Plasma oscillations and zero sound in a degenerate spin-polarized electron system

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The spectrum of collective excitations due to fluctuations of the electron density in a degenerate semiconductor, in the presence of spin polarization of the conduction electrons, is investigated theoretically. Two types of excitation are considered—plasma and zero-sound. It is established that spin polarization does not influence the activation energy of the plasma oscillations, but can noticeably increase the coefficient of their spatial dispersion and enhance the dependence of the plasma frequency on the wave number. The influence of spin polarization on propagation of zero sound is investigated, and it is shown that its velocity increases with increase of the degree of polarization. The temperature corrections to the plasmon and zero-sound spectra at $T = 0$ are obtained and the coefficients of their collisionless damping are calculated. The contributions of these excitations to the dynamic structure factor of the system are calculated. The calculation results are used to determine the rate of the energy loss of the fast electrons in spontaneous emission of plasmons and the cross section for light scattering by zero sound.

INTRODUCTION

Much attention is paid of late to investigations of the properties of various spin-polarized Fermi systems. These include liquid and gaseous ^{3}He↑, ^{3}He↑–He II solutions. ^{3}He↑–^{4}He gaseous mixtures (Refs. 1 and 2), and spin-polarized electrons in metals and in degenerate semiconductors.[3,4] Spin-oriented carriers are produced in the latter either by injecting polarized particles through a ferromagnet-semiconductor tunnel junction or by optical pumping. The second of these methods yields a high degree of spin polarization of the electrons:

$$\alpha=(n_\uparrow-n_\downarrow)/(n_\uparrow+n_\downarrow), \tag{1}$$

which reaches 100% in some crystals at the instant when they are excited with light [4] ($n_\uparrow$ and $n_\downarrow$ are the densities of electron with oppositely directed spins).

Collective excitations in an electron system that is highly spin-polarized should have a number of interesting distinctive features, as is attested, in particular, by the results of Refs. 5 and 6, in which spin waves in such a medium were investigated. These excitations include also plasma waves, whose dispersion properties are altered, as expected, in the presence of spin polarization. This question has nevertheless, to our knowledge, been investigated heretofore neither theoretically nor experimentally.

We study in the present paper the influence of spin polarization on collective excitations produced by electron-density fluctuations in a degenerate electron plasma of a superconductor. We consider only low-energy excitations (with energy lower than the forbidden gap), i.e., excitations in the conduction-electron system. We shall show that the excitation spectrum is sensitive to the spin-polarization state of the system[1)] in the entire investigated wavelength range, which includes both plasma waves of length λ longer than the Fermi-Thomas screening radius λ_{FT} and the zero-sound waves that take their place in the region $\lambda \ll \lambda_{FT}$. At sufficiently low temperatures both modes are quite stable and can be investigated by optical Raman spectroscopy. In addition, we shall show that an increase of the plasmon dispersion coefficient by the spin polarization is manifested in the rates of energy losses of fast electrons scattered by plasmon, an effect likewise verifiable by experiment.

From the standpoint of general physics, the present results can be regarded as a generalization of the classical results of Ref. 7, pertaining to the dispersion and damping of Langmuir zero-sound waves in a degenerate plasma, to the case when the electron component of the plasma is spin-polarized.

1. HYDRODYNAMIC MODEL

We carry out in this section a simple and physically lucid analysis of plasma oscillations in a spin-polarized system, in the framework of the hydrodynamic model. In this model, which is widely used to describe many plasma phenomena, it is assumed that the plasma behaves as a conducting liquid. The system of interest to us must be considered in this case as a mixture of two charged liquids consisting of electrons with oppositely oriented spins $\sigma = \pm 1/2$. The initial equations that describe the behavior of the plasma in this case are the Euler equation and the continuity equation for each of the plasma components:

$$\frac{\partial \mathbf{v}_\sigma}{\partial t}+(\mathbf{v}_\sigma\nabla)\mathbf{v}_\sigma=-\frac{e}{m}\mathbf{E}-\frac{\nabla p_\sigma}{\rho_\sigma}, \tag{2}$$

$$\frac{\partial \rho_\sigma}{\partial t}+\nabla(\rho_\sigma\mathbf{v}_\sigma)=0, \tag{3}$$

and also the Maxwell equation for the electric field $\mathbf{E}$

$$\nabla\mathbf{E}=4\pi\sum_\sigma\rho_\sigma+4\pi\rho_i, \tag{4}$$

where $\mathbf{v}_\sigma$, p_σ, and $\rho_\sigma = -en_\sigma$ are, respectively, the velocity, pressure, and density of the plasma σ component, and ρ_i is the density of the ion charge that cancels the equilibrium electron charge. The condition for the validity of the applicability of the hydrodynamic equations (2) and (3) for the description of the wave processes (with characteristic time scale ω^{-1}) in a degenerate collision-free plasma is smallness of the parameter v_{F_σ}/ω compared with the wavelength λ, where $v_{F_\sigma} = (6\pi^2 n_\sigma)^{1/3}\hbar/m$ is the Fermi velocity of the electrons with spin projection σ.

