Bremsstrahlung in collisions of atoms with slow neutrons

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(Submitted 4 December 1986)
Zh. Eskp. Teor. Fiz. **93**, 1537–1544 (November 1987)

The radiation resulting from the scattering of slow neutrons by atoms and deuterons is considered. It is shown that if the scatterer has internal structure this has an important influence on the form of the radiation spectrum. The part played by different mechanisms in forming the bremsstrahlung is investigated.

Recently, numerous studies have been devoted to the role played by the electron shell of an atom in the bremsstrahlung that arises when atoms scatter electrons, positrons, protons, and heavy atomic particles.[1-7] These studies have shown that the bremsstrahlung of the atoms is due to coherent processes of photon emission by the charge accelerated in the field of the scattering atom and the emission of light by the atomic shell deformed in the collision. At photon frequencies near the characteristic atomic frequencies the bremsstrahlung process that takes place through the virtual excitation of the atomic shell is more probable than direct radiation by the charge. The part played by deformation of the atomic structure is manifested particularly clearly in the process of bremsstrahlung of colliding atoms,[7] since in this case, the particles being neutral, there is no bremsstrahlung due to their charge at all.

An analogous situation arises in the collision of neutrons with atoms. Here, the main source of the radiation is the time-dependent (during the collision process) atom electric dipole moment produced both by the recoil of the nucleus when the neutron collides with it, as well as by the direct interaction of the magnetic moment of the neutron with the atomic electrons. The present paper is devoted to analysis of the emission of light in collisions of slow neutrons with atoms and also with the simplest nuclear system, the deuteron. Study of these phenomena enables one not only to trace the role of internal structure in the formation of the bremsstrahlung spectrum but also to study, while remaining within the framework of the Born approximation, the bremsstrahlung process in the hitherto unconsidered region of low energies of the colliding particles.

1. BREMSSTRAHLUNG IN COLLISIONS OF SLOW NEUTRONS WITH ATOMS

In the first stage, we consider the collision of a neutron with a hydrogen atom, ignoring the interaction of the magnetic moment of the neutron with the atomic electron. The Schrödinger equation describing the scattering of the slow neutron by the hydrogen atom has in the center-of-mass system of the pair $n + H$ the form (we use the atomic system of units with $m = e = \hbar = 1$)

$$\left[-\frac{1}{2}\nabla_{\mathbf{r}}^2-\frac{1}{M}\nabla_{\mathbf{R}}^2-\frac{1}{r}+U\left(\mathbf{R}+\frac{\mathbf{r}}{M}\right)-E\right]\Psi_E(\mathbf{R},\mathbf{r})=0. \qquad (1)$$

Here, $\mathbf{r}$ is the vector joining the proton and the electron, $\mathbf{R}$ is the radius vector joining the center of mass of the atom to the neutron, M is the mass of the nucleon, and $U(\mathbf{R}, \mathbf{r})$ is the interaction potential of the nucleons, which depends on the mutual orientation of its spins.

We seek solutions of Eq. (1) in the form of the series

$$\Psi_E(\mathbf{R},\mathbf{r})=\sum_n a_n(\mathbf{R})\varphi_n(\mathbf{r}). \qquad (2)$$

In this expansion, which contains, as usual, appropriate integrals over the continuum, $\varphi_n(\mathbf{r})$ are the wave functions of the hydrogen atom and satisfy the equation

$$[-{}^1/_2\nabla_{\mathbf{r}}^2-r^{-1}-\varepsilon_n]\varphi_n(\mathbf{r})=0. \qquad (3)$$

Substituting (2) in (1) and using in the usual manner the orthogonality of the functions $\varphi_n(\mathbf{r})$, we obtain an equation for the coefficients in the expansion of $a_n(\mathbf{R})$:

$$\frac{1}{M}(\nabla_{\mathbf{R}}^2+k_n^2)a_n(\mathbf{R})=\int U(\mathbf{R},\mathbf{r})\varphi_n^*(\mathbf{r})\Psi_E(\mathbf{R},\mathbf{r})\,d\mathbf{r}\equiv V_{nE}(\mathbf{R}) \qquad (4)$$

in which the wave vector k_n is related to the energies ε_n and E by $k_n^2/M+\varepsilon_n=E$.

