Mobility of a dissipative quantum system in a periodic potential

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A quantum-mechanical system with an ohmic dissipation and a periodic potential is analyzed. The frequency dependence of the mobility is studied. Over a wide neighborhood ($2/3 < \alpha < 3/2$, where α is the dimensionless viscosity) of the localization phase transition, which occurs at $\alpha = 1$, the frequency dependence is found to be quite different from that predicted by the self-consistent approximation. The temperature dependence of the static mobility at low temperatures has the same form in the cases of strong and weak potentials [in particular, for $\alpha > 1$ it is proportional to $T^{2(\alpha-1)}$].

1. INTRODUCTION

Schmid[1] and Bulgadaev[2] showed that a quantum-mechanical system with an ohmic dissipation and a periodic potential described by an effective Euclidean action

$$S[q(t)]=\int_{-\beta/2}^{\beta/2} dt\left[\frac{m}{2}\left(\frac{\partial q}{\partial t}\right)^2 - V\cos q\right]$$
$$+\frac{\eta}{4\pi}\iint_{-\beta/2}^{\beta/2} dt\,dt'\left\{\frac{q(t)-q(t')}{(\beta/\pi)\sin[\pi(t-t')/\beta]}\right\}^2 \qquad (1)$$

may be in either a localized state or a delocalized state when the temperature $T\equiv 1/\beta$ is zero, depending on the value of the viscosity η. For either a strong or weak potential, the transition from localization to delocalization occurs at the value $\alpha\equiv 2\pi\eta = 1$ of the dimensionless viscosity.[2] In (1) and everywhere below, we are using $\hbar$, $k_B = 1$. In the Fourier representation, the propagator corresponding to the quadratic part of (1) is of the form

$$G_0(\omega)=(\eta|\omega|+m\omega^2)^{-1}. \qquad (2)$$

The integration in (1) is carried out over the imaginary time t, which we will call simply the "time."

At a nonzero temperature the localization does not occur, but it is nevertheless manifested in a power-law decay of the static mobility as the temperature is lowered.[2,3]

Despite the large number of papers[1–10] which have been published on the model (1), we are far from an exhaustive understanding of its properties. In the present paper several new results are derived pertaining to the frequency dependence and temperature dependence of the mobility. We will be comparing our results with those of other investigators as we go along.

A Josephson junction shunted by a normal resistance is usually considered as a particular case of a real physical system describable by an effective action similar to (1) (Refs. 1 and 2). In junction terms, q would be the phase difference, m would be proportional to the capacitance of the junction, V would be the critical current, and η would be the shunting conductance. The mobility corresponds to the slope of the I-V characteristic. The nonlocal term, which was found in the effective reaction of the junction in Refs. 11 and 12 by using the tunneling Hamiltonian, differs from the last term in (1) in being periodic in $q(t)-q(t')$. This circumstance has an extremely important effect on the nature of the localization.[13] On the other hand, according to Ref. 14 a junction with a direct conductance is described by an effective action like (1). As an additional Gaussian heat reservoir one might use a infinite superconducting wire one end of which is connected to one side of the junction. Such a wire would serve as the infinite string in the mechanical analog of a system with a Gaussian dissipation.[15]

An effective action like (1) also arises in a study of the motion of a delocalized line defect in a quantum crystal[16] or in the equivalent problem of quantum-mechanical tunneling in an infinite chain of dissipationless Josephson junctions.[16]

2. WEAK POTENTIAL

a. Frequency dependence of the mobility in the delocalized phase. The partition function of model (1),

$$Z=\int D\{q(t)\}\exp\{-S[q(t)]\},\quad q\left(\frac{\beta}{2}\right)=q\left(-\frac{\beta}{2}\right), \qquad (3)$$

reduces to within a nonsingular factor to the partition function of a one-dimensional logarithmic gas[17]:

$$Z_{LG}=\sum_{p=0}^{\infty}\frac{(V/2)^p}{p!}\prod_{i=1}^{p}\left(\sum_{\sigma_i=\pm1}\int_{-\beta/2}^{\beta/2}dt_i\right)$$
$$\times\exp\left[-\frac{1}{2}\sum_{j,l=1}^{p}\sigma_j G_0(t_j-t_l)\sigma_l\right] \qquad (4)$$

with changes $\gamma_i = \pm 1$ and an interaction

$$G_0(t)=T\sum_{n=-\infty}^{\infty}G_0(\omega_n)\exp(-i\omega_n t),\qquad \omega_n=2\pi Tn,$$

which becomes the correlation function $\langle q(t)q(0)\rangle$ in the absence of a potential. This agreement of the interaction function of test charges and the two-point correlation function for the variable $q(t)$ occurs not only for their seed values but also for their exact values.

