Kinetics of surface structures on solids exposed to laser radiation

B. I. Makshantsev and E. A. Manykin

Institute of Optico-Physical Measurements
(Submitted 28 November 1986)
Zh. Eksp. Teor. Fiz. **93**, 437–449 (August 1987)

An analysis is given of the evolution of a "periodic" surface structure on the surface of solid semiconductors or dielectrics under the influence of laser radiation when the concentration of conduction photoelectrons or electron excitations is a function of position along the plane surface of the material. In particular, it is shown that the region of localization of this structure can be a rapidly varying nonmonotonic function of the laser power density.

The different aspects of irreversible "periodic" structures formed on the surface of solids under the influence of laser radiation now have an extensive literature devoted to them.[1–5] As a rule, this literature is concerned with the conditions under which the medium reaches high temperatures and its surface becomes corrugated.[6,7] In the case of semiconductors and dielectrics, it is also interesting to consider situations in which the laser radiation does not produce much heating of the material and its surface remains planar, but conduction photoelectrons or electron excitations, whose concentration depends on position along the surface of the specimen, are produced in the surface layer around a local and specially introduced inhomogeneity in the dielectric properties of the material.

Since this heating is not accompanied by phenomena such as evaporation, generation of acoustic waves, and so on, it is possible to examine in a relatively simple form the interaction between the electromagnetic radiation and the solid material in which this type of surface structure is discussed in the present paper.

1. To be specific, let us suppose that the electromagnetic wave

$$\mathbf{E}=\mathbf{E}^{(0)}\exp(-ikx\sin\theta-ikz\cos\theta) \tag{1}$$

is incident on a medium (semiconductor or dielectric) that fills a half-space and has a planar surface. In this expression, $k=\omega/c$ is the wave number, θ is the angle of incidence, and the z axis lies along the outward normal to the surface of the medium. For simplicity, we shall suppose that the amplitude $\mathbf{E}^{(0)}$ of the wave has a time dependence of the form $\mathbf{E}^{(0)}(t)=\mathbf{E}^{(0)}\theta(t)$, where $\theta(t)=0$ for $t<0$ and $\theta(t)=1$ for $t>0$. Let us also suppose that the electromagnetic field in the medium is determined by the equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+\tilde{\varepsilon}k^2-\text{grad div}\right)\mathbf{E}=\mathbf{f}, \tag{2}$$

where $\tilde{\varepsilon}=1$ for $z>0$ and $\tilde{\varepsilon}=\varepsilon=$ const for $z<0$. The vector $\mathbf{E}$ has only two (E_x and E_z) nonzero components ; $\mathbf{f}=-k^2\Delta\varepsilon\mathbf{E}$, and $\Delta\varepsilon=0$ for $z>0$ and $\Delta\varepsilon=\Delta\varepsilon(x,z)$ for $z<0$. Equation (2) is complemented by the condition $\text{div}[(\varepsilon+\Delta\varepsilon)\mathbf{E}]=0$.

In the ensuing analysis, we shall have to determine the field $\mathbf{E}$ inside the medium. We shall do this by deriving the integral equation for $\mathbf{E}$ in the region $z<0$. Let us Fourier-transform (2) with respect to the variable x, and solve the resulting equation, assuming that the vector $\mathbf{f}$ is known and using the well-known boundary conditions for the electric field components E_x and E_z. For $z<0$, we obtain

$$E_{xq}=\left(E_{xq}^{(+)}+\int_{-\infty}^{-0}\frac{1}{\lambda_q}\sinh(\lambda_q z')F_{xq}(z')\,dz'\right)e^{-\lambda_q z}+\frac{1}{\lambda_q}\int_{-\infty}^{z}\sinh[\lambda_q(z-z')]F_{xq}(z')\,dz', \tag{3}$$

$$E_{zq}=\left(\frac{1}{\varepsilon}E_{zq}^{(+)}+\frac{1}{\lambda_q}\int_{-\infty}^{-0}\sinh(\lambda_q z')F_{zq}(z')\,dz'-\frac{1}{\varepsilon}(\Delta\varepsilon^{(-)}E_z^{(-)})_q\right)e^{-\lambda_q z}+\frac{1}{\lambda_q}\int_{-\infty}^{z}\sinh[\lambda_q(z-z')]F_{zq}(z')\,dz',$$

where subscript q labels the Fourier components. The symbols + and − indicate that the value of the corresponding variables is taken at $z=+0$ and $z=-0$. Moreover,

$$F_{xq}(z)=\frac{iq}{\varepsilon k^2}\frac{\partial f_{zq}}{\partial z}+\left(1-\frac{q^2}{\varepsilon k^2}\right)f_{xq},$$

$$F_{zq}(z)=f_{zq}+\frac{iq}{\varepsilon k^2}\frac{\partial f_{xq}}{\partial z}+\frac{1}{\varepsilon k^2}\frac{\partial^2 f_{zq}}{\partial z^2},$$

$$\lambda_q=ik(n+i\varkappa)(1-q^2/\varepsilon k^2)^{1/2},\quad \varepsilon^{1/2}=n+i\varkappa.$$

It is well-known that, outside the medium, the electric field satisfies an integral relation between the field at an arbitrary point on the surface and its normal derivatives on the surface.[8] In our case, we are dealing with a plane surface and, after the Fourier transformation, this integral relation assumes

$$E_{x,zq}^{(+)}=2E_{x,zq}^{(0)}-2\pi G_q\frac{\partial E_{x,zq}^{(+)}}{\partial z}, \tag{4}$$

where

$$E_{x,zq}^{(0)}=E_0(\cos\theta,\sin\theta)\delta(k\sin\theta-q),$$

$$G_q=[2\pi((-ik)^2+q^2)^{1/2}]^{-1},$$

and $\delta(\ldots)$ is the delta function. The conditions

$$\text{div}\,\mathbf{E}^{(+)}=\text{div}[(\varepsilon+\Delta\varepsilon^{(-)})\mathbf{E}^{(-)}],$$

$$\mathbf{H}^{(+)}=\mathbf{H}^{(-)},\quad \mathbf{H}=\frac{i}{k}\,\text{curl}\,\mathbf{E}$$

then yield

$$\frac{\partial E_{xq}^{(+)}}{\partial z}=iq\left(1-\frac{1}{\varepsilon}\right)E_{zq}^{(+)}-\lambda_q E_{xq}^{(+)}$$

