Excitation and detection of magnetostatic waves on the boundary of a type-II superconductor of the second kind

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The effect of an alternating homogeneous magnetic field **H** of frequency ω on the boundary of a type-II superconductor located in a homogeneous stationary magnetic field perpendicular to the surface is investigated. It is shown that an inhomogeneous magnetostatic wave with a wavelength equal to the distance d between the layers of the vortex structure of the superconductor is induced. Conversely, an inhomogeneous magnetostatic wave of wavelength d induces a homogeneous alternating magnetic field of frequency ω. In this case the amplitude-transformation coefficient for the exciting and induced waves is $K \approx 4.5\times10^{-2}\varkappa^{-2}$, where $\varkappa$ is the Ginzburg-Landau parameter. It is shown that this property of the boundary of a type-II superconductor can be used to detect surface Rayleigh waves of frequency $\sim 10^{11}$–5×10^{11} s^{-1} in concentrated paramagnetics with an acoustic energy flux density of 10^{-3}–10^{-4} W·cm^{-2}.

INTRODUCTION

In a concentrated paramagnetic material in an external constant magnetic field, the propagation of sound causes oscillations of magnetization such that the acoustic wave is accompanied by a magnetostatic wave of the same frequency. The surface acoustic wave is accompanied also by a magnetostatic wave above the surface of the paramagnetic substance, but its amplitude drops exponentially with increasing distance from the surface at a distance on the order of the acoustic wavelength λ_s. Similar magnetostatic surface waves accompany oscillations in magnetization caused by collective excitations of spin systems in solids (for example, by spin waves, magnons, etc.). At frequencies $\omega \sim 10^{11}$ to 5×10^{11} per second we have $\lambda_s \sim 10^{-5}$, so that excitation of this sort of surface excitation directly by a magnetostatic wave is not possible. In addition, even the generation of similar excitations causes difficulties, since this requires an alternating inhomogeneous magnetic field having an inhomogeneity period λ_s, so that the method of "edge excitation" must basically be used,[1] which is only slightly effective at frequencies $\omega \geqslant 10^{11}$ s^{-1}, and is not used for the excitation of surface waves.

In this study it is proposed that the boundary of a type-II superconductor be used for the excitation and detection of similar surface magnetostatic waves. It is known[2] that in an external constant homogeneous magnetic field **H** perpendicular to the surface of a superconductor ($H_{c1} < H < H_{c2}$, $H_{c1,c2}$ are the first and second critical fields), an ordered two-dimensional vortex structure consisting of currents and a magnetic field is formed in a type-II superconductor. The period of the structure determined by H can easily be selected to be equal to the acoustic wavelength λ_s at frequencies $\omega \sim 10^{11}$–10^{12} s^{-1}. If a disk of this superconductor is laid on the surface, say, of a paramagnetic substance, along which a supersonic surface wave is propagated, then the magnetostatic field accompanying the sound causes a perturbation of the periodic structure of the superconductor. Any small perturbation of the vortex structure can be represented as a superposition of various modes of a reciprocal two-dimensional vortex lattice, in which the various modes interact with one another as a result of the nonlinearity of the nonstationary equations of superconductivity (Ref. 3), so that a nonhomogeneous exciting mode will induce a homogeneous mode which can be detected by microwave radio methods.

In the inverse problem, a homogeneous mode induces an inhomogeneous one, and if λ_s turns out to be equal to the distance between the vortex layers, it may prove to be sufficiently effective to excite sound in the paramagnetic substance.

In order to clarify these possibilities, we solve here two boundary-value problems of the nonstationary superconductivity equations (Ref. 3) for the half-space occupied by the superconductor with: 1) a uniform alternating magnetic field of frequency ω at the superconductor boundary; 2) an inhomogeneous magnetic field of frequency ω, with a period equal to the distance of the superconductor vortex layers, at the superconductor boundary.

