The laser as a source of a quantum electromagnetic field in a compressed state

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When an additional cell working on the parametric principle is inserted into a laser cavity, the cell automatically begins to amplify. The quantum properties of the generated field due to sub-Poisson statistics are then suppressed if, in fact, they are present. However, when the cell efficiency is high enough, other quantum properties of the field due to compressed states may arise. Photodetection shot noise can be almost completely suppressed in this scheme.

At present, the study of quantum electromagnetic fields has reached the stage in which a search is being made for suitable sources. Many papers and several reviews have been published, both in the Soviet Union and abroad. The work reported here is part of this effort. We shall show below that an ordinary laser containing an additional internal cell working on the parametric principle will emit light with well-defined discrete properties. The particular defect of all the models proposed so far is that either they are very difficult to implement in practice or the discrete light effect is small (or even exceedingly small). On the other hand, a laser containing a parametric cell does not contain any unusual elements, and, at the same time, the discrete effect can reach its limiting values.

1. KINETIC EQUATION FOR THE DENSITY MATRIX OF THE GENERATED FIELD

We shall consider that the medium generating the field consists of stationary atoms with a working transition frequency ω (see Fig. 1a). The degeneracy of the levels will not be taken into account and the level widths γ_a, γ_b will be considered to have been formed as a result of transitions to extraneous levels. As far as incoherent excitation of the medium is concerned, we shall suppose that only the upper level is excited and the excitations either obey Poisson statistics or are free from fluctuations.

We shall assume that the parametric cell is also filled with stationary atoms with a transition frequency 2ω (Fig. 1b). The parametric interaction with the generated field at frequency ω is produced by having the cell in an external pump field of frequency 2ω. The working levels of the atoms in the parametric cell have widths equal to γ. They are nondegenerate and have time-independent populations produced by some incoherent excitation system.

We shall solve the problem in terms of the kinetic equation for the density matrix ρ of the generated field. We may write

$$(\dot\rho)_{\text{laser}} + (\dot\rho)_{\text{param}},$$

i.e., the rate of change of the field in the resonator is determined by the usual laser conditions (the working medium and the interaction with the resonator) and the presence of the additional parametric cell. The two contributions can be calculated independently of one another. This means that we can use previously published results. For example, $(\dot\rho)_{\text{laser}}$ can be taken from Refs. 1 and 2. It is given in the Appendix in the antinormal diagonal representation of the density matrix. On the other hand, $(\dot\rho)_{\text{param}}$ was obtained in Ref. 3 and is given by

$$\dot\rho = -i[\lambda a^+a^+ + \lambda^* aa, \rho].$$

The effective constant representing the parametric interaction between the cell and the generated field in an external pump field (photon operators a_H, a_H^+ replaced by the c-numbers α_H, α_H^*)

$$\lambda = {}^1/_2 iqA_H\alpha_H/(1+I_H)$$

is thus expressed in terms of the complex amplitude α_H, the dimensionless pump-field power I_H, and the linear (unsaturated) amplification coefficient A_H of the parametric medium at frequency 2ω. The constant q is equal to the ratio of the strength of the interaction between the cell and the generated field

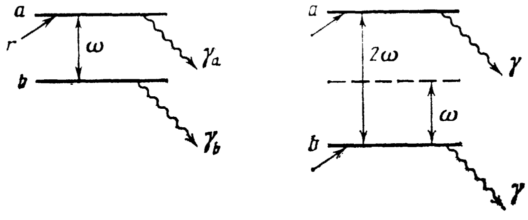
$$f_{ab} = \sum_n g_{an}g_{nb}\left(\frac{1}{\omega_{na}+\omega} - \frac{1}{\omega-\omega_{nb}}\right)$$

to the strength of the interaction between the same atom and the pump field

$$g_{ab} = -i(\omega/2L^3)^{1/2} d_{ab} e^{i\mathbf{kr}}, \quad q = (f_{ab}/g_{ab})^*, \tag{1}$$

where L is the resonator perimeter.

Laser problems can be satisfactorily solved by the method of diagonal representation of the Glauber density matrix (normal diagonal representation). However, in problems in which quantum fields are expected to arise, this is not very convenient because the equations then contain derivatives with respect to the complex amplitudes of all orders. In such cases, one can use other representations that are less convenient than the Glauber representation but, at the same time, do not suffer from these disadvantages. We shall use the antinormal representation that is obtained from the density matrix ρ by taking the diagonal matrix element over coher-



a b

FIG. 1.

