the individual features of each type of carrier.

In the isotropic model the electron contribution to the conductivity is[9]

$$\sigma_{\alpha\beta}^{e}(k)=\frac{3\sigma_e}{4}\frac{\pi\gamma}{\mathrm{sh}(\pi\gamma)}\int_0^{\pi/2} d\Theta\, I_{\alpha\beta}(kR\sin\Theta)\sin^3\Theta,$$

$$\begin{aligned}
I_{xx}(q)&=-2J_{1+i\gamma}(q)J_{1-i\gamma}(q)-J_{1+i\gamma}(q)J_{-1-i\gamma}(q)\\
&\quad-J_{1-i\gamma}(q)J_{-1+i\gamma}(q),\\
I_{yy}(q)&=2J_{1+i\gamma}(q)J_{1-i\gamma}(q)-J_{1+i\gamma}(q)J_{-1-i\gamma}(q)\\
&\quad-J_{1-i\gamma}(q)J_{-1+i\gamma}(q),\\
I_{xy}(q)&=-I_{yx}(q)=iJ_{1+i\gamma}(q)J_{-1-i\gamma}(q)-iJ_{1-i\gamma}(q)J_{-1+i\gamma}(q).
\end{aligned} \tag{2.5}$$

Here,

$$\sigma_e=\frac{ne^2}{m_e(\nu_e-i\omega)},\quad \gamma=\frac{\nu_e-i\omega}{\Omega_e},\quad \Omega_e=\frac{eH_0}{m_ec},\quad R=\frac{cp_F}{eH_0}, \tag{2.6}$$

$J_s(q)$ is a Bessel function of the first kind; m_e is the mass of an electron; Ω_e is the cyclotron frequency of electrons; ν_e is the electron relaxation frequency; n is the density of electrons; e is the absolute value of the charge; p_F is the Fermi momentum and R is the Larmor radius, both of which are the same for electrons and holes.

We can find the electrical conductivity tensor of holes $\sigma_{ab}^{h}(k)$ by replacing the electron characteristics m_e and ν_e in Eqs. (2.5) and (2.6) with the hole parameters $m_h>0$ and ν_h, and by reversing the sign in front of the parameter γ.

1. We shall first analyze the low-frequency situation when in the absence of an external magnetic field H_0 we have the quasistatic anomalous skin effect, i.e.,

$$\delta_n\ll l_e,\ l_h,\quad \omega\ll\nu_e,\ \nu_h. \tag{2.7}$$

The first condition in Eq. (2.7) generalizes the condition of anomalous behavior of Eq. (2.1) to a compensated metal with two different groups of carriers. It sets the lower limit to the frequency of an electromagnetic wave ω. The classical skin depth δ_n is described by the previous formula [(Eq. 1.1)] where the total static conductivity of a bulk sample σ_0 is

$$\sigma_0=ne^2(l_e+l_h)/p_F,\quad l_e=p_F/m_e\nu_e,\quad l_h=p_F/m_h\nu_h. \tag{2.8}$$

The mean free paths of electrons l_e and holes l_h will be assumed to be of the same order ($l_e\sim l_h\sim l$), so as to make the case more definite. Therefore, in writing down the inequalities we shall omit the indices of l_e and l_h and assume that these inequalities are satisfied by both mean free paths. The second condition in Eq. (2.7) ensures that the electromagnetic field in the metal is quasistatic, which sets the upper limit to the wave frequency ω.

We shall need asymptotic expressions for the conductivity $\sigma(k)$ for different values of the wave number k. In calculation of $\sigma(k)$ under the conditions of a strong spatial dispersion ($kR\gg1+|\gamma|$) the Bessel functions in Eq. (2.5) can be replaced by asymptotes at high values of the argument. We thus obtain

$$\sigma_s(k)=\frac{3\pi}{4}\frac{\sigma_0}{k(l_e+l_h)}\left[\mathrm{cth}\left(\pi\frac{R}{l_e}\right)+\mathrm{cth}\left(\pi\frac{R}{l_h}\right)\right] \tag{2.9}$$

for $kR\gg1$ and $kl\gg1$. The expression for the conductivity $\sigma(k)$ in the weak spatial dispersion range ($kR\ll1$), when strong and very strong magnetic fields are applied ($|\gamma|\ll1$), is obtained by replacing the products of the Bessel functions with asymptotes which are valid in the case of low values of the argument q:

$$\sigma_l(k)=\sigma_0\left[\frac{R^2}{l_el_h}+\frac{2}{5}(kR)^2\right] \tag{2.10}$$

if $kR\ll1$ and $R\ll l$. Equation (2.10) has two terms and each of them corresponds to two different mechanisms of the quasistatic conductivity in the weak spatial dispersion case. The first is due to the diffusion of centers of electron orbits in a strong magnetic field (conventional magnetoresistance mechanism). The second mechanism is related to the drift of electrons because of a weak inhomogeneity of the electric field. No restrictions on the relationship between the two terms in Eq. (2.10) are imposed when $\sigma_l(k)$ is calculated.