1. LINEARIZATION OF THE NONSTATIONARY SUPERCONDUCTIVITY EQUATIONS

In Ref. 3, a nonstationary generalization of the Ginzburg-Landau superconductivity equations was obtained for zero-gap superconductors at frequencies $\omega < \tau_s^{-1}$, where τ_s^{-1} is the frequency of collisions with spin flip of the conduction electrons. We introduce the dimensionless radius vector $\mathbf{r} = \mathbf{r}'/\lambda$, the time $t = \gamma^{-1}\xi^{-2}t'$, the reciprocal coefficient of conduction-electron diffusion in the normal state $\gamma_0 = \gamma\lambda$, the electric conductivity coefficient $\sigma_0 = 4\pi\lambda\sigma$, the dimensional coefficient of electrical conductivity and the reciprocal diffusion coefficient σ and γ, the depth of magnetic field penetration into the superconductor λ, the coherence length ξ, the radius vector $\mathbf{r}'$, and the time t' ($k = \hbar = c = 1$). In dimensionless units the nonstationary equations of superconductivity take the following form (Ref. 3):

$$\frac{\partial f}{\partial t} + f^3 - f - \frac{\nabla^2}{\varkappa^2} f + \mathbf{Q}^2 f = 0, \tag{1}$$

$$\gamma_0 f^2 u + \operatorname{div}(f^2\mathbf{Q}) = 0,$$
$$\operatorname{rot}\mathbf{H} = \sigma_0\mathbf{E} - f^2\mathbf{Q}, \tag{2}$$
$$\mathbf{H} = \operatorname{rot}\mathbf{Q}, \quad \mathbf{E} = -\nabla u - \frac{\varkappa^2}{\gamma_0}\frac{\partial \mathbf{Q}}{\partial t}.$$

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Here $f=\Delta'/\Delta_0$; Δ' is the amplitude of the order parameter; Δ_0 is the amplitude of the order parameter in the absence of fields (Ref. 4); **E** and **H** are the dimensionless intensities of the electric and magnetic fields (Ref. 5); $\mathbf{Q}=\mathbf{A}-\varkappa^{-1}\nabla\chi$; $u=\varphi+\varkappa\gamma_0^{-1}\partial\chi/\partial t$; $\varkappa=\lambda/\xi$ is the Ginzburg-Landau parameter; **A** and φ are potentials of the electromagnetic field; χ is the phase of the order parameter.

Let the superconductor under study occupy the half-space $z\geqslant 0$. We apply to the sample a uniform magnetic field $\mathbf{H}^0(0,0,H^0)$. We designate by $\mathbf{Q}_0$ and f_0 the stationary solutions of the equation set (1) in the field $\mathbf{H}_0$ ($u=0$). We assume in addition that weak nonstationary perturbations make their appearance at the boundary $z=0$, where the solution of the equation set (1) can be presented in the form $\mathbf{Q}=\mathbf{Q}^0+\mathbf{q}(\mathbf{r},t)$ and $f=f^0+\delta(\mathbf{r},t)$, where $q\ll Q^0$ and $\delta\ll f^0$. The equations (1) linearized with respect to the nonstationary perturbations **q**, u, and δ take the form

$$\dot{\delta}-\varkappa^{-2}\nabla^2\delta+(3f^{02}-1)\delta=-2f^0\mathbf{Q}^0\mathbf{q}-\mathbf{Q}^{02}\delta,$$
$$\sigma_0\gamma_0^{-1}\varkappa^2\dot{\mathbf{q}}-\Delta\mathbf{q}+\nabla\operatorname{div}\mathbf{q}+\sigma_0\nabla u+f^{02}\mathbf{q}=-2f^0\mathbf{Q}^0\delta,\qquad(3)$$
$$\gamma_0 f^0 u+f^0\operatorname{div}\mathbf{q}+2\nabla f^0\mathbf{q}=-2\mathbf{Q}^0\nabla\delta.$$

It is necessary to add to these equations the boundary conditions at the surface $z=0$ (Ref. 3):

$$\left.\frac{\partial\delta}{\partial z}\right|_{z=0}=0,\qquad q^z|_{z=0}=0,\qquad E^z|_{z=0}=0,\qquad(4)$$

and the continuity of the magnetic field vector $\mathbf{h}(r,t)$ and of the tangential component of the vector **E**.

