Magnetodynamic nonlinearity and the pinch effect in metals

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We present a theoretical investigation of the *I-V* characteristics of thin samples of metal for large values of current *I*, in which the characteristic radius of curvature of an electron orbit in the intrinsic magnetic field of the current is smaller than the sample thickness. We establish that in compensated metals the differential resistance increases as the current goes up, independent of the sample shape: in films, *V* is proportional to the square of the current, while for wires $V \propto I^3$. We show that two spatial regions appear in the sample, whose conductivities differ significantly. In the central region the current is transported by electrons which are "trapped," with a conductivity on the order of the bulk metallic conductivity σ_0. Because of this, a phenomenon can occur in the metal which is analogous to the pinch effect: the current density in the central part of the conductor significantly exceeds the density on the periphery. This is due principally to the fact that the pinch effect in electrically neutral metallic media is not related to any redistribution of the electric charge density. We study the characteristics of the pinch-effect for metallic films and wires.

1. INTRODUCTION

An electric current flowing in a metal gives rise to a magnetic field, which is associated with the electron dynamics and consequently with the sample conductivity. The resulting feedback serves as a primary source of nonlinear electromagnetic effects in metals at low temperatures. In the static situation, this magnetodynamic nonlinear mechanism causes the current-voltage (*I-V*) characteristics of a metal to deviate from Ohm's law. The nonlinearity appears most strikingly in samples with diffuse boundaries under classical size-effect conditions, when at least one of the dimensions of the sample transverse to the current is much smaller than the electron mean free path *l*.

A theoretical analysis of magnetodynamic nonlinearity in the static conductivity was first presented in Refs. 1, 2 for the case of a thin metallic film. In Refs. 1, 2 it was noted that a principal role in the development of these nonlinear effects is played by a group of "trapped" electrons. This group of electrons appears in the sample because of sign alternation in the distribution of the intrinsic magnetic field of the currents transverse to the film. The trapped electrons bend across the plane where the magnetic field changes sign, and do not collide with the diffuse boundaries of the film; in effect, these electrons interact with the electric field over their entire mean free path. For values of the total current *I* small enough to ensure that the inequality

$$d<R, \quad R=cp_F/eH, \tag{1.1}$$

holds, the relative number of trapped electrons is of order $(d/R)^{1/2}$. Here R is a characteristic radius of curvature of the electron trajectory in the current's intrinsic magnetic field, $H = 2\pi I/cD$ is the value of the magnetic field at the film boundary, e and p_F are the charge and Fermi momentum, c is the velocity of light, and d and D are the thickness and width of the film. The conductivity of the trapped electrons

$$\sigma_{\text{захв}} \sim \sigma_0 (d/R)^{1/2} \propto I^{1/2} \tag{1.2}$$

grows with increasing current, and becomes important in the region

$$(Rd)^{1/2} \ll l \tag{1.3}$$

(σ_0 is the static conductivity of a bulk sample). For this reason, the linear part of the *I-V* characteristic, which holds when

$$d \ll l \ll (Rd)^{1/2}, \tag{1.4}$$

becomes a square root in the interval

$$d \ll (Rd)^{1/2} \ll l. \tag{1.5}$$

In metallic wires, because of the finiteness of the sample in two dimensions, the relative number of trapped electrons is proportional to $d/R \propto I$ and the nonlinearity of the *I-V* characteristics is manifest in the form of effective stabilization of the voltage.[3] We note that in contrast to other nonlinear mechanisms, the magnetodynamic nonlinearity leads to a decrease in the resistivity of metallic samples in the low-current region (1.1). This effect has been observed experimentally in zinc[4] and gallium,[5] and was investigated in detail for tungsten.[6,7] In Refs. 6, 7 it was pointed out that as the ratio d/D increases, the maximum change (up to 50%) in the differential resistivity is observed when the sample's shape approximates a wire.

The magnetodynamic nonlinearity leads to various physical consequences even in the weak-current region (1.1). For example, in Refs. 8, 9, a decreasing portion of the *I-V* curve was predicted theoretically; the *I-V* curve itself can have both an *N*-shape and an *S*-shape. It is clearly interesting to study these nonlinear processes at larger current values, where the inequality (1.1) reverses, i.e.,

$$R \ll d. \tag{1.6}$$

In this situation, an increase in the sample resistivity is observed experimentally as the current goes up, accompanied by oscillations in the *I-V* curve[5–7] and by time-dependent voltage oscillations, whose spectral composition is very sensitive to the value of *I*.[7]

Let us note two circumstances which play an extremely important role in the development of nonlinearity under conditions (1.6). We are discussing compensated metals