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ent states

$$P_A(\alpha, t)=\langle\alpha|\rho(t)|\alpha\rangle.$$

Here and in what follows, the subscript $A(N)$ will signify that we are taking the density matrix in the antinormal (normal) diagonal representation. There is a direct integral connection between the matrices in the antinormal and normal representations.[4] The transformation from the equation for ρ to the equation for P_A is defined by the following rules:

$$a\rho\to\left(\alpha+\frac{\partial}{\partial\alpha^*}\right)P_A, \quad a^+\rho\to\alpha^*P_A,$$

$$\rho a\to\alpha P_A, \quad \rho a^+\to\left(\alpha^*+\frac{\partial}{\partial\alpha}\right)P_A.$$

The derivatives with respect to α now appear in cases other than the normal representation, and with a different sign. The expressions for $(\dot{P})_{\rm laser}$ and $(\dot{P})_{\rm param}$ are given in the Appendix.

The equations become simpler still if we assume that the generation phase and the number of photons fluctuate weakly around their stable stationary solutions. To find the stationary solutions, we have to construct the truncated equations of the semiclassical theory. It is well-known that these can readily be deduced from quantum theory if we neglect derivatives with respect to α of orders higher than one in the equation for P_A. The equation of the semiclassical theory is

$$\dot\alpha+\frac{1}{2}\left(C-\frac{A}{1+I}\right)\alpha=-2i\lambda\alpha^*,$$

where α and $I=\beta|\alpha|^2$ are, respectively, the complex amplitude and dimensionless power of the generated field, and A is the linear (unsaturated) amplification coefficient of the working medium producing the generation. Hence we obtain the following equations for the stable solutions $\bar n$ and $\bar\varphi$ $(\alpha=n^{1/2}e^{i\varphi})$:

$$\bar\varphi={}^1/_2\varphi_0, \quad A/(1+\beta\bar n)=C-4|\lambda|\equiv\tilde C,$$
$$\beta=2|g_{ab}|^2(\gamma_a+\gamma_b)/\gamma_a\gamma_b\gamma_{ab},$$

where $\varphi_0=\arg\lambda+\pi/2$ is the phase of the pump field, except for a constant term, and g_{ab} is given by (1), but refers to the generating medium and not to the parametric medium.

As can be seen, the parametric cell automatically assumes the amplifying state and compensates losses from the cavity resonator that were originally determined by the resonator width C.

2. FLUCTUATIONS IN THE GENERATED POWER

Consider an experiment in which light leaving the resonator enters a photodetector, and the output characteristic of the measuring system is the photocurrent spectrum which, in the plane-wave approximation, can be formally written as[5]

$$i_\omega^{(2)}=qC\left(\bar n+2qC\,\mathrm{Re}\int_0^\infty g(\tau)e^{i\omega\tau}d\tau\right) \qquad (2)$$

where q is the quantum yield of the photodetector, C is the resonator width without the parametric cell, and

$$g(\tau)=\overline{a^+a^+(\tau)a(\tau)a}-\overline{a^+a}^2.$$

It is convenient to transform to the normal diagonal representation and introduce the fluctuations in the photon number around the stationary value:

$$|\alpha|^2=\bar n+\varepsilon.$$

We thus obtain

$$g(\tau)=\overline{|\alpha|^2|\alpha(\tau)|}_N{}^2-\overline{|\alpha|_N{}^2}{}^2=\overline{\varepsilon\varepsilon(\tau)}_N.$$

It follows that, to determine the photocurrent spectrum, we must know averages of the form $\overline{\varepsilon\varepsilon}$, which can be found from the equations discussed in the last section.

We shall consider that the fluctuations in n and φ specified by the density matrix in the antinormal representation of P_A are small:

$$n=\bar n+\varepsilon, \quad \varepsilon\ll\bar n, \quad \psi=\varphi-\bar\varphi\ll1.$$

The equation for P_A admits of the separation of variables in the form

$$P_A(\alpha, t)=R_A(\varepsilon, t)\Phi_A(\varphi, t).$$

The equation for R_A is

$$\frac{\partial R_A}{\partial t}=\Gamma\frac{\partial}{\partial\varepsilon}[(\varepsilon-1)R_A]+\Gamma\bar n(\xi+2)\frac{\partial^2R_A}{\partial\varepsilon^2} \qquad (3)$$

and describes fluctuations in the number of photons in the resonator. When this equation is written down, we must remember that

$$\overline{\varepsilon_N}=\overline{\varepsilon_A}-1=0, \quad \overline{\varepsilon_N{}^2}=\overline{\varepsilon_A{}^2}-2\bar n-1=\xi\bar n$$

and bear in mind the conditions for stationary stable generation.