 0038-5646/87/080251-07$04.00

$$+\frac{i}{\varepsilon}q(\Delta\varepsilon^{(-)}E_z^{(-)})_q+\int_{-\infty}^{0}e^{-\lambda_q z}F_{xq}(z)\,dz,$$

$$\frac{\partial E_{zq}}{\partial z}=-\frac{\lambda_q}{\varepsilon}E_{zq}^{(+)}-\frac{\lambda_q}{\varepsilon}(\Delta\varepsilon^{(-)}E_z^{(-)})_q$$

$$+\frac{1}{\varepsilon}\left(E_x^{(-)}\frac{\partial\Delta\varepsilon^{(-)}}{\partial x}+E_z^{(-)}\frac{\partial\Delta\varepsilon^{(-)}}{\partial z}\right)_q$$

$$+\int_{-\infty}^{0}e^{-\lambda_q z}F_{zq}(z)\,dz.$$

After substituting these derivatives in (4), we look upon the resulting set of equations for the quantities $E_{x,zq}^{(+)}$ as a set of two linear algebraic equations and, by solving it, we obtain the above quantities as functions of the electric fields only for $z<0$.

Next, substituting the values of $E_{x,zq}^{(+)}$ found in this way in (3), we obtain what is essentially a set of integral equations for the Fourier components of the electric field inside the medium:

$$E_{xq}(z)=\tilde{E}_{xq}(z)+\Delta E_{xq}(z),$$
$$E_{zq}(z)=\tilde{E}_{zq}(z)+\Delta E_{zq}(z), \tag{5}$$

where

$$\tilde{E}_{xq}(z)=\tilde{E}_{xq}^{(-)}e^{-\lambda_q z},$$

$$\tilde{E}_{xq}^{(-)}=\frac{2}{1-2\pi G_q\lambda_q}\left[E_{xq}^{(0)}-\frac{2\pi iqG_q(1-1/\varepsilon)}{1-2\pi G_q\lambda_q/\varepsilon}E_{zq}^{(0)}\right],$$

$$\tilde{E}_{zq}(z)=\tilde{E}_{zq}^{(-)}e^{-\lambda_q z},\qquad \tilde{E}_{zq}^{(-)}=\frac{2}{1-2\pi G_q\lambda_q/\varepsilon}E_{zq}^{(0)}/\varepsilon,$$

$$\Delta E_{xq}(z)=\left(\Delta E_{xq}^{(-)}+\int_{-\infty}^{0}\frac{1}{\lambda_q}\sinh(\lambda_q z')F_{xq}(z')\,dz'\right)e^{-\lambda_q z}$$

$$+\frac{1}{\lambda_q}\int_{-\infty}^{z}\sinh[\lambda_q(z-z')]F_{xq}(z')\,dz',$$

$$\Delta E_{xq}^{(-)}=-\frac{2\pi G_q}{1-2\pi G_q\lambda_q}\left[\int_{-\infty}^{0}e^{-\lambda_q z}F_{xq}(z)\,dz\right.$$

$$\left.+\frac{iq}{\varepsilon}(\Delta\varepsilon^{(-)}E_z^{(-)})_q-\frac{2\pi G_q iq(1-1/\varepsilon)}{1-2\pi G_q\lambda_q/\varepsilon}D_q\right],$$

$$D_q=\int_{-\infty}^{0}e^{-\lambda_q z}F_{zq}(z)\,dz-\frac{\lambda_q}{\varepsilon}(\Delta\varepsilon^{(-)}E_z^{(-)})_q$$

$$+\frac{1}{\varepsilon}\left(E_x^{(-)}\frac{\partial\Delta\varepsilon^{(-)}}{\partial x}+E_z^{(-)}\frac{\partial\Delta\varepsilon^{(-)}}{\partial z}\right)_q,$$

$$\Delta E_{rq}(z)=\left(\Delta E_{zq}^{(-)}+\frac{1}{\lambda_q}\int_{-\infty}^{0}\sinh(\lambda_q z')F_{zq}(z')\,dz'\right)e^{-\lambda_q z}$$

$$+\frac{1}{\lambda_q}\int_{-\infty}^{z}\sinh[\lambda_q(z-z')]F_{zq}(z')\,dz',$$

$$\Delta E_{zq}^{(-)}=-\frac{2\pi G_q}{1-2\pi G_q\lambda_q/\varepsilon}\frac{D_q}{\varepsilon}-\frac{1}{\varepsilon}(\Delta\varepsilon^{(-)}E_z^{(-)})_q.$$

We note that, when $\Delta\varepsilon=0$ in (5), the quantities $\Delta E_{xq}(z)$, $\Delta E_{zq}(z)$ vanish and $\tilde{E}_{xq}(z)$, $\tilde{E}_{zq}(z)$ are the Fourier components of the electric field in the variable x for a homogeneous medium filling the half-space.

2. We shall suppose in (2) that

$$\Delta\varepsilon=\frac{\partial\Delta\varepsilon}{\partial N}(N(x,z,t)+\Delta N^{(0)}(x,z,t)). \tag{6}$$

In the case of a dielectric or a semiconductor, the quantity $N(x,z,t)$ in this expression is the concentration of electron excitations or photoelectrons in the conduction band that appear in the material as a result of the absorption of energy quanta $\hbar\omega$ of the electromagnetic field. The attendant change in the permittivity of the medium is $(\partial\Delta\varepsilon/\partial N)N(x,z,t)$.

The quantity $(\partial\Delta\varepsilon/\partial N)\Delta N^{(0)}(x,z,t)$ is the change in the permittivity of the medium produced by external factors unrelated to the electromagnetic field (1). It will be convenient to write it in a form in which $\Delta N^{(0)}(x,z,t)$ may not actually be the real excitation concentration. We note that the significant point for the ensuing analysis will be the dependence of $\Delta N^{(0)}(x,z,t)$ on x.