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here, in which the Hall effect has no influence on the phenomenon discussed above. The first circumstance concerns the conductivity of "Larmor" electrons, moving in the strong magnetic field of the current through a near-circular orbit whose radius is on the order of R. It is well-known that a magnetoresistive effect occurs in a strong uniform magnetic field since the conductivity of the Larmor electrons is proportional to the parameter $(R/l)^2$. The nonzero value of the latter is related to electron scattering, which leads to diffusion of the orbit centers in an electric field. The strong gradient in the intrinsic magnetic field of the current causes an additional motion of the orbit centers.[10] Therefore, there must be "gradient" terms in the Larmor electron conductivity, which take into account drift of the orbits in the inhomogeneous magnetic field. The magnitude of these gradient terms is of order $\sigma_0(R/\Delta)^2$, where Δ is the spatial scale of variation of the current's intrinsic magnetic field. Thus, there appears a competition between two conductivity mechanisms for the Larmor electrons, one of which is related to diffusion of the orbit centers due to collisions of electrons with scatterers, while the other is related to drift because of gradients in the magnetic field produced by the current.

The second circumstance which must be taken into account is the fact that under conditions (1.6), despite the high current, the trapped electrons do not disappear. It is clear that for any arbitrarily large value of I there exists a small region near the center of the sample where the magnetic field is quite weak. As the current increases, the size of this region decreases; however, its conductivity, which is determined by the trapped electrons, remains constant. From Eq. (1.2) it is clear that for $R \sim d$, the conductivity of the trapped electrons is of order σ_0. This estimate for the conductivity of the central region of the sample remains valid for $R \ll d$. At the same time, the conductivity of the peripheral region, which is determined by the Larmor electrons, is much smaller than σ_0 and proportional to I^{-2}. In other words, a phenomenon occurs which is analogous to the pinch effect in gaseous discharge plasmas[11] and semiconductors[12]: the current density is found to be highest at the sample center.

This pinch effect in metals possesses a number of interesting features. First of all, we note that the requirement of electrical neutrality, which is specific to metals, ensures that the pinch effect is not accompanied by a redistribution of the electron density. A question which requires a special investigation is the following: in what region does the basic current flow—in a filamentary region with high conductivity, whose size decreases with increasing current, or in the peripheral region whose size increases while its conductivity falls?

In this paper, we will study the spatial distribution of the current density and the intrinsic magnetic field in thin metallic films and wires under conditions (1.6), and also calculate their I-V characteristics. We present an interpolation formula for the I-V characteristics, which is valid both for small (1.1) and large (1.6) values of current. We show that in films the transport is basically due to trapped electrons concentrated near the middle of the sample. In wires, the principal concentration to the current is given by the Larmor electrons. However, the distribution of the intrinsic magnetic field in a wire has a form characteristic of the pinch effect: as the distance from the axis increases, a sharp rise in the field is observed, followed by a decrease. In both films and wires, in the region (1.6) the resistivity of the sample increases as the current I increases.

2. STATEMENT OF THE PROBLEM: ELECTRON DYNAMICS AND CURRENT DENSITY

Let us discuss a film of compensated metal, along which flows a current I. The y-axis we direct along the current while the x-axis is perpendicular to the film edge; we choose the plane $x = 0$ at the film's center. The current's self-magnetic field $H(x)$ is parallel to the z-axis. The dimensions of the sample in the direction D are assumed to be much larger than either its thickness d or the elctron mean free path l.

We will present an analysis of the current density distribution in the film and the resulting magnetic field for the case of an isotropic metal: the electron and hole Fermi surfaces we will take to be identical spheres. The masses and mean free paths of electrons and holes we also take to be the same. In this situation there is no Hall effect in the metal, i.e., the off-diagonal components of the conductivity tensor are identically zero. We note that for an arbitrary electron dispersion law, in general there will appear off-diagonal components of the conductivity which leads to complications in the calculation but which do not influence the final result.

For the geometry chosen here, the magnetostatic equation has the form

$$cH' = -4\pi j(x), \tag{2.1}$$

where $j(x)$ is the current density; the prime denotes a derivative with respect to the coordinate x. The boundary conditions to use with Eq. (2.1) are the following:

$$H(-d/2) = -H(d/2) = H, \quad H = 2\pi I/cD. \tag{2.2}$$

From the Maxwell equation curl $\mathbf{E} = 0$ it follows that the electric field $E_y = E$ within the sample is spatially homogeneous.

