

Angular dependence of the critical field of superconducting superlattices: experiment

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We have investigated the dependence of the critical magnetic field $H_c(\theta)$ on orientation for a V/Si superconducting superlattice (SL) made with equal-thickness vanadium layers and silicon interlayers (for this paper, the Si layer thickness is $s = 30 \text{ \AA}$). In order to study the angular dependence of H_c , we chose samples in which a "crossover" was observed from three-dimensional behavior near the transition temperature to two-dimensional behavior at lower temperatures when the field \mathbf{H} was oriented parallel to the SL layers. We show that crossover behavior is strongly evident in the angular dependence of H_c as well. At low temperatures, the functions $H_c(\theta)$ are well-described by a modified Tinkham equation, which takes into account the transition to two-dimensional behavior. We investigated the derivative $\beta = dH_c/d\theta|_{\theta=0^\circ}$ with respect to angle near the parallel orientation over a wide field interval. In fields which exceed the crossover field, we observe good agreement between the experimental behavior of $\beta(H)$ and theory for a regular SL with Josephson coupling between finite-thickness superconducting layers. Deviations from the theoretical dependence $\beta(H)$ which are observed at low fields are explained by the effects of inhomogeneities in the SL.

INTRODUCTION

Layered compounds and artificial superlattices (SLs), which in recent years have been the object of intensive experimental study, possess a number of properties with regard to superconductivity which distinguish them from ordinary isotropic bulk superconductors. Among these properties, the unique temperature dependence of the upper critical field $H_{c\parallel}(T)$ for fields oriented parallel to the layers is especially noteworthy; this dependence is connected with a "crossover" phenomenon from three-dimensional behavior near the critical temperature T_c to two-dimensional behavior at lower temperatures, and an unusual dependence on the orientation of the critical field.¹⁻⁴ In recent years, much progress has been made in simulating the properties of layered compounds using artificially created superconducting superlattices which consist of periodically alternating layers of superconducting and dielectric or normal materials.²⁻⁴

Artificial superlattices possess definite advantages over natural layered crystals: it is possible to change not only the thickness of the isolating or normal interlayers through which the weak coupling takes place between the superconducting layers, but also the thickness of the metal layers themselves. With existing technology, an SL can possess a much higher degree of structural perfection than layered crystals intercalated by large organic molecules.

The phenomenon of crossover can be observed in these artificial superlattices much more clearly than in layered compounds. The layer thicknesses in an artificial SL can be considerably larger than the atomic spacing, so that in this case there are no restrictions on the value of the critical field connected with the paramagnetic limit. This makes interpretation of the experimental results much simpler.

In a previous paper⁴ on V/Si superlattices, we investigated critical fields for two orientations of \mathbf{H} —parallel and perpendicular to the layers. We observed crossover behavior

in the functions $H_{c\parallel}(T)$ at silicon layer thicknesses $s = 30\text{--}60 \text{ \AA}$. In contrast to the field $H_{c\parallel}$, the critical field $H_{c\perp}$ had no temperature anomalies which might reflect a transition from two-dimensional to three-dimensional behavior.⁴

As far as we know, the dependence of an SL critical field on orientation has been studied experimentally only for the Nb/Cu system (see Ref. 3). The authors of Ref. 3 found that the functions $H_c(\theta)$ they observed did not agree with any of the well-known theories: close to the parallel orientation, these functions were found to vary more rapidly in the three-dimensional regime than theory would predict.⁵ We note that for the majority of layered intercalated compounds, a disagreement of the same kind is observed with the predictions of Ref. 5, although the results of this paper should apply directly.

The intuitive considerations of Bannerjee and Schuller³ regarding the applicability of the Tinkham formula to SL in the region of two-dimensional behavior are not correct, as we will show below.