The asymptotes of the conductivity described by Eqs. (2.9) and (2.10) are governed only by the first term in Eq. (2.3) for $\sigma(k)$. The Hall renormalization of the conductivity can be ignored in the cases of strong and weak spatial dispersion. The smallness of the second term in Eq. (2.3), compared with the first, in the weak spatial dispersion case is ensured by the anisotropy of the compensated metal model. However, even in the anisotropy model this term exceeds the diagonal conductivity $\sigma_{yy}(k)$. Therefore, the structure and order of the asymptote of Eq. (2.10) are retained also in the general case of an anisotropic compensated metal.

We can study the distribution of an electromagnetic field in a metal and calculate the surface impedance if we determine those characteristic values of the wave number k which define the intervals that make the dominant contributions to the integrals (2.1), (2.2), and (2.4). We should therefore find and analyze zero of the denominator in the integrands in Eqs. (2.1), (2.2), and (2.4):

$$k^2-4\pi i\omega c^{-2}\sigma(k)=0,\quad \mathrm{Re}\,k>0. \tag{2.11}$$

Using the asymptote of Eq. (2.9), we can readily obtain the short-wavelength root of the dispersion equation (2.11) in the form

$$\begin{aligned}
&k_s=\delta_s^{-1}(H_0)\exp(i\pi/6)\quad\text{for}\quad \delta_n\ll R,\\
&\delta_s(H_0)=\left[\frac{4}{3\pi}\frac{\delta_n^2(l_e+l_h)}{\mathrm{cth}(\pi R/l_e)+\mathrm{cth}(\pi R/l_h)}\right]^{1/3}.
\end{aligned} \tag{2.12}$$

It follows from the system (2.12) that this root is related to the short-wavelength penetration depth $\delta_s(H_0)$ and it exists for weak and strong [Eq. (1.4)] magnetic fields ($\delta_n\ll R$). The short-wavelength length $\delta_s(H_0)$ decreases monotonically on increase in H_0 (Fig. 1) from the value δ_a (corresponding to $\delta_n\ll l\ll R$) given by

$$\delta_a=[2\delta_n^2(l_e+l_h)/3\pi]^{1/3}, \tag{2.13}$$

to the value $(4/3)^{1/3}\delta_n$. The value of $\delta_s(H_0)$ remains much smaller than the Larmor radius R or the mean free paths l_e and l_h. In strong magnetic fields [Eq. (1.4)] the short-wavelength penetration depth is given by the formula

$$\delta_s(H_0)=(4\delta_n^2R/3)^{1/3}\quad\text{for}\quad \delta_n\ll R\ll l. \tag{2.14}$$

The second (long-wavelength) root of the dispersion equation of (2.11) can be found with the aid of the asymptote (2.10):

$$k_l=\delta_l^{-1}(H_0)\exp\left(-\frac{i\pi}{2}-\frac{i}{2}\operatorname{arctg}\frac{5\delta_n^2}{2R^2}\right)\quad \text{for}\quad R\ll l,$$
$$\delta_l(H_0)=\frac{(l_el_h)^{1/2}}{R}\left|\frac{2}{5}R^2+i\delta_n^2\right|^{1/2}. \tag{2.15}$$

This root determines the long-wavelength scale of the change in an electromagnetic field $\delta_l(H_0)$, which exists in strong [Eq. (1.4)] and very strong magnetic fields. The depth of penetration $\delta_l(H_0)$ remains practically constant in the range of strong magnetic fields [Eq. (1.4)]:

$$\delta_l(H_0)=(2l_el_h/5)^{1/2}\quad \text{for}\quad \delta_n\ll R\ll l, \tag{2.16}$$

and rises monotonically on increase in H_0 in the limit of very strong fields (Fig. 1):

$$\delta_l(H_0)=\delta_n(l_el_h)^{1/2}/R\quad \text{for}\quad R\ll\delta_n\ll l. \tag{2.17}$$

Throughout the range defined here ($R\ll l$) the function $\delta_l(H_0)$ is much larger than the Larmor radius R or the classical thickness δ_n (Fig. 1).