We now discuss the dynamics of charge carriers in a strong inhomogeneous magnetic field $H(x)$. Because within the model we have chosen the electron and hole trajectories differ only in the sign of their radii of curvature, we will discuss only electron dynamics; it should be kept in mind that the contributions of electrons and holes to the diagonal components of the conductivity add together while those to the nondiagonal components compensate each other.

Let us pick a gauge for the vector potential of the form

$$A = \{0; A(x); 0\}, \quad A(x) = \int_0^x dx' H(x'). \tag{2.3}$$

Then inequality (1.6), which determines the maximum value of the field, can be rewritten in the form

$$eA(d/2)/cp_F \gg 1. \tag{2.4}$$

Integrals of the electron motion in the field $H(x)$ give the total energy, which equals the Fermi energy $\varepsilon_F = p_F^2/2m$, as well as the generalized momenta $p_z = mv_z$ and $p_y = mv_y - eA(x)/c$ (m is the electron mass and v_z and v_y the components of its velocity). The component v_x equals

$$v_x = \pm \frac{1}{m}[p_\perp^2 - (p_y + eA(x)/c)^2]^{1/2}, \quad p_\perp = (p_F^2 - p_z^2)^{1/2}. \tag{2.5}$$

From the condition that the quantity under the radical (2.5)

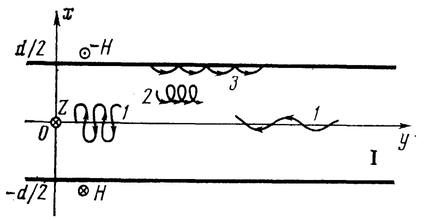

FIG. 1. Coordinate system; trajectories of trapped (1), Larmor (2), and surface (3) electrons in a metal film.

be nonnegative, we find the attainable values of the electron integrals of motion:

$$p_y^-(x)\equiv-\frac{e}{c}A(x)-p_\perp\leqslant p_y\leqslant-\frac{e}{c}A(x)+p_\perp\equiv p_y^+(x). \tag{2.6}$$

In Fig. 2 we illustrate the region (2.6); the coordinates used are p_y,x. From this figure it is clear that the conduction electrons in the sample can be divided into three groups according to the characteristic form of their trajectories:

a) Trapped electrons. These move along trajectories of type 1, as illustrated in Fig. 1. The half-period of their motion along the x-axis is

$$T=\int_{x_1}^{x_2}\frac{dx'}{|v_x(x')|}. \tag{2.7}$$

The turning points $x_1<x_2$ are roots of the equation $p_y^-(x)=p_y$. From Fig. 2 it is clear that the trapped electrons have momenta p_y that lie in the interval

$$p_y^-(x)\leqslant p_y\leqslant p_y^+(0)=p_\perp. \tag{2.8}$$

This group of electrons is present only in the region $|x|<x_0$, where x_0 is the positive root of the equation

$$\frac{e}{c}A(x_0)=2p_F. \tag{2.9}$$

b) Larmor electrons. These have trajectories of type 2 in Fig. 1. The half-period of their motion is determined by Eq. (2.7); however, the turning points $x_1<x_2$ are roots of the equations

$$p_y^-(x)=p_y,\quad p_y^+(x)=p_y.$$

The Larmor electrons occupy the following region:

$$\max\{p_y^-(x),\ p_\perp\}\leqslant p_y<\min\{p_y^+(x),\ p_y^-(d/2)\}. \tag{2.10}$$

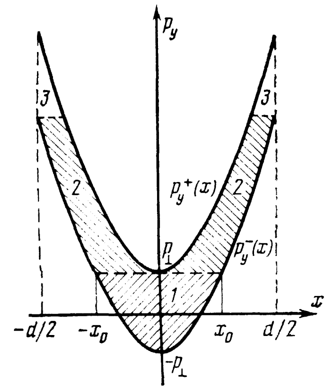

FIG. 2. Phase space (p_y,x) and regions where various groups of electrons exist: 1—trapped electrons, 2—Larmor electrons, 3—surface electrons.

c) Surface electrons. Their characteristic trajectories are illustrated in Fig. 2 above the number 3. The region in p_y they occupy is

$$p_y^-(d/2)\leqslant p_y\leqslant p_y^+(x). \tag{2.11}$$

In the case we are investigating, i.e., diffuse reflection, the contribution of surface electrons to the current is found to be small for $R/l\ll1$. Therefore, we will not include them in the calculation of the current density which follows.

