Formation of surface structure by thermal action of coherent radiation

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We investigate the interference part of the radiation flux (of the Poynting vector) regarded as a thermal source responsible for the formation of periodic surface structures. Various instability mechanisms that lead to growth of the surface structures (similar to stimulated scattering) are considered from a unified point of view by introducing as phenomenological parameters the phase difference between the thermal source and the surface perturbations, and the exponent of the power-law dependence of the growth rate on the spatial period of the periodic structure. The maxima of the growth rate and the corresponding flux extrema determine the parameters of the possible surface structures, viz., the period and the orientation. At least one of the combination spectra glances along the interface. Simple analytic results are obtained in the limit $|\varepsilon| \gg 1$ for elliptically polarized light (at normal incidence), for non-polarized radiation, and for radiation with arbitrary linear polarization (at all incidence angles). The expression for the flux is factorable in these cases and contains a real factor that describes the orientation of the scattering plane. The position of this plane in the nondegenerate situation is determined by the tangential component of the incident-wave magnetic field. The general symmetry properties of the thermal source are also investigated.

Laser radiation, in a definite range of intensities, produces on the surfaces of metals, semiconductors, and dielectrics periodic structures (lattices) (see Ref. 1 and e.g., Refs. 2–7). The lattice period depends on the incidence angle θ and is of the order of the wavelength λ of the applied light. The orientation of the periodic structure is determined by the polarization direction and, in some cases, by the incidence angle. The prevailing opinion is that the periodic structures are due to scattering of the electromagnetic wave by the bare surface roughness of the material and by the subsequent interference of the incident and scattered radiation, and by the reaction of the interference part of the intensity on the slow motions of the medium (boundary), which cause the surface to be unstable. Feedback can be ensured by various mechanisms: evaporation (sublimation), local melting and recrystallization, thermocapillarity and evaporative vapor pressure in the presence of a melt layer, and thermochemical effects.

According to the scheme proposed, the formation of a periodic structure is analogous to stimulated scattering processes that are nonlinear in the field, and is particularly close to the previously considered stimulated scattering by surface waves (SSSW)[8–12] for nondissipative media (transparent[8–11] or ideally conducting[12]), where the buildup mechanism was due to the pressure of the light.[1)] The onset of a periodic structure is a manifestation of SSSW due to thermal mechanisms. It was shown in Ref. 10 that the minimum of the SSSW threshold, which corresponds to the maximum growth rate, is reached in a scattering geometry in which one or several scattered waves glide along the interface.[2)] A similar situation obtains also in the formation of periodic structures on the surfaces of dissipative media.

The experimentally observable similarity of the periods and orientations of the lattices for various substances and mechanisms (see Refs. 1–7 and below) suggests that the periodic-structure parameters are not governed by the details of the actual interaction mechanism, but are determined mainly by the scattering geometry, i.e., they can be determined by considering the electrodynamic part of the problem. It suffices here to consider fields of first order[3)] scattered by a periodic surface structure $\zeta \exp[i(\mathbf{qr} - \Omega t)]$. The reflected ($R$) and refracted ($T$) fields at the combination frequencies $\omega + j\Omega$, $\Omega \ll \omega$, $\mathbf{k}_{jt} = \mathbf{k}_t + j\mathbf{q}$ ($j = \pm$, $\mathbf{k}_t$ is the tangential wave vector of the incident electromagnetic wave $\mathbf{E}_0 \exp[i(\mathbf{kr} - \omega t)]$) are bilinear in the incident field $\mathbf{E}_0$ and in the surface roughness ζ:

$$\mathbf{E}^R = \hat{R}\mathbf{E}_0\zeta,$$

where $\hat{R}$ are generalized Fresnel coefficients.[10] This results in beats of the Poynting vector $[\mathbf{E}_0 + \mathbf{E}^R, \mathbf{H}_0 + \mathbf{H}^R] \sim Qk\zeta$ having the same frequency and wavelength as the surface roughness ζ that leads to the wave scattering. Here $Q = Q(\mathbf{q},\mathbf{k})$ is a dimensionless and suitably normalized interference flux. Energy absorption leads to modulation of the temperature-sensitive parameters of the medium, and as a consequence to a counteraction on the motion of its surface. The appearance of a resonant driving term $\sim Q\zeta$ in the equation of motion for ζ leads to an instability with a growth rate $\Gamma \sim \mathrm{Re}[q^\mu Q e^{i\beta}]$ proportional to the heat-source power,[4)] but generally out of phase with the latter. The phase shift β and the exponent μ depend on the specific mechanism; they are in general slow functions of $\mathbf{q}$. The most effectively excited from among the possible periodic structures are those corresponding to maxima of the growth rate, i.e., in the general case, to extrema of the flux $Q(\mathbf{q},\mathbf{k})$. The behavior of Q is determined in turn by the generalized Fresnel coefficients

$$\hat{R} \sim [\varepsilon k_{jz}{}^R - k_{jz}{}^T]^{-1}, \ [k_{jz}{}^R - k_{jz}{}^T]^{-1},$$

where $k_{jz}^R = -(k^2 - k_{jt}{}^2)^{1/2}$, $k_{jz}^T = (\varepsilon k^2 - k_{jt})^{1/2}$ are the normal components of the wave vectors of the scattered waves, and $\varepsilon = \varepsilon' + i\varepsilon''$ is the effective dielectric constant that depends on the average temperature and on the aggregate state of the substance during the "epoch" of the periodic-structure formation. It follows from Ref. 10 that the determination of the extrema of Q is facilitated by the fact that it is made up of paired differences of generalized Fresnel coefficients corresponding to the Stokes and anti-Stokes[5)] scattering channels. These differences are as a rule small in a wide range of $\mathbf{q}$; the cancellation of the terms, however, breaks down at the non-analyticity points $(\partial Q/\partial q \to \infty)$ at which the z-components of the wave vectors vanish $(k_{jz}^{RT}=0)$. The maxima of the growth rate correspond therefore to excitation of glancing or nearly glancing scattered waves. It must be noted that for metals with $\varepsilon' < -1$, and in general at $|\varepsilon| \gg 1$, an important role is assumed by the resonant character of the Fresnel coefficients (Wood's resonance), which have a sharp extremum at $k_{jz}^{R} \approx 0$, likewise a factor favoring scattering along the interface. The singularity of Q at a point corresponding to the condition $k_{jz}^{T} \approx 0$ at $|\varepsilon| \gg 1$ turns out to be weaker. For transparent media with moderate ε, however, it can turn out to be substantial: it corresponds to lattices with period $\Lambda = \lambda/n$, where n is the refractive index; such lattices are observed, for example, on NaCl surfaces.[14]

Since the function Q depends on the two-dimensional vector $\mathbf{q}$, the condition $|\mathbf{k}_t + \mathbf{q}| = k$ or $|\mathbf{k}_t - \mathbf{q}| = k$ (which follows from $k_{jz}^{R} = 0$) does not by itself determine completely the wave vector $\mathbf{q}$. It turns out, however, that in the limiting case $|\varepsilon| \gg 1$ the function $Q(\mathbf{q},\mathbf{k})$ can be represented by a product of two functions, a slow one F that depends only on the lattice orientation, and a fast one f that depends on k_{jz}^{R} and has a sharp extremum at $k_{jz}^{R} = 0$. As a result, the orientation of the lattices can in many cases be determined independently, by testing the flow function F for an extremum. Allowance for the factor q^{μ} of Q in the growth rate Γ enhances certain extrema of the function W and weakens others. In particular, the extrema will be enhanced in the region of large q if $\mu > 0$ and in the region of small q if $\mu < 0$.