Equations (3) are a set of linear equations, whose coefficients $\mathbf{Q}^0$ and f^0 are periodic functions of the two-dimensional vortex lattice. We define the set of coordinates in such a way that the basis vectors of the triangular vortex lattice have the form $\mathbf{a}=a\mathbf{i}$, $\mathbf{b}=(a/2)\mathbf{i}+(3^{1/2}/2)a\mathbf{j}$. We obtain for the basis vector of the reciprocal lattice:

$$\mathbf{A}=\frac{2\pi}{a}\left(\mathbf{i}-\frac{1}{3^{1/2}}\mathbf{j}\right),\qquad \mathbf{B}=\frac{2\pi}{a}\frac{2}{3^{1/2}}\mathbf{j},$$

$\mathbf{Q}^0$ and f^0 can be represented in the form of an expansion in the vectors of the reciprocal lattice, $\mathbf{G}_{kl}=k\mathbf{A}+l\mathbf{B}$:

$$\mathbf{Q}^0=\sum_{k,l}\mathbf{Q}_{kl}{}^0\exp(i[k\mathbf{A}+l\mathbf{B}]\mathbf{r}),$$
$$f^0=\sum_{k,l}f_{kl}{}^0\exp(i\mathbf{G}_{kl}\mathbf{r}).$$

We further assume that $\varkappa\gg 1$ and $\mathbf{H}^0$ is such that $a\sim\lambda$, that is, $a\gg\xi$. In these equations $\mathbf{Q}^0$ can be represented as a superposition of isolated vortices of a London superconductor, and $f^0\approx 1$. (We note that since f^0 varies from 0 to 1 at distances of $\sim\xi$, it follows that $f^0_{00}\approx 1$ and $f^0_{kl}\approx\varkappa^{-2}$ at $k,l\neq 0$.) The coefficients $\mathbf{Q}^0_{kl}(Q^x_{kl},Q^y_{kl},0)$ are defined in the Appendix, and have an order $\varkappa^{-1}$. On assuming in Eq. (3) $f^0=1$ and restricting ourselves to terms of highest order in $\varkappa^{-1}$ we obtain:

$$\dot{\delta}-\varkappa^{-2}\Delta\delta+2\delta=-2\mathbf{Q}^0\mathbf{q},\qquad(5a)$$
$$\sigma_0\gamma_0^{-1}\varkappa^2\dot{\mathbf{q}}-\Delta\mathbf{q}+\nabla\operatorname{div}\mathbf{q}+\sigma_0\nabla u+\mathbf{q}=-2\mathbf{Q}^0\delta,\qquad(5b)$$
$$\gamma_0 u+\operatorname{div}\mathbf{q}=-2\mathbf{Q}^0\nabla\delta.\qquad(5c)$$

The solution of the equation set (5) should be sought in the form[6]

$$\delta=\sum_{k,l}\delta_{kl}\exp(i\mathbf{G}_{kl}\mathbf{r}),\qquad \mathbf{q}=\sum_{k,l}\mathbf{q}_{kl}\exp(i\mathbf{G}_{kl}\mathbf{r}),$$
$$u=\sum_{k,l}u_{kl}\exp(i\mathbf{G}_{kl}\mathbf{r}),\qquad(6)$$

where δ_{kl}, u_{kl}, $\mathbf{q}_{kl}$ $(q^x_{kl},q^y_{kl},q^z_{kl})$ are functions of z. We substitute these expressions in Eqs. (5). Then the modes k and l are "intermixed" in the right side of Eqs. (5), so that there is a parametric interaction between the various k and l modes. We note that the terms describing the parametric interaction of various modes k and l are $\sim\varkappa^{-1}$, and they can be considered to be small in comparison with the left-hand side of Eq. (5). Then solution of Eq. (5) with the boundary conditions taken into account can be sought by iterations with respect to the weak parametric interaction between the various modes k and l. In the zeroth approximation we assume the right-hand side of Eq. (5) to be equal to zero, and we seek for the homogeneous equations (5) a solution satisfying the boundary conditions. We substitute then the obtained solution in the right-hand side of Eq. (5), and seek a solution of the inhomogeneous equations (again with the boundary conditions taken into account). The solution obtained will take into account the parametric interaction between the modes k and l.

2. SOLUTION OF THE HOMOGENEOUS EQUATIONS

The fundamental solution of the set of equations (5) with zero right-hand side has the following form (we assume δ, **q**, $u\sim e^{i\omega t}$),

