Theory of emission of radiation by an ultrarelativistic charged particle in a longitudinal magnetic field in matter

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A consistent theory of the emission of radiation by an ultrarelativistic classical charged particle is constructed for the case when the particle traveling in a medium subjected to a longitudinal magnetic field is scattered elastically by the atoms in this medium. The spectral and angular distribution of the intensity of the emitted radiation is found for wide ranges of the parameters such as the energy of the particle, field intensity, radiation frequency, etc. The spectrum depends strongly on the duration of motion of the charge in the medium and on its scattering properties. As the particle penetrates the medium, the maximum of the spectral density of the radiation intensity shifts increasingly to the hard range. At sufficiently low frequencies there is a characteristic narrowing of the cone of characteristic angles where synchrotron radiation is concentrated. Bremsstrahlung is then suppressed. However, at very high frequencies the influence of a magnetic field on the spectrum is unimportant and the intensity of the radiation is governed mainly by the bremsstrahlung mechanism, i.e., by the scattering of the charge on the atoms of the medium. The spectral and angular distributions of the radiation energy of such a particle are determined for different ranges of frequencies, angles, and thicknesses of the scattering medium.

1. INTRODUCTION

We shall develop a theory of emission of radiation by an ultrarelativistic classical charged particle traveling in a longitudinal magnetic field **H** in a medium where the particle experiences elastic collisions with atoms.

Synchrotron radiation (SR) in vacuum has been investigated in detail (see, for example, Refs. 1 and 2). However, in a real situation the motion of a radiating particle occurs always in some scattering medium or another. Obviously, collisions of a charge with atoms in this medium alter radically both the frequency and the angular distribution of the intensity of SR.

Moreover, the passage of a charged particle through matter is accompanied by the generation of bremsstrahlung (BS) and the formation time[1)] of BS is macroscopically long in the ultrarelativistic case.[4] Consequently, we encounter the problem of the influence of a magnetic field on the BS spectrum, which may be significant if the field **H** is sufficiently strong.

The case of a longitudinal **H** (when a particle enters matter at a velocity $\mathbf{v}_0$ parallel to the direction of **H**) is special because this is precisely the case in which the mutual influence of the medium and the magnetic field on the spectrum of the emitted radiation is manifested most strikingly. This is due to the fact that for $\mathbf{v}_0 \| \mathbf{H}$ the origin of SR is the scattering which not only disturbs the coherence of the emission of photons in the magnetic field, but also suppresses the radiation.

This is true of bremsstrahlung in a longitudinal field **H**: only if $\mathbf{v}_0 \| \mathbf{H}$ are all the characteristic angles of the problem of the same order of magnitude [the angle ξ in which an ultrarelativistic particle emits as a result of a single collision; the multiple scattering angle $(qt)^{1/2}$, where q is the average of the square of the multiple scattering angle per unit length; the characteristic angle $\Delta\varphi$ of rotation of a particle by a magnetic field].

The radiation process considered by us occurs in practice in astrophysical objects[5] whose the rapid rotation accelerates particles to ultrarelativistic energies along **H**. In particular, this applies to pulsars, the emission spectra of which are formed as a result of interaction of charge carriers with the surface of a star in a longitudinal magnetic field.[6,7]

We shall develop a consistent theory of the emission of radiation by an ultrarelativistic classical particle in a medium subjected to a longitudinal magnetic field. We shall obtain the exact relationships governing the frequency and angular distributions of the intensity $dI_{n,\omega}(t)$ of the radiation emitted by such a particle in the case of elastic multiple scattering. The spectrum found in this way depends strongly on the time t of motion of a charge in a medium and on the scattering properties of this medium throughout the full range of frequencies ω. In particular, as a particle penetrates deeper into a medium, the maximum of the spectral density of the radiation intensity shifts to the range of increasingly harder frequencies. Strongly nonlinear dependences of the spectral and angular distributions of the radiation on the total duration of motion of a particle in a medium are obtained. In the long-wavelength part of the spectrum the cone of characteristic angle where the radiation is concentrated becomes narrower. At sufficiently low frequencies it is found that BS is suppressed because of bending of the particle trajectory by a magnetic field.

However, in the case of very short wavelengths the influence of a magnetic field is unimportant and the radiation intensity is governed mainly by the BS mechanism, which is the scattering of a charge on atoms in a medium.