The representation (4) is particularly convenient in the case of a weak potential. It allows one to assign a more graphic meaning to both the normalization-group transformations and the particle summation of perturbation-theory series.

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The model (1) can be subjected to a Wilson renormalization, as first used for models with a sinusoidal potential by Wiegmann[18] (see also Ohta and Kawasaki[19]). We partition $q(t)$ into slowly and rapidly varying parts: $q(t) = q_0(t) + q_1(t)$. Incorporating a periodic potential by (second-order) perturbation theory in the partition function (3), and integrating over $q_1(t)$, causes the following changes in the original action (1) (Ref. 18):

$$V \to V \exp(-{}^1/_2\langle q_1^2\rangle), \tag{5a}$$

$$G_0^{-1}(\omega) \to G_0^{-1}(\omega) + \Sigma_1(\omega), \tag{5b}$$

where

$$\Sigma_1(\omega) = \frac{V^2}{2}\int dt(1-\cos\omega t)\left\{\exp\left[-\frac{1}{2}\langle[q_1(t)-q_1(0)]^2\rangle\right] - \exp[-\langle q_1^2\rangle]\right\}, \tag{6}$$

and the average over q_1 should be carried out with the help of the unperturbed action

$$S_0[q_1(t)] = \frac{1}{2}\iint dt\,dt'\,G_0(t-t')q_1(t)q_1(t').$$

In this section of the paper we are considering only the zero-temperature case, but the renormalization-group transformations found in Subsec. 2c also apply at nonzero temperatures (as long as the instantaneous cutoff frequency is much higher than T).

In general we would need to apply the recursion transformation (5) many times in succession, giving rise to a transition to progressively lower frequencies. A situation is possible, however, in which the correction to $G_0^{-1}(\omega)$ is so small that the recursion can be ignored in the following steps. In this case we can assume that q_1 in (6) includes harmonics with arbitrarily low frequencies, i.e., we can set

$$\langle q_1(t)q_1(0)\rangle = G_0(t)$$

and, correspondingly,

$$\Sigma_1(\omega) = \frac{V^2}{2}\int dt(1-\cos\omega t)\exp[-G_0(t=0)+G_0(t)]. \tag{7}$$

In a study of the analytic behavior of expression (7), it is convenient to replace (2) by

$$G_0(\omega) = [\eta|\omega|\exp(\tau_0|\omega|)]^{-1}, \quad \tau_0 = m/\eta, \tag{8}$$

after making the high-frequency cutoff exponential. Doing so changes neither the behavior of the correlation function at long times nor the order of magnitude of the results. For $G_0(\omega)$ as in (8) we have

$$G_0(t=0) - G_0(t) \equiv \int\frac{d\omega}{2\pi}(1-\cos\omega t)G_0(\omega) = \frac{1}{\alpha}\ln\left(1+\frac{t^2}{\tau_0^2}\right), \quad \alpha = 2\pi\eta, \tag{9}$$

$$\Sigma_1(\omega) = \frac{\pi^{1/2}\tau_0 V^2}{4\Gamma(\nu+{}^1/_2)}\left[\Gamma(\nu) - \left(\frac{\tau_0\omega}{2}\right)^\nu \cdot 2K_\nu(\tau_0\omega)\right], \tag{10}$$

where $\nu = 1/\alpha - 1/2$, and $K_\nu(z)$ is the modified Bessel function. For high frequencies, the correction given by (10) tends toward a constant, while at low frequencies we have the asymptotic behavior

$$\Sigma_1(\omega) \approx \frac{\pi^{1/2}\tau_0 V^2}{4\Gamma(\nu+{}^1/_2)}\begin{cases}\Gamma(\nu-1)(\tau_0\omega/2)^2, & \nu > 1 \quad (\alpha < {}^2/_3),\\ (\tau_0\omega/2)^2\ln(4/\gamma\tau_0\omega), & \nu = 1 \quad (\alpha = {}^2/_3),\\ -\Gamma(-\nu)(\tau_0\omega/2)^{2\nu}, & 0<\nu<1 \quad ({}^2/_3 < \alpha < 2),\end{cases} \tag{11}$$

where $\gamma \approx 1.781$ is the reciprocal of Euler's constant. It follows from (10) that, aside from numerical factors of order unity, we have

$$\max[\Sigma_1(\omega)/G_0^{-1}(\omega)] \sim (mV)^2/\eta^{3/2},$$

for $\alpha < 1$ $(\nu > 1/2)$, and for $mV \ll \eta^{3/4}$ correction (7) is indeed small in comparison with $G_0^{-1}(\omega)$, and our approach is justified.