Our analysis of the dispersion equation (2.11) demonstrates that the distribution of an electromagnetic field in a metal is determined mainly by the contributions made to the integrals of Eq. (2.1) and (2.2) by integration domains characterized by $k\sim\delta_l^{-1}(H_0)$ and $k\sim\delta_s^{-1}(H_0)$. Each of these two contributions exists in its own range of external magnetic fields H_0. Since in weak ($\delta_n\ll l\ll R$) and very strong ($R\ll\delta_n\ll l$) magnetic fields Eq. (2.11) has only one root, it follows that the electromagnetic field has in this case only one scale of variation ($\delta_s=\delta_a$ when $\delta_n\ll l\ll R$ and δ_l obtained from Eq. (2.17) when $R\ll\delta_n\ll l$). In contrast to these known familiar situations, both contributions are important in the intermediate range of strong fields [Eq. (1.4)]. Therefore, the electromagnetic field is of two-component nature and the mixed skin effect is observed. It is important to stress that the values of the functions $\delta_s(H_0)$ and $\delta_l(H_0)$ differ from one another for any value of H_0 (Fig. 1), so that

$$\delta_s(H_0)\ll R\ll\delta_l(H_0) \tag{2.18}$$

[see also the inequalities of Eq. (1.7)]. Consequently, the contributions of large ($k\sim\delta_s^{-1}$) and small ($k\sim\delta_l^{-1}$) wave number can easily be distinguished. We shall consider in greater detail the distributions of the electric $E(x)$ and magnetic $H(x)$ fields under the mixed skin effect conditions of Eq. (1.4).

Using the asymptotic behavior of the conductivity $\delta_s(k)$ described by Eq. (2.9) and of $\delta_l(k)$ described by Eq. (2.10), and bearing in mind the specific nature of the sine Fourier expansion of Eq. (2.1), we obtain the magnetic field distribution in a compensated metal:

$$\frac{H(x)}{H(0)}=\frac{2}{\pi}\int_0^\infty\frac{k^2\sin(kx)\,dk}{k^3-i\delta_s^{-3}},\quad x\ll R, \tag{2.19}$$

$$\frac{H(x)}{H(0)}=\frac{5i}{2}\left(\frac{\delta_n}{R}\right)^2\exp\left(-\frac{x}{\delta_l}\right),\quad x\gg R. \tag{2.20}$$

It is clear from these expressions that a considerable reduction in the amplitude of the magnetic field occurs already at the anomalous skin depth $\delta_s\approx(\delta_n^2R)^{1/3}$ In the range $x\sim R$ the short-wavelength component (2.19) becomes comparable with the long-wavelength component (2.20), but a further increase of the coordinate x results in predominance of the exponential decay of $H(x)$. The long-wavelength component of the magnetic field (2.20) is $(R/\delta_n)^2\gg1$ times less than $H(0)$. However, it penetrates a sample to a depth $\delta_l=(l_el_h)^{1/2}$ which is of the order of the mean free path of carriers and is considerably greater than δ_s. Therefore, the contributions of the asymptotes (2.19) and (2.20) to the integral depth of the skin layer of Eq. (1.8) are in competition. The same competition appears also in the case of the electric field $E(x)$ in the range $x\ll R$, since it represents an integral of the magnetic field:

$$E(x)=-\frac{i\omega}{c}\int_x^\infty dx'\,H(x'). \tag{2.21}$$

In fact, a direct calculation of $E(x)$ by the substitution of the conductivities of Eqs. (2.9) and (2.10) into the cosine expansion of Eq. (2.2) shows that the electric field consists of two components:

$$E(x)=E_s(x)+E_l(x). \tag{2.22}$$

The first component in Eq. (2.22) is determined by the contribution of large wave numbers $k\sim\delta_s^{-1}$ to the integral of Eq. (2.2). It represents the short-wavelength component of the electric field which exists for $x\ll R$ and is damped out at the anomalous skin depth $\delta_s\approx(\delta_n^2R)^{1/3}$ given by Eq. (2.14):

$$\frac{E_s(x)}{H(0)}=-\frac{i\omega}{c}\frac{2}{\pi}\int_0^\infty\frac{k\cos(kx)\,dk}{k^3-i\delta_s^{-3}}. \tag{2.23}$$