It is well-known that the discreteness of the SL structure gives rise to characteristic singularities in the temperature dependence of the parallel critical field. In the presence of a weak coupling between thin superconducting layers, the order parameter is practically unchanged out to the edges of each metallic layer, and exhibits a discontinuity when the layers meet. In the low-field region as defined by the inequality

$$L_H \gg D$$

($L_H = c\hbar/eH$ is a magnetic length which characterizes the size of the superconducting nucleus, while D is the period of the superlattice), the order parameter varies slowly from layer to layer, and the discreteness of the SL structure is only weakly evident; the function $H_{c\parallel}(T)$ matches closely the normal linear dependence of a bulk superconductor. As the magnetic field increases, the ratio D/L_H grows, and with it

the deviation from a linear dependence. The discreteness of the SL structure, which gives rise to these deviations, entirely dominates the behavior of the function $H_{c\parallel}(T)$ in the strong-field ($L_H < D$) limit. In this limit, the superconducting nucleus is found to be localized in an individual metallic layer; because of this, the form of the $H_{c\parallel}(T)$ curve is similar to that for a thin layer. Thus, the discreteness of the structure gives rise to a crossover from three-dimensional to two-dimensional behavior in the function $H_{c\parallel}(T)$.

As the angle of inclination of the magnetic field with respect to the superlattice layers is varied, the discreteness of the SL structure must appear quite clearly. Let us first examine the two-dimensional situation: although a nucleus is concentrated in one layer for parallel fields, for fields inclined away from this orientation the situation should be analogous to a system of inclined vortices: the nuclei permeate a large number of layers, but the size of the nucleus in the plane of each layer $L_{H\perp}$ becomes finite, determined by the perpendicular component of the field

$$L_{H\perp} = (2\pi\Phi_0/H \sin \theta)^{1/2}.$$

Because the magnetic field lines intersect the layers at an angle θ , the centers of the nuclei in neighboring layers are shifted by a distance $D \cot \theta$ in the plane of the layers. From this it is clear that for small θ such that the inequality

$$\sin \theta / \cos^2 \theta \leq HD^2 / 2\pi\Phi_0, \quad (1)$$

holds, the nucleus of each layer borders on portions of neighboring layers (or weak coupling) which are found in the normal state, as in the case of a parallel field. In this sense, the situation remains two-dimensional, just as for $\theta = 0^\circ$. In this angular region, the function $H_{c\parallel}(\theta)$ can be described by a modified Tinkham equation, which differs from the usual equation for an isolated thin layer. This difference is based to the fact that the superconducting transition temperature T_c of a sample differs from the extrapolated temperature \tilde{T}_c determined by the properties of one SL layer "adjacent" on neighboring normal layers.⁴ The required modification of the Tinkham formula consists of replacing the measured value of the field $H_{c\parallel}$ by a value which leads to the extrapolated temperature \tilde{T}_c :

$$\tilde{H}_{c\perp} = H_{c\perp}(\tilde{T}_c - T) / (T_c - T).$$

When inequality (1) is violated, the conditions at the junction between two layers change (the superconducting nuclei in neighboring layers contact each other). As a consequence, there occurs an unusual crossover as a function of the angle as the size of a nucleus in a cross-section perpendicular to the layers becomes larger than the SL period. This causes H_c to vary more smoothly with angle than in the small-angle region.

In the region of three-dimensional behavior (i.e., for temperatures close to T_c), the discrete structure of a superconducting nucleus in the SL evolves in the same way. There exists a characteristic angle θ^* which separates the small-angle region, in which the function $H_c(\theta)$ varies rapidly, from a region of fairly large angles over which the smoother dependence of H_c on θ can be described by a formula for anisotropic continuous superconductors.

Thus, measuring the orientation dependence of H_c in a SL should be a richer source of information about the cross-

over than measuring the critical field in the parallel orientation only. In addition, by measuring the functions $H_c(\theta)$ we can determine not only the functions $H_c(T)$ but also the temperature dependence of the derivative $dH_c/d\theta|_{\theta=0}$, which should be more sensitive to a restructuring of the spatial distribution of the order parameter than the critical field itself; this follows from an analogy with the results of Saint-James⁹ and Thompson¹⁰ for a single layer.¹⁾ We can expect that non-ideality of the SL structure is also more clearly apparent in the function $H_c(\theta)$ than in the behavior of the dependence of the critical field $H_{c\parallel}$ on temperature.