The form of (3) is exactly the same as in the absence of the parametric cell[2] but, of course, the coefficients are different:[1)]

$$\xi=\xi_0+2|\lambda|/\tilde C, \quad \xi_0=I^{-1}-\gamma_b/2(\gamma_a+\gamma_b), \quad \Gamma=\tilde CI/(1+I).$$

The required average $\overline{\varepsilon\varepsilon(\tau)}_N$ can now be written in the explicit form:

$$\overline{\varepsilon\varepsilon(\tau)}_N=\overline{\varepsilon_N{}^2}e^{-\Gamma\tau}=\xi\bar ne^{-\Gamma\tau},$$

and, consequently, the noise spectrum (2) can be written explicitly in the form

$$i_\omega^{(2)}=qC\bar n(1+2\xi qC\Gamma/(\Gamma^2+\omega^2)),$$

which, as can be seen, is precisely the same as for a laser without the parametric cell. The character of the field (quantum or classical) is determined by the sign of ξ. The effect of the cell on ξ is represented by the term $2|\lambda|/\tilde C$, which is manifestly positive. It follows that the cell always gives rise to an increase in ξ. In particular, if for the original laser $\xi_0=-1/2$ (for $\gamma_a=0$), then the generation exhibits Poisson statistics for $C=8|\lambda|$. For arbitrary ratios of C and $|\lambda|$, the depth of the valley is equal to

$$qC|C-8|\lambda||(C-4|\lambda|)^{-2}.$$

The general conclusion of this Section is that the parametric cell will always increase the fluctuations in the number of photons (generated power) and will tend to suppress sub-Poisson statistics if, indeed, it is present in the original laser without the cell.

3. COMPRESSED STATES OF THE GENERATED FIELD

We shall now consider that the laser radiation is mixed at the photocathode with a reference radiation which has the same frequency and, for simplicity, is in a coherent state. The mixing can be formally represented by a unitary transformation,[2] the result of which is that the photon annihilation operator a in the generated field and the photon annihilation operator a_0 in the reference wave transform into the photon annihilation operator in the resultant wave:

$$Ta+Ra_0, \quad |T|^2+|R|^2=1,$$

where $R=|R|e^{i\varphi_R}$ and $T=|T|e^{i\varphi_T}$ are the complex reflection and transmission coefficients of the mixing mirror. Formula (2) then retains its form except that $\bar{n}$ must be replaced with $|T|^2\bar{n}+|T_0|^{-2}|R|^2\bar{n}_{\rm ref}$, where $|T_0|\exp(i\varphi_{T_0})$ is the transmission coefficient of the exit mirror of the laser. It is related to C and to the resonator parameter L by $C=|T_0|^2/L$. The function $g(\tau)$ is given by

$$g(\tau)=|T|^4\overline{(a^+a^+(\tau)a(\tau)a-\overline{a^+a}^2)} +|T|^2|T_0|^{-2}|R|^2\bar{n}_{\rm ref}\{\overline{(aa(\tau)-\bar{a}^2)}e^{-2i\Phi_0}+\overline{(a^+a(\tau)-|\bar{a}|^2)}+\text{c.c.}\}.$$

The first term in this expression is independent of the reference signal power $\bar{n}_{\rm ref}$ and is essentially the same term that was considered in the last Section, i.e., it carries information about fluctuations in the generated power. It is readily verified that the second term has a different physical meaning for different values of the phase

$$\Phi_0=\varphi_{\rm ref}+\varphi_R-\varphi_T-\varphi_{T_0}.$$

When $\Phi_0=0$, it is proportional to the mean square fluctuations in the generalized coordinate of the field oscillator $(\overline{\Delta q^2}-1)$

$$\hat{q}=(\omega/2)^{1/2}(a^++a),$$

and for $\Phi_0=\pi/2$ it is proportional to the fluctuations in generalized momentum $(\overline{\Delta q^2}-1)$

$$\hat{p}=i(2\omega)^{-1/2}(a^+-a).$$

Thus, by specifying the external parameter Φ_0, we can follow the fluctuations in the coordinate, the momentum, or something intermediate.