The second long-wavelength component $E_l(x)$ is due to the contribution of small wave numbers $k\sim\delta_l^{-1}$. This component penetrates the metal to a great depth $\delta_l\approx(l_el_h)^{1/2}$ [see Eq. (2.16)] which is considerably greater than the depth δ_s [see Eqs. (1.7) and (2.18)]:

$$\frac{E_l(x)}{H(0)}=\frac{5\omega\delta_l}{2c}\left(\frac{\delta_n}{R}\right)^2\left(1-\frac{5i}{4}\frac{\delta_n^2}{R^2}\right)\exp\left(-\frac{x}{\delta_l}\right). \tag{2.24}$$

As pointed out in the Introduction the presence of the field $E_l(x)$ in a metal is not associated with the ballistic electron transport mechanisms, which in the case of the conventional anomalous skin effect are responsible for the Sondheimer component.[1] The appearance of the long-wavelength component $E_l(x)$ in our situation can be explained as follows. According to Eq. (2.10) for the conductivitiy $\sigma_l(k)$, the response of a metal to an inhomogeneous electric field can be described by the following expression in the x representation:

$$j(x)=\sigma_0\left[\frac{R^2}{l_el_h}E(x)-\frac{2}{5}R^2E''(x)\right]. \tag{2.25}$$

We can easily show that the substitution of the field $E_l(x)$ of Eq. (2.24) into Eq. (2.25) causes the current $j(x)$ to vanish. This means that there is a strong compensation of the magnetoresistance by the drift conductivity in a weakly inhomogeneous electric field $E_l(x)$. In other words, the long-wavelength component of the electric field described by Eq. (2.24) is essentially of the zero-current nature. A characteristic self-consistent pattern is then established: the electric

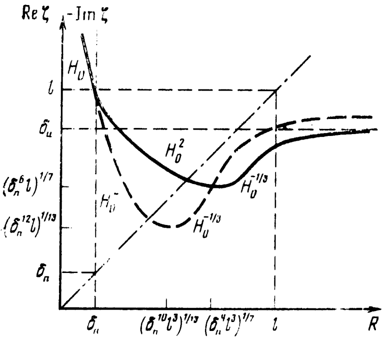

FIG. 2. Schematic dependences of the real (continuous curve) and imaginary (dashed curve) parts of the impedance $\zeta = c^2 Z/4\pi\omega$ on the reciprocal of the external magnetic field H_0 ($R \propto H_0^{-1}$) in the case of diffuse reflection of electrons.

field $E_l(x)$ varies with the coordinate x in such a way as to cause the current to vanish; on the other hand, the smallness of the current density is the necessary condition for such a slow decay of the electric field with depth in the sample.

These special features of the skin effect in a compensated metal are reflected in the magnitude of the surface impendance of Eq. (2.4). Figure 2 shows schematically the behavior of $Z(H_0)$ in a wide range of magnetic fields H_0. In weak magnetic fields ($\delta_n \ll l \ll R$) an electromagnetic field is represented solely by the short-wavelength component which is damped out over the anomalous skin depth $\delta_s = \delta_a \approx [\delta_n^2(l_e + l_h)]^{1/3}$. It follows from Eqs. (2.4), (2.9), (2.12), and (2.13) that the impedance of the metal is then given by

$$Z_s(H_0) = \frac{16\pi\omega\delta_a}{3^{3/2}c^2}\exp\left(-\frac{i\pi}{3}\right), \quad \delta_n \ll l \ll R. \tag{2.26}$$

To avoid misunderstanding, we must mention that we are not considering here nonmonotonic variation of $Z_s(H_0)$ observed in weak fields when $(8R\delta_a)^{1/2} \sim l$ and known in the literature as the background signal.[12,13]