In this paper, experimental investigation of the orientation dependence of H_c was carried out on V/Si superlattices with thin ($s = 30 \text{ \AA}$) silicon interlayers and differing vanadium-layer thicknesses. SLs consisting of metal and semiconductor are easier to study than superlattices of superconductors/normal metals, since in the former the coupling strength between layers does not depend on temperature. The choice of $s = 30 \text{ \AA}$ is dictated by the fact that a crossover in parallel-oriented fields is clearly observed in all samples with this thickness of the semiconducting interlayers. This allows us to investigate the angular dependences of H_c in the regions of three-dimensional and two-dimensional behavior in one and the same sample.

The experimental data obtained in this paper agree qualitatively with a physical picture describing superconducting nuclei in the SL in oblique fields, which will be illustrated below. We also give a detailed comparison with a SL theory¹¹ which appears in this issue of JETP.

DESCRIPTION OF SAMPLES AND EXPERIMENTS

We studied metal-semiconductor superlattices prepared by sequential deposition of layers of vanadium and silicon onto various substrates, both orienting (fluorophlogopite, sapphire) and amorphous (glass). The conditions for preparing the samples are described in detail in Ref. 4. All samples investigated consisted of ten V/Si periods with 30 \AA silicon interlayers. The metal layer thickness was varied from one SL to another.

In these SLs we carried out detailed measurements of the angular dependence of the critical magnetic field in the temperature interval from 4.2 K down to 1.6 K. The temperature at a given point was stable to 0.003 K. In order to vary the orientation of the SL in an external magnetic field, the samples were affixed to stocks provided with special devices which allowed them to be rotated. A given orientation could be established to better than 0.5° . In order to determine the derivative $dH_c/d\theta|_{\theta=0}$ (θ is the angle between the SL plane and the direction of the magnetic field), the most detailed investigation was of the angular region close to the parallel orientation. The parallel orientation was identified by finding the minimum in the SL resistance. Thanks to the strong angular dependence of H_c near the point $\theta = 0^\circ$, it could be identified to within 0.1° , while the relative error in determining H_c was less than 1%. The value of H_c was determined by using that point on the curve $R(H)$ at which the sample resistance equaled half its normal resistance R_n . In recording the dependence $R(H)$, we used the standard four-probe method. The measured current came to $100 \mu\text{A}$.

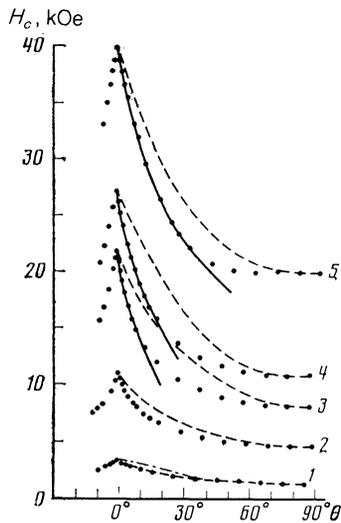


FIG. 1. Angular dependence of the critical magnetic field for SL-6 ($d = 290 \text{ \AA}$) at the following temperatures in degrees Kelvin: 1—3.822; 2—3.698; 3—3.601; 4—3.302; 5—2.996. Continuous curve—calculated from Eq. (3); dashed curve—from Eq. (2); dotted-dashed curve—from Eq. (4).