Thus, information on the possible types of periodic structures is contained mainly in the expression for the interference flux. We therefore investigate below in detail the function $Q(\mathbf{q},\mathbf{k})$. Owing to the substantial simplification achieved at $|\varepsilon| \gg 1$ it was possible to consider analytically the experimentally realized general cases of elliptic polarization (for normal incidence) and of arbitrarily linear polarization (at any incidence angle), and also the case of unpolarized radiation. We shall show that the different instability mechanisms discussed in the literature can be analyzed in a unified manner by taking phenomenologically into account the slow dependence of the growth rate on the lattice period and the phase shift of the thermal source.

The plan of the article is the following. In Sec. 1 we introduce the notation and cite equations for the scattered fields. In Sec. 2 we derive a general expression for the interference energy flux, consider the limiting case $|\varepsilon| \gg 1$, and investigate the fast function f. In Sec. 3 we investigate the slow function F for different cases of polarization and conditions of incidence of the acting radiation, and obtain the periods and orientations of the possible types of periodic structures. The influence of specific mechanisms of periodic-structure formation on the instability growth rate and on the lattice parameters are considered briefly in Sec. 4.

1. FIELD TRANSFORMATION ON A CORRUGATED SURFACE

To calculate the interference radiation flux through an interface, we must know the scattered (reflected and transmitted) fields. The amplitudes of the scattered spectra were calculated back in the classical papers of Mandel'shtam, Andronov, and Leontovich on spontaneous scattering of light by oscillations of a boundary, and later in many papers by others. We introduce here the notation and cite the corresponding results, following Ref. 10.

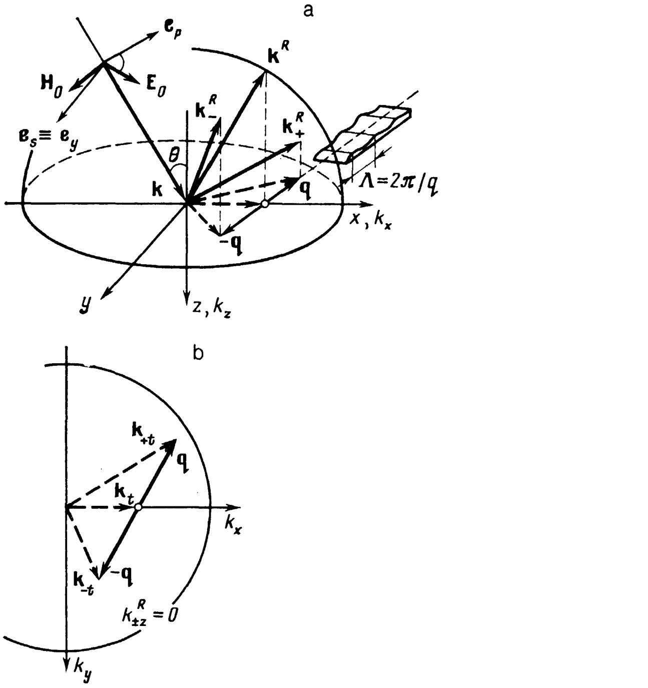

FIG. 1. Geometry of scattering of an electromagnetic wave by a corrugated surface: a) general view, b) view in the interface plane.

Let the surface-roughness height be given by the equation $z = \zeta(\mathbf{r},t)$; by virtue of the linearity in ζ, we can confine ourselves to one Fourier component

$$\zeta(\mathbf{r},\, t) = \mathrm{Re}[\zeta_{\mathbf{q},\,\Omega} \exp(i\mathbf{q}\mathbf{r} - i\Omega t)]. \tag{1.1}$$

We put $\zeta_j = \zeta_{j\mathbf{q},\, j\Omega}$, $j = \pm$; since ζ is real, we have $\zeta_- = \zeta_+^*$. A plane electromagnetic wave is incident on the surface (1.1) (Fig. 1)

$$\left\{ \begin{matrix} \mathbf{E}_0 \\ \mathbf{H}_0 \end{matrix} (\mathbf{r}, t) \right\} = \mathrm{Re} \left\{ \begin{matrix} \mathbf{E}_0 \\ \mathbf{H}_0 \end{matrix} e^{-i\varphi} \right\}, \quad \varphi = \omega t - \mathbf{k}\mathbf{r}. \tag{1.2}$$

We resolve the field amplitudes into components perpendicular (s) and parallel (p) to the incidence plane:

$$\begin{gathered} \mathbf{E}_0 = E_s \mathbf{e}_s + H_p \mathbf{e}_p, \qquad \mathbf{H}_0 = H_p \mathbf{e}_s - E_s \mathbf{e}_p, \\ E_s = E_0 \sin\psi, \qquad H_p = E_0 \cos\psi e^{-i\delta}, \end{gathered} \tag{1.3}$$

where $\mathbf{e}_s \equiv \mathbf{e}_y$, $\mathbf{e}_p = [\mathbf{e}_y \times \mathbf{k}]/k$, $\psi = \mathbf{E}_0 \hat{\mathbf{e}}_p$ is the polarization angle, $E_0 = |\mathbf{E}_0|$, and δ is the phase shift between the s- and p-components of the fields. The end of the vector $\mathbf{E}_0$ traces in the $(\mathbf{e}_s, \mathbf{e}_p)$ plane a polarization ellipse whose principal-axes orientation is given by the equation

$$\mathrm{tg}\, 2\gamma = \cos\delta\, \mathrm{tg}\, 2\psi, \tag{1.4}$$

where γ is the angle between the major axis of the ellipse and the vector $\mathbf{e}_p$.

Scattering of the incident field (1.2) by the surface (1.1) produces, in first order in $k\zeta, q\zeta \ll 1$, fields $\mathbf{E}_j^{R,T}(\mathbf{r},t)$

having combination frequencies $\omega_j = \omega + j\,\mathrm{Re}\Omega(q)$ and wave vectors $\mathbf{k}_{jt} = \mathbf{k}_t + j\mathbf{q}$. The amplitudes of the scattered waves are bilinear in the amplitudes of the surface wave and of the incident field:

$$\mathbf{E}_j^{R,T} = -\frac{i}{2}(\varepsilon-1)\zeta_j[\mathbf{C}_j^{R,T}E_s{}^T+\mathbf{B}_j^{R,T}H_p{}^T], \quad j=\pm, \quad (1.5)$$

where $\varepsilon \equiv \varepsilon(\omega) = \varepsilon' + i\varepsilon''$ is the relative dielectric constant of the medium, $E_s^T = T_s E_s$, $H_p^T = T_p H_p$; T_s and T_p are the Fresnel coefficients:

$$T_s = \frac{2k_z}{k_z+k_z{}^T}, \quad T_p = \frac{2\varepsilon k_z}{\varepsilon k_z+k_z{}^T},$$

$k_z^T = (\varepsilon k^2 - k_t^2)^{1/2}$. The conversion coefficients $\mathbf{C}^{R,T}$ and $\mathbf{B}^{R,T}$ are given in Ref. 10 in a form convenient for us and will not be written out here explicitly. We note only that the notation for the scattered waves is different here from that in Ref. 10, viz., the subscripts 2 and 3 in Ref. 10 correspond here to $j = -$ and $+$, respectively.