The simplest kinetic equation for N is[9,10]

$$\frac{\partial N}{\partial t}=-\frac{N}{\tau}+k\left|\operatorname{Im}\frac{\partial\Delta\varepsilon}{\partial N}\right|(N_0-N)I, \tag{7}$$

where $\operatorname{Im}(\partial\Delta\varepsilon/\partial N)$ is the imaginary part of the derivative $\partial\Delta\varepsilon/\partial N$, N_0 is a constant equal to the concentration of the particles of the medium at the inversion threshold,

$$I=\frac{c}{8\pi}|\mathbf{E}(x,z,t)|^2/(\hbar\omega),$$

$\mathbf{E}(x,z,t)$ is determined by (5), and τ is the intrinsic lifetime of an electron excitation in the medium. We note that, in approximate estimates, it is convenient to remember that $\operatorname{Im}(\partial\Delta\varepsilon/\partial N)\simeq-2\sigma/k$ (Ref. 10), where σ is the characteristic cross section for the absorption of a photon $\hbar\omega$ by a particle in the medium.

The set of equations given by (5) and (7) is self-consistent and its solution determines both the space-time structure of E and the concentration N of the excitations. This will, in fact, be the solution for the evolution of the surface structure produced under the influence of laser radiation on solids.

We shall assume in (2) that the quantity $\Delta\varepsilon$ given by (6) satisfies the condition

$$|\Delta\varepsilon|\ll|\varepsilon|, \tag{8}$$

so that the solution of (5) can be sought by the method of successive approximations, in which we continue our attention to the zeroth and first iterations. Moreover, we shall consider only media for which

$$|\varepsilon|\gg 1. \tag{9}$$

By virtue of (9), and for angles of incidence that are not too large ($|\varepsilon|^{\frac{1}{2}}\cos\theta\gg 1$), the field component E_x in the medium is significantly greater than E_z, so that the latter will be omitted from the ensuing discussion.

In view of the foregoing, and recalling (6), (8), and (9), we find from (5) that

$$E_x(x,z,t)\approx\frac{2E_0}{\varepsilon^{1/2}}\exp(-\lambda z+ikx\sin\theta)[1-F(x,z,t)],$$

$F(x,z,t)$

$$=\frac{|\mathrm{Im}(\partial\Delta\varepsilon/\partial N|}{\mathrm{Im}\varepsilon}\int_{-\infty}^{\infty}dq e^{iqx}K_q\int_{-\infty}^{0}(N_q+\Delta N_q^{(0)})e^{-2\lambda z}dz,$$

$$K_q=k\frac{\mathrm{Im}\,\varepsilon}{|\mathrm{Im}(\partial\Delta\varepsilon/\partial N)|}\frac{\partial\Delta\varepsilon/\partial N}{\varepsilon}\times\frac{(\sin\theta+q/k)^2}{[(-i)^2+(\sin\theta+q/k)^2]^{1/2}-2^{1/2}(a+ib)},\qquad(10)$$

where

$$\lambda=ik(n+i\varkappa),\quad a=\varkappa/[2^{1/2}(n^2+\varkappa^2)],\quad b=n/[2^{1/2}(n^2+\varkappa^2)],$$

and N_q and $\Delta N_q^{(0)}$ are the Fourier components of $N(x,z,t)$ and $\Delta N^{(0)}(x,z,t)$ in the variable x.

In the expression for the field E_x given by (10) we retain only that part of the correction $F(x,z,t)$ in $\Delta\varepsilon$ that leads to a more complicated dependence of E_x on x than is indicated by the factor $\exp(ikx\sin\theta)$. It is precisely this term that is responsible for the periodic structure; the remaining terms, which are of the same order in $\Delta\varepsilon$, provide no contribution to this structure.

We note that the presence of the quantity $(-i)^2$ in the expression for the function $F(x,z,t)$ indicates that the integration with respect to q in the complex plane of q is performed along the real axis over a contour lying below this axis. For the complex conjugate function $F^*(x,z,t)$, the contour integration lies above this axis.

We now substitute into (7) the solution N in the form of the two terms $N(x,z,t)=\tilde{N}(z,t)+\Delta N(x,z,t)$, where the first term represents the x-independent part of the solution. In view of the expression for the electromagnetic field given by (10) and the inequality (8), and using the approximation to (7) that is linear in $\Delta N(x,z,t)$, we obtain the following integrodifferential equation:

$$\frac{\partial\Delta N}{\partial t}=-\frac{1}{\tau}\left(1+\frac{Q}{\tilde{Q}}e^{2k\varkappa z}\right)\Delta N-\frac{1}{\tau}\frac{Q}{\tilde{Q}}e^{2k\varkappa z}\int_{-\infty}^{\infty}dq e^{iqx}K_q\int_{-\infty}^{0}(\Delta N_q+\Delta N_q^{(0)})e^{-2\lambda z}dz\qquad(11)$$

where

$$Q=\frac{c}{8\pi}E_0^2,\quad \tilde{Q}=\frac{\hbar\omega}{2k\varkappa(1-R)}\mathrm{Im}\,\varepsilon\Big/\left|\mathrm{Im}\frac{\partial\Delta\varepsilon}{\partial N}\right|,$$

and $R\approx1-4n/(n^2+\varkappa^2)$

is the light reflection coefficient of the half-space occupied by the medium.