We shall derive formulas describing the spectral and angular distributions of the energy of the radiation emitted by a particle in various ranges of the frequencies, angles, and thicknesses of the scattering medium.

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2. FORMULATION OF THE PROBLEM. FREQUENCY AND ANGULAR DISTRIBUTIONS OF THE RADIATION INTENSITY

An ultrarelativistic ($E \gg m$) classical ($E \gg \omega$) particle (E, m, and e are, respectively, the energy, mass, and the charge of the particle; $\hbar = c = 1$) enters at a moment $t = 0$ a material medium with randomly distributed scatterers and the velocity of this particle in $\mathbf{v}_0$ parallel to a homogeneous magnetic field $\mathbf{H}$. Elastic scattering by atoms in the medium causes the particle to emit BS and to acqiure a velocity component transverse to the direction of $\mathbf{H}$ and this gives rise to SR.

The energy $E_{\mathbf{n},\omega}$ emitted by a charge traveling in vacuum along the direction n within an angle $d\Omega$ at frequencies in the range $d\omega$ is described by[8]

$$E_{\mathbf{n},\omega}\, d\Omega\, d\omega = \frac{e^2\omega^2}{4\pi^2} \int dt_1 \int dt_2 \exp\{i\omega(t_1-t_2) - i\mathbf{k}[\mathbf{r}(t_1)-\mathbf{r}(t_2)]\} \times [\mathbf{n}, \mathbf{v}(t_1)][\mathbf{n}, \mathbf{v}(t_2)]\, d\Omega\, d\omega, \quad (1)$$

where $\mathbf{r}(t)$ is the radius vector of this charge (particle) at a time t; $\mathbf{v}(t)$ is its velocity; $\mathbf{k} = \mathbf{n}\omega$ is the wave vector of the radiation field. Following Ref. 4, we shall introduce new variables $t = t_1$ and $\tau = t_2 - t_1$. The times t and τ have quite clear physical meaning: t is the moment of emission of a photon and τ is the photon formation time. The intensity of the emitted radiation at any momtent t is given by the integral with respect to t, obtained after this substitution is made in Eq. (1).

The observed radiation intensity is found, as in Ref. 9, by averaging $dI_{\mathbf{n},\omega}$ over all possible trajectories of a particle in a scattering medium with conditional probabilities $W_1(t)$ and $W_2(t+\tau)$ (Ref. 10) and its value is given by

$$dI_{\mathbf{n},\omega} = \frac{e^2\omega^2}{2\pi^2} \operatorname{Re} \int_0^{T-t} d\tau \int d\mathbf{v}\, d\mathbf{v}'\, d\mathbf{r}\, d\mathbf{r}' \exp[-i\omega\tau + i\mathbf{k}(\mathbf{r}'-\mathbf{r})] \times [\mathbf{n}\mathbf{v}][\mathbf{n}\mathbf{v}'] \quad (2)$$
$$\times W_1(\mathbf{r}, \mathbf{v}, t) W_2(\mathbf{r}', \mathbf{v}', t+\tau; \mathbf{r}, \mathbf{v}, t)\, d\Omega\, d\omega.$$

Here, $W_2(\mathbf{r}', \mathbf{v}', t+\tau; \mathbf{r}, \mathbf{v}, t)$ is the probability that a particle (scattered elastically by atoms in a medium in a homogeneous magnetic field) has the coordinate $\mathbf{r}'$ and the velocity $\mathbf{v}'$ at a moment $t+\tau$ on condition that at a moment t these quantities had the values $\mathbf{r}$ and $\mathbf{v}$, respectively. We then obtain $W_1 = W_2(t=0; \tau \to t)$. The functions W_1 and W_2 satisfy the Fokker-Planck equation. The solution of this equation is given in the Appendix. In Eq. (2) the quantity t is the total duration of motion of the investigated particle in the medium. We shall assume that $\tau \ll T \to \infty$ and replace the upper limit in Eq. (2) with infinity.