$$\begin{bmatrix}q^x_{kl}\\ q^y_{kl}\\ q^z_{kl}\\ u_{kl}\end{bmatrix}=A_{kl}\begin{bmatrix}-p_{kl}\cos\varphi_{kl}\\ -p_{kl}\sin\varphi_{kl}\\ G_{kl}\\ 0\end{bmatrix}\exp(ip_{kl}z)+B_{kl}\begin{bmatrix}-\sin\varphi_{kl}\\ \cos\varphi_{kl}\\ 0\\ 0\end{bmatrix}\exp(ip_{kl}z)+C_{kl}\begin{bmatrix}-i\dfrac{\sigma_0}{\beta}G_{kl}\cos\varphi_{kl}\\ -i\dfrac{\sigma_0}{\beta}G_{kl}\sin\varphi_{kl}\\ -i\dfrac{\sigma_0}{\beta}p_{kl}\\ 1\end{bmatrix}\exp(ip'_{kl}z),\qquad(7)$$

$$\delta_{kl}=D_{kl}\exp(id_{kl}z),\quad d_{kl}=i(2\varkappa^2+i\omega\varkappa^2+G_{kl}{}^2)^{1/2},$$
$$p_{kl}=i(\beta+G_{kl}{}^2)^{1/2},\quad p_{kl}'=i(\beta\gamma_0/\sigma_0+G_{kl}{}^2)^{1/2},\quad \beta=1+i\omega\sigma_0\varkappa^2/\gamma_0,\qquad(8)$$
$$\operatorname{Im}d_{kl}>0,\ \operatorname{Im}p_{kl}>0,\ \operatorname{Im}p_{kl}'>0,\ \varphi_{kl}=\operatorname{arctg}(G_{kl}{}^y/G_{kl}{}^x),$$

where $\mathbf{q}_{00}$ and u_{00} are obtained from Eq. (7) at $\varphi_{00}=0$. A_{kl}, B_{kl}, C_{kl}, and D_{kl} are arbitrary constants.

3. SOLUTION OF THE BOUNDARY-VALUE PROBLEM FOR THE CASE OF AN ALTERNATING UNIFORM FIELD

We assume that a uniform alternating magnetic field $\mathbf{h}=\mathbf{h}_{00}e^{i\omega t}$, $\mathbf{h}_{00}(0,h_{00},0)$ is incident on the surface of the superconductor. It causes a perturbation in the periodic structure of the superconductor, and in the vicinity of the boundary there appears an alternating electromagnetic field with different modes k and l, so that the exact value of $\mathbf{h}$ at the boundary $x=0$ cannot be specified, but in the zeroth approximation in the interaction between modes it is possible to put $\mathbf{q}=\mathbf{q}_{00}e^{i\omega t}$, $\mathbf{q}_{00}(q_{00},0,0)$ at $z=0$. We note that the boundary conditions of Eq. (4) should be met exactly at any step of the iterations, since they indicate the absence of a normal current through the boundary of the superconductor (Ref. 3). In the zeroth approximation in the parametric interaction of modes we then obtain, using the solutions (7) and (8), subject to the boundary conditions

$$\begin{gathered} q_{00}^{(0)x}=q_{00}\exp(ip_{00}z), \quad q_{00}=ih_{00}/p_{00}, \\ q_{kl}^{(0)x}=0 \quad (k,l\neq 0), \\ q_{kl}^{(0)y}=q_{kl}^{(0)z}=\delta_{kl}^{(0)}=u_{kl}^{(0)}=0. \end{gathered} \tag{9}$$

We substitute Eq. (9) in the first equation of set (5), and find a general solution of the inhomogeneous equation that is obtained:

$$\delta_{kl}^{(1)}(z)=D_{kl}\exp(id_{kl}z)-\frac{2Q_{kl}^{x}q_{00}\varkappa^2}{d_{kl}^2-p_{00}^2}\exp(ip_{00}z). \tag{10}$$

We find the constants $D_{k,l}$ from the boundary condition $\partial\delta_{kl}^{(1)}/\partial z|_{z=0}=0$. We obtain

$$\delta_{kl}^{(1)}(z)=\frac{2Q_{kl}^{x}q_{00}\varkappa^2}{d_{kl}^2-p_{00}^2}\left[-\frac{p_{00}}{d_{kl}}\exp(id_{kl}z)+\exp(ip_{00}z)\right]. \tag{11}$$