For subsequent calculations, it is convenient to write the solution of (11) in the form

$$\Delta N(x,z,t)=\frac{e^{2k\varkappa z}}{4\pi^2 i}\int_0^t dt'\int_{\gamma-i\infty}^{\gamma+i\infty}dp e^{p(t-t')}R(p,z)\cdot\int_{-\infty}^{\infty}dx'\delta N^{(0)}(x',t')\int_0^{\infty}dy e^{-y}\int_{-\infty}^{\infty}dq e^{iq(x-x')}P(p,q)e^{-yP(p,q)},\quad(12)$$

where γ is a positive constant, chosen so that the integrand has no singularities to the right of the contour of integration with respect to the complex variable p. Moreover,

$$R(p,z)=-1\Big/\left[\left(1+\frac{e^{2k\varkappa z}Q}{\tilde{Q}(p\tau+1)}\right)\times F\left(1,2-\frac{b}{a}i,3-\frac{b}{a}i;-\frac{Q}{\tilde{Q}(p\tau+1)}\right)\right],$$

$$\delta N^{(0)}(x,t)=2k(2\varkappa-in)\int_{-\infty}^{0}dz e^{-2k(\varkappa-in)z}\Delta N^{(0)}(x,z,t),$$

$$P(q,p)=\frac{Q}{\bar{Q}(p,Q)}\times\frac{2(a+ib)(\sin\theta+q/k)^2}{[(-i)^2+(\sin\theta+q/k)^2]^{1/2}-2^{1/2}(a+ib)}\frac{1}{p\tau+1}$$

$$\bar{Q}(p,Q)=\bar{Q}/F\left(1,2-\frac{b}{a}i,3-\frac{b}{a}i;-\frac{Q}{\tilde{Q}(p\tau+1)}\right),$$

where

$$\bar{Q}=\frac{\tilde{Q}}{\rho},\quad \rho=\frac{a^2+b^2}{(a+ib)(2a-ib)}r,$$

$$r=\frac{1}{2\cdot2^{1/2}}\frac{\mathrm{Im}\,\varepsilon}{\varepsilon}\frac{\partial\Delta\varepsilon/\partial N}{|\mathrm{Im}(\partial\Delta\varepsilon/\partial N)|},$$

and $F[1,2-(b/a)i,3-(b/a)i;-Q/\tilde{Q}(p\tau+1)]$ is the hypergeometric function.

To determine the quantity $\Delta N(x,z,t)$ in (12), we recall that, by virtue of the inequality (9), the integrand $P(p,q)$ in the integral with respect to q contains the small parameters $a^2, b^2\ll1$. Hence, as follows from (12), the main contribution to the integral with respect to q is provided by the following ranges of values:

$$q=\pm q_1=\pm k(1-\sin\theta),\quad q=\pm q_2=\pm k(1+\sin\theta).$$

Let us divide the integral with respect to q in (12) into a sum of integrals over the indicated regions. Integrals of the form

$$\int_{-\infty-i0}^{\infty-i0}dr\frac{e^{-ik\varkappa r}}{r^{1/2}-a-ib}\exp\left(-\frac{Ay}{r^{1/2}-a-ib}\right),\qquad(13)$$

are then found to arise, where $A=2^{\frac{1}{2}}(a+ib)Q/\bar{Q}(p,Q)$. These integrals can be evaluated using the series expansion

$$\frac{Ay}{r^{1/2}-a-ib}\exp\left(-\frac{Ay}{r^{1/2}-a-ib}\right)=-\sum_{m=1}^{\infty}\frac{(-Ay)^m}{(m-1)!^2(a+ib)^{m-1}}\cdot\frac{d^{m-1}}{d\mu^{m-1}}\frac{1}{r^{1/2}-\mu(a+ib)}$$

and the expressions for the Hermite polynomials

$$H_n(x)=(-1)^n e^{x^2}\frac{d^n}{dx^n}e^{-x^2},\quad H_n(x)=\frac{2^{n/2}}{\pi^{1/2}}\int_{-\infty}^{\infty}dt e^{-t^2}(x+it)^n.$$

The final results are

$$\Delta N(x,z,t)=e^{2k\varkappa z}\int_0^t dt'\int_{\gamma-i\infty}^{\gamma+i\infty}dp e^{p(t-t')}R(p,z)\times\int_{-\infty}^{\infty}dx'\delta N^{(0)}(x',t')\Phi_p(x-x')+\text{c.c.},\qquad(14)$$

$$\Phi_p(x)=\frac{k}{(2\pi)^{1/2}}\frac{Q}{\bar{Q}(p,Q)}\frac{(a+ib)^2}{p\tau+1}$$

$$\times[e^{iq_1x}\theta(x)\varphi(p,x)+e^{-iq_2x}\theta(-x)\varphi(p,-x)],$$

$$\varphi(p,x)=L(p,Q)\exp[ikx(a+ib)^2L^2(p,Q)]$$
$$\cdot\Gamma(-{}^1/_2,-ikx(a+ib)^2L^2(p,Q))$$
$$+2(2\pi)^{1/2}\theta(a-b)\cdot L(p,Q)$$
$$\times\exp[ikx(a+ib)^2L^2(p,Q)],$$

$\theta(x)=1$ for $x>1$, $\theta(x)=0$ for $x<0$,

$$L(p,Q)=1-\frac{1}{p\tau+1}\frac{Q}{\bar{Q}(p,Q)},\quad \Gamma\left(-\frac{1}{2},y\right)=\int_y^\infty r^{-3/2}e^{-r}\,dr;$$

where $\Gamma(-\frac{1}{2},y)$ is the incomplete gamma-function. We note that the first term in the expression for $\varphi(p,x)$ is due to electromagnetic waves propagating within the medium, whereas the second is due to the appearance of surface electromagnetic waves.[7] Accordingly, the surface structure (14) will be a superposition of two structures, which we shall denote by SS1 and SS2.

Thus, (14) and (10) together constitute a solution of our problem and determine the space-time "periodic" structure of the light field and the concentration of electron excitations (or nonequilibrium photoelectrons in the conduction band of the semiconductor) along the surface of the specimen.

3. We now turn to an analysis of (14). To do this, we first evaluate the integral with respect to the complex variable p. This can hardly be done in the general case because the expression for $L(p,Q)$ includes the hypergeometric function of the variable p [cf., (12) and (14)]. We can glean some information about the form of $L(p,Q)$ as a function of p from the asymptotic expression

$$L(p,Q)\approx 1-\frac{1}{p\tau+1}\frac{Q}{\bar{Q}},\tag{15}$$

if

$$\frac{Q}{\bar{Q}}\leqslant 1,\tag{16}$$

and $L(p,Q)\approx 1-r$, if

$$\frac{1}{|p\tau+1|}\frac{Q}{\bar{Q}}\gg 1.$$

According to (14), the function $L(p,Q)$ should be determined on the right half-plane. It is clear from (15) that when (16) is satisfied, the quantity $L(p,Q)$ as a function of p is determined on the entire plane of the complex variable p. The form of $L(p,Q)$ is then so simple that the integral with respect to p in (14) can be evaluated. Since this is possible only in this case, and this situation is of interest in practice, we shall confine our attention to it.