Conversion to the comoving reference system, in which the velocity vector $\mathbf{v}(T)$ of a particle is at any moment directed along the Z axis, makes it possible to carry out integration with respect to $\mathbf{r}$, $\mathbf{r}'$, $\mathbf{v}$, and $\mathbf{v}'$ in accordance with Eq. (2). Consequently, the frequency and angluar distributions of the intensity of the radiation are described by

$$\frac{dI_{\mathbf{n},\omega}}{d\omega\, d\theta^2} = \frac{e^2\omega^2}{2\pi q t} \operatorname{Re} \int_0^\infty d\tau\, F(\tau) \times \exp\left\{-\frac{i\omega\xi^2}{2}\tau - \theta^2 \frac{K(\tau)}{4\alpha(a^2+\omega_H^2)^2}\right\}, \quad (3)$$

where θ is the angle of emission of a photon measured from the $\mathbf{H} \| \mathbf{v}_0$ direction; $(qt)^{1/2}$ is the average value of the square of the angle of the multiple scattering at a moment t (Ref. 11); $\omega_H = eH/E$ is the cyclotron frequency, $\xi = m/E$; $\alpha = (qt)^{-1} + i\omega \operatorname{th} a\tau/2a$; $a = (i\omega q/2)^{1/2}$. The functions $K(\tau)$ and $F(\tau)$ are described by

$$K(\tau) = -\frac{2i\omega}{qt}\Big[-\omega_H^2(a^2+\omega_H^2)\tau + a(\omega^2-a^2)\operatorname{th} a\tau - \frac{2a^2\omega_H}{\operatorname{ch} a\tau}\sin\omega_H\tau\Big] + \frac{\omega^2\omega_H^2}{a}[-(a^2+\omega_H^2)\tau \operatorname{th} a\tau + 2a(1-\cos\omega_H\tau/\operatorname{ch} a\tau)], \quad (4)$$

$$F(\tau) = \Big\{\cos\omega_H\tau + \theta^2\Big[\alpha\frac{a^2\cos\omega_H\tau + \omega_H^2\operatorname{ch} a\tau}{(a^2+\omega_H^2)} - \frac{\omega^2\cos\omega_H\tau}{4\alpha(a^2+\omega_H^2)^2\operatorname{ch}^2 a\tau} \times (2\omega_H^2 + (a^2+\omega_H^2)\operatorname{sh}^2 a\tau - 2\omega_H^2\cos\omega_H\tau\operatorname{ch} a\tau + 2a\omega_H\sin\omega_H\tau\operatorname{sh} a\tau)$$
$$- i\frac{\omega}{(a^2+\omega_H^2)\operatorname{ch} a\tau}(\omega_H^2\operatorname{ch} a\tau + a^2\cos\omega_H\tau) \times (\omega_H\sin\omega_H\tau + a\operatorname{sh} a\tau)\Big]\Big\}(\alpha\operatorname{ch} a\tau)^{-2}. \quad (5)$$

Integration with respect to angles θ in Eq. (3) gives an expression for the frequency distribution of the radiation intensity:

$$\frac{dI_\omega}{d\omega} = \frac{e^2\omega^2}{2\pi q t} \operatorname{Re} \int_0^\infty d\tau \frac{\exp(-{}^1/_2 i\omega\xi^2\tau)}{K^2(\tau)} G(\tau), \quad (6)$$

$$G(\tau) = \frac{16(a^2+\omega_H^2)^2}{\operatorname{ch}^2 a\tau}\Big\{\frac{i\omega\omega_H^2}{2}(a^2+\omega_H^2)\tau\cos\omega_H\tau - i\omega\omega_H^3\sin\omega_H\tau + \frac{i\omega\omega_H^2}{2a}(\omega_H^2-a^2)\operatorname{sh} a\tau + \frac{(a^2+\omega_H^2)}{qt}(a^2\cos\omega_H\tau + \omega_H^2\operatorname{ch} a\tau)\Big\}. \quad (7)$$

The relationships (3)–(7) describe completely the spectral and angular distributions of the radiation of an ultrarelativistic charged classical particle in a longitudinal magnetic field in matter for arbitrary values of $\mathbf{H}$ and $E \gg \omega$.

The integrand in Eq. (7) has poles in the plane of the complex variable τ which are zeros of the function $K(\tau)$. These singularities govern the formation time of a photon.