To investigate small plasma oscillations we represent each of the variables $\mathbf{v}_\sigma$, p_σ and ρ_σ in the form of an equilibri-

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um value and a perturbation due to the oscillations:

$$\mathbf{v}_\sigma=\mathbf{v}_\sigma^{(0)}+\delta\mathbf{v}_\sigma,\quad p_\sigma=p_\sigma^{(0)}+\delta p_\sigma,\quad \rho_\sigma=\rho_\sigma^{(0)}+\delta\rho_\sigma. \quad (5)$$

We substitute Eq. (5) in Eqs. (2)–(4) and linearize the latter with respect to the perturbations, assuming that the dependence of the latter on the time and on the coordinates is determined by the factor $\exp[i(\mathbf{q}\cdot\mathbf{r}-\omega t)]$. As a result we obtain a system of linear algebraic equations, the condition for their solvability yields a dispersion equation for the natural oscillations of the plasma. The solution of this equation takes for an immobile plasma ($\mathbf{v}_\sigma^{(0)}=0$) the form

$$\omega^2(q)=\omega_p^2+\left[(1+\alpha)\frac{\partial p_\uparrow^{(0)}}{\partial\rho_\uparrow^{(0)}}+(1-\alpha)\frac{\partial p_\downarrow^{(0)}}{\partial\rho_\downarrow^{(0)}}\right]\frac{q^2}{2}. \quad (6)$$

The first term in Eq. (6), which does not depend on the wave number q, is the square of the langmuir frequency $\omega_p=(4\pi ne^2/m)^{1/2}$. Oscillations with this frequency are due entirely to the action of the electric field produced as a result of charge decompensation, so that the spin polarization of the electrons is in no way manifested at this frequency. The second term in (6), which establishes the dependence of ω on q at small $q(qv_{F_\sigma}\ll\omega_p)$, is connected with allowance for the influence of the usual acoustic effect on plasma oscillations. It is determined in the absence of spin polarization by the ratio of the electron pressure drop in the plasma to the plasma density, which is the same for both σ components of the plasma. The spin polarization, causing the Fermi surfaces of electrons with opposite spin orientations to move apart, makes the electron pressure and density of each of the σ components differ from each other. This is in final analysis the cause of the dependence of the oscillation dispersion on the degree α of the spin polarization.

Let us determine the dependence of ω on α at zero temperature, when the process of propagation of plasma oscillations is adiabatic. One-dimensional adiabatic compression of a degenerate electron gas is described by the equation $p=\mathrm{const}\cdot\rho^3$, which is easily obtained by a method similar to the one used in Ref. 8 to derive the adiabat equation for bulk compression. Calculating the derivatives in Eq. (6) with the aid of this equation and using the equation of state for each of the two plasma components, we obtain ultimately the plasma-oscillation spectrum in the form

$$\omega_p(q)=\omega_p\left[1+\frac{3}{10}P(\alpha)\,(qv_F^{(0)}/\omega_p)^2\right], \quad (7)$$

where

$$P(\alpha)=\frac{1}{2}\left[(1+\alpha)^{5/3}+(1-\alpha)^{5/3}\right], \quad (8)$$

$v_F^{(0)}=(3\pi^2n)^{1/3}\hbar/m$ is the Fermi velocity of the electrons in the absence of spin polarization and is determined by their total density $n=n_\uparrow+n_\downarrow$.

Comparison of expression (7) with the dispersion relations for plasma oscillations in an unpolarized system[7] shows that allowance for the spin polarization produces in the dispersion coefficient an additional component $P(\alpha)$ whose value, as follows from Eq. (8), increases by approximately 60% when α changes from 0 to 1. Thus, the spin polarization can increase the dispersion coefficient noticeably and, consequently, strengthen the dependence of the plasma frequency on the wave number q. Figuratively speaking, at finite q a plasmon in a spin-polarized electron system is more "rigid" than in an unpolarized one. We note also that although the entire preceding analysis pertrained to plasma oscillations of a free electron gas, the final result (7) is applicable also to the oscillations of interest to us, of an electron plasma of a degenerate semiconductor, if e^2 in the expression for ω_p is replaced by e^2/ε_0 (ε_0 is the lattice dielectric constant of the crystal), and take n and m to mean respectively the density and the effective mass of the electrons in the conduction band.

The hydrodynamic approach used above, which enabled us to analyze the influence of the spin polarization on the dispersion properties of plasma wave, is not suitable, however, for the investigation of more subtle effects that are sensitive to details of interactions in a system, such as collisionless damping of waves, the influence of exchange on plasmon dispersion, and others. We construct therefore in the next section a rigorous theory of plasma waves in a spin-polarized system, based on kinetic concepts and permitting a complete analysis of the problem. We shall verify with the aid of this theory, in particular, the validity of the results that follow already from the hydrodynamic approach.

2. SPECTRUM AND DAMPING OF PLASMONS IN THE SELF-CONSISTENT-FIELD APPROXIMATION

To develop a microscopic theory of plasma waves it is convenient to use a linear-response-function formalism within the framework of which the spectrum of the collective excitations due to the fluctuations of the electron density is determined from the condition that the longitudinal dielectric constant $\varepsilon(\omega,\mathbf{q})$ vanish. The function $\omega(\varepsilon,\mathbf{q})$ can in turn be obtained by a standard procedure based on a generalized random-phase approximation that takes into account the electron exchange interaction. We consider the later in a simple model in which the Fourier component of the exchange potential $V(\mathbf{k}-\mathbf{k}')$ is replaced by an effective coupling constant $J\sim e^2n^{1/3}/\varepsilon_0$ that is independent of the particle momenta. It is known that this model leads in many cases to results that agree satisfactorily with experiment.