Bearing in mind (12), (15), and (16), we find from (14) that

$$\Delta N(x,z,t)=e^{2kxz}\int_0^t\frac{dt'}{\tau}e^{-(t-t')/\tau}\int_{-\infty}^{\infty}dx'\,\delta N^{(0)}(x',t')$$
$$\Psi(t-t',x-x')+\text{ c.c.},\tag{17}$$

$$\Psi(t,x)=-\frac{i}{(2\pi)^{1/2}}\frac{Q}{\bar{Q}}k(a+ib)^2[e^{iq_1x}\theta(x)\psi(t,x)$$
$$+e^{-iq_2x}\theta(-x)\psi(t,-x)],$$

$$\psi(t,x)=\xi(t,x)+\eta(t,x),$$

$$\xi(t,x)=\int_0^\infty\frac{dz\exp[-z-(t/\tau)u(x,z,Q)]}{[z-ikx(a+ib)^2]^{3/2}}\left[\alpha'(t,x,z,Q)\right.$$
$$\left.\cdot I_0\left(\frac{t}{\tau}v(x,z,Q)\right)+\beta'(t,x,z,Q)I_1\left(\frac{t}{\tau}v(x,z,Q)\right)\right],$$

$$u(x,z,Q)=\frac{Q}{\bar{Q}}\frac{ikx(a+ib)^2}{z-ikx(a+ib)^2}$$

$$v(x,z,Q)=-\{u(x,z,Q)[u(x,z,Q)+Q/\bar{Q}]\}^{1/2},$$
$$\alpha'(x,z,Q,t)=1-[2u(x,z,Q)+Q/\bar{Q}]t/\tau,$$

$$\beta'(t,x,z,Q)=-[2u(x,z,Q)$$
$$+Q/\bar{Q}]t/\tau+2[u(x,z,Q)+v(x,z,Q)]$$
$$\cdot[u(x,z,Q)+v(x,z,Q)+Q/\bar{Q}]\frac{t}{\tau}\left[\frac{t}{\tau}+\frac{1}{2v(x,z,Q)}\right],$$

$I_0(\ldots)$, and $I_1(\ldots)$ are Bessel functions of an imaginary argument, and

$$\eta(x,t)=2\cdot 2^{1/2}\theta(a-b)\exp[ikx(a+ib)^2]$$

$$\times\int_{-\infty}^{\infty}dy\,e^{-y^2}\left(1+\frac{1}{(ikx)^{1/2}}\frac{y}{a+ib}\right)I_0(M(t,-ikx(a+ib)^2,y)).$$

$$M(t,-ikx(a+ib)^2,y)$$
$$=2\left\{2\frac{Qt}{\bar{Q}\tau}(-ikx)^{1/2}(a+ib)[(-ikx)^{1/2}(a+ib)+iy]\right\}^{1/2}.$$

Since the main contribution to the integral with respect to z in the expression for the function $\xi(t,x)$ is provided by the region $z\ll kx(a^2+b^2)$, and since $\lim u(x,z,Q)=-Q/\bar{Q}$, $z/(k|x|(a^2+b^2))\to 0$

$$\lim_{z/(k|x|(a^2+b^2))\to 0} v(x,z,Q)=0,$$

the above integration can be carried out and $\xi(t,x)$ assumes the more convenient form

$$\xi(t,x)\approx\exp\left[\frac{t}{\tau}\frac{Q}{\bar{Q}}+\frac{1}{4}\left(\frac{Q}{\bar{Q}}\frac{t}{\tau}\right)^2\frac{1}{-ikx(a+ib)^2+Qt/\bar{Q}\tau}\right]$$
$$\cdot\left[1+\frac{Q}{\bar{Q}}\frac{t}{\tau}+\frac{1}{4}\left(\frac{Q}{\bar{Q}}\frac{t}{\tau}\right)^2\frac{1}{-ikx(a+ib)^2+Qt/\bar{Q}\tau}\right]$$
$$\cdot\{(-ikx)^{1/2}(a+ib)[-ikx(a+ib)^2+Qt/\bar{Q}\tau]\}^{-1}.\tag{18}$$

Let us consider the most interesting situation, in which $|\rho|\gg 1$ and $\text{Re}\,\rho>0$. These conditions do not exclude any of the possible cases of the evolution of the space-time structure (17). The first inequality means that (17), i.e., condition (16), is valid for power densities Q for which $Q/\bar{Q}\gg 1$. We then have a threshold value $Q=Q_t$ above which there is no stationary surface structure (17). The quantity Q_t is determined from the behavior of (17) as $t\to\infty$. It follows from (17) and (18) that, when $Q>Q_t$,

$$Q_t={}^4/_5\bar{Q}/\text{Re}\,\rho,\tag{19}$$

and the quantity given by (17) is proportional to $\exp[Q/Q_t-1)t]$, i.e., it increases exponentially with time, which corresponds to absolute instability.[11] We note that the increase is associated with SS1.

Let us begin by substituting $\delta N^{(0)}(x,t)=(N^{(0)}/k)\delta(x)$ in (17) [$N^{(0)}$ is a constant and $\delta(x)$ is the delta-function] and let us investigate the Q dependence of $\Delta N(x,z,t)$ for large t, such that, in the terms in which the corresponding integrals with respect to t converge, the upper limit of integration can be considered infinite, whereas, in those in which a divergence is possible, the limit is taken to be t.