2. INTERFERENCE FLUX

We consider the component, normal to the surface, of the radiation flux $\vec{\mathbf{S}} = c[\vec{\mathscr{E}} \times \vec{\mathscr{H}}]/4\pi$ on the boundary $z = \zeta$, where $\mathscr{E}$ and $\mathscr{H}$ are the resultant (total) fields. Accurate to terms of first order in ζ, we have

$$\mathbf{S}\mathbf{n} = \mathbf{S}_0\mathbf{n}_0 + \mathbf{S}_0\mathbf{n}' + \mathbf{S}'\mathbf{n}_0, \quad (2.1)$$

where $\mathbf{n} = \mathbf{n}_0 + \mathbf{n}' \approx \mathbf{e}_z - \nabla \times \zeta$ is the inward normal to the surface (1.1) of the medium, $16\pi\mathbf{S}_0/c = [\mathbf{E} \times \mathbf{H}^*] + [\mathbf{E}^* \times \mathbf{H}]$ is the unperturbed flux, $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_0^R$, $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_0^R$; $\mathbf{E}_0^R$, $\mathbf{H}_0^R$ are the reflected waves at the fundamental frequency,

$$\frac{16\pi}{c}\mathbf{S}' = [\mathbf{E}\mathbf{H}_-{}^{R*}] + [\mathbf{E}^*\mathbf{H}_+{}^R] + [\mathbf{E}_-{}^{R*}\mathbf{H}] + [\mathbf{E}_+{}^R\mathbf{H}^*] + \text{c.c.}$$

is the increment to $\mathbf{S}_0$ due to the excitation of the combination spectra. With account taken of (1.5), (2.1), and the equations for the conversion of the main radiation into combination spectra,[10] the flux increment proportional to $\exp(i\mathbf{q}\cdot\mathbf{r}) - i\Omega t)$ and responsible for the onset of feedback can be represent in the form ($S_n' = \mathbf{S}_0\mathbf{n}' + \mathbf{S}'\mathbf{n}_0$)

$$S_n' = \frac{cE_0{}^2}{16\pi}\frac{k\zeta_+}{2}Q\exp(i\mathbf{q}\mathbf{r} - i\Omega t) + \text{c.c.}, \quad (2.2)$$

where

$$Q \equiv Q(\mathbf{q}, \mathbf{k}, \psi, \delta) = \sin^2\psi Q_s + \cos^2\psi Q_p + \frac{\sin 2\psi}{2}Q_{sp} \quad (2.3)$$

is the dimensionless interference flux. Explicit expressions for the partial fluxes Q_s, Q_p, and Q_{sp} (corresponding respectively to the pure contributions of the s and p components and to their joint influence; the particular case Q_p [see Ref. 15]) is obtained after straightforward but rather cumbersome transformations; we present here only the final result:

$$Q_s = \frac{i|T_s|^2}{k^2}\Big\{k_xk_{+x} - k_{+z}{}^Tk_{+z}{}^R - k_z{}^{T*}(k_{+z}{}^T + k_{+z}{}^R) + \frac{k_{+y}{}^2}{\varepsilon} + \frac{(\varepsilon-1)k_{+y}{}^2[k_z{}^{T*} + (1+1/\varepsilon)k_{+z}{}^T]}{k_{+z}{}^T - \varepsilon k_{+z}{}^R} - \begin{pmatrix}\text{c.c.}\\ + \to -\end{pmatrix}\Big\}, \quad (2.4)$$

$$Q_p = \frac{i|T_p|^2}{k^2}\Big\{-\frac{k_xk_{+x}}{\varepsilon} - \frac{k_{+y}{}^2|k_z{}^T|^2}{k^2|\varepsilon|^2\varepsilon} - \frac{k_z{}^T}{\varepsilon}(k_{+z}{}^T + k_{+z}{}^R) + \frac{(\varepsilon-1)}{(k_{+z}{}^T - \varepsilon k_{+z}{}^R)}\Big[\frac{|k_z{}^T|^2k_{+z}{}^T}{|\varepsilon|^2}\Big(1 - (1+1/\varepsilon)\frac{k_{+y}{}^2}{k^2}\Big) - k_xk_{+x}\Big(\frac{k_z{}^{T*}}{\varepsilon^*} + \frac{k_{+z}{}^T}{\varepsilon}\Big) + \frac{k_z{}^Tk_{+x}{}^2}{\varepsilon}\Big] - \begin{pmatrix}\text{c.c.}\\ + \to -\end{pmatrix}\Big\}, \quad (2.5)$$

$$Q_{sp} = \frac{i}{k^2}\Big\{T_sT_p{}^*e^{i\delta}k_{+y}\Big\{-\frac{k_x(k_z{}^T - k_z{}^{T*}) + (1-1/\varepsilon)k_{+x}k_z{}^{T*}}{\varepsilon^*k} + \frac{(\varepsilon-1)k_{+x}}{k_{+z}{}^T - \varepsilon k_{+z}{}^R}\Big[\Big(1 + \frac{1}{\varepsilon}\Big)\frac{k_z{}^{T*}k_{+z}{}^T}{\varepsilon^*k} + k\Big]\Big\} - T_s{}^*T_pe^{-i\delta}k_{+y} \times\Big\{\Big(1 - \frac{1}{\varepsilon}\Big)\frac{k_{+x}k_z{}^T}{\varepsilon k} + \frac{(\varepsilon-1)}{\varepsilon k(k_{+z}{}^T - \varepsilon k_{+z}{}^R)}[k_x(\varepsilon k^2 + k_z{}^{T*}k_{+z}{}^T) - k_{+x}k_z{}^T(k_z{}^{T*} + (1+1/\varepsilon)k_{+z}{}^T)]\Big\} - \begin{pmatrix}\text{c.c.}\\ + \to -\end{pmatrix}\Big\}. \quad (2.6)$$

Equations (2.2)–(2.6) hold for light incident from vacuum; for incidence from a transparent dielectric it is necessary to multiply (2.2) by the dielectric constant ε_1 of the medium from which the light is incident, make the substitution $T_p \to \varepsilon_1{}^{1/2}T_p$, and take ε to mean the relative dielectric constant $\varepsilon_2/\varepsilon_1$, in which case $k^2 = \varepsilon_1\omega^2/c^2$.

The nondimensional flux Q as a function of its arguments has symmetry properties which are listed in Table I. We note, in particular, that the properties of the observed periodic structures formed by left- and right-circularly polarized light[16] agree fully with the symmetry of Q with respect to reflections σ_x or σ_y with simultaneous reversal of the direction of rotation of $\mathbf{E}_0$.

In view of the difference structure of the flux, the contributions of the Stokes and anti-Stokes components cancel

TABLE I. Symmetry properties of interference flux.