The coefficients $\delta_{kl}^{(1)}$ determine the nonstationary part of the order parameter $\delta^{(1)}$ to a first order in the parametric interaction of modes. This part determines the right-hand sides of the equations for $\mathbf{q}^{(1)}$ and $u^{(1)}$ in Eq. (5). Further, it is possible to find a general solution of the inhomogeneous equations obtained from Eq. (5) for $u_{kl}^{(1)}(z)$ and $q_{kl}^{(1)}(z)$ with arbitrary constants A_{kl}, B_{kl}, C_{kl}, which are then determined from the boundary conditions. We find the coefficients $u_{01}^{(1)}$ and $\mathbf{q}_{01}^{(1)}$, since just they will determine the alternating inhomogeneous electromagnetic field having an inhomogeneity period equal to the distance between vortex layers of the superconductor. To do this it is necessary to separate out in the right-hand sides of Eqs. (5) the expansion terms corresponding to the mode 01:

$$(Q_x^0\delta^{(1)})_{01}=-\sum_{k,l}Q_{kl-1}^{x}\delta_{kl}^{(1)}, \tag{12}$$

$$(Q_y^0\delta^{(1)})_{01}=-\sum_{k,l}Q_{kl-1}^{y}\delta_{kl}^{(1)}=0, \tag{13}$$

$$(\mathbf{Q}^0\nabla\delta^{(1)})_{01}=-i\sum_{k,l}(\mathbf{Q}_{kl-1}\mathbf{G}_{kl})\delta_{kl}^{(1)}=0. \tag{14}$$

To prove Eqs. (13) and (14) we note that these expressions contain sums of the form:

$$S=\sum_{k,l=-\infty}^{\infty}kg(k^2,2l-k), \tag{15}$$

where q is some function of the arguments k^2 and $2l-k$. Then S is equal to:

$$S=\sum_{l=-\infty}^{\infty}\left(\sum_{k=0}^{\infty}kg(k^2,2l-k)-\sum_{k=0}^{\infty}kg(k^2,2l+k)\right).$$

We make the replacement $l=l'-k$ in the second sum. In this case the summation with respect to l' for each k will be from $-\infty$ to ∞ so that we have:

$$S=\sum_{l=-\infty}^{\infty}\sum_{k=0}^{\infty}kg(k^2,2l-k)-\sum_{l'=-\infty}^{\infty}\sum_{k=0}^{\infty}kg(k^2,2l'-k)=0.$$

Thus, the right-hand sides of Eqs. (5) for $u_{01}^{(1)}$ and $q_{01}^{(1)}$ are determined by Eqs. (12) to (14). Taking into account the boundary conditions $q^{(1)}z|_{z=0}=0$ and $E^{(1)}z|_{z=0}=0$, the general solution of the obtained equations takes the form.

$$\begin{gathered} u_{01}^{(1)}=0, \quad q_{01}^{(1)y}=0, \quad q_{01}^{(1)z}=0, \\ q_{01}^{(1)x}=-B_{01}\exp(ip_{01}z)+q_{00}\sum_{k,l}a_{kl}\exp(ip_{00}z) \\ +q_{00}\sum_{k,l}b_{kl}\exp(id_{kl}z), \end{gathered} \tag{16}$$

$$a_{kl}=-\frac{4Q_{kl}^{x}Q_{kl-1}^{x}\varkappa^2}{(p_{00}^2-d_{kl}^2)(p_{00}^2-p_{01}^2)}, \tag{17}$$

$$b_{kl}=\frac{4Q_{kl}^{x}Q_{kl-1}^{x}p_{00}\varkappa^2}{(d_{kl}^2-p_{00}^2)d_{kl}(d_{kl}^2-p_{01}^2)}. \tag{18}$$

We find B_{01} from the remaining boundary condition—the continuity of the magnetic field intensity $\mathbf{h}_{01}$ at $z=0$. We assume for simplicity that the superconductor borders a vacuum; it is then easy to show that $h_{01}^{y}/h_{01}^{z}=i$ at $z=0$. This relation determines B_{01}. The term with the sum b_{kl} need not be taken into account in the calculations, since it will give a contribution $\sim\varkappa^{-1}$ relative to the previous term with a_{kl}. Finally we obtain at $z=0$

$$h_{01}^{y}=-ih_{00}^{y}\frac{G_{01}(p_{01}-p_{00})}{p_{00}(p_{01}+iG_{01})}\sum_{k,l}a_{kl}\exp(iG_{01}y). \tag{19}$$