The current densities for the corresponding groups of particles are rather simple to calculate with the help of standard methods of solving the kinetic equation. This equation is linearized with respect to the electric field, while the entire nonlinearity is due to the magnetic field $H(x)$ in the Lorentz force. Leaving aside the calculations, we present the equation for the current density of trapped and Larmor electrons:

$$j_{t,l}(x)=\frac{4e^2}{(2\pi\hbar)^3}\int_{O_{t,l}}\frac{d\varepsilon_\perp\,dp_y\,v_y}{|v_z|\,|v_x|}\times\left\{\int_{x_1}^{x}dx'\frac{v_y(x')E(x')}{|v_x(x')|}\operatorname{sh}(\nu\tau(x;x'))+\frac{\operatorname{ch}(\nu\tau(x_1;x))}{\operatorname{sh}\nu T}\int_{x_1}^{x_2}dx'\frac{v_y(x')E(x')}{|v_x(x')|}\operatorname{ch}(\nu\tau(x_2;x'))\right\} \tag{2.12}$$

Here $\nu=v_F/l$ is the collision frequency, and O_t, l is the region in phase space occupied by trapped (2.8) or Larmor (2.10) electrons, $\varepsilon_\perp=\varepsilon_F-p_z^2/2m=p_\perp^2/2m$,

$$\tau(x_1;x)=\int_{x_1}^{x}\frac{dx'}{|v_x(x')|}.$$

In Eq. (2.12) we retain the dependence of the electric field E on the coordinate x, so that calculation of the current density will be useful when we also investigate the dynamic situation.

If in the Larmor electron current denisty (2.12) the small parameter $\nu\tau\sim R/l\ll1$ is set equal to zero, and if we neglect the x-dependence of the electric and magnetic fields, then the quantity $j_L(x)$ reduces to zero. Expanding (2.12) in powers of $\nu\tau$ and including the x-dependence of the fields shows that the Larmor electron current under condition (2.4) can be cast in the form of a sum of two terms:

$$j_\Pi(x)=\sigma_0\frac{R^2(x)}{l^2}E-\frac{2}{5}\sigma_0R^2(x)H\left(\frac{E}{H}\right)'', \tag{2.13}$$

$$R(x)=cp_F/eH(x),\quad H'\equiv dH/dx.$$

Expression (2.13) was obtained under the assumption of weak field gradients, in which the characteristic scales of variation of E and H are small compared to $R(x)$:

$$RH'/H,\ R^2H''/H,\ RE'/E,\ R^2E''/E\ll1. \tag{2.14}$$

Equation (2.13) reflects the competition of the two conductivity mechanisms for Larmor electrons described in the Introduction. The first term in (2.13) corresponds to the magnetoresistance effect and is caused by the diffusion of the centers of electron orbits in the crossed electric and magnetic fields. The second gradient term is related to drift of the electron orbits due to the field gradients.

The asymptotic expansion of the current density of the trapped electrons begins with a term which in the case of strong currents (1.6) depends neither on the quantity $v\tau \sim R/l \ll 1$ nor on the size of the field gradients. Thus, in contrast to the weak-field case (1.1), the contribution to the p_y-integral in (2.12) is determined by electrons at arbitrary angles with respect to the plane $x = 0$. For this reason, the conductivity of trapped electrons does not contain a small parameter, and its magnitude is on the order of σ_0:

$$j_3 \approx \sigma_0 E. \tag{2.15}$$

Thus, the asymptotic behavior of the current density in the case of (1.6) has the form

$$j(x) = \sigma_0 E(x) \begin{cases} 1, & |x| < x_0 \\ \dfrac{R^2(x)}{l^2} - \dfrac{2}{5} R^2(x) \dfrac{H}{E}\left(\dfrac{E}{H}\right)'', & x_0 < |x| < d/2. \end{cases} \tag{2.16}$$

We note that in the static situation where the electric field is homogeneous, Eq. (2.9) for the boundary x_0 of the region where the trapped electrons are located can be replaced by a simpler equation. Actually, according to (2.16) the current density in the region $|x| < x_0$ does not depend on position. Consequently, the magnetic field in this region varies linearly with x, while the vector potential satisfies $A(x) = H(x)x/2$. As a result, (2.9) can be written in the form

$$x_0 = 4R(x_0). \tag{2.17}$$

3. SOLUTION TO THE MAGNETOSTATIC EQUATION: *I-V* CHARACTERISTICS OF A FILM

In the static situation the distribution of magnetic field in the film is found by solving the Maxwell equation (2.1) with the current density (2.16), (2.17) and the boundary conditions (2.2). Thus, from the Maxwell equation curl $\mathbf{E} = 0$ it follows that the electric field E_y is homogeneous, and the derivatives E' and E'' in (2.16) must be set equal to zero:

$$H = -\frac{4\pi}{c}\sigma_0 E \begin{cases} 1, & |x| < x_0 \\ \dfrac{R^2(x)}{l^2} + \dfrac{2}{5} R^2(x) \left(\dfrac{H''}{H} - 2\dfrac{H'^2}{H^2}\right), & x_0 < |x| < d/2. \end{cases} \tag{3.1}$$