№	Symmetry properties*	Operation	Note
1	$Q(-q_y, \delta\pm\pi) = Q(q_y, \delta)$	σ_x	Reflection in xz plane with change of E_0 rotation direction
2	$Q(-q_x, -k_x, \delta\pm\pi) = Q(q_x, k_x, \delta)$	σ_y	Reflection in yz plane with change of E_0 rotation direction
3	$Q(-q_x, -q_y, -k_x) = Q(q_x, q_y, k_x)$	$C_2 = \sigma_x\sigma_y$	Rotation through π in the xy plane
4	$Q^*(-q_x, -q_y) = Q(q_x, q_y)$	R	Real character of flux
5	$Q^*(-q_x, \delta\pm\pi) = Q(q_x, \delta)$	$R\sigma_x$	Superposition of 1 and 4
6	$Q^*(-q_y, -k_x, \delta\pm\pi) = Q(q_y, k_x, \delta)$	$R\sigma_y$	Superposition of 2 and 4
7	$Q^*(-k_x) = Q(k_x)$	RC_2	Superposition of 3 and 4

*Only the arguments subject to transformations are given for Q.

each other in the general case. If $q \ll k_z$, in particular, by virtue of the cancellation of the terms in (2.4)–(2.6) the flux is small: $Q \sim \mathbf{q} \cdot \mathbf{k}_t$. For large wave numbers $q \gg k_z$, k_z^T the flux is a sufficiently smooth function ($Q \sim q$, and the surface temperature depends little on the wave number). However, in the vicinity of the lines $|\mathbf{k}_t + j\mathbf{q}| = k$, $nk (n = \mathrm{Re}\sqrt{\varepsilon})$ on the plane $\mathbf{q}$, which correspond to conversion of any one of the scattered waves (reflected or refracted) into one glancing along the interface, the flux increases and becomes altogether nonmonotonic. This singular behavior of Q in transparent media is due to the different complex character of the contributions, and hence to the absence of cancellation of the Stokes and anti-Stokes components. In metals, furthermore, come into play the proximity of the wave scattered in the glancing direction to the natural surface electromagnetic wave (SEW) of the medium (Wood's resonance).

Investigation of the function $Q(\mathbf{q},\mathbf{k})$ for arbitrary ε calls generally speaking for numerical computations. We consider hereafter the limiting case $|\varepsilon| \gg 1$, which is of interest for most applications and is at the same time still simple enough, and includes both semiconductors and metals. In this case the expression for the flux is factorable (accurate to small terms ε^{-1}, $k^R_{\pm z}/k\sqrt{\varepsilon}$):

$$Q = 4i(F_+ f_+ - F_-^* f_-^*), \quad |\varepsilon| \gg 1, \tag{2.7}$$

where the factor

$$F_j \equiv F(\mathbf{k}_{jt}) = [\mathbf{k}_{jt} - \mathbf{k}_t, \mathbf{H}_0]_z [\mathbf{k}_{jt}\mathbf{H}_0]_z^* / k^2 E_0^2 \tag{2.8}$$

describes the slow angular dependence that determines the orientation of the periodic structure, and the factor

$$f_j \equiv f(\lambda_j) = \frac{2n}{|\varepsilon| (\lambda_j + 1/\sqrt{\varepsilon})} \tag{2.9}$$

describes the fast ("resonant") function that determines the period of the structure; $\sqrt{\varepsilon} = n + im$, $-\lambda_j \equiv k^R_{jz}/k = (k^2 - |\mathbf{k}_t + j\mathbf{q}|^2)^{1/2}/k$, Re, $\mathrm{Im}\lambda_j \geqslant 0$, and since $\lambda_j^2 = (\lambda' + i\lambda'')^2$ is real, we have either $\lambda' \geqslant 0$ and $\lambda'' = 0$ or else $\lambda' = 0$ and $\lambda'' \geqslant 0$.

The function f_j stems from the principal (resonant) part of the Fresnel coefficients at combination frequencies, and is therefore itself resonant: when $|\lambda_j| \ll 1/\sqrt{|\varepsilon|}$, the function f_j increases by $\sqrt{|\varepsilon|} \gg 1$ times compared with the values outside this region. Note that although exact Wood's resonance takes place only for metals at $\varepsilon' < -1$, $\varepsilon'' \ll \varepsilon'$, the "memory" of the pole corresponding to it should remain in the general case of arbitrary ε for $|\varepsilon| \gg 1$, as is indeed manifest in the behavior of f_j. Figure 2 shows plots of the modulus $|f|$ and of the phase $\arg f$. It can be seen that the maximum of $|f|$ is reached at $\lambda'' = m/|\varepsilon|$ ($\ll 1$ at $|\varepsilon| \gg 1$), and the width of the maximum is of the order of $n/|\varepsilon| \ll 1$. Actually, in different feedback mechanism contributions to the growth rate can be made by Re f or Im f or by a linear combination of the two (depending on the phase difference between the "driving" force $\sim Q$ and the surface roughnesses ζ). Plots of Re f and Im f are given in Ref. 1. Accurate to $\sim 1/\sqrt{|\varepsilon|}$, the location of the extrema of Re f and Im f accord with the glancing-scattering condition; the difference in the positions of the extrema of Im f influence little the period of the periodic structure, although it can be recorded[17] and, in principle, used to identify the probable mechanism whereby the periodic structure is formed.

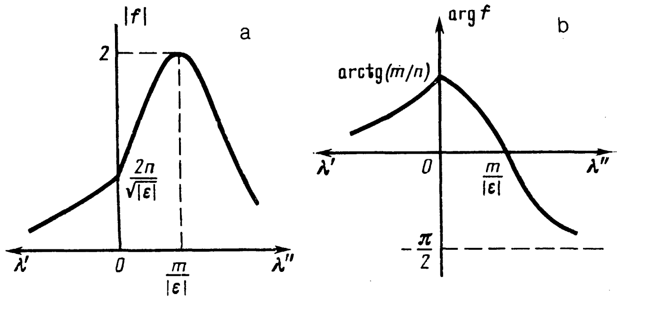

FIG. 2. Modulus $|f|$ and phase $\arg f$ of the resonant function f as functions of $\lambda = -k^{(R)}_{jz}/k$.

Within the framework of perturbation theory, a growth rate that is small compared with the frequency is proportional to the flux. The maxima of the growth rate correspond to the fastest growing periodic structure, which can become predominant as a result. The resonance condition in (2.9) is close to the glancing-propagation condition $k^R_{jz} \to 0$ or

$$|\mathbf{k}_t + j\mathbf{q}| = k \tag{2.10}$$

and singles out a one-parameter set of wave vectors $\mathbf{q}$ corresponding to the possible periodic structure. The condition (2.10) does not determine completely the lattice periods, since the orientation of the wave vector $\mathbf{q}$ remains unknown. This orientation can be determined by testing the slow function (2.8) for an extremum. It is seen from the form of (2.8) that the extrema of F_j are reached either at $\mathbf{q} \perp \mathbf{H}_{0t}$ or at $\mathbf{k}_{jt} \perp \mathbf{H}_{0t}$, where $\mathbf{H}_{0t}$ is the tangential component of the magnetic field in the $z = 0$ plane. We note, however, that the vector products contained as factors in (2.8) are not on a par, since the length of the vector $\mathbf{k}_{jt}$ is fixed [$= k$, see (2.10)], and the modulus of $\mathbf{q}$ changes when $\mathbf{k}_{jt}$ is rotated. It follows also from (2.8) that it is more natural to define the orientation of the lattices (of the vector $\mathbf{q}$ relative to the magnetic rather than the electric field; this is of importance of arbitrary linear polarization (see below).