Nevertheless, for $2/3 < \alpha < 1$ this correction is of fundamental importance, since it alters the frequency dependence of the mobility. The basic frequency dependence of the mobility in this case does not stem from the mass term, and takes a different form:

$$\mu(\omega) \approx [\eta + a(i\omega)^{2/\alpha-2}]^{-1}, \quad a \propto V^2(m/\eta)^{2/\alpha},$$

where we have transformed to real time. This functional dependence can also be expected to prevail at nonzero temperatures.

In analyzing the frequency dependence of the mobility, Bulgadaev[4,5] was concerned not with the analytic properties of the correlation functions at $T = 0$ but with the temperature dependence of the coefficients in an expression of the form

$$G(\omega) = [V_R(T) + \eta|\omega| + m_R(T)\omega^2]^{-1} \tag{12}$$

used to approximate the correlation function. For this reason, the nontrivial frequency-dependent correction which we have found here escaped his attention.

b. Neglect of higher-order corrections. Expression (7) could also be derived by perturbation theory. Amit *et al.*[20] have described in detail a diagram technique which makes it possible to write in the expression for $G(\omega)$ all the terms of the perturbation-theory series in the amplitude of the sinusoidal potential V:

$$G(\omega) = G_0(\omega) - G_0^2(\omega)[\Sigma_1(\omega) + \Sigma_2(\omega) + \Sigma_3(\omega) + \ldots]. \tag{13}$$

The first term of the series (Σ_1), of order V^2, is the same as (7). The next nonvanishing correction is of fourth order in V and is given by the integral

$$\Sigma_2(\omega) = \left(\frac{V}{2}\right)^4\iiint dt_1\,dt_2\,dt_3(1-e^{-i\omega t_1}+e^{-i\omega t_2}-e^{-i\omega t_3}) \cdot\exp[-2G_0(t=0)+G_0(t_1)+G_0(t_2-t_3)]\{\exp[G_0(t_2)-G_0(t_3) - G_0(t_1-t_2)+G_0(t_1-t_3)]-1\}. \tag{14}$$

If the correction Σ_1 can be interpreted as the result of the presence of bound pairs of charges, the correction Σ_2

describes the effect of the binary part of the interaction of these pairs with each other. When the average distance between pairs is much larger than their size ($\alpha \ll 1$), this and all the following corrections are obviously negligible in comparison with Σ_1.

At low frequencies the function (14) has the asymptotic behavior

$$\Sigma_2(\omega) \propto \begin{cases} \omega^2, & \alpha \leqslant {}^4/_5 \\ \omega^{4/\alpha - 3}, & {}^4/_5 \leqslant \alpha < {}^4/_3 \end{cases},$$

since the dominant correction at low frequencies is $\Sigma_1(\omega)$ over the entire range of nontrivial behavior $\Sigma_1(\omega)$ ($2/3 < \alpha < 1$) of interest here, while $\Sigma_2(\omega)$ and all the succeeding corrections contain the frequency raised to a higher power. At this point we are talking exclusively about the case $\alpha < 1$, since (13) diverges for $\alpha > 1$.

The series (13) can be rewritten as a series for the self-energy,

$$\Sigma(\omega) \equiv G^{-1}(\omega) - G_0^{-1}(\omega)$$
$$= \Sigma_1(\omega) + [\Sigma_1^2(\omega) G_0(\omega) + \Sigma_2(\omega)] + \ldots,$$

where the term of fourth order in V consists of two terms, the first of which contains the frequency raised to the same power as, or higher than, that in the second.