By means of the Green's function of the homogeneous equation (4), we write the wave functions of the system before and after emission of the photon in the form

$$\Psi_{E_i}^+(\mathbf{R},\mathbf{r})=e^{i\mathbf{k}_i\mathbf{R}}\varphi_0(\mathbf{r})+\sum_n\varphi_n(\mathbf{r})\int G_{k_in}^+(\mathbf{R};\mathbf{R}')V_{nE_i}(\mathbf{R}')\,d\mathbf{R}',$$
$$\Psi_{E_f}^-(\mathbf{R},\mathbf{r})=e^{i\mathbf{k}_f\mathbf{R}}\varphi_0(\mathbf{r})+\sum_n\varphi_n(\mathbf{r})\int G_{k_fn}^-(\mathbf{R};\mathbf{R}')V_{nE_f}(\mathbf{R}')\,d\mathbf{R}'. \qquad (5)$$

Asymptotically, these functions contain plane and spherical waves: in $\Psi_{E_i}^+$ the spherical wave is an outgoing wave, in $\Psi_{E_f}^-$ an incoming one; $k_i^2/M=E_i-\varepsilon_0$, $k_f^2/M=E_f-\varepsilon_0$. It is assumed that after emission of the photon the hydrogen atom remains in the ground state $\varphi_0(\mathbf{r})$.

The cross section of the bremsstrahlung process in which the $n + H$ system goes over from the state $\Psi_{E_i}^+$ to the state $\Psi_{E_f}^-$ with emission of a photon of frequency $\omega = E_i - E_f$ and polarization e in the solid angle $d\Omega$, is determined by the expression[8]

$$d\sigma=\frac{\omega^3}{(2\pi)^4c^3v_i^2}|\mathbf{er}_{fi}|^2\,d\mathbf{k}_f\,d\Omega. \qquad (6)$$

Here, c is the velocity of light, v_i is the relative velocity of the atom and neutron before the collision, and dk_f is the interval of states containing the momentum of the atom after emission of the photon. The dipole matrix element in (6) is determined by the integral

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$$\mathbf{er}_{f_i}=\iint \Psi_{E_f}^{-*}(\mathbf{R},\mathbf{r})\,\mathbf{er}\,\Psi_{E_i}^{+}(\mathbf{R},\mathbf{r})\,d\mathbf{R}\,d\mathbf{r}. \qquad (7)$$

Substituting (5) in (7) and integrating, we obtain under the condition $\omega \lesssim \varepsilon_i = k_i^2/M$ the following expression for the matrix element:

$$\mathbf{er}_{f_i}=\frac{4\pi f}{M}\sum_n\left\{\frac{(\mathbf{er})_{0n}[\exp(-i\mathbf{qr}/M)]_{n0}}{\varepsilon_0-\varepsilon_n+\omega}+\frac{[\exp(-i\mathbf{qr}/M)]_{0n}(\mathbf{er})_{n0}}{\varepsilon_0-\varepsilon_n-\omega}\right\}. \qquad (8)$$