In the approximation indicated, the function $\varepsilon(\omega,\mathbf{q})$ is expressed in terms of a polarization operator $\Pi_\sigma(\omega,\mathbf{q})$ that describes the renormalization of the Coulomb interaction on account of the dynamic screening, in the form

$$\varepsilon(\omega,\mathbf{q})=1-\frac{4\pi e^2}{\varepsilon_0q^2}\left[\Pi_\uparrow(\omega,\mathbf{q})+\Pi_\downarrow(\omega,\mathbf{q})+2\left(\frac{J}{n}\right)\Pi_\uparrow(\omega,\mathbf{q})\Pi_\downarrow(\omega,\mathbf{q})\right]\times\left[1+\left(\frac{J}{n}\right)\Pi_\uparrow(\omega,\mathbf{q})\right]^{-1}\left[1+\left(\frac{J}{n}\right)\Pi_\downarrow(\omega,\mathbf{q})\right]^{-1}, \quad (9)$$

where

$$\Pi_\sigma(\omega,\mathbf{q})=\frac{1}{2}\int\frac{\mathrm{th}[(\varepsilon_{\mathbf{k}-\mathbf{q}}-\varepsilon_{F\sigma})/2T]-\mathrm{th}[(\varepsilon_{\mathbf{k}}-\varepsilon_{F\sigma})/2T]}{\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}-\mathbf{q}}-\hbar\omega-i0}\frac{d^3k}{(2\pi)^3}, \quad (10)$$

$\varepsilon_{\mathbf{k}}$ is the unperturbed electron energy near the edge of the conduction band, $\varepsilon_{F\uparrow}=(1+\alpha)^{2/3}\varepsilon_F^{(0)}$ and $\varepsilon_{F\downarrow}(1-\alpha)^{2/3}\varepsilon_F^{(0)}$ are the Fermi quasilevels for electrons with oppositely oriented spins, $\varepsilon_F^{(0)}$ is the Fermi level corresponding to the total electron density and to zero degree of spin polarization ($\alpha=0$), and T is the temperature in energy units.

Equation (9) is in essence a trivial generalization of the known Lindhard formula for the dielectric constant of an equilibrium electron plasma to the case of a plasma in a quasiequilibrium spin-polarized state. In p-type semiconductors, in which optical pumping orients only the spins of the nonequilibrium electrons, such a state is realized if the energy relaxation of the electron is faster than the spin relaxation, and the time of the electron spin relaxation exceeds their lifetime. The results of experiments on optical orientation of spins in the indicated semiconductors (see, e.g., Ref. 4) is evidence that both these conditions can be relatively easily met, so that the use of Eq. (9) for real systems of this type is quite legitimate.

We proceed now to solve the dispersion equation $\varepsilon(\omega,\mathbf{q}) = 0$ at zero temperature. We seek the solution in the quasiclassical region of q values ($q \ll m v_{F\sigma}/\hbar$), assuming for the sake of argument that $v_{F\uparrow} > v_{F\downarrow}$, and regarding for simplicity the conduction band as isotropic and parabolic. Calculation of the polarization operator (10) yields in this case[7]

$$\Pi_\sigma(\omega, q) = \frac{m^2 v_{F\sigma}}{2\pi^2\hbar^3}\left[\frac{\omega}{2qv_{F\sigma}}\ln\left|\frac{\omega+qv_{F\sigma}}{\omega-qv_{F\sigma}}\right| - 1 - \frac{i\pi}{2}\frac{|\omega|}{qv_{F\sigma}}\Theta(qv_{F\sigma}-|\omega|)\right], \tag{11}$$

where $\Theta(qv_{F\sigma} - |\omega|)$ is the Heaviside function. Using expressions (9) and (11) we find that in the region $\omega > qv_{F\uparrow} > qv_{F\downarrow}$, in which $\mathrm{Im}\,\Pi_\sigma(\omega,q) = 0$ the solution of the dispersion equation for small values of $q(qv_{F_\sigma} \ll \omega_p)$ is of the form

$$\omega_p(q)=\omega_p\left\{1+\frac{3}{10}P(\alpha)\left[1-\frac{5}{12}\frac{1+\alpha^2}{P(\alpha)}\frac{J}{\varepsilon_F^{(0)}}\right]\left(\frac{qv_F^{(0)}}{\omega_p}\right)^2\right\}. \tag{12}$$

This expression determines the spectrum of the plasma oscillations with account taken of spin polarization and exchange. If exchange interaction is neglected, this expression coincides with the dispersion relation (7) obtained for plasmons within the framework of the hydrodynamic approach, and goes over in the limit as $\alpha \to 0$ into the known expression for the spectrum of the plasmons in an unpolarized electron system.[9]

It is obvious from a comparison of Eq. (12) with Eq. (7) that in the case considered the exchange interaction leads to an effect that is quite obvious from the viewpoint of the Pauli effect, viz., weakening of the plasmon dispersion. It is easy to verify that the polarization factor $(1+\alpha^2)/P(\alpha)$ in the exchange term in Eq. (12) remains of the order of unity in the entire range of values of α, and therefore the spin polarization does not manifest itself at all in this effect. Therefore on satisfaction of the inequality $J \ll \varepsilon_F^{(0)}$, which is one of the conditions for the validity of relation (12), the exchange correction to the plasmon dispersion is small to the extent that the parameter $J/\varepsilon_F^{(0)}$ is small, just as in an unpolarized system. Thus, the conclusion drawn in Sec. 1 that the plasmon dispersion is substantially enhanced if the degree of polarization is high, remains in force in this case, too. Allownace for exchange leads only to insignificant renormalization of the enhancement. Bearing this in mind, we shall henceforth, for simplicity, neglect the exchange.