The general solution (3.1) to this second-order nonlinear differential equation contains two unknown constants. In addition, it is necessary to determine the value of the electric field which enters into (3.10) as a parameter. There are only the two boundary conditions (2.2) available to find these three unknown quantities. The role of the third condition is played by requirement (2.14) that the magnetic field distribution be smooth in the region $x_0 < |x| < d/2$, which is necessary in order to replace the integral conductivity operator (2.12) by the differential operator (2.13).

Equation (3.1) can be solved by the method of successive approximations. In the zeroth-order approximation the magnetic field in the region $|x| > x_0$ is homogeneous, and all the variation in $H(x)$ comes from the central part of the film $|x| < x_0$:

$$H(x) = -\begin{cases} \dfrac{4\pi}{c}\sigma_0 E x, & |x| < x_0 = \dfrac{2}{\pi}\dfrac{c^2 p_F D}{el} \\ H \operatorname{sign} x, & x_0 < |x| < d/2 \end{cases}, \tag{3.2}$$

$$E = \frac{\pi}{4}\frac{el^2}{\sigma_0 c^2 p_F D^2}. \tag{3.3}$$

This solution exhibits the pinch effect: the current density for $|x| > x_0$ equals zero, and all the current I is transported by the trapped electrons, which are present in the region $|x| < x_0$ (Fig. 3). Including the magnetoresistive effect gives rise to small corrections of order Rd/l^2 in the magnetic field and current distribution, while the gradient terms in (3.1) are manifested in higher-order terms in the expansion. We note that the corrected current ΔI transported by Larmor electrons in the region $x_0 < |x| < d/2$ does not depend on I:

$$\Delta I = I\frac{Rd}{8l^2} = \frac{c^2 p_F dD}{12\pi e l^2}. \tag{3.4}$$

The *I-V* characteristic of the metallic film in the strong current region is parabolic. From (3.3) it follows that

$$V = \frac{\pi}{4}\frac{eL}{\sigma_0 c^2 p_F D^2} I^2, \tag{3.5}$$

where L is the sample length and V the voltage.

Let us recall that we have obtained Eqs. (3.2) for the current's magnetic field distribution and (3.5) for the *I-V* characteristic using an isotropic model, in which the off-diagonal components of the conductivity tensor are identically equal to zero. Let us discuss what possible influence the Hall effect, which occurs for arbitrary dispersion laws for the charge carriers, will have on the phenomena discussed here. The presence of a Hall component to the conductivity

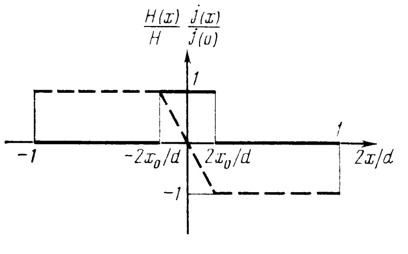

FIG. 3. Schematic form of the magnetic field distribution (dashed curve) and current density (continuous curve) in a film.

leads to the appearance of a normal component to the electric field E_x, and also to renormalization of the diagonal conductivity σ_{yy} due to the quasineutrality condition $j_x = 0$

$$\sigma_{yy} \to \tilde{\sigma}_{yy} = \sigma_{yy} + \sigma_{xy}^2/\sigma_{xx}. \quad (3.6)$$

We note that the presence of E_x does not change our conclusion that the electric field E_y is homogeneous, becuase $\partial E_x/\partial y = 0$. In the current filament region $|x| < x_0$, the conductivity renormalization plays no role. Here the quantities σ_{yy} and σ_{xx} are determined by the trapped particles, and, as we showed above, are of order σ_0; the nondiagonal components σ_{xy} and σ_{yx} in a compensated metal are always much smaller than σ_0.[13] On the periphery $x_0 < |x| < d/2$, where the conductivity is due to Larmor particles, the magnetic field (3.2) is large ($R \ll d \ll l$) and homogeneous. In such a situation, it is well-known[13] that the off-diagonal conductivity components σ_{xy} and σ_{yx} also cannot exceed σ_{yy} and σ_{xx}:

$$\sigma_{xy} \sim \sigma_{yx} \leqslant \sigma_{xx} \sim \sigma_{yy} \sim \sigma_0 R^2/l^2. \quad (3.7)$$

Thus, in the worst case the renormalization (3.6) changes the numerical coefficient in front of the R^2/l^2 term in the current (2.16). This variation in the current on the periphery obviously is not dependent on the homogeneity of the magnetic field (3.2) in the region $x_0 < |x| < d/2$ and will not affect the character of the pinch effect. Thus, the Hall effect in a compensated metal with an anisotropic current-carrier dispersion law is not a controlling factor in the phenomena we discuss here.