c. Renormalization-group transformations. Applied to a logarithmic gas, the recursion procedure (5) corresponds to the incorporation of the effect of bound pairs of charges of small size on the interaction of well-separated charges. If we do not assume at the outset that the total effect of this renormalization is negligible then we should initially restrict the discussion to the effect of pairs of charges smaller than some prespecified size τ_1. The integration in (7) should then be restricted to the interval $(-\tau_1, \tau_1)$, and $\Sigma_1(\omega)$ turns out to be an analytic function of ω:

$$\Sigma_1(\omega) = M\omega^2 + \ldots .$$

To find M, it is sufficient to replace $1 - \cos \omega t$ by $(\omega t)^2/2$ in the integral. As a result we find

$$M = \frac{V^2}{4} \int_{-\tau_1}^{\tau_1} dt \frac{t^2}{(1 + t^2/\tau_0^2)^{1/\alpha}}$$
$$\approx \frac{V^2 \tau_0^3}{2(3 - 2/\alpha)} \left(\frac{\tau_1}{\tau_0} \right)^{3 - 2/\alpha}, \quad \alpha > {}^2/_3, \quad \tau_1 \gg \tau_0$$

(for $\alpha < 2/3$, the correction to the mass M remains finite even as $\tau_1 \to \infty$). If we are interested in the correction to $G_0^{-1}(\omega)$ at the frequency ω, we should take the frequency ω itself as the lower cutoff frequency for $q_1(\tau_1 = 2\pi/\omega)$. According to (11) we would then have

$$M(\omega)\omega^2 \propto \omega^{2/\alpha - 1}.$$

How far down the frequency scale does this approach remain valid? After eliminating harmonics with frequencies above $2\pi/\tau_1$, we can assume that we have transformed to a system with a propagator

$$G_R^{-1}(\omega) = \eta|\omega| + m_R\omega^2, \quad m_R = \xi m, \quad \xi = \tau_1/\tau_0 \qquad (15a)$$

and a potential

$$V_R = V \exp[-{}^1/_2 \langle q_1^2 \rangle] = \xi^{-1/\alpha} V. \qquad (15b)$$

If the condition

$$M \ll m_R \qquad (16)$$

holds, then we need not be concerned with any additional renormalization of the propagator. For $mV \ll \eta^{3/2}$, condition (16) holds automatically for arbitrary τ_1 under the condition $\alpha < 1$, while in the case $\alpha > 1$ it holds only for

$$\tau_1 \ll \tau_0 (\eta^{3/2}/mV)^{\alpha/(\alpha - 1)}.$$

This nevertheless leaves a broad range in which this condition can be used.

The form of the transformation (15b) assumes that only the linear term is retained in the renormalization-group equation for V (Ref. 17). Generally speaking, this circumstance may lead to the appearance of additional restrictions on τ_1. We will return to the applicability of the transformations (15) in Sec. 4, after we have dealt with the strong-potential case.

We wish to stress that the renormalized mass m_R which appears in (15a) is somewhat indefinite—it is essentially only a notation for the cutoff frequency. For the original system and also for the system obtained after this renormalization, parts of the macroscopic characteristics will be the same. Specifically, these parts are those for which m_R is important only as a parameter which specifies the high-frequency cutoff (the free energy and so forth), while the frequency dependence of the correlation functions will be slightly different. In Sec. 4 we will use transformations (15) to study the temperature dependence of the static mobility.

3. STRONG POTENTIAL; SEMICLASSICAL APPROXIMATION

In the Gaussian approximation, the mean square amplitude of the fluctuations near the potential minimum is given by the integral

$$\langle q^2 \rangle = \int \frac{d\omega}{2\pi} g_0(\omega), \quad g_0(\omega) = \frac{1}{V + \eta|\omega| + m\omega^2},$$

and if at least one of the conditions

$$mV \gg 1, \quad \eta \gg \max[1, \ln(\eta^2/mV)]$$

holds then this amplitude is much smaller than the period of the potential, so that we can use a semiclassical approximation.

In this case instantons—extremal trajectories connecting neighboring potential minima—play the major role in the partition function (3) (Ref. 1). Depending on the tunneling direction (to the right or to the left), we can associate a charge $\varepsilon = \pm 1$ with each instanton.