In deriving (8) we have used as interaction potential of the nucleons the zero-range potential $U(\mathbf{R}) = -(4\pi f/M)\delta(\mathbf{R})$ (f is the singlet or triplet proton-neutron scattering amplitude), which in the Born approximation describes satisfactorily nucleon scattering at energies $\varepsilon_i \lesssim 20$ MeV.[9] The matrix elements in the expression (8) are determined by the integrals

$$(\mathbf{er})_{0n}=\int \varphi_0^*(\mathbf{r})\,\mathbf{er}\,\varphi_n(\mathbf{r})\,d\mathbf{r},$$
$$[\exp(-i\mathbf{qr}/M)]_{n0}=\int \varphi_n^*(\mathbf{r})\exp(-i\mathbf{qr}/M)\varphi_0(\mathbf{r})\,d\mathbf{r}, \qquad (9)$$

where $\mathbf{q} = \mathbf{k}_i - \mathbf{k}_f$ is the collision momentum transfer. In the case of collision of a neutron with a many-electron atom the matrix element $[(\exp)(-i\mathbf{q}\cdot\mathbf{r}/M)]_{n0}$ is replaced in (8) by

$$\left[\exp\left(-iM_N^{-1}\mathbf{q}\sum_{j=1}^{N}\mathbf{r}_j\right)\right]_{n0}$$

($\mathbf{r}_j$ is the coordinate of the jth electron, and M_N is the mass of the nucleus), and $(\mathbf{r})_{0n}$ is replaced by the matrix element of the dipole moment $\mathbf{D}_{0n}$ of the atom. Further,

$$r_N=-M_N^{-1}\sum_{j=1}^{N}\mathbf{r}_j,$$

which appears in the argument of the exponential, is the coordinate of the nucleus in the center-of-mass system of the atom.

It follows from (8) that at photon energies equal to the energies of the dipole transitions in the discrete spectrum of the hydrogen atom the dipole matrix element is infinite (if the line width is ignored). Therefore, singularities must also be observed in the bremsstrahlung cross section at these frequencies.

We consider emission of a photon at small momentum transfers, $q \ll M$. In this case, the differential cross section of the bremsstrahlung, integrated over all directions of emission of the photon and summed over its polarizations, is given by the expression

$$\frac{d\sigma}{d\omega\,dq}=\frac{4}{3}\frac{\omega^3}{c^3k_i^2M_N^2}|f_N\alpha(\omega)|^2q^3, \qquad (10)$$

in which $\alpha(\omega)$ is the dynamic polarizability of the atom in the ground state. The expression (10), obtained for hydrogen ($M = M_N$), also describes the bremsstrahlung cross section in the case of a many-electron atom. In this case, f_N is the amplitude for scattering of the neutron by the nucleus.

The replacement of f_N by $zZ_N 2\mu/q^2$ (z is the charge of the incident particle) corresponds to replacement of the nuclear interaction between the particles by the Coulomb interaction. In this case, the expression (10) describes the bremsstrahlung of a charged particle on an atom, due to the recoil of its nucleus in the process of Coulomb interaction in the collision.[7]

Integrating (10) over all the momentum transfers, we obtain the bremsstrahlung spectrum:

$$\frac{d\sigma}{d\omega}=\frac{16}{3}\frac{\omega^3}{c^3}|f\alpha(\omega)|^2\frac{2\varepsilon_i^2}{v_i^2M_N^2}\left(1-\frac{\omega}{\varepsilon_i}\right)^{1/2}\left(2-\frac{\omega}{\varepsilon_i}\right). \qquad (11)$$

The expression (11) is valid under the conditions $k_i \ll M/2$ and

$$\omega \ll \varepsilon_i = v_i^2 M_N M/2(M_N+M) = \mu v_i^2/2.$$

In accordance with (11), the bremsstrahlung cross section in the region of low photon energies ($\omega \ll 1$) is proportional to the cube of the frequency of the emitted light; at resonance frequencies, it becomes infinite; and in the high-frequency region of the spectrum ($\omega \gg 1$) decreases as ω^{-1}, vanishing at $\omega = \varepsilon_i$.

The averaging of the cross section (11) over the spin states of the nucleons leads to the substitution

$$|f|^2={}^3/_4|f_t|^2+{}^1/_4|f_s|^2,$$

where f_t and f_s are, respectively, the triplet and singlet np scattering amplitudes.