From (17), we have

$$\Delta N(x,\ z,\ t)=-i(2\pi)^{-1/2}e^{2kxz}(Q/\bar{Q})N^{(0)}(a+ib)^2 \cdot[e^{iq_1x}\theta(x)f(x,\ t)+e^{-iq_2x}\theta(-x)f(-x,\ t)]+\text{c.c.},$$

$$f(x,\ t)=f_1(x,\ t)+f_2(x),\quad f_1(x,\ t)=\int_0^t dt'\xi(x,\ t')/\tau, \tag{20}$$

where $\xi(x,t)$ is defined by (18) and

$$f_2(x)=2(2\pi)^{1/2}\theta(a-b)\exp[kx(\mu(q)+i\nu(q))] \cdot\{[(1+B/2A)q-B/2A](1+i\chi)-i\chi\}. \tag{21}$$

We note that the function $f_2(x)$ in (21) does not depend on the time t, i.e., in principle, a stationary SS2 will always be present, its appearance being due to surface electromagnetic waves. In (21),

$$\mu(q)=\alpha q^2+\beta,\quad \alpha=A(1+B/2A)^2,\quad \beta=-B^2/4A+C,$$
$$A=-2[ab(1-\chi^2)+(a^2-b^2)\chi],\quad \chi=\operatorname{Im}\rho/\operatorname{Re}\rho,$$
$$B=2\chi(a^2-b^2-2ab),\quad C=2ab\chi^2,\quad q=1-Q/Q_r,$$
$$Q_r={}^5/_4(1+B/2A)Q_t,$$

where Q_t is defined by (19) and

$$\nu(q)=D\left(1+\frac{B}{2A}\right)^2q^2+\left(E-\frac{BD}{A}\right)\left(1+\frac{B}{2A}\right)q$$
$$+\frac{1}{4}\frac{B^2}{A^2}D-\frac{1}{2}\frac{B}{A}E+G,$$
$$D=(a^2-b^2)(1-\chi^2),\quad E=2(\chi^2+2ab),\quad G=(b^2-a^2)\chi^2.$$

For low power densities $Q/Q_t\ll 1$, we have

$$\frac{t}{\tau}\gg\frac{Q}{|\bar{Q}|}k|x|(a^2+b^2)\gg\left(\frac{Q}{|\bar{Q}|}\right)^2\frac{t}{\tau},$$
$$k|x|(a^2+b^2)\frac{Q}{|\bar{Q}|}\ll 1,$$

the expressions for $f_1(x,t)$ and $f_2(x)$ show that the surface structure $\Delta N(x,z,t)$ in (20) is stationary and the function $f(x,t)=f(x)$ that determines it is given by

$$f(x)=\frac{i^{3/2}}{(kx)^{3/2}(a+ib)^3}$$
$$+2(2\pi)^{1/2}\theta(a-b)\exp[kx(\mu(q=1)+i\nu(q=1))], \tag{22}$$

where $\mu(q=1)=\alpha+\beta=-2ab$, $\nu(q=1)=a^2-b^2$.

When the power density Q is high enough, so that $|1-Q/Q_t|\ll 1$, and

$$\frac{t}{\tau}\gg\frac{Q}{|\bar{Q}|}k|x|(a^2+b^2)$$

it follows from the expressions for $f_1(x,t)$ and $f_2(x)$ that (20) is determined by the function

$$f(x,\ t)=i\gamma(2,\ (1-Q/Q_t)t)/[4(kx)^{1/2}(a+ib)(1-Q/Q_t)^2]+f_2(x).$$

where

$$\gamma(\alpha,x)=\int_0^x e^{-r}r^{\alpha-1}\,dr$$

is the incomplete gamma-function and $f_2(x)$ is defined by (21). When $1-Q/Q_t>0$ and $(1-Q/Q_t)t\gg 1$, we find from the last expression that we have a stationary surface structure, i.e., stationary SS1 and SS2, determined by the function $f((x,t)=f(x)$, given by

$$f(x)=\frac{i}{4(kx)^{1/2}(a+ib)(1-Q/Q_t)^2}+f_2(x). \tag{23}$$

When $1-Q/Q_t<0$ and $(Q/Q_t-1)t\gg 1$, we obtain from this expression a nonstationary surface wave for which

$$f(x,t)\approx\frac{i(t/\tau)\exp[(Q/Q_t-1)t/\tau]}{4(kx)^{1/2}(a+ib)(Q/Q_t-1)}+f_2(x), \tag{24}$$

i.e., SS1 is nonstationary and SS2 is stationary.

Let us compare (22) and (23), (24). First, we note that the SS1 amplitude due to the bulk electromagnetic waves varies significantly as Q increases. The reduction in this amplitude with increasing $|x|$ is described by $|x|^{-3/2}$ for $Q\ll|\bar{Q}|$ and by $|x|^{-1/2}$ for $Q\gtrsim|\bar{Q}|$.

As far as the analogous relation for SS2 due to surface electromagnetic waves is concerned, everything is determined by the function $\mu=\mu(Q)$ [cf. (21)]. Here, we have a large number of possibilties. Let us consider some of them. To be specific, let us suppose that

$$1>(-\beta/\alpha)^{1/2},\quad B>0,\quad 1+B/2A>0. \tag{25}$$

The corresponding function $\mu=\mu(Q)$ is shown in the figure, from which it is clear that for $A<0$, the SS2 amplitude decays with $|x|$ for $Q<Q_1$ and $Q>Q_2$, where $Q_{1,2}=[1\pm(-\alpha/\beta)^{1/2}]Q_r$ [cf., (21)]. On the other hand, when $Q_1<Q<Q_2$, the amplitude increases.[11] For fixed x, the amplitude at first increases as a function of the power density Q, reaching its maximum at $Q=Q_r$, and then decreases.

When $A>0$, there are two possibilities, depending on the ratio of B^2 and $4AC$. When $B^2>4AC$, we have curve 2, i.e., for small values of Q, the SS2 amplitude increases with increasing $|x|$, but this is replaced by a reduction for $Q>Q_1$ and, eventually, when $Q>Q_2$, the amplitude increases again. Here, the SS2 amplitude for fixed x but varying Q at first decreases, reaching a minimum at $Q=Q_r$, and then increases again.

For $B^2<4AC$, we have curve 3, i.e, only an increase in the SS2 amplitude is possible and there is a minimum at $Q=Q_r$ for fixed x.