For an isolated instanton, the effect diverges logarithmically and is finite only for a pair of instantons of opposite sign. The instantons form a logarithmic gas with an interaction (at long range)

$$G_0(t=0) - G_0(t) = 2\alpha \ln(\omega_0 t) \qquad (17)$$

and an activity y. For $mV \gg \eta^2$ we have[1]

$$\omega_0 = (V/m)^{1/2}, \quad y = 4(V^3/\pi^2 m)^{1/4} \exp[-8(mV)^{1/2}], \qquad (18)$$

while for $mV \ll \eta^2$ we have

$$\omega_0 = V/\eta, \quad y = (\eta^{3/2}/m)\exp(-\alpha). \tag{19}$$

The possibility of a description of this sort suggests a duality between the cases of strong and weak potentials, which holds under the substitutions[1]

$$\alpha \leftrightarrow 1/\alpha, \quad V\tau_0 \leftrightarrow 2y/\omega_0 \tag{20}$$

and which makes it possible to use the results derived previously. The duality persists at nonzero temperatures.[4–6]

In the semiclassical approximation, the correlation function $\langle q(t)q(0)\rangle$ breaks up into two terms, the first associated with small oscillations near the potential minimum and the second associated with instantons:

$$\langle q(t)q(0)\rangle = g_0(t) + \iint dt_1\, dt_2\, Q(t-t_1)Q(t-t_2)F(t_1-t_2). \tag{21}$$

Here $Q(t)$, the trajectory which corresponds to an isolated instanton, is displaced in such a way that its center coincides with the point $t=0$ $[Q(t) \equiv -Q(-t)]$, and

$$F(t_1-t_2) = \langle e(t_1)e(t_2)\rangle \tag{22}$$

is the correlation function for the instanton charges e.

The exact form of the instanton trajectory for an arbitrary viscosity can be found only in the case of a piecewise-parabolic potential

$$V(q) = V\min(q-2\pi n)^2,$$

where n is an integer. In this case we have

$$Q(\omega) = -\frac{2\pi}{i\omega} V g_0(\omega). \tag{23}$$

For a sinusoidal potential, the exact form of the trajectory can be found only for $m=0$ (Ref. 10):

$$Q(\omega) = -\frac{2\pi}{i\omega}\exp\left(-\frac{\eta}{V}|\omega|\right). \tag{24}$$

Comparison of (23) and (24) indicates the universality of the first two terms in the expansion of $Q(\omega)$ in powers of $|\omega|$, which determine the behavior of $Q(t)$ at long times.

If only the binary interaction of instantons is taken into consideration, the correlation function (22) can be expressed exactly in terms of the bare function $[G_0(\omega)]$ and the exact [i.e., renormalized, $G(\omega)$] instanton interaction function:

$$G(\omega) = G_0(\omega) - G_0^2(\omega)F(\omega) \tag{25}$$

[see, for example, Ref. 21 or 22, where relation (25) was derived for a two-dimensional logarithmic gas]. The perturbation-theory series discussed in Subsec. 2b, (13), is specifically an expansion for $F(\omega)$ in which one need consider only the first term in brackets in the case of strong coupling of instantons.

Approximating the bare instanton interaction function, (17), by

$$G_0(\omega) = \frac{4\pi^2\eta}{\omega\exp(\omega/\omega_0)},$$

and making use of the duality relations (20) and the results of Sec. 2, we find the following asymptotic behavior for $F(\omega)$ at low frequencies:

$$F(\omega) \propto \begin{cases} \omega^2, & \alpha \geqslant 3/2, \\ \omega^{2\alpha-1}, & 1 < \alpha < 3/2. \end{cases} \tag{26}$$

Substituting (23) and (26) into (21), we immediately see that in the limit $\omega \to 0$ we have

$$\langle |q(\omega)|^2\rangle \to \text{const}, \tag{27}$$

for $\alpha \geqslant 3/2$, while for $1 < \alpha < 3/2$ we have

$$\langle |q(\omega)|^2\rangle \propto \omega^{2\alpha-3}. \tag{28}$$

So far, both the self-consistent approximation of the Debye-Hückel type (Ref. 6, for example) and the renormalization-group analysis[4,5] have led to functional dependences $\langle |q(\omega)|^2\rangle$ having the property (27) over the entire region $\alpha > 1$. The self-consistent approximation is valid only for $\alpha \gg 1$. Furthermore, it is like the renormalization-group analysis carried out by Bulgadaev[4,5] in that its very structure does not allow the possible appearance of a functional dependence of the form (28).

Since α is (rigorously) renormalizable in the regions of both weak and strong potential,[2,6] it can be assumed that the functional dependence (28) will hold over the entire region $1 < \alpha < 3/2$, even outside the range of applicability of the semiclassical approximation. It follows from duality considerations that this applicability is equivalent to retaining the form of the leading frequency-dependent correction to the mobility for $2/3 < \alpha < 1$ in the region of strong fields.