To conclude this section, we present an interpolation formula for the *I-V* characteristics of a metal film, which is valid both for small (1.1) and for large (1.6) currents:

$$\frac{V}{u} = \frac{I/i_1}{(^3/_4)\ln(l/d) + (I/i_1)^{1/2}} + \left(\frac{i_2}{i_1}\right)^{1/2}\left(\frac{I}{i_2}\right)^2, \quad (3.8)$$

$$u \approx 0.97 c^2 p_F L/\sigma_0 e l d, \quad i_1 \approx 0.97 c^2 p_F D d/e l^2, \quad i_2 \approx 1.2 i_1 (l/d)^2 \gg i_1.$$

For small currents $I \ll i$ Ohm's law holds. Here the conductivity is caused by the so-called "drift" electrons, which move in almost straight lines and scatter diffusively off the film boundaries. Their conductivity is described by the well-known Fuchs equation.[14] As the current continues to increase, an ever-larger number of electrons are trapped by the intrinsic magnetic field $H(x)$. In the region $i_1 \ll I \ll i_2$ or $d \ll (Rd)^{1/2} \ll l$, the distance transversed by the trapped electrons significantly exceeds the path $(Rd)^{1/2}$ of the transit electrons. Therefore, the conductivity of the film is entirely determined by the trapped electrons, and the *I-V* characteristics (3.8) originate from the square-root portion $V/u = (I/i_1)^{1/2}$. Starting with currents $I = i_2$, the radius of curvature of the electron trajectories become smaller than the film thickness. Thus, the transit electrons disappear, and a group of Larmor electrons appear in the sample. Because the trapped electron current is pinched the *I-V* characteristics coincide with the parabola (3.5).

4. MAGNETOSTATICS OF A METALLIC WIRE

When condition (1.6) is fulfilled in a metallic wire with diameter $d \ll l$, two distinct spatial regions form, just as in a thin film. Near the axis of the wire, the conductivity is determined by a group of trapped electrons, while on the periphery it is determined by Larmor electrons. The calculations for the asymptotic behavior of the current density are analogous to the procedure described in Sec. 2. In a cylindrical coordinate system (r, φ, z) with the z-axis along the wire axis the current density $j_z(r) = j(r)$ is described by the expressions

$$j(r) = \sigma_0 E \begin{cases} 1, & r < r_0 \\ \dfrac{R^2(r)}{l^2} + \dfrac{R^2(r)}{5}\left[\dfrac{3}{r^2} + \dfrac{2}{r}\left(\dfrac{H'}{H} - \dfrac{E'}{E}\right) - \dfrac{2H}{E}\left(\dfrac{E}{H}\right)''\right], & r_0 < r < d/2, \end{cases} \quad (4.1)$$

where

$$r_0 = 2R(r_0), \quad H' \equiv dH/dr, \quad H \equiv H_\varphi(r), \quad E \equiv E_z(r).$$

The current density of the Larmor electrons (4.1) in the wire contains additional terms compared with the analogous expression (2.16) for the film, terms caused by the curvature of the lines of force of the current's self-magnetic field. We remark that these terms clearly depend on r, and reduce to zero as $r \to \infty$. Passing to this limit implies that the conductivity in a sample having the shape of a tube with large radius $\rho \gg l$ with a wall thickness $d \ll l$ coincides with the conductivity of a film.

Substituting the current density (4.1) into the Maxwell equation leads to a nonlinear magnetostatic equation

$$\frac{1}{r}(rH)' = \frac{4\pi}{c}\sigma_0 E \begin{cases} 1, & r < r_0 \\ \dfrac{R^2(r)}{l^2} + \dfrac{R^2(r)}{5}\left[\dfrac{3}{r^2} + \dfrac{2}{r}\dfrac{H'}{H} + 2\dfrac{H''}{H} - 4\dfrac{H'^2}{H^2}\right], & r_0 < r < d/2 \end{cases} \quad (4.2)$$