4. SOLUTION OF THE BOUNDARY-VALUE PROBLEM FOR THE CASE OF AN ALTERNATING INHOMOGENEOUS FIELD

Let a magnetostatic wave with a wavelength equal to the distance between vortex layers be propagated along the boundary of a superconductor. In the zeroth approximation in the parametric interaction it is then possible to put at $z=0$:

$$\mathbf{q}(\mathbf{r},t)=\mathbf{q}_{01}\exp(i\omega t+iG_{01}y),$$
$$\mathbf{q}_{01}(q_{01},0,0),\ \mathbf{q}_{kl}=0(k\neq 0,l\neq 1),$$

and the solution will be determined by Eqs. (6) and (7). Using the boundary conditions we obtain for the zeroth approximation:

$$\begin{gathered} q_{01}^{(0)x}=q_{01}\exp(ip_{01}z), \quad q_{kl}^{(0)x}=0 \quad (k\neq 0,\ l\neq 1), \\ q_{kl}^{(0)y}=q_{kl}^{(0)z}=u_{kl}^{(0)}=\delta_{kl}^{(0)}=0. \end{gathered} \tag{20}$$

We substitute Eq. (20) in Eq. (5a) and we obtain for $\delta_{kl}^{(1)}$ inhomogeneous equations whose solution with allowance for the boundary conditions $\partial\delta^{(1)}/\partial z|_{z=0}=0$ takes the form

$$\delta_{kl}^{(1)}=-\frac{2q_{01}Q_{kl-1}^{x}\varkappa^2}{d_{kl}^2-p_{01}^2}\left(\frac{p_{01}}{d_{kl}}\exp(id_{kl}z)-\exp(ip_{01}z)\right). \tag{21}$$

The solution of Eq. (21) determines the right-hand side of

Eqs. (5b) and (5c) to first order in the parametric interaction of modes $\mathbf{q}_{kl}^{(1)}$ and $u_{kl}^{(1)}$. Let us find $\mathbf{q}_{00}$ and $u_{00}^{(1)}$, since it is they which will determine the homogeneous electromagnetic field induced by the surface of the superconductor. To do this it is necessary to separate out the 00 mode in the right-hand sides of Eqs. (5b) and (5c):

$$(Q_x{}^0\delta^{(1)})_{00}=\sum_{k,l}Q_{-k-l}^{x}\delta_{kl}^{(1)},\tag{22}$$

$$(Q_y{}^0\delta^{(1)})_{00}=\sum_{k,l}Q_{-k-l}^{y}\delta_{kl}^{(1)}=0,\tag{23}$$

$$(\mathbf{Q}^0\nabla\delta^{(1)})_{00}=i\sum_{k,l}\mathbf{Q}_{-k-l}\mathbf{G}_{kl}\delta_{kl}^{(1)}=0.\tag{24}$$

Note that the sum encountered in Eq. (23) has the form of Eq. (15), while the sum in Eq. (24) has the following as a multiplier:

$$Q_{-k-l}^{x}G_{kl}{}^x+Q_{-k-l}^{y}G_{kl}{}^y=0.$$

Equations (22)-(24) determine the right-hand sides of the set of equations (5) for the components $u_{00}^{(1)}$ and $q_{00}^{(1)}$. Solution of these equations with the use of boundary conditions in Eq. (4) gives:

$$q_{00}^{(1)z}=q_{00}^{(1)y}=u_{00}^{(1)}=0,$$

$$q_{00}^{(1)x}=q_{01}\sum_{k,l}a_{kl}'\exp(ip_{01}z)+q_{01}\sum_{k,l}b_{kl}'\exp(id_{kl}z),\tag{25}$$

$$a_{kl}'=-\frac{4Q_{kl}{}^xQ_{kl-1}^{x}\varkappa^2}{(p_{01}{}^2-d_{kl}{}^2)(p_{01}{}^2-p_{00}{}^2)},$$

$$b_{kl}'=\frac{4Q_{kl}{}^xQ_{kl-1}^{x}p_{01}\varkappa^2}{(p_{01}{}^2-d_{kl}{}^2)(d_{kl}{}^2-p_{00}{}^2)d_{kl}}.\tag{26}$$

From this we obtain the alternating uniform magnetic field induced on the boundary of the superconductor (the term with b'_{kl} is dropped, since it gives a contribution on the order of $\varkappa^{-1}$ relative to the preceding one):

$$h_{00}{}^x=h_{00}{}^z=0,\qquad h_{00}{}^y=h_{01}{}^y\sum_{k,l}a_{kl}',\tag{27}$$

where h_{01}^{y} is the amplitude of the y-component of the magnetic field of the inhomogeneous wave.