Following Ref. 6, we clarify the part played by the bremsstrahlung mechanism considered above in the formation of the total cross section

$$d\sigma^n=\sum_m d\sigma_{m0},$$

which is the sum of the partial cross sections of the emission processes in which the atom remains in the ground state and the processes with excitation and ionization of the target atom. The partial contributions $d\sigma_{m0}$ are calculated in accordance with formula (6), in which the matrix element (8) is replaced by

$$\mathbf{er}_{fi}^{m0}=\frac{4\pi f}{M}\sum_n\left\{\frac{(\mathbf{er})_{mn}[\exp(-i\mathbf{qr}/\mathbf{M})]_{n0}}{\varepsilon_m-\varepsilon_n+\omega}+\frac{[\exp(-i\mathbf{qr}/M)]_{mn}(\mathbf{er})_{n0}}{\varepsilon_0-\varepsilon_n-\omega}\right\}. \qquad (12)$$

At high frequencies ($\omega \gg 1$), er_{fi}^{m0} has the simple form

$$\mathbf{er}_{fi}^{m0}=-\frac{4\pi f}{M^2}i\frac{\mathbf{eq}}{\omega^2}\left[\exp\left(-\frac{i}{M}\mathbf{qr}\right)\right]_{m0}. \qquad (13)$$

It follows that in the region of small momentum transfers or low energies of the neutron the dominant process is bremsstrahlung without excitation of the atom, since

$$\mathbf{er}_{fi}^{m0}\approx -i\frac{4\pi f}{M^2}\frac{\mathbf{eq}}{\omega^2}\delta_{m0}, \qquad q \ll M.$$

In the region $\omega \ll 1$, the cross sections of the bremsstrahlung processes with and without excitation are, in accordance with (12), of the same order. The bremsstrahlung processes with large (on atomic scales) momenta of the ionized electron in the final state are suppressed by the oscillations of the electron wave function in the matrix element (12) when $q \ll M$.

Thus, when $\omega \gtrsim 1 (k_i \ll M)$ the bremsstrahlung processes without excitation of the target atom play the decisive role in forming the total spectrum of the bremsstrahlung in the neutron-atom collisions.

We estimate the cross section in the region of frequencies characteristic of the hydrogen atom, $\omega \approx 0.5$, at neutron energy $\varepsilon_i \approx 5$. In this case $|f|^2 = 0.16 \cdot 10^{-23}$ cm^2 (Ref. 10), the polarizability of the atom is $\alpha(\omega) \approx \alpha(0) = 4.5$, and the cross section $d\sigma \sim 5 \cdot 10^{-32}$ cm^2. This is about 20 times greater than the bremsstrahlung cross section at these frequencies in the case of the scattering of neutrons by protons.[9] (In our estimates, $d\omega \sim 0.1$.) The cross section of the process increases appreciably at the resonance frequencies for which the polarizability can be represented in the form

$$|\alpha(\omega_n)| \approx f_n/\Gamma_n\omega_n,$$

where f_n is the oscillator strength of the transition to the nth level of the hydrogen atom, and Γ_n is the width of this level. As an example, we consider the frequency corresponding to the transition $1s \to 2p$. For it $f_n = 0.42$, $\Gamma_n = 0.15 \times 10^{-7}$ (Ref. 11), and, therefore, $|\alpha(\omega_{1s2p})| \approx 7.5 \times 10^7$. This is seven orders of magnitude greater than the static polarizability $\alpha(0)$. At resonance, all the radiation is concentrated in an interval of frequencies $d\omega \sim \Gamma_n$, and therefore the bremsstrahlung cross section for this transition is $d\sigma \sim 0.8 \cdot 10^{-25}$ cm^2. It should be noted that in the laboratory frame the resonance singularities in the bremsstrahlung spectra will be significantly smeared by the Doppler effect. The resonances in the *np* amplitudes also lead to an increase of the bremsstrahlung cross section at the corresponding nucleon energies, as in the case of bremsstrahlung absorption by a neutron scattered by an atom.[12]

The bremsstrahlung cross sections of many-electron atoms can exceed the hydrogen value by several orders of magnitude, since the static polarizability of atoms such as potassium, rubidium, and cesium are almost two orders of magnitude greater than the polarizability of the hydrogen atom. In addition, an important role will also be played in this case by the resonances in the neutron-nucleus scattering, since their number increases appreciably with increasing charge of the nucleus.