The boundary conditions for Eq. (4.2) have the form

$$H(0) = 0, \quad H(d/2) = 4I/cd. \quad (4.3)$$

The solution to the problem (4.2), (4.3) is given by the expressions

$$H(r) = \begin{cases} \dfrac{2\pi}{c}\sigma_0 E r, & r < r_0 \sim R(R/d)^{1/2} \\ H(d/2)(2r/d)^{-1/2}, & r_0 < r < d/2 \end{cases}, \quad (4.4)$$

$$E = \frac{72}{5\pi}\left(\frac{e}{p_F c^2}\right)^2 \frac{1}{\sigma_0 d^2} I^3. \quad (4.5)$$

The magnetic field distribution (4.4) has a form characteristic of the pinch effect in a gas discharge plasma: a rapid rise of the field in the region $r < r_0$ is followed by a decrease for $r_0 < r < d/2$ (see Fig. 4). However, the current I_0 following in the central region $r < r_0$ is small compared to the total current I when the parameter $R/d \ll 1$. Thus, the magnitude

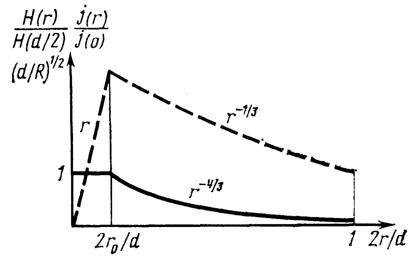

FIG. 4. Schematic form of the magnetic field distribution (dashed curve) and current density (continuous curve) in a wire.

of the current I_0 does not depend on the total current I:

$$I_0=c^2p_F/2e. \tag{4.6}$$

Let us recall that in the metallic film, thanks to the pinch effect, the total current I is basically concentrated in the central portion, while at the periphery a small current ΔI flows which does not depend on the magnitude of I. In the case of the wire the opposite holds: in its central region a small current I_0 flows which does not depend on I, while the main part of the current I is carried by the Larmor elecrons at the periphery. Thus, in contrast to the film, the principal terms in the Larmor electron current density (4.1) are the gradient terms.

Hence, the I-V characteristic of a wire in the high-current region is entirely determined by the gradient terms in the conductivity of Larmor electrons. From (4.5) it follows that

$$V=\frac{18}{5\pi}\frac{LI}{\sigma_0 d^2}\left(\frac{I}{I_0}\right)^2. \tag{4.7}$$

We introduce an interpolation formula for the I-V characteristic which is valid both for small and for large currents:

$$\frac{V}{U}=\frac{I/I_1}{1+I/I_1}+0.55\left(\frac{I}{I_0}\right)^3, \tag{4.8}$$

$$U\approx 1.04c^2p_FL/\sigma_0ed^2,\quad I_1\approx 0.82c^2p_Fd/el,\quad I_0\approx 0.61I_1l/d\gg I_1.$$

In a metal wire, the linear part of the I-V characteristic shifts to the saturated portion for currents $I>I_1$, at which the characteristic radius of curvature of the electron trajectories becomes smaller than the mean free path l. In the region $I>I_0$ Larmor electrons appear in the sample with orbital radii $R<d$ and the I-V characteristic (4.8) reduces to the cubic portion (4.7).

5. CONCLUSION

The pinch-effect phenomena in metals which we have investigated in this paper is a consequence only of the magnetodynamic nonlinearity mechanism. As far as we know, this is the first example of a pinch effect in an electrically-neutral medium which does not result from redistribution of the charge carrier density.

1. The results obtained in the previous sections for the I-V characteristic of thin films and wires agree qualitatively with the experimental data of Refs. 5–7. In these experiments, an increase in the differential resistivity of the sample is observed in the current region (1.6); the dependence of dV/dI on current is linear for the film and becomes approximately quadratic as the parameter d/D increases. We note that the growth in resistivity begins at a fairly small value of current $I\gtrsim I_0$, where the nonlinearity is connected only with the magnetodynamic mechanism. It is not difficult to verify that under the experimental conditions of Refs. 5–7, the current I_0 equals 10 amperes in order of magnitude. Thus, the Joule heating per unit area of the sample surface comes to 0.05 W/cm^2, and can easily be removed in liquid helium.

2. Let us turn our attention to a number of circumstances which play an important role in the creation of the nonlinear I-V characteristic, and which determine the current distribution in the metal. First of all, consider the character of electron reflection off the sample surface: in this paper, we have discussed the most interesting case of diffuse scattering. In the weak current region (1.1) an increase in the degree of specular reflection off the sample boundary results in an increase in the "transit" electron conductivity. For this reason, the nonlinearity, which is connected with the set of trapped electrons, appears at larger values of the current.[15] Thus, an increase in the specular-reflection parameter leads to constriction of the interval $i_1<I<i_2$ (for a wire, $I_1<I<I_0$), over which the resistivity decreases with an increase in the current.