As already noted, in the region $\omega > qv_{F\uparrow} > qv_{F\downarrow}$ at $T=0$ the imaginary part of $\varepsilon(\omega,\mathbf{q})$ is equal to zero, so that there is no collisionless Landau damping of the plasma oscillations. The physical reason is that at $T=0$ the system has no particles with velocities $v > v_{F\uparrow}$ and capable of resonantly interacting with plasmons. At low but nonzero temperatures such particles do exist, and this leads to the onset of collisionless plasmon damping. In the case of weak damping, the real part of the frequency ω and the damping coefficient $\gamma(\gamma \ll \omega)$ of the plasmons are determined from the equations

$$\mathrm{Re}\,\varepsilon(\omega, \mathbf{q})=0, \tag{13}$$

$$\gamma(q) = \left.\frac{\mathrm{Im}\,\varepsilon(\omega,\mathbf{q})}{\partial\,\mathrm{Re}\,\varepsilon(\omega,\mathbf{q})/\partial\omega}\right|_{\omega=\omega_p(q)} \tag{14}$$

Calculating the integral in Eq. (10) with the aid of the usual rule for bypassing the poles, and separating the real and imaginary parts of the polarization operator, we find that at $T \ll \varepsilon_{F\sigma}$

$$\mathrm{Re}\,\Pi_\sigma(\omega, q) = -\frac{m^2 v_{F\sigma}}{2\pi^2\hbar^3}\left\{1-\frac{\omega}{2qv_{F\sigma}}\ln\left(\frac{\omega+qv_{F\sigma}}{\omega-qv_{F\sigma}}\right)-\frac{\pi^2}{24}\left(\frac{T}{\varepsilon_{F\sigma}}\right)^2 \times\left[\frac{3\omega}{2qv_{F\sigma}}\ln\left(\frac{\omega+qv_{F\sigma}}{\omega-qv_{F\sigma}}\right)-\frac{\omega^2(3\omega^2-5q^2v_{F\sigma}^2)}{(\omega^2-q^2v_{F\sigma}^2)^2}\right]\right\}, \tag{15}$$

$$\mathrm{Im}\,\Pi_\sigma(\omega, q) = -\frac{m^2\omega}{8\pi\hbar^3 q}\left\{1-\mathrm{th}\left[\frac{m}{2q^2T}(\omega^2-q^2v_{F\sigma}^2)\right]\right\}. \tag{16}$$

Substituting in Eq. (13) the expression (9) at $J=0$ and using Eq. (15) we obtain an equation that determines the spectrum of the plasma oscillations with allowance for the thermal motion of the electrons. The solution of this equation in the region $q \ll \omega_p/v_{F\sigma}$ is of the form

$$\omega_p(q)=\omega_p+\left[\frac{3}{10}P(\alpha)+\frac{\pi^2}{3}\left(\frac{T}{\varepsilon_F^{(0)}}\right)^2\mathfrak{P}(\alpha)\right]\frac{(qv_F^{(0)})^2}{\omega_p}, \tag{17}$$

where

$$\mathfrak{P}(\alpha)=\frac{1}{2}[(1+\alpha)^{1/3}+(1-\alpha)^{1/3}]. \tag{18}$$

It follows from Eq. (17) and (18) that the value of the temperature correction to the plasmon dispersion decreases with increase of degree of the spin polarization. Estimates show that in the limiting case of total polarization of the electrons this decrease is approximately 40% of the value of the correction at $\alpha = 0$.

Using Eqs. (9) and (14)–(16), we obtain the coefficient of collisionless damping of the plasmons

$$\gamma_p(q) = \frac{\pi}{8\cdot 3^{1/2}}\frac{\omega_p}{(q\lambda_{FT})^3}\left\{\exp\left[-\frac{1}{3(q\lambda_{FT})^2}\frac{\varepsilon_F^{(0)}}{T}(1+f^+)\right]+\exp\left[-\frac{1}{3(q\lambda_{FT})^2}\frac{\varepsilon_F^{(0)}}{T}(1+f^-)\right]\right\}, \tag{19}$$

where

$$f^\pm=\frac{9}{5}\left\{P(\alpha)+\frac{\pi^2}{3}\left(\frac{T}{\varepsilon_F^{(0)}}\right)^2\mathfrak{P}(\alpha)-\frac{5}{3}(1\pm\alpha)^{4/3}\right\}(q\lambda_{FT})^2, \quad \lambda_{FT}=v_F^{(0)}/3^{1/2}\omega_p. \tag{20}$$

It is seen from Eq. (19) that at $q\lambda_{FT} \ll 1$ the collisionless damping of the plasmons is exponentially small. The physical cause is the exponential smallness (in terms of the parameter $q\lambda_{FT}$) of the phase space of the states for which energy exchange is possible between the electrons and the plasma mode. Of course, there is always collisional damping of plasmon, due to incoherent quasiparticle-scattering processes, but at $ql \gg 1$ (l is the electron mean free path it is smaller than the collision-governed damping.