EXPERIMENTAL RESULTS

In Fig. 1, we show the experimentally-determined function $H_c(\theta)$ at various temperatures for sample SL-6 with $d \approx 290 \text{ \AA}$ (d is the width of the V-layers) deposited on a glassy substrate. The analogous functions for SL-2 with $d \approx 200 \text{ \AA}$ (on a substrate of fluorophlogopite) are shown on Fig. 2. It is apparent from Figs. 1 and 2 that the qualitative behavior of these angular dependences is the same over all the temperature regions investigated, despite the presence of the crossover. In all cases, a finite value of the derivative $dH_c/d\theta|_{\theta=0^\circ}$ is observed near the angle $\theta = 0^\circ$ (recall that this angle corresponds to \mathbf{H} oriented parallel to the SL layers). In the range of angles between 0° and 5° , the critical field varies almost linearly with angle. The dome-shaped de-

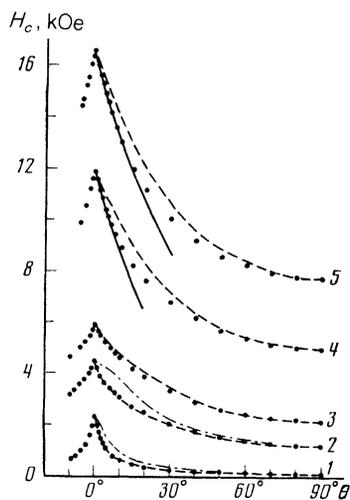


FIG. 2. Angular dependence of H_c for SL-2 ($d = 200 \text{ \AA}$) for the following temperatures in degrees Kelvin: 1—4.200; 2—3.797; 3—3.323; 4—3.00; 5—2.038. Notation for the calculated dependences is as in Fig. 1.

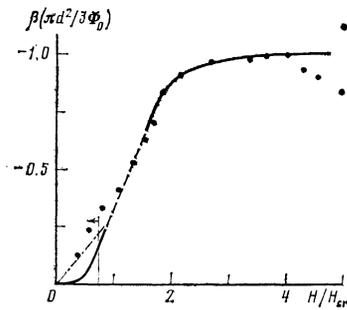


FIG. 3. Dependence of the normalized derivative of the critical field with respect to angle for $\theta = 0^\circ$ on the normalized field H/H_{cr} . The continuous curves are theoretical calculations based on Eqs. (6) and (7); the dotted-dashed curves are from Eq. (9). Data from SL-2.

pendence of $H_c(\theta)$ near $\theta = 0^\circ$, which is a characteristic of anisotropic three-dimensional superconductors,⁵ and which is also observed experimentally both in artificial SLs made with Nb and Cu layers³ and in certain layered transition-metal dichalcogenides,⁷ is not observed in our experiments.

Figures 3 and 4 illustrate the way the quantity $\beta = dH_c/d\theta|_{\theta=0^\circ}$ varies as the field H is varied. For the majority of samples investigated the quantity β increases as H increases in fields smaller than the crossover field (Fig. 3). In larger fields, we observe a tendency for the function $\beta(H)$ to saturate. The interval of fields where the variation of β is insignificant corresponds to a field region in which two-dimensional behavior is observed in the dependence of $H_c(T)$ for the parallel orientation of \mathbf{H} . In very large fields, as is apparent from Fig. 3 and the inset in Fig. 4, the derivative of the field with respect to angle near $\theta = 0^\circ$ once again begins to change markedly as the field increases. In this region, we observe a deviation from linearity in the dependence of $H_{c\parallel}^2(T)$ which is characteristic of the region of two-dimensional behavior of the critical parallel field, and which is connected with the penetration of a single chain of vortices into a particular layer of the SL. In some samples (see Fig. 4) for $H < H_{cr}$ we observe a decreasing portion of the curve $\beta(H)$.