In the high-current region (1.6), as the degree of specular reflection increases the surface electrons play an even greater role (for diffuse scattering we neglected these electrons; see Sec. 2).

In the case of ideally specular reflection, the surface electrons lead to a significant redistribution of the current in the sample, and are found to influence the form of the I-V characteristic. In metallic films the surface electrons are concentrated in a layer of thickness $2R$ near its boundary, and the conductivity in this region is on the order of σ_0. For this reason, when the reflection is specular the picture of the pinch effect in the film becomes extremely peculiar. The current flows essentially in three regions which are considerably separated from one another: in the central region, it is due to trapped electrons, while near the boundaries it is due to the surface electrons. Thus the I-V characteristic remains parabolic: $V\propto I^2$.

In the wire, the role of the surface electrons is found to be still more important: namely, they carry most of the current for the case of specular reflection. The magnitude of this current is of order $\sigma_0E\cdot 2\pi dR$, which exceeds the Larmor electron current $\sigma_0E(R/d)^2d^2$ by a factor $d/R\gg 1$. For this reason the increase in specular reflection at the boundary of a wire indicates that the I-V characteristic changes over from cubic $V\propto I^3$ to parabolic $V\propto I^2$.

3. In the weak-current region (1.1), the current density distribution in the sample and the form of the I-V characteristic do not depend on whether or not the metal is compensated. For strong currents (1.6), however, the question of compensation is important. In this paper, we have studied the case of a compensated metal; in an uncompensated metal the condition that the current normal to the boundary equal zero leads to a significant renormalization of the longitudinal conductivity σ_{yy} (for the wire, σ_{xx}).

As a result, in the region (1.6) the effective conductivity is on the order of σ_0, and the I-V characteristic is linear in lowest approximation. Thus, in an uncompensated metal, for $I<i_1$, I_1 the I-V characteristic is linear, while for i_1, $I_1<I<i_2$, I_0 a decrease in the sample resistivity is observed; for $I>i_2$, I_0 the I-V characteristic again goes over to a linear portion.

4. In this paper we have investigated the pinch effect in

thin metallic samples with $d \ll l$. In a bulk compensated metal the magnetodynamic nonlinearity arises only in the strong-current region for $R < l \ll d$. Here the conductivity of the Larmor electrons is determined by the magnetoresistive effect: $\sigma = \sigma_0 R^2/l^2$. As in thin films, the bulk current distribution is quite inhomogeneous: in the central part of the sample, where the intrinsic magnetic field is small, and also near the sample boundary (for near-specular reflection) the current density is large. However, it is clear that for sufficiently large sample dimensions the role of these regions of increased conductivity becomes insignificant, and the *I-V* characteristic is entirely determined by the group of Larmor electrons. Independent of the sample shape, the *I-V* characteristic has the form of a cubic parabola $V \propto I^3$.

5. The principal means of affecting the shape of the nonlinear *I-V* characteristic and the current distribution is an external magnetic field. For example, if an external field h_0 is parallel to the self-magnetic field of the current in the film, then h_0 will shift the plane where the total magnetic field changes sign. As a result, it becomes possible to shift the current filament within the sample.

6. Superposed on the increasing portion of the *I-V* characteristic, one observes in experiment[7] a small (on the order of a few percent) time-dependent voltage oscillation in the fixed current regime, and also an oscillation in the *I-V* characteristic which is related to this phenomenon. In a theoretical paper (Ref. 16) it was shown that it is possible in principle for an instability to appear in the static current distribution of the metal. However, the conclusions of Ref. 16 are inapplicable in the case we investigate here, since in Ref. 16 the metallic conducivity is assumed to depend locally only on the value of the current's intrinsic magnetic field, not on its gradient. In addition, the stability analysis given in Ref. 16 was carried out in the fixed-voltage regime.

The authors of Ref. 7 made an experimental attempt to explain the instability of their static solutions by invoking motion of the boundaries of the current filament. However, in their analysis they did not include the variations in electric field in the filament region which must necessarily accompany such motion; these authors were thus led to an erroneous conclusion. We investigated the static solutions (3.2), (3.3) for the film and (4.4), (4.5) for the wire for stability in the fixed-current regime, and obtained a negative resistance decrement. In other words, the system is found to be stable relative to infinitesimal fluctuations.

We propose that the time-dependent voltage oscillations observed in experiment arise either as a result of "hard switching" (that is, the oscillations develop when the fluctuations exceed some critical value) or are a result of processes we have not included in our model (e.g., inhomogeneous sample heating). To solve the problem of such oscillations would require a special investigation.

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