3. DECELERATING ABILITY OF SPIN-POLARIZED PLASMA

The change of the plasmon dispersion coefficient as a result of spin polarization should influence the decelerating ability of the plasma. We consider here this effect, assuming that the plasma is in thermodynamic equilibrium, and that the charged particle losing energy in scattering from plasmons is an electron excited by light high into the conduction band. In experiment, these assumptions are valid in the case of optical orientation of the spins in an n-type semiconductor with a straight conduction band that is doubly degenerate in spin. Actually, as first shown by D'yakonov and Perel',[10] and later confirmed experimentally in Refs. 11 and 12, an equilibrium spin-polarized plasma can exist in semiconductors of this type under certain conditions. An appreciable degree of polarization of the equilibrium electrons is reached already at exciting-light intensities at which the density of nonequilibrium carriers is still low compared with the density of the equilibrium ones.

The general expression for the loss of energy loss of a rapid "probing" electron interacting with the electric field of the plasma-charge-density field fluctuations is of the form (the plasma volume is $V = 1$)

$$-\frac{d\mathscr{E}}{dt} = \int (\mathscr{E}_{\mathbf{p}} - \mathscr{E}_{\mathbf{p}-\hbar\mathbf{q}})\, W(\mathbf{p} \to \mathbf{p} - \hbar\mathbf{q})\, \frac{d^3q}{(2\pi)^3}, \tag{21}$$

where $W(\mathbf{p} \to \mathbf{p} - \hbar\mathbf{q})$ is the transition porbability of an electron of energy $\mathscr{E}_{\mathbf{p}}$ from the state with monentum $\mathbf{p} = M\mathbf{u}$ into a state with a momentum $\mathbf{p} - \hbar\mathbf{q}$, a probability connected with the spectral distribution $S(\omega,\mathbf{q})$ of the density fluctuations by the relation

$$W(\mathbf{p} \to \mathbf{p} - \hbar\mathbf{q}) = \frac{2\pi}{\hbar^2}\left(\frac{4\pi e^2}{\varepsilon_0 q^2}\right)^2 S(\omega, \mathbf{q}). \tag{22}$$

Equation (21) is valid if the following inequality holds:

$$e^2/\varepsilon_0 \hbar u \ll 1.$$

It is assumed here, as already noted, that $u \gg v_{F\sigma}$.

$S(\omega,\mathbf{q})$ can be calculated by using the fluctuation-dissipation theorem, which permits $S(\omega,\mathbf{q})$ to be expressed in terms of $\varepsilon(\omega,\mathbf{q})$ in the form

$$S(\omega, \mathbf{q}) = -\frac{\hbar\varepsilon_0 q^2}{4\pi^2 e^2}\left[1 - \exp\left(-\frac{\hbar\omega}{T}\right)\right]^{-1} \operatorname{Im}\left[\frac{1}{\varepsilon(\omega, \mathbf{q})}\right]. \tag{23}$$

We confine ourselves hereafter to plasma-particle densities such that the plasma frequency ω_p is far enough from the frequency ω_{LO} of the longitudinal optical phonons. In this case it is possible to neglect the interaction of the corresponding modes, which leads to their hybridization, and retain in the complete expression for the dielectric constant of the crystal only the terms that describe the plasmon contribution. We obtain then with the aid of Eqs. (9), (15), (16), and (23)

$$S_p(\omega, q) = \frac{\varepsilon_0 \hbar q^2}{8\pi^2 e^2}\left[1 - \exp\left(-\frac{\hbar\omega}{T}\right)\right]^{-1} \omega_p \times \left\{\frac{\gamma_p(q)}{[\omega - \omega_p(q)]^2 + \gamma_p^2(q)} + \frac{\gamma_p(q)}{[\omega + \omega_p(q)]^2 + \gamma_p^2(q)}\right\}. \tag{24}$$

The spectral distribution of the fluctuations of the electron density is thus a sum of two Lonrentz distributions centered near $\omega = \pm\omega_p$ and having a width determined by the plasmon damping decrement γ_p. The relation between the intensities of the maxima of these distributions depends on the temperature; at low temperatures ($T \ll \hbar\omega_p$) the main contribution to $S_p(\omega,q)$ is made by a maximum located in the Stokes region.

We proceed now to calculation of the energy losses. At $T = 0$ there is no plasmon damping in first order in the parameter $e^2/\varepsilon_0 \hbar v_F^{(0)}$, and according to (24) $S_p(\omega,q)$ takes the form of a δ-function peak located at the point $\omega = \omega_p(q)$. The energy losses due to spontaneous emission of plasmons can be found in this case not with logarithmic accuracy but accurate to terms of order $e^2/\varepsilon_0 \hbar v_F^{(0)}$. Taking in Eq. (23) the limit as $\operatorname{Im}\varepsilon(\omega,\mathbf{q}) \to 0$ and using Eqs. (9), (11), (21), and (22), we find after simple calculations

$$\left(-\frac{d\mathscr{E}}{dt}\right)_p = \frac{e^2\omega_p^2}{\varepsilon_0 u}\left(\frac{4}{3} + \ln\frac{u}{2v_F^{(0)}}\right) + \frac{e^2\omega_p^2}{\varepsilon_0 u} F(\alpha), \tag{25}$$

where

$$F(\alpha) = \frac{1}{2}\left\{(\alpha - 1)(1 - \ln 2) + (1-\alpha)^{1/3}(1+\alpha)^{2/3} + \alpha \ln\frac{(1+\alpha)^{1/3} - (1-\alpha)^{1/3}}{(1+\alpha)^{1/3}} - \ln[(1+\alpha)^{2/3} + (1-\alpha^2)^{1/3}]\right\}. \tag{26}$$