If we now consider small fluctuations in amplitude and phase, we obtain

$$g(\tau)=|T|^2|T_0|^{-2}|R|^2\bar{n}_{\rm ref}\left\{\frac{\overline{\varepsilon\varepsilon(\tau)_N}}{\bar{n}}\Delta + \frac{\overline{\varepsilon\varepsilon(\tau)_N}}{\bar{n}}\cos^2\Phi+4\bar{n}\overline{\psi\psi(\tau)_N}\sin^2\Phi\right\},$$

where Δ is the ratio of the generated laser power to the power carried by the reference signal (in the mixing channel), $\Phi=\Phi_0-\bar{\varphi}(\bar{\varphi}=(1/2)\varphi_0)$ is the phase difference between the laser and reference signals (in the mixing channel), and $\psi=\varphi-\bar{\varphi}$ is the fluctuation in the phase of the generated laser power around its stationary value $\bar{\varphi}$.

To obtain the explicit expression for $\overline{\psi\psi(\tau)_N}$, we must write down the equation for $\Phi_A(\varphi,t)$:

$$\frac{\partial\Phi_A}{\partial t}=4|\lambda|\frac{\partial}{\partial\varphi}[(\varphi-\bar{\varphi})\Phi_A]+4|\lambda|\left(\overline{\psi_N^2}+\frac{1}{2\bar{n}}\right)\frac{\partial^2\Phi_A}{\partial\varphi^2}. \quad (4)$$

In the coefficient of the second derivative, we used the fact that

$$\bar{\psi}_N=\bar{\psi}_A=0, \quad \overline{\psi_N^2}=\overline{\psi_A^2}-1/2\bar{n}.$$

These equations can readily be deduced from the condition $\overline{aa}=\overline{\alpha_N^2}=\overline{\alpha_A^2}$ and the assumption that the amplitude and phase fluctuations can be factorized. The quantity $\overline{\psi_N^2}$ can be expressed in terms of the laser parameters as follows:

$$\overline{\psi_N^2}=\frac{C}{16\bar{n}|\lambda|}\left(1+I\frac{\gamma_a}{\gamma_a+\gamma_b}-\frac{2|\lambda|}{C}\right).$$

Once we know the properties of (4), we can write down the required average:

$$\overline{\psi\psi(\tau)_N}=\overline{\psi_N^2}e^{-4|\lambda|\tau}.$$

We now have all that is necessary to enable us to write down the photocurrent spectrum with the reference signal. If

$$\frac{\overline{\varepsilon_N^2}}{\bar{n}}\frac{\Delta}{\Gamma}\ll 4\bar{n}|\overline{\psi_N^2}|\frac{1}{4|\lambda|}, \quad \Delta\ll 1, \quad (5)$$

the spectrum assumes the simple form

$$i_\omega^{(2)}\propto 1+qC|T|^2\cdot 8\bar{n}\overline{\psi_N^2}\frac{4|\lambda|}{16|\lambda|^2+\omega^2}.$$

From our point of view, the most interesting cases are those where the shot component (represented by the unity here) is compensated by the additional term. This means that the excess noise must be negative, which occurs when (compare with $\overline{\psi_N^2}$)

$$C<2|\lambda|\frac{3+2I_0}{1+I_0}, \quad I_0=I\frac{\gamma}{\gamma_a+\gamma_b}.$$

This condition cannot be satisfied for $I_0\gg 1$ because it is then transformed to the condition $C<4|\lambda|$, i.e., it is in conflict with the condition for stationary generation. When $I_0\ll 1$, it is equivalent to $C<6|\lambda|$. The depth of the valley is then given by

$$qC|T|^2\frac{1}{4|\lambda|}\frac{6|\lambda|-C}{2|\lambda|}$$

and is a maximum, almost completely compensating the shot noise for $C-4|\lambda|\ll C$ and $q|T|^2\sim 1$. Condition (5), for which all this is valid when $C\sim 4|\lambda|$ can be rewritten in the explicit form

$$\Delta\ll\begin{cases}\dfrac{1}{2}I^2\dfrac{C-4|\lambda|}{4|\lambda|}\ll 1, & I\ll 1\\[2mm] \left(\dfrac{C-4|\lambda|}{4|\lambda|}\right)^2\ll 1, & I\gg 1\end{cases}$$

Thus, to optimize the suppression of the photodetector shot noise, we must satisfy three conditions: first, the mixing mirror must completely transmit the laser radiation, i.e., $|T|^2\sim 1$, and weakly reflect the reference radiation, i.e., $|R|^2\ll 1$; second, the power carried by the reference signal in the mixing channel must be much greater than the generated laser power, which is readily assured when the original reference signal power is high enough; and, third, resonator losses must be compensated as far as possible by amplification in the parametric cell $(4|\lambda|\to C)$.