DISCUSSION OF RESULTS

Let us first investigate the orientation dependence of H_c in the region of two-dimensional behavior, i.e., at fairly

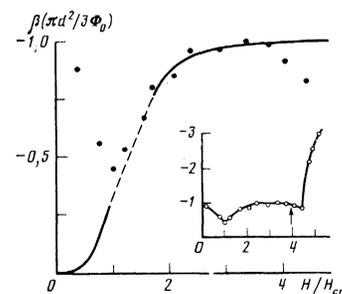


FIG. 4. The same as Fig. 3, data from SL-6. In the inset we present the form of the function $\beta(H)$ over a wide field range, including the field H_{FE} where a chain of vortices is introduced into a single layer of superconducting vanadium.

strong fields which exceed the crossover field ($H/H_{cr} \gtrsim 2.5$). In Figs. 1 and 2, we show a comparison of the experimental data with the usual Tinkham formula¹² for a few temperatures:

$$\left(\frac{H_c \cos \theta}{H_{c\parallel}(T)}\right)^2 + \frac{H_c \sin \theta}{H_{c\perp}(T)} = 1. \quad (2)$$

In the majority of samples, we observed significant disagreement between the experimental data and Eq. (2) for all the intervals of temperature we investigated, similar to the results of experiments³ on Nb/Cu superlattices.²⁾ In agreement with theoretical formalisms which take into account the discreteness of the SL structure (see the Introduction), the experimental angular dependence of H_c is found to be stronger than predicted by Eq. (2). If we take into account renormalization of the perpendicular critical field

$$\tilde{H}_{c\perp} = H_{c\perp}(T_c - T)/T_c - T,$$

then for the two-dimensional region we obtain in place of Eq. (2) the angular dependence

$$\left(\frac{H \cos \theta}{H_{c\parallel}(T)}\right)^2 + \frac{H \sin \theta}{H_{c\perp}(T)} \frac{T_c - T}{T_c - T} = 1. \quad (3)$$

We note that this relation which follows from simple physical considerations, can be obtained directly by solving the linearized Landau-Ginzburg equations for SLs.¹¹ The numerical functions calculated using Eq. (3) are shown in Figs. 1 and 2 for various temperatures as continuous curves. The extrapolated temperature \tilde{T}_c is determined from the functions $H_{c\parallel}(T)$ (see Ref. 4). Equation (3) should be correct in the region of angles determined by inequality (1). From Figs. 1 and 2, it is apparent that expression (3) describes the experimental functions $H_c(\theta)$ rather well for fairly small angles. In agreement with (1), as H increases (i.e., with decreasing temperatures) the region of angles where we see good agreement between the theoretical and experimental functions $H_c(\theta)$ broadens. For example, for SL-2 the angular interval changes from 9° at $T = 3.323$ K to 31° for $T = 2.038$ K (see Fig. 1).

For weak fields $H < H_{cr}$, where we observe three-dimensional behavior, according to the theory in Ref. 11 the angular dependence of H_c should be described by the formula for an anisotropic bulk superconductor⁵

$$\left(\frac{H_c \cos \theta}{H_{c\parallel}(T)}\right)^2 + \left(\frac{H_c \sin \theta}{H_{c\perp}(T)}\right)^2 = 1 \quad (4)$$

only for angles $\theta > \theta^*$, where

$$\theta^* = \left(\frac{32}{\pi}\right)^{1/4} \frac{D^2 H_{cr}}{\Phi_0} \left(\frac{H}{H_{cr}}\right)^{1/4} \exp\left\{-2\frac{H_{cr}}{H}\right\}. \quad (5)$$

Both in the region $H > H_{cr}$ and in the region $H < H_{cr}$, as $\theta \rightarrow 0^\circ$ the behavior of the angular dependence of H_c ought to be singular, i.e., the quantity $\beta = dH_c/d\theta|_{\theta=0^\circ}$ is different from zero.¹¹ In agreement with this statement, experiments at small θ do not show the dome-shaped function $H_c(\theta)$ given by (4) in the region $H < H_{cr}$ in practice. However, these results also do not agree with Eq. (4) for angles $\theta > \theta^*$, i.e., in the region of three-dimensional behavior [see Figs. 1 and 2, in which an approximation to Eq. (4) is shown by the dashed curve].