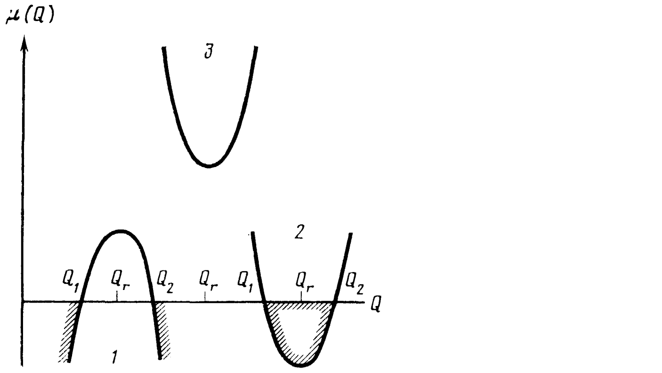

FIG. 1. Different cases of the dependence $\mu=\mu(Q)$. Curve 1 corresponds to $A<0$ and curves 2 and 3 to $A>0$ and, correspondingly, to $B^2>4AC$ and $B^2<4AC$. The shaded regions represent values of Q for which the spatial structure produced by surface electromagnetic wave decays with $|x|$.

Thus, the size of the region in which the SS2 is localized in space can be a relatively complicated, nonmonotonic function of the power density Q. Both the SS2 amplitude and its period may depend significantly on the power density Q [cf. (21), (23), and (24)].

We note that condition (25) signifies the validity of the inequality $Q_r < Q_t$ for curve 1 and $Q_r > Q_t$ for curves 2 and 3. It follows that, for $A < 0$ (curve 1) and $Q < Q_r$, the surface wave is completely stationary and is described by (23). On the other hand, when $A > 0$ holds (curves 2 and 3), the surface structure is described by (24) for $Q > Q_r$, i.e., it is nonstationary, because the SS1 amplitude then increases exponentially with time. Because of this increase, it may not be possible to observe the stationary part of the surface structure. However, for power densities Q for which the amplification of the SS2 amplitude is possible, and for values of Q for which amplification is replaced by attenuation, the conditions for the observation of the stationary part of the structure become more favorable. This is so because the region within which the stationary SS2 is localized may now be significantly greater than the corresponding size for the nonstationary part of the structure [cf., (24)].

Let us now consider another case, i.e., let us substitute $\delta N^{(0)}\tau = \delta N^{(0)}\tau k\delta(t)\delta(x)$ in (17). In the most interesting situation, in which

$$a>b, \quad \operatorname{Re}\rho>0, \quad Q<Q_t,$$
$$k|x|(a^2+b^2)\gg Qt/Q_t\tau, \quad k|x|(a^2+b^2)\gg 1,$$

the surface structure is determined exclusively by SS2, for which we have

$$\Delta N(x,z,t)\approx -\frac{i}{(2\pi)^{1/2}}N^{(0)}e^{2k\varkappa z}\frac{Q}{\tilde{Q}}(a+ib)^2$$
$$\cdot[e^{iq_1x}\theta(x)\psi(t,x)+e^{-iq_2x}\theta(-x)\psi(t,-x)]+\text{ c.c.}, \tag{26}$$

where

$$\psi(t,x)=A(t,x)\exp[i\Phi(t,x)],$$
$$A(t,x)=2^{1/4}\exp\left[-\frac{t}{\tau}-2abk|x| +2^{3/2}s(a+b)\left(\frac{Qt}{Q_t\tau}k|x|\right)^{1/2}\right],$$
$$\cdot\left\{\left(\frac{Qt}{Q_t\tau}k|x|\right)^{1/4}[(a+b)(s^2+u^2)]^{1/2}\right\}^{-1},$$
$$s=\left(\frac{2}{5}\right)^{1/2}\left[\cos\left(\frac{1}{2}\arctan\chi\right)+\frac{a-b}{a+b}\sin\left(\frac{1}{2}\arctan\chi\right)\right],$$
$$u=\left(\frac{2}{5}\right)^{1/2}\left[\sin\left(\frac{1}{2}\arctan\chi\right)-\frac{a-b}{a+b}\cos\left(\frac{1}{2}\arctan\chi\right)\right],$$
$$\Phi(t,x)=k|x|(a^2-b^2)+4u(a+b)\left(\frac{Qt}{Q_t\tau}k|x|\right)^{1/2} -\frac{1}{2}\arctan\frac{u}{s}.$$

It follows from the expressions for $A(x,t)$ and from (26) that the SS2 amplitudes with spatial periods $d_1 = 2\pi/q_1$ and $d_2 = 2\pi/q_2$ that lie, respectively, to the right and left of the origin, are equal and given by

$$\bar{A}(x,t)=\frac{4}{5}(1+\chi^2)\frac{a^2+b^2}{(2\pi)^{1/2}}\frac{Q}{Q_t}N^{(0)}e^{2k\varkappa z}A(x,t)\cdot$$

Their maxima occur at

$$x_{max}=\pm\frac{1}{2k}\frac{Qt}{Q_t\tau}\frac{(a+b)^2s^2}{a^2b^2}.$$

Hence, it follows that these points move with constant velocities given by

$$v=\pm\frac{1}{2k\tau}\frac{Q}{Q_t}\frac{(a+b)^2s^2}{a^2b^2}, \tag{27}$$

to the right and left of the point $x = 0$, respectively.

At $x = x_{\max,}$ the amplitude is

$$\bar{A}_{max}(t)=\frac{8}{5}\cdot 2^{1/4}(1+\chi^2)\frac{a^2+b^2}{a+b}\left[\frac{Q\tau}{Q_t t}\frac{ab}{2\pi s(s^2+u^2)}\right]^{1/2}e^{2\varkappa kz}$$
$$\cdot\exp\left\{\frac{t}{\tau}\left[\frac{(a+b)^2}{ab}s^2\frac{Q}{Q_t}-1\right]\right\}.$$

It is clear from this expression that the maxima of the SS2 amplitude (26) for

$$Q>Q_c=Q_t\frac{abs^{-2}}{(a+b)^2} \tag{28}$$

increase exponentially with time and move in space with velocity v given by (27). The evolution of the space-time structure (26) then corresponds to the evolution of a convective instability.[11] We note that, since $s\sim 1$, we have $Q_c \simeq Q_t bs^2/a \ll Q_t$ for $a \gg b$, i.e., the convective instability can occur for power densities much lower than the absolute instability threshold.