In summary, we have studied the frequency dependence of the mobility in various parts of the phase diagram. We have shown that in a substantial neighborhood ($2/3 < \alpha < 3/2$) of the localization transition line this frequency dependence is of a form quite different from that far from the transition. We turn now to a study of the temperature dependence of the mobility.

4. TEMPERATURE DEPENDENCE OF THE STATIC MOBILITY

In the range of applicability of the semiclassical approximation, the localization which occurs for $\alpha > 1$, $T=0$, may be interpreted as the vanishing of the probability W for tunneling to a neighboring potential minimum. At a finite temperature, W becomes nonzero.[2,23,24] We wish to emphasize that we are talking specifically about a totally incoherent tunneling,[24] which can be described by means of a probability (rather than an amplitude), so that it becomes a trivial matter to express the static mobility μ in terms of W (Ref. 3):

$$\mu = 4\pi^2 W/T. \tag{29}$$

The tunneling probability can be calculated through an analytic continuation of the instanton interaction function at a nonzero temperature[24]:

$$W = y^2 \int_{-i\infty}^{i\infty} dt\, \exp[-G_0(t=0) + G_0(t)]. \tag{30}$$

From this expression, after we substitute into (29) and assume $mV \gg \eta^2 > (2\pi)^{-2}$, we find

$$\mu = \frac{8\pi^{3/2}\Gamma(\alpha)}{\Gamma(\alpha+1/2)}\left(\frac{\Delta}{\omega_c}\right)^2\left(\frac{\pi T}{\omega_c}\right)^{2\alpha-2}$$

(Ref. 3). Here Δ is the tunneling amplitude in the absence of dissipation. For

$$mV \ll \eta^2, \quad \ln(\eta^2/mV) \ll \eta, \tag{31}$$

where the integration in (30) can be carried out by the method of steepest descent, we find[10]

$$\mu = \frac{(2\pi)^3 (2\eta^9)^{1/2}}{(mV)^2} e^{-2\alpha} \left(\frac{\alpha T}{V}\right)^{2\alpha-2}. \tag{32}$$

The temperature dependence of μ is equivalent in the two cases.

When the product mV becomes so small that the second of inequalities (31) is violated, the semiclassical approximation must be abandoned. In this case we cannot treat the motion of the system as sequential hops between neighboring minima of a periodic potential. Nevertheless, the temperature dependence of the mobility can still be found.

For this purpose we use the renormalization-group transformations (15), which make it possible to transform from a system with arbitrarily small mV to a renormalized system with $m_R V_R$ satisfying conditions (31). We then use (32). Since the combination $m^2 V^{2\alpha}$ which appears in (32) is invariant under the transformations (15),

$$m_R^2 V_R^{2\alpha} = m^2 V^{2\alpha},$$

the final result for the static mobility turns out to be independent of where we terminate the renormalization specifically in region (31); the final result is again given by (32).

A universality of this sort stems from the circumstance that the transformations (15), which we derived in the weak-field approximation, remain valid under the conditions of the semiclassical approximation over the entire region (31). That this is the case can be seen by using expression (19) for the parameters of the instanton gas and the duality relations (20), which make it possible to go from region (31) to the region

$$\alpha \ll 1, \quad mV \ll \eta^{1/2},$$

where there is no doubt regarding the applicability of (15).

Expression (32) thus gives a correct description of the temperature dependence of the static mobility not only in region (31), where the semiclassical approximation holds, but also at arbitrarily small values of the product mV (at least in the case $\alpha \gg 1$). Its range of applicability is limited by temperatures which become progressively lower as mV decreases.

It follows from the results derived here and from the duality relations that in a delocalized phase ($\alpha < 1$) the temperature dependence of the mobility can be described at low temperatures by

$$\mu(T) = 1/\eta - cT^{2(1-\alpha)/\alpha},$$

not only for a weak potential[5,6] but also for a strong potential.

Bulgadaev has made a previous attempt[5] to determine the temperature dependence of the mobility for $\alpha > 1$ in the case of a weak potential. He approximated the dynamic correlation function by an expression like (12), in which he inserted $\omega_1 = 2\pi T$ in place of the frequency. As a result he found $\mu \propto T^2$. It seems to us that the method used in the present paper has a stronger physical foundation.

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