APPENDIX

In the antinormal diagonal representation of the density matrix, the expression for $(\dot{P}_A)_{\text{laser}}$ can be written in the following form:

$$(\dot{P}_A)_{\text{laser}} = (\dot{P}_A)_{\text{resonator}} + (\dot{P}_A)_{\text{work.med.}} \quad a|\alpha\rangle=\alpha|\alpha\rangle, \quad \alpha=n^{1/2}e^{i\varphi},$$

$$(\dot{P}_A)_{\text{resonator}} = C\left(\frac{\partial}{\partial n}n + \frac{\partial}{\partial n}n\frac{\partial}{\partial n} + \frac{1}{4\bar{n}}\frac{\partial^2}{\partial n^2}\right)P_A,$$

$$(\dot{P}_A)_{\text{work.med.}} = r\left(\hat{S} - \frac{1}{2}\hat{S}^2\right)P_A,$$

$$r\hat{S} = -A\frac{\partial}{\partial n}n\left(\frac{1}{1+\beta n} - \frac{\beta n}{(1+\beta n)^2}\frac{\partial}{\partial n}\right) - \frac{A\beta n}{(1+\beta n)^2}\frac{\delta\gamma_b}{\gamma_{ab}(\gamma_a+\gamma_b)}\frac{\partial^2}{\partial n\partial\varphi} + \frac{A}{1+\beta n}\frac{\delta}{2\gamma_{ab}}\frac{\partial}{\partial\varphi}$$
$$+\frac{1}{4n}\frac{A\beta n}{1+\beta n}\left[\left(1+\frac{\delta^2}{\gamma_{ab}^2}\right)\frac{\gamma_a}{\gamma_a+\gamma_b} - \frac{1}{1+\beta n}\frac{\delta^2}{\gamma_{ab}^2}\frac{\gamma_a-\gamma_b}{\gamma_a+\gamma_b}\right]\frac{\partial^2}{\partial\varphi^2}$$

$$A = \frac{2r|g_{ab}|^2\gamma_{ab}}{\gamma_a(\gamma_{ab}^2+\delta^2)}, \quad \beta = \frac{2|g_{ab}|^2(\gamma_a+\gamma_b)\gamma_{ab}}{\gamma_a\gamma_b(\gamma_{ab}^2+\delta^2)}$$

$$\delta = \omega - \omega_{ab}.$$

This representation for $r\hat{S}$ was obtained in Ref. 2. The term $\sim\hat{S}^2$ is significant for the noise-free excitation of the working atoms and is absent in the case of Poisson excitation statistics.

The corresponding expression for the effect of the parametric medium on the field is

$$(\dot{P}_A)_{\text{param}} = \left(2i\lambda\alpha^*\frac{\partial}{\partial\alpha} + i\lambda\frac{\partial^2}{\partial\alpha^2} + \text{c.c.}\right)P_A$$
$$= 4|\lambda|\left\{\cos 2\psi\frac{\partial}{\partial n}n - \frac{1}{2}\cos 2\psi n\frac{\partial^2}{\partial n^2} + \frac{1}{2}\frac{\partial}{\partial\varphi}\sin 2\psi + \cos 2\psi\frac{1}{8n}\frac{\partial^2}{\partial\varphi^2} + \frac{1}{4}\sin 2\psi\frac{\partial^2}{\partial n\,\partial\varphi} + \frac{1}{4}\sin 2\psi n\frac{\partial^2}{\partial n\,\partial\varphi}\frac{1}{n}\right\}P_A,$$

$$\psi=\varphi-\bar{\varphi}, \quad \bar{\varphi}=\frac{1}{2}\varphi_0=\frac{1}{2}\arg\lambda+\frac{\pi}{4}.$$

1) The second term in ξ_0 occurs only in the case of excitation of the working medium without fluctuations; for Poisson excitation statistics, $\xi_0 = I^{-1}$.

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