The tendency of the superconducting nucleus to localize in one layer of the SL (i.e., the transition to two-dimen-

sional behavior) should leave a clear signature in the dependence of the derivative $\beta = dH_c/d\theta|_{\theta=0^\circ}$ on H . For $H > H_{cr}$, according to Ref. 11:

$$\beta = -\frac{3\Phi_0}{\pi d^2} \left\{ 1 - \frac{1}{4} \left(6 \frac{D^2}{d^2} + 1 \right) \left(\frac{H_{cr}}{H} \right)^4 \right\}. \quad (6)$$

In weak fields $H < H_{cr}$, this dependence has the form¹¹

$$\beta = -\frac{2^{3/4}\Phi_0}{\pi^{1/4}D^2} \left(\frac{H}{H_{cr}}\right)^{3/4} \exp\left\{-2\frac{H}{H_{cr}}\right\}. \quad (7)$$

The theoretical function corresponding to Eqs. (6), (7) is shown in Figs. 3 and 4, plotted in the coordinates $(\pi d^2/3\Phi_0)\beta$ versus H/H_{cr} . The dashed curve in the vicinity of $H/H_{cr} = 1$ corresponds to an interpolation between Eqs. (6) and (7). In large fields, the dependence (6) of β on H asymptotically approaches the value $\beta = -3\Phi_0/\pi d^2$. For comparison with theory, the thickness d of the metallic layer is used as a fitting parameter. We note that the fitting parameter values of d for the various samples differ from the values of d obtained from the functions $H_{c\parallel}(T)$ in the region of two-dimensional behavior by no more than 8%. From Figs. 3, 4 it follows that in the interval of H/H_{cr} from ~ 1 to ~ 4 , we observe very good agreement between the theoretical and experimental functions. A small decrease in β and its subsequent sharp increase for $H \gtrsim 4H_{cr}$ is related to the introduction of a one-dimensional chain of vortices in an isolated superconducting film. This follows first of all from an estimate of the critical thickness $d_c = (5/2)^{1/2}\xi(T)$.¹³ The field H_{FE} corresponding to this equation is denoted in the inset of Fig. 4 by an arrow for sample SL-6. Secondly, the singular behavior of $\beta(H)$ in large fields is in qualitative agreement with the data of Ref. 9, where the form of the function $\beta(H)$ is determined theoretically in the vicinity of the field H_{FE} for an isolated film. The character of this function should also be preserved for the SL in the high-field region, where the critical fields are defined by the properties of an individual layer.

In the region $H < H_{cr}$, for all samples under investigation we observed a significant disagreement between experimental data and the theoretical function (7). This disagreement can be due to a number of causes in general.

First of all, there is a trivial possibility related to the fact that the region of angles over which the quantity β is determined may be too narrow as defined by Eq. (5). Thus, for SL-6 in a field $H = 0.88H_{cr}$ we have $\theta^* = 2.8^\circ$, whereas for $H = 0.42H_{cr}$ the value of θ^* falls to 0.2° . Because the value of θ^* decreases sharply as $H \rightarrow 0$, the accuracy with regard to limitations in angular resolution in determining β from experiment decreases as we approach T_c . However, the function $H_c(\theta)$ observed in experiment differs significantly from the form (4), which is characteristic of an anisotropic superconductor, over a wide range of angles ($\theta \gg \theta^*$). Therefore, we maintain that the difference between the theoretical values (7) and the experimental values of β are not connected with the limits of measurement precision.