The result (25) differs from the well-known result for the energy lost by a fast electron passing through a plasma (see, e.g., Ref. 13) by its last term, which is due entirely to spin polarization. It follows from a numerical analysis of Eq. (26) that this term is negative at all nonzero values of α, i.e., spin polarization decreases the decelerating ability of the plasma. A quantitative estimate of this effect, based on Eq. (25) for $\alpha = 60$–70%, when the factor $F(\alpha)$ reaches its maximum absolute value, shows that for electrons having a velocity higher by a decade than the Fermi velocity the decreases of $(-d\mathscr{E}/dt)_p$ is approximately 10% of its value at $\alpha = 0$.

4. ZERO SOUND

We have considered so far plasma waves whose phase velocity ω/q greatly exceeds the electron Fermi velocities $v_{F\uparrow}$ and $v_{F\downarrow}$. It is well known, however, that besides these waves there can exist in a plasma, at sufficiently low temperature, also waves of another type viz., zero-sound wave with phase velocity close to the Fermi velocity.[7] In contrast to ordinary hydrodynamic electron-sound waves, which propagate when the sound wavelength is appreciably larger than the electron mean free path ($ql \ll 1$), these waves propagate in the opposite limiting case ($ql \gg 1$), when particle pair collisions are insignificant.

We investigate in the present section the dispersion properties of zero-sound waves in a plasma in the presence of spin polarization. We begin the analysis with the case of zero temperature, when the dispersion equation $\varepsilon(\omega,\mathbf{q}) = 0$ can be written in the form

$$\frac{\omega}{4qv_F^{(0)}} \ln\frac{(\omega + qv_{F\uparrow})(\omega + qv_{F\downarrow})}{(\omega - qv_{F\uparrow})(\omega - qv_{F\downarrow})} = \frac{1}{3}\left(\frac{qv_F^{(0)}}{\omega_p}\right)^2 + \mathfrak{P}(\alpha). \tag{27}$$

Just as above, we assume that $v_{F\uparrow} > v_{F\downarrow}$. At large q ($q \gg \omega_p/$

$v_{F\sigma}$, but $q \ll mv_{F\sigma}/\hbar$ as before), Eq. (27) has a single real root

$$\frac{\omega}{q}=\frac{1}{2}(v_{F\uparrow}+v_{F\downarrow})+\left\{\left[\frac{1}{2}(v_{F\uparrow}-v_{F\downarrow})\right]^2+2v_{F\uparrow}(v_{F\uparrow}+v_{F\downarrow})\times\exp\left[-\frac{\varepsilon_0 m}{6\pi n_\uparrow}\left(\frac{qv_{F\uparrow}}{e}\right)^2-2\frac{v_{F\downarrow}}{v_{F\uparrow}}-2\right]\right\}^{1/2}. \quad (28)$$

This root determines the propagation velocity of a zero-sound mode that is not damped at $T=0$ and whose existence is due the presence of a self-consistent interaction between the particles. Under the condition that the second term in the curly brackets of (28) is much less than the first, Eq. (28) takes the form

$$\frac{\omega}{q}=v_{F\uparrow}\left\{1+2\frac{v_{F\uparrow}+v_{F\downarrow}}{v_{F\uparrow}-v_{F\downarrow}}\exp\left(-\frac{2q^2v_{F\uparrow}^2}{3\omega_{p\uparrow}^2}-2\frac{v_{F\downarrow}}{v_{F\uparrow}}-2\right)\right\}, \quad (29)$$

where $\omega_{p\uparrow}^2=4\pi n_\uparrow e^2/m\varepsilon_0$. It is easy to verify that this condition is violated only at values of α that are small compared with the quantity $\frac{1}{2}\exp(-q^2v_F^{(0)2}/3\omega_p^2)$, which is itself small compared with unity in the considered range of wavelengths. Relation (29) is therefore actually valid in the entire significant range of α.

It follows from Eq. (28) and (29) that at $T=0$, neglecting the exponentially small dispersion, the zero-sound phase velocity u_{zs} is equal to

$$u_{zs}^{(0)}(\alpha)=v_F^{(0)}(1+\alpha)^{1/3}. \quad (30)$$

The zero-sound velocity increases thus with increase of α and exceeds, at total polarization of the electrons its value in an unpolarized system by a factor $2^{1/3}$.