So far, we have considered the cross section for bremsstrahlung by a neutron on an atom without allowance for the magnetic moment of the neutron, the presence of which creates a long-range field proportional to $1/R^2$ that polarizes the atom and thus gives rise to bremsstrahlung. We analyze now the part played by this mechanism.

The bremsstrahlung cross section in the collision of an atom with a neutron, with allowance for the magnetic field of the latter, can be calculated as follows. We introduce in the original Schrödinger equation (1), in addition to $U(\mathbf{R} + \mathbf{r}/M)$, the operator of the interaction of the magnetic momentum of the neutron with the atom:

$$\hat{V} = -\frac{\hat{\mu}}{Mc}[\hat{\mathbf{E}}\hat{\mathbf{p}}] - \hat{\mu}\hat{\mathbf{H}}, \qquad (14)$$

where $\hat{\mu} = \mu\hat{s}/s$ is the operator of the magnetic moment and is proportional to the neutron spin $\mathbf{s}$, and $\hat{\mathbf{p}}$ is the neutron momentum operator; $\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}$ are the operators of the electric and magnetic fields of the electrons and of the nucleus of the atom. The first term in (14) describes the Schwinger scattering of the neutron in the electric field of the atom,[8] and the second describes the scattering in the magnetic field, which is produced mainly by the orbital motion of the atomic electrons. Calculations similar to those in the derivation of (10) and (11) yield

$$\frac{d\sigma}{d\omega\, dq} = \frac{16\omega^3}{3c^3 v_i^2}\frac{|\alpha(\omega)|^2}{q}\mu^2 \times\left\{\left(\frac{\omega}{c}\right)^2 + \frac{v_i^2 q^2}{c^2}\left[1 - \frac{\omega}{2\varepsilon_i} - \frac{q^2}{4k_i^2}\right]\right\}. \qquad (15)$$

Formula (15) describes the bremsstrahlung due to the polarization of the electron shell of the atom by the magnetic moment of the neutron in the center-of-mass system of the $n + H$ pair. This expression is obtained in the region of small momentum transfers, $q \ll 1$. It describes collisions that take place at large (on atomic scales) distances, where the long-range interaction of the magnetic moment of the neutron with the atom is capable of producing an appreciable polarization of the atom. The region of small impact parameters, or large q, is not of interest, since in this region the cross section of the process is appreciably smaller than (15). This is due to the fact that the generalized polarizability $\alpha(\omega, q)$ of the atom, which describes the polarization at arbitrary q, decreases rapidly with increasing q in the region $q \gg 1$.

We consider the bremsstrahlung spectrum, calculated by integrating (15) in the case $k_i \ll 1$:

$$\frac{d\sigma}{d\omega} = \frac{16\omega^3}{3c^3 v_i^2}|\alpha(\omega)|^2\mu^2\left\{\left(\frac{\omega}{c}\right)^2 \ln\frac{1+(1-\omega/\varepsilon_i)^{1/2}}{1-(1-\omega/\varepsilon_i)^{1/2}} + 4\left(\frac{\varepsilon_i}{c}\right)^2\left(1-\frac{\omega_i}{\varepsilon_i}\right)^{1/2}\left(1-\frac{\omega}{2\varepsilon_i}\right)\right\}. \qquad (16)$$