A second long-wavelength component of the electric field $E(x)$ appears in strong magnetic fields defined by Eq. (1.4) However, the total current and the alternating magnetic field are still concentrated in the anomalous skin depth $\delta_s \approx (\delta_n^2 R)^{1/3}$, which now begins to depend on H_0 [see Eq. (2.14)]. It follows from Eqs. (2.4) and (2.22)–(2.24) that under the conditions of the mixed skin effect the impedance $Z(H_0)$ is equal to the sum of the short- and long-wavelength impedances:

$$Z(H_0) = \frac{16\pi\omega\delta_s(H_0)}{3^{3/2}c^2}\exp\left(-\frac{i\pi}{3}\right) + \frac{10\pi\omega\delta_l}{c^2}\left(\frac{\delta_n}{R}\right)^2\left(1 - \frac{5i}{4}\frac{\delta_n^2}{R^2}\right), \quad \delta_n \ll R \ll l. \tag{2.27}$$

The competition between the long- and short-wavelength components of the electric field is manifested particularly in the real part of the impedance Re $Z(H_0)$:

$$\operatorname{Re} Z(H_0) = \frac{8\pi}{3^{3/2}}\left[\frac{\omega^2 p_F^2}{3\pi n e^3 c^3 (l_e + l_h) H_0}\right]^{1/3} + \left(\frac{5}{2}\right)^{1/2}\frac{(l_e l_h)^{1/2}}{l_e + l_h}\frac{H_0^2}{n p_F c^2}, \quad \delta_n \ll R \ll l. \tag{2.28}$$

It follows from Eqs. (2.23) and (2.24) that this competition can be described by the magnitude of the following parameter:

$$|E_s(0)/E_l(0)| \sim R^2\delta_s(H_0)/\delta_n^2\delta_l. \tag{2.29}$$

Therefore, in the range of magnetic fields corresponding to

$$\delta_n \ll (\delta_n^4 l^3)^{1/7} \ll R \ll l \tag{2.30}$$

Eq. (2.28) is dominated by the first (short-wavelength) term. When H_0 is increased, the two terms become comparable and in the range where

$$\delta_n \ll R \ll (\delta_n^4 l^3)^{1/7} \ll l, \tag{2.31}$$

Eq. (2.28) is dominated by the second (long-wavelength) term. The impedance of the metal then becomes practically real. In fact, its imaginary part

$$-\operatorname{Im} Z(H_0) = \frac{8\pi}{3}\left[\frac{\omega^2 p_F^2}{3\pi n e^3 c^3 (l_e + l_h) H_0}\right]^{1/3} + \frac{5}{16\pi}\left(\frac{5}{2}\right)^{1/2}\frac{(l_e l_h)^{1/2}}{(l_e + l_h)^2}\frac{H_0^4}{\omega n^2 c^2 p_F^2}, \quad \delta_n \ll R \ll l \tag{2.32}$$

is much smaller than Re $Z(H_0)$ throughout the full range defined by Eq. (2.31). Moreover, in the imaginary part of the impedance described by Eq. (2.32) there is again competition between the long- and short-wavelength components of the electric field. The second (long-wavelength) term of Eq. (2.32) predominates in the range corresponding to

$$\delta_n \ll R \ll (\delta_n^{10} l^3)^{1/13} \ll l. \tag{2.33}$$

Finally, in very strong magnetic fields ($R \ll \delta_n \ll l$) the short-wavelength component disappears and only the long-wavelength components of the electric and alternating magnetic fields remain. The asymptote of Eq. (2.10) is dominated by the magnetoconductivity, which results in the normal skin depth of penetration $\delta_l(H_0) \approx \delta_n l/R$ [see Eq. (2.17)]. It follows from Eqs. (2.4), (2.10), (2.15), and (2.17) that the impedance of a metal is now described by the expression

$$Z_l(H_0) = \frac{4\pi\omega\delta_l(H_0)}{c^2}\exp\left(-\frac{i\pi}{4}\right) = \frac{4\pi\omega\delta_n}{c^2}\frac{(l_e l_h)^{1/2}}{R}\exp\left(-\frac{i\pi}{4}\right), \quad R \ll \delta_n \ll l. \tag{2.34}$$

2. The dependence of the surface impedance on an external magnetic field of frequency corresponding to the

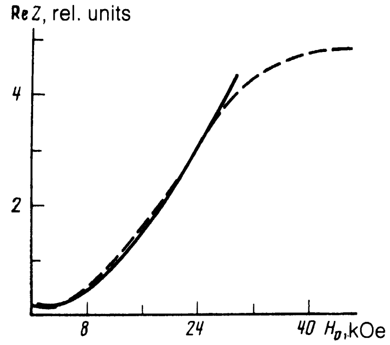

FIG. 3. Theoretical (continuous curve) and experimental (dashed curve, taken from Ref. 14) dependences of the real part of the surface impedance on the external magnetic field applied under mixed skin effect conditions.