A second source of disagreement between experiment and Eq. (7) is the nonideal behavior of the SL structure. Irregularities in the SL are felt most strongly in fields $H < H_{cr}$, where in place of the exponential dependence (7) for $\beta(H)$ we obtain a power-law dependence⁷:

$$\beta = -\frac{2^{3/4}\Phi_0}{\pi^{3/4}D^2} \left(\frac{\Delta d}{d}\right)^{1/4} \left(\frac{H}{H_{cr}}\right)^{3/4}. \quad (8)$$

Here Δd is the difference in thickness of one of the layers from the d of an ideal SL. In order to derive Eq. (8), it was assumed in Ref. 11 that $\Delta d \ll d$. A comparison of the values of β obtained in experiment along with Eq. (9) leads to the value $\Delta d \sim d$. Such values of Δd could occur in a real SL because of the disruption in continuity of one of the dielectric interlayers, leading to direct contact between adjacent metallic layers.

Note that a defect "packet" in the SL which causes β to grow markedly in the weak-field region leaves the dependence of $H_{c||}$ on T practically unaffected. The reason for this is that, although the nucleus is localized near the inhomogeneity, its size greatly exceeds the SL period. Unfortunately, a detailed comparison of the experimental data with Eq. (8) is not possible, since apparently the condition $\Delta d/d \ll 1$ is not fulfilled in our experiments; in addition, our investigation was carried out with insufficient precision in the region $H < 0.5H_{cr}$.

A further cause of disagreement between the theoretical function (7) and experiment could be the fact that in the three-dimensional region the superconducting nucleus can be localized at the external boundary of the SL, i.e., we have a case of surface superconductivity. In this case, for the weak-field region where the discrete character of the SL does not enter in, the function $\beta(H)$ should have the form¹¹:

$$\beta = -1.30(\Phi_0/\pi D^2 H_{cr})^{1/2} H. \quad (9)$$

Equation (9) is correct under the condition

$$H \ll H_{cr}(1 + 1/4 \ln(\Phi_0/D^2 H_{cr}))^{-1}. \quad (10)$$

The theoretical dependence (9) for sample SL-2 is shown as the dotted-dashed curve in Fig. 3; it lies higher than the function (7), which corresponds to localization of the superconducting nucleus in the volume of the SL (for the case of a regular structure). However, it is considerably below the experimental data in any case. Condition (10) is fulfilled within the field interval marked by the arrow in Fig. 3. Despite the fact that the experimental points lie considerably closer to the theoretical curve in this region (the curve which corresponds to surface superconductivity), the formation of a nucleus at the surface is nonetheless quite improbable, since such an assumption contradicts the observed temperature dependence of the field $H_{c||}(T)$. The whole curve $H_{c||}(T)$ (including the crossover region) in our experiments is described with high accuracy by a theory constructed without including surface effects.⁴ The value of the extrapolated length l between layers can be determined using certain independent methods over various temperature ranges of the function $H_{c||}(T)$.⁴ These methods give the same value of l only if the field $H_{c||}(T)$ in the limit $T \rightarrow T_c$ corresponds to the field H_{c2} , not H_{c3} .

On the basis of our analysis of all the possible causes of disagreement between the experimental function $\beta(H)$ for $H < H_{cr}$ and the theoretical relation (7) for an ideal SL, we are inclined to the point of view that most likely this disagreement is due to the defect structure of the SL.

In conclusion, we would like to point out still another peculiarity in the low-field behavior observed in certain samples—a growth of β with decreasing H in very small fields (see Ref. 4). This behavior is observed in samples with a "double" crossover, as we mentioned above. The double

crossover is related to the fact that the SLs under investigation consisted of a limited number of layers (10 periods), and for very high temperatures the total thickness L of the whole sample is found to be smaller than $\xi(T)$. For example, for SL-6 (see Ref. 1) and $T = 3.698$ K we have $\xi(T) = 3880$ Å, whereas L for this sample was ≈ 3000 Å. The appearance of an additional crossover is observed both in the critical field for parallel orientation of H and in the angular dependence of H_c . Phenomena of this kind can be observed only when the temperature corresponding to the condition $L \sim \xi(T)$ is not too close to the transition temperature T_c . For $L \lesssim \xi(T)$, the whole SL sample behaves like a two-dimensional film of thickness $L = nD$ (n is the number of metal-semiconductor periods in the SL). As we already mentioned in the Introduction, close to T_c the discreteness of the SL structure should have only a small effect on the behavior of the critical fields. From experiment it follows that in this temperature interval we have $H_{c||}^2 \sim (T_c - T)$, while the angular dependences agree with the usual thin-film Tinkham formula. The presence of a second crossover is additional evidence that the role of the structural discreteness is smaller the closer the sample temperature is to T_c .