Note that owing to the resonant character of the functions f_j, the main contribution to Q is made in general only by one term in (2.7); in degenerate cases, however, when the resonance condition is met for both the Stokes ($j = -$) and anti-Stokes ($j = +$) components, both terms must be taken into account.[1,3] Degeneracy sets in, first at normal incidence ($\mathbf{k}_t = 0$) and, second, for a geometry in which the Stokes and anti-Stokes waves are specularly scattered symmetrically about the incidence plane, i.e., at $\mathbf{q} \perp \mathbf{k}_t$; both cases are covered by the condition $\mathbf{q} \cdot \mathbf{k}_t = 0$.

3. INVESTIGATION OF SLOW DEPENDENCE

We analyze below those cases in which the function F is real; this enables us to locate the extrema of Q under condition (2.10) independently of the complex character of the fast function f. The restriction $|\varepsilon| \gg 1$ leads in this case to simple analytic equations.

1. Normal incidence of elliptically polarized beam. This case is degenerate, since the resonance condition (2.10) is simultaneously met for the Stokes and anti-Stokes waves. Recognizing that $\mathbf{k}_t = 0$, $\mathbf{k}_{jt} = j\mathbf{q}$, we get from (2.7) and (2.8)

TABLE II. Values of F_j (or Q) at extremal points.

Extremum points	Extremal values	Note
	Non-polarized radiation	
$k_{jx}=-k$	$\langle F\rangle=(1+\sin\theta)/2$	Maximum
$k_{jx}=k$	$\langle F\rangle=(1-\sin\theta)/2$	Maximum
$k_{jx}=k/2\sin\theta$	$\langle F\rangle=(4\cos^2\theta-1)/8$	Minimum at $\theta\geqslant 30°$
$k_{jx}=k_x$	$\langle Q\rangle=-4\cos^4\theta\ \mathrm{Im}\, f$	Degenerate case $\mathbf{q}\perp\mathbf{k}_t$
	Arbitrary linear polarization	
$\chi_j=3\pi/2$	$F_j=A(1+a)$	Maximum
$\chi_j=\pi/2$	$F_j=A(1-a)$	Maximum
$\sin\chi_j=a/2$	$F_j=-Aa^2/4$	Minimum
$\sin\chi_j=\mp\cos(\theta\mp\psi_1)$	$Q=-8\sin^2\psi\cos^4\theta\ \mathrm{Im}\, f$ $+i4\sin 2\psi\cos^2\theta\sin\theta\ \mathrm{Re}\, f$	Degenerate case $\mathbf{q}\perp\mathbf{k}_t$

$$Q=-8|[\mathbf{q}\mathbf{H}_0]|^2\ \mathrm{Im}\, f/k^2E_0^2. \quad (3.1)$$

It is obvious from symmetry considerations that the singled-out orientations of the periodic structures should accord with the principal axes of the polarization ellipse. It follows directly from (3.1) that the maximum of Q corresponds to the direction (1.4) of the major axis. This conclusion agrees with experimental data.[6] In the limiting case of linear polarization ($\delta=0$) this leads to the well known fact[1] that the lattice wave vector is directed along $\mathbf{E}_0$: $\mathbf{q}\|\mathbf{E}_0$; the lattice period is $\Lambda=\lambda+O(1/\sqrt{|\varepsilon|})$, where λ is the wavelength of the light. Note that the lattice orientation does not depend at all on the mechanism, in contrast to the period Λ, which changes by the relative value $\lesssim 1/\sqrt{|\varepsilon|}$, depending on which extremum of $\mathrm{Im}\, f$, the positive or negative, the positive feedback is realized.

2. Unpolarized light. At $\mathbf{k}_t=0$, after averaging (3.1) over all directions of $\mathbf{H}_0$, we obtain an obvious result—the increment does not depend on the orientation of $\mathbf{q}$ (Ref. 3). At $\mathbf{k}_t\neq 0$, we transform (2.8) into

$$k^2E_0^2F_j=(\mathbf{k}_{jt}-\mathbf{k}_t)\mathbf{k}_{jt}|\mathbf{H}_{0t}|^2-(\mathbf{k}_{jt}-\mathbf{k}_t,\ \mathbf{H}_{0t}^*)(\mathbf{k}_{jt}\mathbf{H}_{0t}). \quad (3.2)$$

Averaging over the orientations of $\mathbf{H}_{0t}$ and recognizing that $\langle H_{0\alpha}H^*_{0\beta}\rangle=\langle|H_{0\alpha}|^2\rangle\delta_{\alpha\beta}$, we get

$$k^2E_0^2\langle F_j\rangle$$
$$=k^2\langle|H_{0x}|^2\rangle+k_{jx}^2(\langle|H_{0y}|^2\rangle-\langle|H_{0x}|^2\rangle)-k_xk_{jx}\langle|H_{0y}|^2\rangle. \quad (3.3)$$

It follows from (3.3) that the principal maximum $\langle F_j\rangle$ corresponds to backscattering $k_{jx}=-k$ (see Table II, where all the singular points of $\langle F_j\rangle$ are listed). Comparing the maximum value of $\langle F_j\rangle$ with $\langle Q\rangle/(-4\ \mathrm{Im} f)$ $=2k_z^2\langle|H_{0x}|^2\rangle/k^2E_0^2$ in the degenerate case, we find that the absolute maximum of the slow flux component is reached at

$$k_{jx}=-k,\quad \text{if}\quad 1+\sin\theta>2\cos^4\theta\quad (\theta>\theta_0\approx 24°), \quad (3.4)$$

$$k_{jx}=k_x,\quad \text{if}\quad 1+\sin\theta<2\cos^4\theta\quad (\theta<\theta_0). \quad (3.5)$$

The periods of the induced lattice are respectively $\Lambda=\lambda/(1+\sin\theta)$ and $\Lambda=\lambda/\cos\theta$ i.e., the same as in the case of pure p- and s-polarizations (cf. Ref. 1).

Since $\langle Q\rangle$ is real in the degenerate[6)] case, the degenerate and nondegenerate cases were compared for $\mathrm{Re}\langle Q\rangle$, i.e., for a phase shift $\beta=0$. If, however, the mechanism is such that $\beta=\pi/2, 3\pi/2$, we have $\Gamma=0$ for degenerate geometry, and the increment has the extremum at the point $k_{jx}=-k$.

In an experiment with unpolarized radiation,[3] lattices $\Lambda=\lambda/\cos\theta$ were recorded (degenerate configuration, fundamental type), and lattices $\Lambda=\lambda/(1-\sin\theta)$ are reported. The latter correspond to a local maximum $k_{jx}=k$. The mechanism in this case apparently such that the local maximum $k_{jx}=k$ of the function $\langle F\rangle$ turns out to be singled out in the instability growth rate, owing to the additional slow dependence on the wave number (see Sec. 4).