anomalous skin effect $\delta_n \ll l$ was determined experimentally for tin in Ref. 14. Figure 3 shows the $\mathrm{Re}Z(H_0)$ curve taken from Ref. 14 (dashed curve). The continuous curve represents the results of a numerical calculation of the real part of the surface impedance:

$$\mathrm{Re}\, Z(H_0) = \frac{8\pi\omega}{3^{3/2}c^2}\left\{\left(\frac{4}{3}\delta_n^2 R\right)^{1/3} + 3{,}75\left(\frac{\delta_n}{R}\right)^2 l\left[1+\frac{25}{4}\left(\frac{\delta_n}{R}\right)^4\right]^{-1/4} \times \cos\left[\mathrm{arctg}\frac{5}{2}\left(\frac{\delta_n}{R}\right)^2\right]\right\}, \qquad (2.35)$$

which holds applies under the experimental conditions under discussion ($l_e = l_n = l = 0.05$ cm, $\sigma_0 = 10^{21}\ \mathrm{s}^{-1}$, $p_F = 3\times 10^{-19}\ \mathrm{g\cdot cm\cdot s^{-1}}$, $\omega = 4\pi\times 10^5\ \mathrm{s}^{-1}$, $\delta_n = 2.4\times 10^{-4}$ cm). This expression differs from Eq. (2.28) because terms of the order of δ_n/R are retained. Such a modification extends the range of its validity to the boundary $R\sim\delta_n$ between strong and very strong magnetic fields, which is reached in the experimental conditions of Ref. 14 at $H_0 \approx 80$ kOe. The $\mathrm{Re}\, Z(H_0)$ depedences are plotted in Fig. 3 using arbitrary units and are normalized to ensure coincidence of the theoretical and experimental curves in a field $H_0 = 22$ kOe. The minimum of the real part of the surface impendance corresponds to $H_0 = 2$ kOe, which is in agreement with the condition $R \approx 3.64(\delta_n^4 l^3)^{1/7}$ deduced from Eq. (2.35) (see also Fig. 2). The magnetic field in which $\mathrm{Re}Z(H_0)$ assumes its minimum value separates two regions: in lower fields the impedance is governed by the anomalous skin (first) term in Eq. (2.35), whereas in higher fields it is governed by the long-wavelength (second) term in Eq. (2.35). It is pointed out in Ref. 14 that in magnetic fields $H_0 \gtrsim 2$ kOe the real part of the impedance is proportional to the square of the external field H_0. Moreover, in the range H_0 where $\mathrm{Re}Z \propto H_0^2$ is obeyed, there is no frequency dependence $\mathrm{Re}Z(\omega)$. These conclusions are in agreement with our theoretical predictions for the case when $\delta_n \ll R \ll (\delta_n^4 l^3)^{1/7}$ and the second term in Eq. (2.35) predominates.

As H_0 is increased (and R approaches δ_n) the depth of the skin layer increases in accordance with the law described by Eq. (2.15). Since in the experiments reported in Ref. 14 the thickness of a sample was $d \approx 0.1$ cm, its "bleaching" (when the skin layer depth δ_l became comparable with d) occurred in fields of the order of 40 kOe. This accounts for the discrepancy between the theoretical dependence $\mathrm{Re}Z(H_0)$ of Eq. (2.35), derived for a semiinfinite metal ($d \gg \delta_l$), and the experimental curve obtained for $H_0 \gtrsim 40$ kOe.

3. The long-wavelength component of an electromagnetic field observed by us at high frequencies

$$\omega \gg \nu_e,\ \nu_h \qquad (2.36)$$

represents a weakly damped wave. In investigating this situation it is sufficient to modify all the formulas in the preceding section by adding to the relaxation frequencies of electrons ν_e and holes ν_h an imaginary part amounting to $-i\omega$ ($\nu_{e,h} \to \nu_{e,h} - i\omega$). In the high-frequency limit the range of strong magnetic fields of Eq. (1.4) corresponds to

$$\delta_n = c/\omega_p \ll R \ll l_\omega, \qquad (2.37)$$

whereas the range of very strong magnetic fields corresponds to

$$R \ll c/\omega_p \ll l_\omega, \quad \omega_p = [4\pi n e^2 (m_e^{-1} + m_h^{-1})]^{1/2}. \qquad (2.38)$$

Here, ω_p is the plasma frequency and l_ω is the effective path of carriers during one period $2\pi/\omega$ of an electromagnetic wave (($l_\omega \sim p_F/m_e\omega \sim p_F/m_h\omega$). The requirement that the skin effect should be anomalous, $\delta_n \ll l_\omega$, now sets the upper limit to a frequency ω.