5. DISCUSSION OF RESULTS

As is evident from Eqs. (19) and (27), the ratio of the amplitudes of the induced and exciting waves is determined for both boundary-value problems by the sums Σa_{kl} and $\Sigma a'_{kl}$. To estimate these sums we note that a sufficiently large class of type-II superconductor exists with $\varkappa = 10$–50 and $\lambda \sim 1000$–2000 Å, for which $\tau_s^{-1}\sim 5\times10^{11}$ to 10^{13} s^{-1} (Ref. 7), so that these superconductors can be used for the detection and excitation of magnetostatic waves with $\omega\sim10^{11}$–10^{12} s^{-1}. On the assumption that $\sigma=10^{17}$ s^{-1} (see, for example, Ref. 8), we obtain $\beta\approx1$ for the frequencies under consideration. Then a_{kl} and a'_{kl} are approximately equal, and for $\Sigma a_{kl}\sim\Sigma a'_{kl}=K$ we have

$$K=\frac{1}{\pi^2\varkappa^2}\sum_{k,l}\frac{R_{kl}{}^yR_{kl-1}}{R_{kl}{}^2R^2_{kl-1}[2+(4\pi^2/a^2\varkappa^2)R_{kl}{}^2]}\times J_0\left(\frac{2\pi}{\varkappa a}R_{kl}\right)J_0\left(\frac{2\pi}{\varkappa a}R_{kl-1}\right),\tag{28}$$

$\mathbf{G}_{kl}=(2\pi/a)\mathbf{R}_{kl}$. (In calculations of K the value 1 in the denominator of $\mathbf{Q}_{kl}$ is neglected in comparison with $(4\pi^2/a^2)R^2_{kl}$ so that terms with $k=l=0$ and with $k=0$ and $l=1$ should be put equal to zero in K.

The sum in Eq. (28) converges rapidly and to estimate this sum it is sufficient to retain terms in k,l up to the first root of the Bessel function, that is, $(2\pi/\varkappa a)R_{kl}\leqslant2.5$. At $\varkappa=10$ and $a=1$ it is enough to take into account the first two or three coordination layers and the coefficient $K=4.5\times10^{-2}\varkappa^{-2}$. Note that the parametric interaction is strongest between modes with neighboring k and l, for when calculating the amplitude ratio of exciting and induced waves whose k and l differ by more than 1, sums analogous to Eq. (28) obviously appear, but with a still large dephasing in the arguments of the Bessel functions, so that the values of these sums decrease sharply.

At a surface-wave velocity $\sim1.5\times10^5$ cm·s^{-1} we have $\lambda_s\approx1.5\times10^{-5}$ cm at frequencies $\omega=6\times10^{10}$ s^{-1} (at $a=1$ this corresponds to an external field $H^0\approx1$ kOe). A superconducting slab of width 0.3 cm will span of $N=2\times10^4$ vortex layers, so that although the amplitude of the inhomogeneous magnetic wave, as follows from Eq. (20), is a $2\pi K$ times less than the amplitude of the uniform alternating field, nevertheless the strength of the induced magnetic excitation for such a slab will be $(2\pi KN)^2$ times greater than the "edge excitation," that is, larger by three orders of magnitude for the slab in question.

A slab with these parameters can be also used to detect Rayleigh surface waves in concentrated, rare earth (RE) paramagnetic materials. It is known (Ref. 9) that in a magnetic field H^0 a significant magnetostriction is observed in concentrated rare-earth paramagnetic substances and is due to the strong spin-phonon interaction of the RE ions. A term $\sim0.5\,deH^2$ appears therefore in the free-energy density of these substances, where e is the strain tensor with components $|d|\approx0.5$. It can be shown that the magnetostatic field, accompanying a Rayleigh wave is $h_{01}\sim8\pi^2dH\lambda_s^{-1}U_0$ (U_0 is the amplitude of the displacements in the sound wave). Thus for the Van Vleck paramagnet $LiTmF_4$ at a Rayleigh-wave acoustic-energy flux density $\sim1W$·cm^{-2} ($\omega=6\times10^{10}$ s^{-1}) we have $h_{01}\approx0.1$ Oe. This means that a superconducting slab with $\varkappa=10$ placed on the surface of a paramagnetic substance will induce a homogeneous alternating field $\sim10^{-5}$ Oe. Such a field at these frequencies is easily measured by the ordinary radio devices used, for example, in spin-echo work. The preceding arguments show that a superconducting disk can detect a rather weak surface wave $\sim10^{-3}$ to 10^{-4} W·cm^{-2}. For higher frequencies it may even be possible to use superconductors with $\varkappa=50$ in stronger external fields ($a=0.2$).