Let us now consider possible materials for which the above results are valid. The main requirement is that the inequalities (8) and (9) be satisfied. This is possible, first, for nonluminescing dielectrics that absorb electromagnetic radiation of frequency ω and contain emission centers that radiate under the influence of this radiation. It is clear that, when the concentration of these centers is high enough, so that $N_c \gg k^3$, holds the surface structure will take the form of luminescing bands of different intensity. Second, the inequalities (8) and (9) are satisfied for sufficiently highly doped n-type semiconductors. Depending on the light frequency ω, the surface structure is determined by the variation in the concentration of excitons or free photoelectrons along the surface of the sample. For excitons, the surface structure can be observed in the form of luminescing bands of different intensity. On the other hand, in the case of free photoelectrons, this structure can be observed by recording the scattering of light at the frequency that is not absorbed by the semiconductor.

Finally, let us estimate the order of magnitude of the principal quantities encountered above, and consider whether the conditions for which these effects occur are satisfied. Take the case where $a \gtrsim b$, $a^2 \gg b^2$. In the spectral region in which light absorption is strong, we can put $|\varepsilon| \sim 10$, which corresponds to (9). Since

$$\varepsilon=[-{}^1/_2(a^2-b^2)+abi]/(a^2+b^2)^2,$$

it follows from the foregoing that $a^2 \sim 0.1$, $b^2 \sim 0.01$.

From the definition of $\tilde{Q}$ [cf., (11)] and the fact that $|\operatorname{Im}(\partial\Delta\varepsilon/\partial N)| \sim \sigma/k$ (see above), we have

$$\tilde{Q}\approx 2^{-3/2}\frac{\hbar\omega}{(1-R)\sigma\tau}\frac{b}{a^2+b^2},$$

so that, for the typical situation for which $\hbar\omega \sim 1$ eV, $\tau \sim 10^{-8}$, $1-R \sim 1$, and $\sigma \sim 10^{-16}$ cm^2, it turns out that $\tilde{Q} \sim 10^5$ W/cm^2.

From the definition of ρ [cf., (12)] and the inequality $\mathrm{Re}\,\rho \gg 1$ adopted above, it follows that

$$\mathrm{Re}\,\rho = -[(2a^2-b^2)ab/2^{1/2}(4a^2+b^2)(a^2+b^2)] \times \mathrm{Re}(\partial\Delta\varepsilon/\partial N)/|\mathrm{Im}(\partial\Delta\varepsilon/\partial N)|$$

and that

$$\mathrm{Re}(\partial\Delta\varepsilon/\partial N) < 0 \quad \text{and} \quad |\mathrm{Re}(\partial\Delta\varepsilon/\partial N)|/|\mathrm{Im}(\partial\Delta\varepsilon/\partial N)| \gg 1.$$

The last two conditions are fully realistic, so that we may take $\mathrm{Re}\,\rho \sim 10$. From the expressions for Q_r, Q_t, and Q_c [cf., (19), (21), and (28)], we then have

$$Q_r \sim Q_t \sim Q_c \sim \tilde{Q}/\mathrm{Re}\,\rho \sim 10^4 \ \mathrm{W/cm^2}.$$

In particular, the velocity of propagation of the SS2 amplitude maximum (27) turns out to be $v \sim 1/(k\tau b^2) \sim 10^5$ cm/s if we suppose that $k \sim 10^5$ cm^{-1} for $Q \sim Q_c$.

We note in conclusion that, for the parameter values chosen above, all the other inequalities that were assumed to be valid are satisfied. In particular, the increase in temperature due to heating by the laser radiation is small, i.e., about 10 K.

[1]D. C. Emmony, R. P. Howson, and L. J. Willis, J. Appl. Phys. Lett. **23**, 598 (1973).

[2]M. K. Kochengina, M. N. Libenson, V. S. Makin, and S. D. Putkov, Abstracts of Papers read to the Fifth All-Union Conf. on the Interaction between Optical Radiation and Matter [in Russian], Nauka, L., 1981, p. 381.

[3]S. I. Anisimov, S. M. Gol'berg, B. A. Malomed, and M. H. Tribel'skii, Dokl. Akad. Nauk SSSR **262**, 1117 (1982) [Sov. Phys. Dokl. **27**, 130 (1982)].

[4]V. I. Emel'yanov and V. N. Seminogov, Zh. Eksp. Teor. Fiz. **86**, 1026 (1984) [Sov. Phys. JETP **59**, 598 (1984)].

[5]A. M. Prokhorov, A. V. Svakhin, V. A. Sychugov, *et al.*, Kvantovaya Elektron. (Moscow) **10**, 906 (1983) [Sov. J. Quantum Electron. **10**, 568 (1983)].

[6]E. Keilmann and Y. H. Bai, J. Appl. Phys. A **29**, 9 (1982).

[7]Yu. V. Vigant, A. A. Kovalev, O. L. Kulikov, *et al.*, Zh. Eksp. Teor. Fiz. **91**, 213 (1986) [Sov. Phys. JETP **64**, 308 (1986)].

[8]E. L. Feinberg, Studies on the Propagation of Radio Waves [in Russian], USSR Academy of Sciences, M.-L., 1948, p. 97.

[9]A. P. Bogatov and P. G. Eliseev, Kvantovaya Elektron. (Moscow) **12**, 465 (1985) [Sov. J. Quantum Electron. **15**, 308 (1985)].

[10]R. H. Pantell and H. E. Puthoff, *Fundamentals of Quantum Electronics*, Wiley, 1969 [Russ. transl., Mir, M., 1972, pp. 75, 62].

[11]E. M. Lifshitz and L. P. Pitaevskiĭ, *Physical Kinetics*, Pergamon Press, Oxford (1981) [Russ. original, Nauka, Moscow, 1979, pp. 324, 330.

Translated by S. Chomet