The dispersion equation (2.11) and the conductivity of Eq. (2.10) yield a linear spectrum of the wave in question:

$$k = \omega (V_F^2 + V_A^2)^{-1/2},$$
$$V_F^2 = \frac{2}{5}\frac{p_F^2}{m_e m_h}, \quad V_A^2 = \frac{H_0^2}{4\pi n (m_e + m_h)}. \qquad (2.39)$$

In strong magnetic fields of Eq. (2.37) we have $V_A \ll V_F$ and the phase velocity of the wave becomes equal to the Fermi velocity V_F. The depth of penetration of oscillations into a metal is then $2V_F/(\nu_e + \nu_h)$ and is of the same order of magnitude as the mean free path of carriers l. In very strong fields of Eq. (2.38) the wave with the spectrum of Eq. (2.39) reduces to a familiar magnetoplasma wave[7] with the Alfven velocity V_A. However, one should point out that this is true only in the case of semimetals, when V_A reduces to V_F in realistically attainable magnetic fields H_0.

We shall now give the expression for the amplitude of a wave of Eq. (2.39) excited in a metal. It follows from Eq. (2.2) that

$$\frac{E_l(0)}{H(0)} = \frac{1}{c}\frac{V_A^2}{(V_F^2 + V_A^2)^{1/2}}. \qquad (2.40)$$

It is clear from Eq. (2.40) that an increase in H_0 increases the wave amplitude and the conditions for observing it become easier.

3. INFLUENCE OF SURFACE SCATTERING OF ELECTRONS ON THE MIXED SKIN EFFECT

As established in the preceding section, the surface impedance of a compensated metal derived for strong magnetic fields of Eq. (1.4) is a sum of the "short-wavelength" Z_s and "long-wavelength" Z_l terms. In the preceding section the impedance was calculated ignoring a group of surface electrons colliding with the boundary of the sample. However, it is well known that the impedance Z_s is sensitive to the nature of the interaction of electrons with the surface. In the case of diffuse reflection an allowance for the surface scattering simply alters slightly the numerical coefficient in the short-wavelength (first) term in Eq. (2.27), whereas in the case of near-specular reflection the impedance Z_s becomes very different. It is shown in Refs. 15, 10, and 11 that in the specular reflection case a new group of grazing electrons appears in a sample and these collide with the surface boundary of a metal, but still move all the time in a skin layer of depth δ_s. Such electrons dominate the conductivity and determine the impedance Z_s. According to Ref. 10, the impedance is then

$$Z_s = 4.1\left(\frac{\omega^3 R}{c^6 \sigma_0^2}\right)^{1/5} \exp\left(-i\frac{3\pi}{10}\right) = 11.2\frac{\omega(\delta_n^4 R)^{1/5}}{c^2}\exp\left(-i\frac{3\pi}{10}\right). \qquad (3.1)$$

We shall now consider the influence of the surface scattering of electrons on the long-wavelength term Z_l of the impedance [second term in Eq. (2.27)]. In the calculation of Z_l we need to find the distribution of the long-wavelength electric field $E_l(x)$. The equation for the cosine transform $\mathscr{E}_l(k)$ of the electric field can be derived from the general expressions of Ref. 10:

$$k^2\mathscr{E}_l(k)+2E'(0)=\frac{4\pi i\omega}{c^2}[\sigma_l(k)\mathscr{E}_l(k)-Q(0,0)E_l(0)]. \quad (3.2)$$

Equation (3.2) is derived bearing in mind that the integral kernel of the conductivity $Q(k, k')$ has characteristic values k and k' of the order of R^{-1}. On the other hand, the long-wavelength electric field $\mathscr{E}_l(k)$ varies for values of (k) which are considerably smaller and amount to δ_l^{-1} For this reason the current of surface electrons [representing the last term in the brackets of Eq. (3.2)] is governed by the electric field at the boundary of the metal $E_l(0)$.