3. Arbitrary linear polarization. For linear polarization the magnetic-field amplitude can be chosen real, and the complex conjugate term in (2.8) can be omitted. We introduce the notation

$$[\mathbf{k}_{jt}\mathbf{H}_{0t}]_z/kH_{0t}=\sin\chi_j, \quad (3.6)$$

$$a=[\mathbf{k}_t\mathbf{H}_{0t}]_z/kH_{0t}=\sin\theta\sin\psi_1, \quad (3.7)$$

where χ_j is the angle between $\mathbf{k}_{jt}$ and $\mathbf{H}_{0t}$ (we took into account that $k_{jt}=k$), ψ_1 is the angle between $\mathbf{k}_t$ and $\mathbf{H}_{0t}$, and $\sin\psi_1=\cos\psi/(\sin^2\psi\cos^2\theta+\cos^2\psi)^{1/2}$. Since Q is periodic in ψ, it suffices to consider only the interval $0\leqslant\psi_1<\pi$. We obtain them for $F(\chi_j)$ a parabolic quadratic form in $\sin\chi_j$:

$$F_j=A\sin\chi_j(\sin\chi_j-a), \quad (3.8)$$

where $A=H_{0t}^2/E_0^2=\cos^2\theta/(1-a^2)\leqslant 1$. The principal maximum F_j (3.8) is reached at $\chi_j=3\pi/2$; other extremal points together with the values of f_j in them are listed in Table II (see also Fig. 3). It remains to compare the value F_j at the principal maximum with the value of the slow function F at the point corresponding to the degenerate case $\mathbf{q}\perp\mathbf{k}_t$. To be specific, we assume that the formation of the periodic structure is determined by the real part of Q (i.e., that the phase shift $\beta=0$; this applies, e.g., to the evaporative mechanism). From Table II we have

$$\mathrm{Re}\, Q=-4F\ \mathrm{Im}\, f,\quad F=2A\cos^2\theta\cos^2\psi_1,\quad \mathbf{q}\perp\mathbf{k}_t. \quad (3.9)$$

It follows then from a comparison of the extrema that:

a) if $-1\leqslant\cos 2\psi_1<0$ (the radiation is close to p-polarized), the absolute maximum of the slow function F_j is reached, independently of the incidence angle, at $\chi_j=3\pi/2$ (Fig. 3a), in which case the lattice period is equal to $\Lambda=\lambda/(1+\sin^2\theta+2\sin\theta\sin\psi_1)^{1/2}$;

b) if $0<\cos 2\psi_1\leqslant 1$ (the radiation is close to s-polarized, then at $z_1<\sin\theta<1$, where

$$z_1=[-\sin\psi_1+(\sin^2\psi_1+8\cos^2\psi_1\cos 2\psi_1)^{1/2}]/4\cos^2\psi_1,$$

the maximum of F_j is again reached at $\chi_j=3\pi/2$ (Fig. 3b).

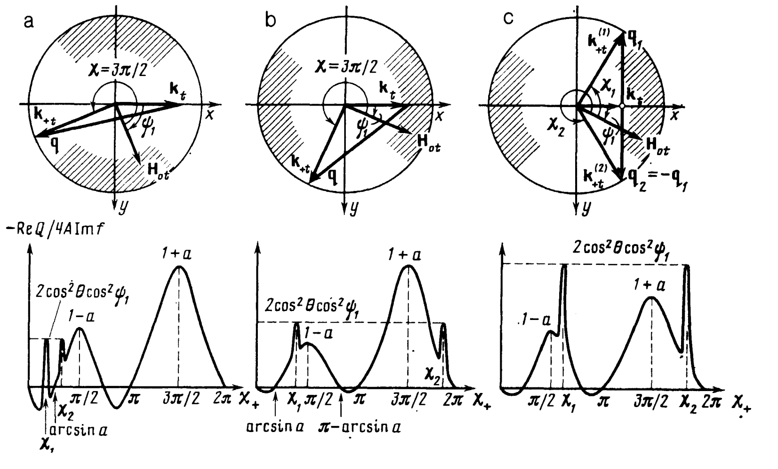

FIG. 3. Formation of periodic structure by arbitrary linearly polarized radiation: top—scattering pattern in the interface (the sectors in which the vector $\mathbf{H}_{0t}$ lies are shaded); bottom—plot of Re Q vs χ_t; a) beam close to p-polarized; b,c) beam close to s-polarized at large (b) and small (c) incidence angles; a degenerate pair of lattices $\mathbf{q}\perp\mathbf{k}_t$ is formed in case (c).

At small incidence angles $\theta < \sin\theta < z_1$ the maximum of the slow function is reached at $\sin\chi_{1,2} = \pm\cos(\theta \mp \psi_1) = a \pm \cos\theta\cos\psi_1$, corresponding to formation of a pair of degenerate $\mathbf{q}\perp\mathbf{k}_t$ lattices with period $\Lambda = \lambda/\cos\theta$ (Fig. 3c).

Note that the fact that the wave vector $\mathbf{k}_{jt}$ of the scattered-wave is perpendicular to the projection $\mathbf{H}_{0t}$ of the magnetic field does not mean at the same time that $\mathbf{k}_{jt}$ (and all the more the lattice wave vector $\mathbf{q}$) is parallel to the projection $\mathbf{E}_{0t}$ of the electric field on the interface, since the right angle between the vectors $\mathbf{E}_0$ and $\mathbf{H}_0$ is not projected into a right angle between their tangential components (except in the particular cases of pure s- and p-polarizations).

It can be shown that the slow dependence of the growth rate on the wave vector, of the type q_μ with $\mu < -1$, singles out a maximum $\chi_j = \pi/2$ that can become the principal one. We know of no published reports of experiments with arbitrary linear polarization.

The equations presented lead also to known results for s- and p-polarized beams,[1] which shall recall to facilitate the comparison.

s-polarization ($\psi_1 = 0$). The absolute maximum of the slow function F_j is reached at

$$\chi_j=\pi/2,\ 3\pi/2, \quad \text{if} \quad 2\cos^2\theta<1, \qquad \cos\chi_j=\sin\theta, \quad \text{if} \quad 2\cos^2\theta>1. \tag{3.10}$$

Oblique lattices $\mathbf{k}_{jt}\perp\mathbf{H}_{0t}$ with periods $\Lambda = \lambda/(1+\sin^2\theta)^{1/2}$ are formed in the first case, and a degenerate pair $\mathbf{q}\perp\mathbf{k}_t$ with period $\Lambda = \lambda/\cos\theta$ in the second.[1,2]

p-polarization ($\psi_1 = \pi/2$):

$$F_j=\sin\chi_j(\sin\chi_j-\sin\theta). \tag{3.11}$$

The maximum of F_j is reached at $\chi_j = 3\pi/2$ and the periods of the corresponding lattices are $\Lambda = \lambda/(1+\sin\theta)$. The local maximum $\chi_j = \pi/2$ will be treated in detail in Sec. 4.

4. PHENOMENOLOGICAL ACCOUNT OF THE PERIODIC-STRUCTURE FORMATION MECHANSIM

We consider some instability mechanisms. The simplest is the evaporative one[1] caused by local evaporation of the material in places where the field intensity is a maximum. The dynamics of the surface is described in this case by the equation

$$\rho L\frac{\partial\zeta}{\partial t} = CS_n', \quad C=\text{const}>0, \tag{4.1}$$

where ρ is the density, L the specific heat of the evaporation, $S_n' = I_0k\zeta Q, I_0 = cE_0^2/32\pi$. Obviously, for ζ to increase exponentially it is necessary to have Re $Q>0$. It follows hence that $\beta = 0$ and $\mu = 0$.