3. FREQUENCY SPECTRUM OF SYNCHROTRON RADIATION IN MATTER

We shall expand Eqs. (4), (6), and (7) in powers of a small parameter $|a|\tau \ll 1$. Then, the zeroth term of the expansion of Eq. (6) determines the frequency spectrum of SR in the case when a particle traveling up to the moment t is scattered in a medium and the scattering is ignored during the photon emission time τ. In this approximation, we have

$$K(\tau)=\omega^2\omega_H^2[i\beta x-x^2+2(1-\cos x)],$$
$$\beta=2\omega_H/\omega qt,\quad x=\omega_H\tau.$$

The poles of the integrand of Eq. (6) [zeros of $K(\tau)$] are governed by the equation

$$\beta z_0+z_0^2+2(1-\mathrm{ch}\, z_0)=0,\quad z_0=ix,\quad z_0\neq 0, \tag{8}$$

an analysis of which shows (see Fig. 1) that there is only one root of Eq. (8) and it is located on the imaginary axis in the complex plane τ. Taking the residue of Eq. (6) at the point z_0 we obtain the general expression for the spectral density of SR in the case when the scattering of particles by atoms in a medium does not affect the formation of an SR photon:

$$\frac{dI_\omega}{d\omega}=\frac{2e^2\omega_H\beta\exp(-\xi^2z_0/\beta qt)}{(2z_0+\beta-2\,\mathrm{sh}\, z_0)^2}\left\{\frac{\xi^2}{qt\beta}(\beta+z_0+z_0\,\mathrm{ch}\, z_0-2\,\mathrm{sh}\, z_0)\right.$$
$$\left.+\mathrm{ch}\, z_0-z_0\,\mathrm{sh}\, z_0-1+\frac{2(1-\mathrm{ch}\, z_0)(\beta+z_0+z_0\,\mathrm{ch}\, z_0-2\,\mathrm{sh}\, z_0)}{2z_0+\beta-2\,\mathrm{sh}\, z_0}\right\}. \tag{9}$$

The root of Eq. (8) obtained for small values of $\beta\ll 1$ and, consequently, for $z_0\ll 1$ it is $z_0=(12\beta)^{1/2}$, i.e., the SR photon formation time is $\tau_{H_s}=2\cdot 3^{1/3}(\omega\omega_H^2qt)^{-1/3}$ and the intensity of SR radiation is

$$\frac{dI_\omega}{d\omega}=\frac{1}{3}e^2\omega\xi\exp\left[-\gamma+\frac{2}{15}\frac{\xi^2}{qt}\right]\left\{1+2\gamma^{-1}+\frac{2\cdot 3^{2/3}}{15}\left(\frac{\omega_H}{\omega qt}\right)^{2/3}\right.$$
$$\left.-\frac{14\cdot 3^{1/3}}{45}\left(\frac{\omega_H}{\omega qt}\right)^{4/3}\left(\frac{qt}{\xi^2}\right)\right\}, \tag{10}$$

where

$$\frac{\omega_H}{qt}\ll\omega\ll\frac{\omega_H(\omega_Ht)^3}{qt},\quad \gamma=\left(\frac{\omega}{\omega_t}\right)^{2/3},\quad \omega_t=3^{-1/2}\omega_H\xi^{-3}(qt)^{3/2}.$$

The spectral distribution of Eq. (10) corresponds to the emission of an SR photon by a charge in a small section of the trajectory of motion in a medium subjected to a magnetic field ($\Delta\varphi\sim\omega_H\tau_{H_s}\ll 1$) and can be found by averaging Eq. (2.21) of Ref. 1 using the Fermi distribution function for multiple low-angle scattering.[11] The maximum of Eq. (10) lies at a frequency $\omega_{ms}\approx 0.78\omega$ and it shifts increasingly to harder frequencies in accordance with the law $(qt)^{1/2}$ as the particle crosses matter.

It is interesting to note that the dependence of ω_{ms} on H and t in Eq. (10) is the same as in the case of SR in vacuum,[1,2] whereas the spectral density of the radiation intensity ($dI_\omega/d\omega$), described by Eq. (10) and found in Refs. 1 and 2, depends differently on H and t. Moreover, whereas approximate averaging over the distribution of atoms in a medium gives a spectral distribution of the intensity proportional to $\exp(-2\omega/3^{1/2}\omega_t)$ (Ref. 12), a consistent averaging using conditional probabilities found from the transport equation gives the dependence $\sim\exp[-(\omega/\omega_t)^{2/3}]$.

If $\gamma\gg 1$, we find that

$$dI_\omega/d\omega\propto\xi\omega\exp[-(\omega/\omega_t)^{2/3}],$$

but if $\gamma\ll 1$, then

$$dI_\omega/d\omega\propto\xi\omega^{1/3}\omega_t^{2/3}.$$

The frequency dependence of $dI_\omega/d\omega$ given by Eq. (10) is shown in Fig. 2.