Let us compare the contributions of the two bremsstrahlung mechanisms. For this, we form the ratio of the spectra (16), $d\sigma_2/d\omega$, and (11), $d\sigma_1/d\omega$:

$$\xi = \left[\frac{d\sigma_2}{d\omega}\right] \Big/ \left[\frac{d\sigma_1}{d\omega}\right] \sim \frac{\mu^2(\varepsilon_i/c)^2}{|f_N|^2(\varepsilon_i/M_N)^2} \approx \left(\frac{M_N}{M}\right)^2 \frac{1}{|f_N|^2 c^4}. \qquad (17)$$

In the estimate (17), we have used the value of the neutron magnetic moment: $\mu = -1.91\mu_B$ ($\mu_B = (2mc)^{-1}$). It follows from (17) that at the lowest neutron energies $\varepsilon_i \ll M^{-1}$ the relative importance of the two mechanisms is determined by the properties of the target nucleus. As an example of a light target we have hydrogen, for which $|f_N|^2 = 1.6 \times 10^{-24}$ cm^2 (Ref. 13) and $M_N = M$. Then $\xi_H \sim 4 \times 10^{-2}$, i.e., the bremsstrahlung due to the recoil of the nucleus when it collides with the neutron is dominant. For a heavy target, the amplitude $|f_N|$ can be estimated as $|f_N| \sim R_N$, where R_N is the radius of the nucleus, which increases with increasing mass number A: $R_N \sim A^{1/3}$. Therefore, we obtain the estimate $\xi \sim \xi_H A^{4/3}$. For $A \sim 200$, the ratio $\xi_{200} \sim 50 \gg 1$. Thus, with increasing mass number of the nucleus the importance of recoil in the formation of the bremsstrahlung spectrum decreases appreciably, and for sufficiently large A the main emission mechanism is the bremsstrahlung due to the magnetic moment of the neutron. We have considered the region of very low neutron energies: $\varepsilon_i \ll M^{-1}$. At higher energies, the ratio ξ acquires a dependence on ε_i, and this enhances the importance of the mechanism associated with the recoil of the nucleus. This is readily seen by noting that the integration of the cross section (15) over q in the case $k_i \gg 1$ ($\varepsilon_i \gg M^{-1}$) is cut off at values $q \sim 1$.

The relatively large importance of the neutron magnetic moment in a collision with an atom is due to the fact that the nuclear force of the neutron, polarizing the atom, acts on the heavy particle, the nucleus, whereas the magnetic moment acts on the mobile electron shell. In the process of bremsstrahlung of a neutron on a nuclear target, the importance of the magnetic moment is much less than in the case of scattering by an atom, since in the nuclear system there is no light electron component. We note however that in the differential (with respect to q) cross section of bremsstrahlung at sufficiently small $q \to 0$ the neutron magnetic moment will undoubtedly be manifested in the same way as occurs in Schwinger scattering of a neutron in the electric field of a nucleus.

2. BREMSSTRAHLUNG OF A DEUTERON IN A COLLISION WITH A NEUTRON

In Ref. 14, the polarization bremsstrahlung of a fast nucleon on a heavy nucleus was considered without allowance for the recoil of the nucleus. In this part of our paper, we study bremsstrahlung in a collision of a slow neutron with a deuteron. Since the masses of the neutron and deuteron are comparable, it is necessary to take into account the dynamics of all the particles that participate in the process. In accordance with what we have said above, the magnetic moments of the particles can be ignored. Then the Schrödinger equation describing the scattering of the neutron by the deuteron in the center-of-mass system of the particles has the form (in this section, we use a system of units with $\hbar = e = M = 1$)

$$[-\nabla_{\mathbf{r}}^2 - {}^3/_4\nabla_{\mathbf{R}}^2 + U_{pn}(\mathbf{r}) + U_{pn}(\mathbf{R} - {}^1/_2\mathbf{r}) + U_{nn}(\mathbf{R} + {}^1/_2\mathbf{r}) - E] \times \Psi(\mathbf{R}; \mathbf{r}) = 0. \quad (18)$$

Here, $\mathbf{r}$ is the relative radius vector of the neutron and proton in the deuteron, $\mathbf{R}$ is the vector between the center of mass of the deuteron and the neutron scattered by it, and U_{pn} and U_{nn} are, respectively, the pn and nn scattering potentials. As we aim to obtain only a qualitative picture of the phenomenon, we express these potentials, as in the first section, in the zero-range approximation.