If a molten layer appears on the surface, the possible feedback mechanisms are assumed to be the evaporative vapor pressure and the thermocapillary effect.[18,1] The equations for the growth rates of these mechanisms are obtained from the hydrodynamics and heat-conduction equations (for a surface heat source) in analogy with Ref. 18. The dispersion equation for small surface perturbations is

$$(q^2+b^2)^2 + \frac{\Omega_0^2}{\nu^2} + 4q^3b = \frac{I_0kqQ}{c_v\chi\nu^2d}\left[p' + \frac{\alpha' q\nu(q-b)^2}{i\Omega}\right], \tag{4.2}$$

where $b = (q^2 - i\Omega/\nu)^{1/2}$, $d = (q^2 - i\Omega/\chi)^{1/2}$, Re $b, d \geqslant 0$, ν is the viscosity, χ the thermal diffusivity, c_v the specific heat, $\Omega_0 = (\alpha q^3/\rho)^{1/2}$ the capillary-wave dispersion law, and $p' = \partial p_{ev}/\partial T$, $\alpha' = \partial\alpha/\partial T$ the evaporative-pressure and surface-tension temperature coefficients. Assuming the perturbation to be small, we solve the dispersion equation by successive approximations: in the limiting case of low viscosity $\nu q^2 \ll \Omega_0$ we have

$$\Omega=\pm\Omega_0-2i\nu q^2 \mp \frac{I_0kq(p'-q\alpha')Q}{2c_v\chi\Omega_0d_\pm}, \quad d_\pm=(q^2\mp i\Omega_0/\chi)^{1/2}; \tag{4.3}$$

usually $\alpha' < 0$ and $p' > 0$, so that $p' - q\alpha' > 0$. The phenomenological parameters β and μ which characterize the mechanism are obtained from (4.3) or from analogous equations in other limiting cases, and are gathered in Table III.

We consider now the influence of the factor $(q/k)^\mu$ for the case of p-polarization. Noting that $\varkappa^2 \equiv (q/k)^2 = 1 + \sin^2\theta - 2\sin\theta\sin\chi_j$, we find that $\varkappa^\mu F_j$ reaches absolute maximum at

$$k_{jx}=k\ (\mu<-1), \quad k_{jx}=\pm k\ (\mu=\pm1), \quad k_{jx}=-k\ (\mu>-1). \tag{4.4}$$

The mechanisms suggested in the literature provide a rather diverse spectrum of μ (see Table III); in most cases $\mu \leqslant 0$, and frequently μ does not differ greatly from -1. This means that the lattices $k_{jx} = \pm k$, $\Lambda = \lambda/(1\mp\sin\theta)$ should be approximately effectively excited (especially at not too large incidence angles), as is indeed observed in experiments.[2] It is also reported in Ref. 2 that with increase of

TABLE III. Phase shifts β and exponents μ for different instability mechanisms.

№	Mechanism	β *	μ	Validity region
1	Evaporation	0	0	
2	Thermocapillarity	$\pm\pi/2$	$-1/2$	$\nu q^2 \ll \Omega_0 \ll \chi q^2$
		$\pm 3\pi/4$	$-1/4$	$\nu q^2,\ \chi q^2 \ll \Omega_0$
		π	-2	$\nu q^2,\ \chi q^2 \gg \Omega_0$
3	Evaporative pressure	$\pm\pi/2$	$-3/2$	$\nu q^2 \ll \Omega_0 \ll \chi q^2$
		$\pm 3\pi/4$	$-5/4$	$\nu q^2,\ \chi q^2 \ll \Omega_0$
		0	-2	$\nu q^2,\ \chi q^2 \gg \Omega_0$

*Two signs corresponds to two branches of the unperturbed dispersion law $\Omega = \pm\Omega_0$.

incidence angle (at $\theta > 30\ldots45°$) lattices with $\mathbf{q}\perp\mathbf{E}_0$, $\Lambda = \lambda/\cos\theta$ rather than $\mathbf{q}\|\mathbf{E}_0$ begin to be excited. The presence of such lattices does not follow from the foregoing: at $\mathbf{q}\perp\mathbf{E}_0$ we have $\sin\chi_j = \sin\theta$ and $F_j = 0$ according to (3.11) (a similar result is obtained also for actual mechanisms[1]). It is possible, however, that the observed lattice correspond to a local maximum of $\varkappa^\mu F_j$:

$$k_{jx}/k = \frac{(\mu+4)z^2+2+[4+\mu(\mu+4)z^4]^{1/2}}{2(\mu+4)z}, \tag{4.5}$$

where $z = \sin\theta$, which takes place for mechanisms with exponent $\mu > 0$ at sufficiently large incidence angles with $\sin\theta > \{\mu + 3 - [(\mu+3)^2 - 8]^{1/2}\}/2$.

Let us discuss the consequences of the differences between the phase shifts in different mechanisms. If the slow function F_j is real, the phase shift influences only the fast function. For the evaporative mechanism ($\beta = 0$) the role of the latter is played by $-\operatorname{Im} f$, which has a maximum at $\lambda'' = (m+n)/|\varepsilon|$. This maximum corresponds to lattices with wave numbers (at $\mathbf{q}\|\mathbf{k}_t$)

$$q = k\left(1 + \frac{(m+n)^2}{2|\varepsilon|^2}\right) \pm k_t. \tag{4.6}$$

For the thermocapillary mechanism at $\beta = -\pi/2$ the corresponding rapid dependence of the growth rate is given by the function $\operatorname{Re} f$, which reaches a maximum at $\lambda'' = m/|\varepsilon|$, corresponding to lattices

$$q = k\left(1 + \frac{m^2}{2|\varepsilon|^2}\right) \pm k_t. \tag{4.7}$$

Thus, depending on the mechanism (on the phase shift), the locations of the maxima of the fast function are shifted in the considered examples by $\Delta\lambda'' = n/|\varepsilon|$; as a result, the relative differences between the wave numbers and periods of the corresponding lattices are given by

$$\frac{\Delta q}{q} = \frac{|\Delta\Lambda|}{\Lambda} = \frac{k}{q}\frac{n(2m+n)}{2|\varepsilon|}. \tag{4.8}$$

In the general case of complex values of F_j, the shift $\Delta\Lambda$ of the period depends also on the lattice orientation.

CONCLUSION

A sufficiently definite connection exists thus between the singularities of the interference flux and the parameters of the generated periodic states. This enables us to find the lattice parameters without resorting to an analysis of the specific mechanisms whereby the specific periodic states are formed in the general case of arbitrary polarization of the applied radiation. The results of such a procedure agree with the experimental data and with numerical calculations.[2,15]

We dwell in conclusion on the limitations of the employed model. It is assumed here that the energy is absorbed locally on the $z = 0$ boundary, i.e., a surface interference thermal source is postulated [see (4.1)]. Since actually the field penetrates into the medium to the skin-layer depth, this means that this depth must be small compared with the lattice period $\Lambda \sim \lambda$.

The medium was assumed homogeneous; the possibility of the onset of a surface layer with other properties as a result of the heating (oxide film etc.) and of the trapping of the radiation in a surface waveguide[5] was not considered. If the heating causes stratification of the optical properties of the medium, this imposes a limit $(\chi\tau)^{1/2} \gg \lambda$ or $(\chi\tau)^{1/2} \ll \lambda$ on the heating depth (τ is the duration of the action). Finally, we have neglected the frequency dispersion; this is legitimate if the width $\Delta\omega$ of the resonance of ε is much larger than the frequency shift by scattering: $\Delta\omega \gg \Omega$, or if the operating frequency is far from the resonances of ε.