Note that spin polarization influences the propagation velocity of the sound oscillations also in the hydrodynamic frequency range. This effect was first considered by Meyerovich[14] in the framework of the Fermi-liquid theory, and was later discussed in a number of papers (see Ref. 15 and the literature cited there). In the case of interest to us, that of a weakly nonideal Fermi gas consisting of polarized particles, the expression for the first-sound velocity u_s can be obtained directly from Eq. (25) of Ref. 14 (or from the equivalent Eq. (31) of Ref. 15), by setting in it all the Fermi-liquid parameters equal to zero. As a result we get for the real part of u_s

$$u_s(\alpha)=\frac{u_s(0)}{(\mathfrak{P}(\alpha))^{1/2}}=\frac{v_F^{(0)}}{3^{1/2}}\frac{1}{(\mathfrak{P}(\alpha))^{1/2}}. \quad (31)$$

It can be concluded on the basis of Eqs. (30) and (31) that $u_{zs}^{(0)}(\alpha)>u_s(\alpha)$, and the dependence of the velocity ratio $u_{zs}^{(0)}/u_s$ on α is nonmonotonic. A numerical analysis of the expression for $u_{zs}^{(0)}/u_s$ shows that as α varies from 0 to 1 this relation first increases, reaches at $\alpha\approx 0.8$ a maximum approximately equal to 2, and the begins to decrease and tends in the limit as $\alpha\to 1$ to the value $3^{1/2}$ typical of an unpolarized system. Thus, at all value of α except $\alpha=1$ we have the inequality

$$u_{zs}^{(0)}(\alpha)/u_s(\alpha)>u_{zs}^{(0)}(0)/u_s(0). \quad (32)$$

At finite temperatures meeting the condition $T\ll\varepsilon_{F\sigma}$ the solution of the dispersion equation $\varepsilon(\omega,\mathbf{q})=0$ corresponding to zero sound is complex. The expression for the real part of u_{zs}, obtained from Eq. (13) with allowance for Eq. (15), takes in this case the form

$$u_{zs}=\frac{\omega}{q}=u_{zs}^{(0)}+\frac{\pi^2}{48}\frac{v_{F\uparrow}-v_{F\downarrow}}{v_{F\uparrow}+v_{F\downarrow}}\left(\frac{T}{\varepsilon_{F\uparrow}}\right)^2 v_{F\uparrow}\times\exp\left(\frac{2q^2v_{F\uparrow}^2}{3\omega_{p\uparrow}^2}+2\frac{v_{F\downarrow}}{v_{F\uparrow}}+2\right), \quad (33)$$

where $u_{zs}^{(0)}$ is given by Eq. (29). It must be emphasized that the range of validity of Eq. (33) is bounded by the condition

$$\frac{T}{\varepsilon_{F\uparrow}}\ll\frac{4\cdot 6^{1/2}}{\pi}\frac{v_{F\uparrow}+v_{F\downarrow}}{v_{F\uparrow}-v_{F\downarrow}}\exp\left(-\frac{2q^2v_{F\uparrow}^2}{3\omega_{p\uparrow}^2}-2\frac{v_{F\downarrow}}{v_{F\uparrow}}-2\right), \quad (34)$$

which is more stringent than simply that the plasma be degenerate. In this region, the second term of Eq. (33), which determines the temperature dependence of the dispersion of the zero-sound velocity, is much less than the first.

As already noted, the zero-sound solution of the dispersion equation contains at $T\neq 0$ an imaginary part. Its appearance is due to the allowance for the contribution made to the polarization atom by the bypass of the pole of the integrand in Eq. (10), and corresponds to the onset of collisionless damping of the zero sound. If this pole is close to the real axis in the complex ω plane, the damping decrement can be obtained from the general Eq. (14). Putting in it $\omega=\omega_{zs}=qu_{zs}$ and using Eq. (29), we get after simple calculations

$$\gamma_{zs}(q)=2\pi qv_{F\uparrow}\frac{v_{F\uparrow}+v_{F\downarrow}}{v_{F\uparrow}-v_{F\downarrow}}\exp\left(-\frac{2q^2v_{F\uparrow}^2}{3\omega_{p\uparrow}^2}-2\frac{v_{F\downarrow}}{v_{F\uparrow}}-2\right)\times\exp\left\{-\frac{2mv_{F\uparrow}^2}{T}\frac{v_{F\uparrow}+v_{F\downarrow}}{v_{F\uparrow}-v_{F\downarrow}}\exp\left(-\frac{2q^2v_{F\uparrow}^2}{3\omega_{p\uparrow}^2}-2\frac{v_{F\downarrow}}{v_{F\uparrow}}-2\right)\right\}. \quad (35)$$

The criterion for the validity of this equation coincides with the condition (34). If the latter is met we find, from a comparison of Eqs. (33) with (35), that $\gamma_{zs}\ll\omega_{zs}$. This agrees with the initial assumption made in the derivation of Eq. (35). The smallness of the collisionless damping is due in this case to the smallness of the number of particles that move in phase with the wave and participate in the energy exchange with it. Thus, at sufficiently low temperature satisfying the condition (34), zero sound is a long-lived and hence well-defined collective excitation propagating with a velocity close to $v_{F\uparrow}$.

5. LIGHT SCATTERING BY ZERO SOUND

Zero sound can be investigated experimentally by a method based on observing the Brillouin scattering of light from an electron plasma photoinduced in a semiconductor. It is therefore of interest to calculate the cross section for light scattering with simultaneous excitation of zero sound, to which the present section is devoted.

The differential cross section for scattering of an unpolarized electromagnetic wave by the fluctuations of the electron density is given in terms of the dynamic structure factor $S(\omega,\mathbf{q})$ by

$$d\Sigma=\frac{1}{4\pi}\left(\frac{e^2}{mc^2}\right)^2\left(\frac{\omega'}{\omega}\right)^2\left[\frac{\varepsilon(\omega')}{\varepsilon(\omega)}\right]^{1/2}\times(1+\cos^2\vartheta)S(\Delta\omega,\mathbf{q})\,d\omega'\,do', \quad (36)$$

where $\Delta\omega=\omega-\omega'$ is the difference between the frequencies of the incident and scattered waves, ϑ is the scattering angle, and do' is the solid-angle element in the direction of the scattered wave.