The wave functions of the $n + d$ system can be expressed in the form (5), and the dipole matrix element of the transition from the state $\Psi_{E_i}^+$ to the state $\Psi_{E_f}^-$ can be represented in the form

$$\mathbf{e}\hat{\mathbf{p}}_{fi} = \frac{\mathbf{eq}}{2\omega} V_{00}(\mathbf{q}) - i\sum_n \left\{ \frac{(\mathbf{e}\nabla)_{0n} V_{n0}(\mathbf{q})}{\varepsilon_0 - \varepsilon_n + \omega} + \frac{V_{0n}(\mathbf{q})(\mathbf{e}\nabla)_{n0}}{\varepsilon_0 - \varepsilon_n - \omega} \right\}, \quad (19)$$

where $\hat{\mathbf{p}}$ is the operator of the proton momentum, and the matrix elements

$$(\mathbf{e}\nabla)_{0n} = \int \varphi_0^*(\mathbf{r})\,\mathbf{e}\nabla\varphi_n(\mathbf{r})\,d\mathbf{r},$$
$$V_{n0}(\mathbf{q}) = \int \varphi_n^*(\mathbf{r})\,[U_{pn}(\mathbf{R} - \mathbf{r}/2) + U_{nn}(\mathbf{R} + \mathbf{r}/2)]\,e^{i\mathbf{qR}}\varphi_0(\mathbf{r})\,d\mathbf{R}\,d\mathbf{r} \quad (20)$$

are calculated with the deuteron eigenfunction $\varphi_n(\mathbf{r})$. The first term in (19) is due to the motion of the deuteron as a whole, while the second is due to the oscillations of the proton relative to the neutron in the deuteron. Summing in (19) over the intermediate states of the deuteron, we represent the total bremsstrahlung cross section in the form

$$\frac{d\sigma}{d\omega} = \frac{16}{3}\frac{I^2}{c^3 v_i^2 \omega} \int \left| \frac{4}{\beta}(f_{nn} + f_{np}) \tan^{-1}\frac{\beta}{2} + (f_{nn} - f_{np})\,\omega^2 \alpha(\gamma, \beta) \right|^2 \beta^3 d\beta, \quad (21)$$

where f_{nn} and f_{np} are the nn and np scattering amplitude $\gamma = \omega/I$, $\beta = q/2\varkappa$, and $I = \varkappa^2$ is the deuteron ionization potential. The integration in (21) is over the interval

$$3^{-1/2}[(\varepsilon_i/I)^{1/2} - (\varepsilon_i/I - \gamma)^{1/2}] \leqslant \beta \leqslant 3^{-1/2}[(\varepsilon_i/I)^{1/2} + (\varepsilon_i/I - \gamma)^{1/2}].$$

The "generalized" dynamic polarizability $\alpha(\gamma; \beta)$ of the deuteron is determined by the expression ($\gamma \leqslant 1$)

$$\alpha(\gamma; \beta) = \frac{2}{\gamma I^2 \beta^3}\left\{ \frac{\beta}{\gamma}((1+\gamma)^{1/2} + (1-\gamma)^{1/2} - 2) - \frac{2\beta^2}{\gamma}\tan^{-1}\frac{\beta}{2} + \left(1 + \frac{\beta^2}{\gamma}\right)\operatorname{arctg}\frac{\beta}{\gamma}(1 - (1-\gamma)^{1/2}) + \left(1 - \frac{\beta^2}{\gamma}\right)\tan^{-1}\frac{\beta}{\gamma}(1 - (1+\gamma)^{1/2}) \right\}. \quad (22)$$

The analytic continuation of $\alpha(\gamma; \beta)$ beyond the ionization threshold ($\gamma > 1$) is done in such a way that $\operatorname{Im}\alpha(\gamma; \beta)$ in the limit $\beta \to 0$ is positive. In this frequency interval, the polarizability $\alpha(\gamma; \beta)$ is determined by (22), in which the substitution $(1-\gamma)^{1/2} \to -i(\gamma - 1)^{1/2}$ is made.