Modulation of the dielectric constant on account of temperature modulation or as a result of nonlinearity in the field can be treated similarly, although this involves a somewhat different electrodynamic problem.[15] Since our analysis can be based to a considerable degree on general considerations of the mutual cancellation of the Stokes and anti-Stokes scattering channels (with $|\varepsilon| \gg 1$ as the example), the conclusions drawn concerning the periodic structures can apparently be extended to include also the case of scattering by a periodic profile of ε.

APPENDIX

We present for reference explicit expressions for the coefficients of conversion of an incident wave into first-order Raman spectra on scattering by a corrugated surface. In accordance with (1.3), we expand the amplitudes of the scattered fields into components perpendicular (s) and parallel (p) of the scattering plane, and express them respectively in terms of the normal components $E_{j\perp}^{R,T} \equiv E_{js}^{R,T}$, $H_{j\perp}^{R,T} \equiv H_{jp}^{R,T}$ of the electric and magnetic fields

$$\mathbf{E}_j^R = E_{js}^R \mathbf{e}_{js} + H_{jp}^R \mathbf{e}_{jp}^R, \qquad \mathbf{H}_j^R = H_{jp}^R \mathbf{e}_{js} - E_{js}^R \mathbf{e}_{jp}^R,$$
$$\mathbf{E}_j^T = E_{js}^T \mathbf{e}_{js} + H_{jp}^T \mathbf{e}_{jp}^T/\varepsilon^{1/2}, \qquad \mathbf{H}_j^T = H_{jp}^T \mathbf{e}_{js} - \varepsilon^{1/2} E_{js}^T \mathbf{e}_{jp}^T, \tag{A.1}$$

where $\mathbf{e}_{js} \equiv \mathbf{e}_{j\perp} = [\mathbf{e}_z \mathbf{k}_{jt}]/k_{jt}$, $\mathbf{e}_{jp}^R \equiv \mathbf{e}_{j\|}^R = [\mathbf{e}_{js}\mathbf{k}_j^R]/k$, $\mathbf{e}_{jp}^T \equiv \mathbf{e}_{j\|}^T = [\mathbf{e}_{js}\mathbf{k}_j^T]/k^T$ (the subscripts s and p in this notation correspond to the usual definition of s- and p-polarizations relative to the scattering plane). Introducing the two-component vectors

$$\mathbf{X}=(E_s{}^T, H_p{}^T), \quad \mathbf{X}_j^{R,T}=(E_{js}^{R,T},\ H_{jp}^{R,T}\ ), \tag{A.2}$$

we represent the scattered fields in the form

$$X_{j\alpha}{}^R=\frac{ik\zeta_j}{2}\sum_\beta R_j{}^{\alpha\beta}X_\beta; \quad \alpha, \beta=s, p; \tag{A.3}$$

the equations for the refracted fields $\mathbf{X}_j^T$ are obtained from (A.3) by the substitution $R\to T$. The coefficients $R_j^{\alpha\beta}, T_j^{\alpha\beta}$ are equal to

$$R_j{}^{ss}=g\mathbf{C}_j{}^R\mathbf{e}_{js}=-k_{jx}(k_{jz}{}^T+k_{jz}{}^R)/kk_{jt}, \tag{A.4}$$

$$R_j{}^{sp}=g\mathbf{B}_j{}^R\mathbf{e}_{js}=k_{jy}k_z{}^T(k_{jz}{}^T+k_{jz}{}^R)/\varepsilon k^2 k_{jt}, \tag{A.5}$$

$$R_j{}^{ps}=g\mathbf{C}_j{}^R\mathbf{e}_{jp}{}^R=(\varepsilon-1)k_{jy}k_{jz}{}^T/d, \tag{A.6}$$

$$R_j{}^{pp}=g\mathbf{B}_j{}^R\mathbf{e}_{jp}{}^R=(\varepsilon-1)(k_{jx}k_z{}^T k_{jz}{}^T-\varepsilon k_x k_{jt}{}^2)/\varepsilon kd, \tag{A.7}$$

$$T_j{}^{ss}=R_j{}^{ss}, \quad T_j{}^{sp}=R_j{}^{sp}, \tag{A.8}$$

$$T_j{}^{ps}=\varepsilon^{1/2}g\mathbf{C}_j{}^T\mathbf{e}_{jp}{}^T=\varepsilon(\varepsilon-1)k_{jy}k_{jz}{}^R/d, \tag{A.9}$$

$$T_j{}^{pp}=\varepsilon^{1/2}g\mathbf{B}_j{}^T\mathbf{e}_{jp}{}^T=(\varepsilon-1)\ (k_{jx}k_z{}^T k_{jz}{}^R-k_x k_{jt}{}^2)/kd, \tag{A.10}$$

where $g=-(\varepsilon-1)/k$, $d=k_{jt}(k_{jz}^T-\varepsilon k_{jz}^R)$, and the coefficients $C_j^{R,T}$ and $B_j^{R,T}$ are defined in (1.5) and are given in Ref. 10. Note the resonant character of the scattering into the p-component after the onset of glancing spectra ($k_{jz}^R\to 0$) for $\operatorname{Re}\varepsilon<-1$ and for $|\varepsilon|\gg 1$. Substitution of (A.3)–(A.10) in (2.10) leads after suitable transformations to (2.2)–(2.6).

1)The SSSW threshold, however, is significant, and the competition of the thermal effects is so substantial that no SSSW in pure form have been reliably recorded in pure form in nondissipative media.

2)In the case of metals, waves diffracted by periodic structures can be close to the natural surface electromagnetic waves (SEW) of the media (Wood's resonance).

3)This approximation is, at any rate, sufficient to describe the initial stage of the instability, and by the same token to determine the surface states that grow at the fastest rate; the surface structure interaction[13] is not taken into account here. In exceptional cases, for example for an ideally conducting medium, an essentially nonlinear problem must be considered, with diffraction spectra of order higher than the first invoked.[12]

4)This is valid if the growth rate is small compared with the characteristic reciprocal times of the natural motions of the medium. In the general case, however, the extrema of Q should be substantial.

5)This terminology is convenient, and we shall use it hereafter, although in a number of cases it loses its initial meaning (e.g., in the case of static structures).

6)In the degenerate case ($k_{jx}=k_x$, $k_{jy}=jk_z$, $f_+=f_-$) for non-polarized radiation we have $\langle F_+\rangle=\langle F_-\rangle$, so that the contribution of $\langle F_+\rangle$ to $\operatorname{Re}\langle Q\rangle$ is doubled, and $\operatorname{Im}\langle Q\rangle=0$. (A similar result holds also for s-polarization, see Ref. 1 and below.) In general, however, this is not the case: in particular, for arbitrary linear polarization we have

$$F_j\sim k_z{}^2H_{0x}{}^2+jk_xk_zH_{0x}H_{0y}, \quad Q\sim -8k_z^{-2}H_{0x}{}^2\operatorname{Im} f+i\cdot 8k_xk_zH_{0x}H_{0y}\operatorname{Re} f^-,$$

i.e., the part of F_j that is even in j enters in Re Q and is doubled, while the odd part enters in Im Q.

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