CONCLUSION

In this paper we have presented experimental data on the angular dependence of the upper critical field of an SL consisting of alternating metallic and semiconductor layers. The thickness of the metallic V layers was varied from sample to sample, while the thickness of the silicon interlayers was chosen so that Josephson contacts were established between the layers; a crossover was observed from $3D$ to $2D$ in a field parallel to the SL layers.

It was shown that measurements of the function $H_c(\theta)$ are a much richer source of information about the SL structure than investigation of the critical field at a fixed orientation, i.e., parallel or perpendicular to the layers. The structure of an ideal SL is characterized by the following parameters: the period D of the superstructure, the thickness of the metal layers d , and the coupling strength between superconducting layers \hbar^2/mdl . It is not possible to determine any one of these quantities from experiments involving measurements in perpendicular fields alone. By making measurements on one and the same sample in parallel and perpendicular fields, we can reliably determine the coupling strength between layers⁴ if the quantities D and d are known.

From measurements made in oblique fields, especially of the function $\beta(H)$, we can determine the quantity D for the case of an ideal SL.³ In the region of two-dimensional behavior both the function $H_c(\theta)$ and the function $\beta(H)$ are in good agreement with theory,¹¹ which allows us to find the ratio D/d (especially when D and d are sufficiently different).

It follows from our experiments that not one of the well-known formulae describing the angular dependence of these quantities (e.g., the formula for an anisotropic superconductor or the Tinkham formula) agrees with the experimental data. The lack of agreement with the Tinkham formula is because this formula does not take into account the effect of the neighboring normal layers, which is quite significant despite the weak interlayer coupling. The Lawrence-Doniach formula was derived for continuous anisotropic supercon-

ductors; the discreteness of the structure leads to the much stronger angular dependence of H_c near $\theta = 0^\circ$.

Investigations of $\beta(H)$ are especially informative, in that they allow us to see the nonideal features of the SL structure. For low fields $H < H_{cr}$, i.e., in the region of 3D behavior, values of β are obtained in experiment which significantly exceed the calculated values of this parameter for an ideal SL. This fact attests unambiguously to the nonideal behavior of the superstructure in the samples under study. At the same time, in the 3D region the function $H_{c\parallel}(T)$ is practically insensitive to the presence of defects in the SL.⁴ We have also shown that study of the function $\beta(H)$ allows us not only to detect deviations from ideality in the SL structure but also to study effects which arise from the finite thickness of the individual metallic layers (i.e., the introduction of single vortex chains) and the finite thickness of the entire SL sample (double crossover).

In conclusion, the authors are grateful to L. I. Glazman, whose collaboration with us in this work was very useful interpreting the experimental data in terms of the rigorous theory, and to A. N. Stetsenko for help in preparing the samples.

¹For single layers, this function exhibits a kink at fields for which a single chain of vortices enters, although the quantity $dH_c/d\theta|_{\theta=0}$ has a sharp extremum of this field.^{9,10}

²In some samples, good agreement is observed with the Tinkham formula (which at first glance is unexpected) for very high temperatures and

very close to the superconducting transition temperature (see Fig. 1). In these samples, a "double" crossover is observed in the parallel field. Besides the usual SL crossover, which appears in the transition to two-dimensional behavior at low temperatures, there is also a transition to two-dimensional behavior at high temperatures near T_c . This crossover will be discussed in detail below.

³See Eq. (7) for the case $H < H_{cr}$. The corresponding experimental data, together with an x-ray investigation of the SL structure, will be presented in a separate paper.

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