We follow the behavior of $\alpha(\gamma; \beta)$ in some limiting cases In the limit $\beta \to 0$, the "generalized" polarizability goes over into the dynamic polarizability of the deuteron[8]:

$$\alpha(\gamma; \beta) \underset{\beta \to 0}{\longrightarrow} \frac{2}{\omega^2}\left[-1 - \frac{8}{3\gamma^2} + \frac{4}{3\gamma^2}\{(1+\gamma)^{3/2} + (1-\gamma)^{3/2}\} \right], \quad (23)$$

while in the opposite limit $\alpha(\gamma;\beta) \to (2/\omega^2\beta^2)\{(1+\gamma)^{1/2} - (1-\gamma)^{1/2} - 2\}(\beta \to \infty)$. In the static case and in the limit of high frequencies,

$$\alpha(\gamma; \beta) \underset{\gamma \to 0}{\longrightarrow} \frac{2}{I^2(4+\beta^2)^2},$$
$$\alpha(\gamma; \beta) \underset{\substack{\gamma \to \infty \\ \beta \to (\gamma/3)^{1/2}}}{\longrightarrow} \frac{2 \cdot 3^{1/2}}{I^2 \gamma^{5/2}}\left[3^{1/2} - 4\pi + i\left(3^{1/2} + \ln\frac{2+3^{1/2}}{2-3^{1/2}}\right)\right]. \quad (24)$$

We now consider the frequency dependence of the bremsstrahlung cross section. In accordance with (21) and (24), at small $\omega \ll I$ the spectrum $d\sigma/d\omega \propto \omega^{-1}$, and the main contribution to the cross section is made by the bremsstrahlung of the deuteron as a whole. The second term, due to the internal structure of the deuteron, is small. With increasing ω, the contribution of the structure increases and the bremsstrahlung cross section reaches a maximum, after which it again decreases as ω^{-1}. Thus, the excitation of the internal structure of the deuteron in the collision leads to a nonmonotonic dependence of the cross section on the photon frequency.

We estimate in accordance with (21) the cross section at $\omega \approx I$ and $\omega \approx 2I$ and collision energy $\varepsilon_i \approx 10$ MeV, using for f_{nn} and f_{np} the singlet and triplet neutron-proton scattering lengths.[10] We obtain, respectively, $3.9 \cdot 10^{-30}$ cm^2 and

$2.8\cdot10^{-30}$ cm^2 ($d\omega\sim$ MeV); in the first case, the contribution to the cross section of the "structure" mechanism is $\sim14\%$, while in the second it is $\sim35\%$ of the total cross section. These values are of the same order as the cross section for emission of a photon by a free proton.

In the deuteron there are no excited ground states, and $\alpha(\gamma;\beta)$ as a function of ω does not have poles. For this reason, the nature of the singularities in the bremsstrahlung cross section of the deuteron differs appreciably from the behavior of the cross section for the hydrogen atom. In the latter case, an infinite number of resonances, which accumulate at the ionization threshold, is observed as the limit $\gamma\to1$ is approached. But in the deuteron the internal structure is manifested in the form of a broad smeared peak.

The results obtained in this second section of the paper can be applied to investigate the bremsstrahlung of deuterons that collide with neutral mesons or fast neutrons with energy much greater than the depth of the nucleon well.

We thank M. Yu. Kuchiev for helpful discussions during this work.

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Translated by Julian B. Barbour