The Maxwell equation in the form of Eq. (3.2) is algebraic and is easily solved. In the case of the long-wavelength impedance, we obtain the expression

$$Z_l=\frac{Z_l^{(0)}}{1+Z_l^{(0)}|Q(0,0)|/2}, \quad (3.3)$$

where $Z_l^{(0)}$ is the long-wavelength impedance calculated without allowance for the surface electrons [second term in Eq. (2.27)]. The asymptotes of $Q(0, 0)$ are different for the cases of diffuse and specular reflections of electrons. We shall therefore consider these two situations separately.

1. In the diffuse scattering case the asymptote $|Q(0,0)|$ obtained from the general expression of Ref. 10 is

$$|Q(0,0)|=3\sigma_0\frac{R^2}{l_e+l_h}=3\frac{ne^2}{p_F}R^2. \quad (3.4)$$

Substituting Eq. (3.4) into Eq. (3.3), we obtain the following expression for the impedance:

$$Z_l=Z_l^{(0)}\left[1+\frac{3}{2}\left(\frac{5}{2}\right)^{1/3}\frac{(l_e l_h)^{1/2}}{l_e+l_h}\right]^{-1}, \quad \delta_n\ll R\ll l. \quad (3.5)$$

It is clear from Eq. (3.5) that in the diffuse reflection case when strong magnetic fields are applied, the impedance Z_l differs only by a numerical factor from $Z_l^{(0)}$. In the range of very strong fields ($R\ll\delta_n\ll l$) the skin effect is normal so that the surface scattering plays no significant role and Z_l is described by the previous formula (2.34). Therefore, in the diffuse scattering case the group of surface electrons has no significant influence on the short-wavelength Z_s or the long-wavelength Z_l parts of the impedance, which does not change the conclusions reached in the preceding section.

2. In the specular reflection case the asymptote of $|Q(0,0)|$ is given by[10]

$$|Q(0,0)|=\frac{27}{32}\left[\frac{\pi}{3}+\mathrm{ci}(\pi)-\mathrm{ci}(3\pi)\right]\sigma_0R\approx0.93\sigma_0R, \quad (3.6)$$

where ci is the integral cosine. In view of such a high conductivity of surface electrons [$|Q(0,0)|Z_l^{(0)}\sim l/R\gg1$] the long-wavelength impedance of Eq. (3.3) is governed mainly by the surface current:

$$Z_l=2|Q(0,0)|^{-1}\approx2.15/\sigma_0R\propto H_0. \quad (3.7)$$

Impedance of Eq. (3.7) is $l/R\gg1$ times less than $Z_l^{(0)}$. Such

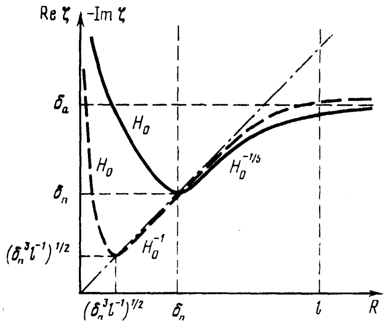

FIG. 4. Schematic dependences of the real (continuous curve) and imaginary (dashed curve) parts of the impedance $\zeta = c^2Z/4\pi\omega$ on the reciprocal of the external magnetic field H_0 ($R\propto H_0^{-1}$) in the case of specular reflection of electrons

a reduction in the long-wavelength impedance compared with the diffuse reflection case occurs because of the screening of the long-wavelength electric field $E_l(x)$ by the current of surface electrons. Such screening is so strong that throughout the range of magnetic fields defined by Eq. (1.4) the amplitude of the long-wavelength electric field $E_l(0)$ is small compared with the short-wavelength field $E_s(0)$. Consequently, the impedance of Eq. (3.7) is small compared with Z_s of Eq. (3.1):

$$Z(H_0)\approx Z_s(H_0)\propto H_0^{-1/3}, \quad \delta_n\ll R\ll l. \quad (3.8)$$

In very strong magnetic fields the short-wavelength component of the electric field vanishes. The impedance is now practically real: its real part is described by Eq. (3.7) and the imaginary part is a small correction in terms of the parameters $(R/\delta_n)^2\ll1$ and $\delta_n/l\ll1$:

$$-\mathrm{Im}\,Z\approx\frac{\omega}{c^2}R+0.83\frac{\delta_n}{\sigma_0R(l_el_h)^{1/2}}. \quad (3.9)$$

Figure 4 shows schematically the dependence of the surface impedance of a compensated metal on the reciprocal of the magnetic field in the case of specular reflection of electrons.

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