Using Eqs. (23), (15), and (16) and recognizing that at low temperatures ($T \ll \hbar q v_{F\uparrow}$) the intensity of the anti-Stokes component in the Brillouin doublet is negligibly small compared with the intensity of the Stokes component, we obtain for the contribution of zero sound to the structure factor the expression:

$$S_{zs}(\omega, q) = \frac{q^3}{2\pi}(qa_B)^2 \frac{v_{F\uparrow}+v_{F\downarrow}}{v_{F\uparrow}-v_{F\downarrow}} \times \frac{\gamma_{zs}}{(\omega-\omega_{zs})^2+\gamma_{zs}^2} \exp\left(-\frac{2q^2 v_{F\uparrow}^2}{3\omega_{p\uparrow}^2} - 2\frac{v_{F\downarrow}}{v_{F\uparrow}} - 2\right), \quad (37)$$

where $a_B = \varepsilon_0 \hbar^2/me^2$ is the effective Bohr radius. Substituting (37) in (36) and recognizing that the scattering involves a small change of the frequency ($\Delta\omega \ll \omega, \omega'$), we obtain after integrating by parts the total cross section for scattering into a solid angle do':

$$d\Sigma_{zs}^{\text{tot}} = \frac{q}{8\pi}(qr_0)^2(qa_B)^2(1+\cos^2\vartheta)\frac{v_{F\uparrow}+v_{F\downarrow}}{v_{F\uparrow}-v_{F\downarrow}} \times \exp\left(-\frac{2q^2 v_{F\uparrow}^2}{3\omega_{p\uparrow}^2} - 2\frac{v_{F\downarrow}}{v_{F\uparrow}} - 2\right) do', \quad (38)$$

where

$$q = 2\frac{\omega}{c} n(\omega) \sin\frac{\vartheta}{2}, \quad r_0 = \frac{e^2}{mc^2}, \quad (39)$$

$n(\omega)$ is the refractive index at the incident-radiation frequency. Integrating (38) over the angles in a coordinate frame with a polar axis directed along the wave vector $\varkappa$ of the incident light, we obtain the total cross section for light scattering (per unit volume)

$$\Sigma_{zs}^{\text{tot}} = \frac{\varkappa}{2^{1/2}}(\varkappa r_0)^2(\varkappa a_B)^2 \frac{v_{F\uparrow}+v_{F\downarrow}}{v_{F\uparrow}-v_{F\downarrow}} \times \mathcal{F}\left(\frac{2\varkappa^2 v_{F\uparrow}^2}{3\omega_{p\uparrow}^2}\right) \exp\left(-2\frac{v_{F\downarrow}}{v_{F\uparrow}} - 2\right), \quad (40)$$

where

$$\mathcal{F}(\beta) = \beta^{-11/2}[2\beta^2\gamma(7/2, 2\beta) - 2\beta\gamma(9/2, 2\beta) + \gamma(11/2, 2\beta)], \quad (41)$$

and $\gamma(a,b)$ is the incomplete gamma function.

We estimate numerically, on the basis of Eq. (38), the cross section for light scattering by zero sound in GaAs crystals, which are customarily used in experiments on optical orientation of spins. Putting $m = 0.07m_0$ (m_0 is the free-electron mass), $\varepsilon_0 = 12.9$, $\alpha = 50\%$, $n = 10^{18}$ cm^{-3}, $\omega = 3.8\cdot 10^{15}$ s^{-1}, we obtain in 180° (backscattering) geometry $d\Sigma_{zs}^{\text{tot}}/do' \sim 10^{-7}$ cm^{-1}, i.e., a value of the same order as the cross section for light scattering by acoustic plasmons in these semiconductors.[9,16] Note also that owing to the high (close to $v_{F\uparrow}$) velocity of the zero sound the frequency shift in the scattering turns out to be larger by approximately two orders than in scattering by acoustic modes of the lattice. The identification of the zero-sound peak in the spectrum of the scattered light should therefore not be particularly difficult.

CONCLUSION

The analysis presented shows that spin polarization of electrons influences substantially the properties of the collective excitations produced in degenerate semiconductors by fluctuations of the electron density. The most important effects in this case are the enhancement of the spatial dispersion of the plasma oscillations and the increase of the velocity of the zero sound. These two predicted effects are easily studied by the light-scattering method. The pertinent experiment should consist of a simultaneous action, on the semiconductor, by circularly polarized radiation that generates a degenerate spin-polarized electron plasma, and by a probing unpolarized (or linearly polarized) radiation that causes no additional spin disequilibrium. The structure of the spectrum of the considered collective excitations can then be investigated by a standard procedure based on the study of scattering of probing radiation by a photo-induced plasma.[17]

One more result of our study is establishement of the fact that spin polarization of electrons decreased the decelerating ability of a plasma. This effect is due to the dependence of the plasmon dispersion on α, and at a high degree of polarization ($\alpha \sim 60$–70%) it is large enough (on the order of 10%) to be observable in measurements of the rate of energy loss of electrons excited by light high into the conduction band.[18]

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