Finally we note two more important properties of the system under study: the possibility of continuously varying the frequency of the excited sound (by changing the value of a by means of H^0), and the absence of acoustic contact between the superconductor and the paramagnetic material, a very important property at such high frequencies.

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APPENDIX

Finding the coefficients Q^x_{kl}, Q^y_{kl}

We introduce the following notation: $K_n(x)$ is a Hankel function of imaginary argument and of order n, and $J_n(x)$ is a Bessel function of order n.

At $\varkappa \gg 1$ and $a\varkappa > 1$ a model can be used of a vortex with a normal core, so that the magnetic field of single vortex centered about the z axis has the form:

$$\mathbf{h}(0, 0, h), \quad h=\varkappa^{-1}K_0(\rho)$$

at $\rho > \varkappa^{-1}$ and $h = \varkappa^{-1}\ln\varkappa$ at $\rho \leqslant \varkappa^{-1}$ ($\rho = (x^2+y^2)^{1/2}$). For the vector $\mathbf{q}$ of a single vortex ($\mathbf{h} = \text{curl}\,\mathbf{q}$) we obtain

$$\mathbf{q}=\mathbf{q}^n\left(-\frac{\ln\varkappa}{\varkappa}y, \frac{\ln\varkappa}{\varkappa}x, 0\right), \quad \rho\leqslant\varkappa^{-1},$$

$$\mathbf{q}=\mathbf{q}^s\left(\frac{K_1(\rho)y}{\varkappa\rho}, -\frac{K_1(\rho)x}{\varkappa\rho}, 0\right), \quad \rho>\varkappa^{-1}. \qquad \text{(A1)}$$

In the approximation considered, the vector Q^0 is equal to

$$\mathbf{Q}^0(\rho)=\sum_{n,m=-\infty}^{\infty}\mathbf{q}(|\boldsymbol{\rho}-n\mathbf{a}-m\mathbf{b}|),$$

a and b are defined in Sec. 1, and n and m are whole numbers and 0. For the Fourier-components $\mathbf{Q}_{kl}$ of the vector $\mathbf{Q}^0$ we have then

$$\mathbf{Q}_{kl}=S_0^{-1}\iint\mathbf{q}(\rho)\exp(-i\mathbf{G}_{kl}\boldsymbol{\rho})\,dx\,dy, \qquad \text{(A2)}$$

S_0 is the area of the surface $z=0$ per vortex. It is convenient to continue the calculations in the polar coordinates ρ and φ. We note that

$$\mathbf{G}_{kl}\boldsymbol{\rho}=\rho G_{kl}\cos(\varphi-\varphi_{kl}).$$

Integration with respect to the angles is easily carried out (see, for example, Ref. 10). The integration with respect to ρ is expressed through $J_2(G_{kl}\varkappa^{-1})$, for the normal part of the vortex and through $J_1(G_{kl}\varkappa^{-1})$, $J_2(G_{kl}\varkappa^{-1})$, $K_1(\varkappa^{-1})$ $K_2(\varkappa^{-1})$ for the superconducting part (see, for example, Ref. 11). Using the asymptote $K_{1,2}$ at small values of the argument, the second integral can be expressed through $J_0(G_{kl}\varkappa^{-1})$. After carrying out the calculations it is evident that the contribution to $Q^{x,y}_{kl}$ from the normal part of the vortex $\sim\varkappa^{-2}\ln\varkappa$ is smaller than the contribution from the superconducting part and can be neglected. Finally we obtain

$$Q_{kl}{}^x=-\frac{2\pi i}{\varkappa S_0}\frac{G_{kl}{}^y}{1+G_{kl}{}^2}J_0\left(\frac{G_{kl}}{\varkappa}\right), \qquad \text{(A3)}$$

$$Q_{kl}{}^y=\frac{2\pi i}{\varkappa S_0}\frac{G_{kl}{}^x}{1+G_{kl}{}^2}J_0\left(\frac{G_{kl}}{\varkappa}\right). \qquad \text{(A4)}$$

We note that $Q^x_{kl} = -Q^x_{-k-l}, Q^y_{kl} = -Q^y_{-k-l}$.

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