If $\beta\gg 1$, then retaining in Eq. (8) the main term for the $z_0\gg 1$ case, we obtain

$$z_0=\tau_{Hl}\omega_H=\ln[\beta\ln\beta],\ \text{i.e.,}\ \Delta\varphi=\omega_H\tau_{Hl}\gg 1.$$

If in the braces of Eq. (9) we retain only the highest terms of the expansion $\propto\exp(-z_0)(z_0\gg 1)$, we find that the radiation intensity is described by

$$\frac{dI_\omega}{d\omega}=\frac{e^2\omega\xi^2}{2}\{\beta\ln\beta\}^{-\xi^2/qt\beta},\quad \omega\ll\min\left\{\frac{\omega_H}{qt};\frac{\omega_H^2}{q\ln^2\beta}\right\}. \tag{11}$$

The distribution (11) has a maximum at a frequency ω_{ml} defined by the equation

$$\ln(2\omega_H/qt\omega_{ml})=qt\xi^{-2}(2\omega_H/qt\omega_{ml}),$$

and in the limit $\omega\to 0$ the SR intensity vanishes in accordnace with the linear law $dI_\omega/d\omega\propto\omega$ (see Fig. 2).

It should be noted that the condition $|a|\tau\ll 1$ is always satisfied if the frequency is sufficient low. However, since $|a|=2^{-1/2}(q\omega)^{1/2}\propto\tau_q^{-1}$, where τ_q is the formation time of radiation in a dense medium (Ref. 4), it follows that in the limit $\omega\to 0$ we have $\tau_{Hl}\ll\tau_q$, i.e., the radiation is determined solely by the magnetic-field-induced bending of the particle trajectory.

An analysis of the general expression (9) shows that the spectral density of the radiation intensity always has one maximum the position of which (and, consequently, the nature of the spectrum) is governed by the parameter ξ^2/qt. If $\xi^2/qt\ll 1$, the maximum of Eq. (9) lies at a frequency $\omega_m=\omega_{ms}\approx 0.87\omega_t$. Then, in the range of high frequencies $\omega\gtrsim\omega_{ms}$, we have

$$dI_\omega/d\omega\propto\omega\exp[-(\omega/\omega_t)^{2/3}],$$

whereas if $\omega\lesssim\omega_{ms}$, then

$$dI_\omega/d\omega\propto\omega^{1/3}\omega_t^{2/3}.$$

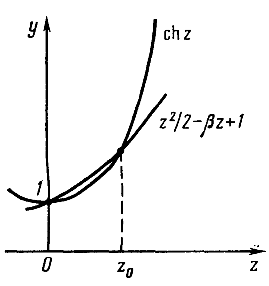

FIG. 1. Point of intersection of graphs representing the solution of Eq. (8).

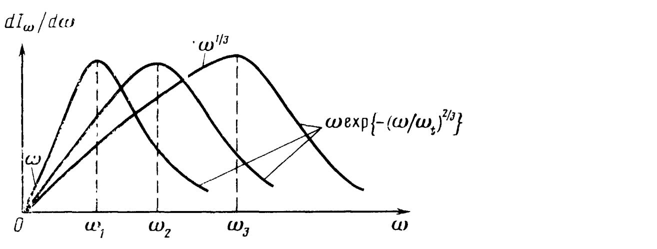

FIG. 2. Frequency dependences of $dI_\omega/d\omega$. The points $\omega_1=0.04\omega_H/qt$, $\omega_2=0.64\omega_H/qT$, and $\omega_3=14.22\omega_H/qt$ correspond to maxima of the spectral distributions of the SR intensities in the cases $\xi^2/qt=10$, $\xi^2/qt=1$, and $\xi^2/qt=0.1$, respectively.

The long-wavelength part of the spectrum is ($\omega \ll \omega_{ms}$) is however described by the distribution (11).

In the opposite limiting case when $\xi^2/qt \gg 1$ the spectrum of Eq. (9) has a maximum at a frequency $\omega_m = \omega_{ml}$ and the distribution in the vicinity of $\omega \approx \omega_{ml}$ and right down to low frequencies $\omega \lesssim \omega_H/qt$ is given by Eq. (11). On the other hand, the short-wavelength part of the spectrum ($\omega \gtrsim \omega_t$) is described by Eq. (10).

If $\xi^2 \sim qt$, the distribution of the SR intensity is given by the general relationship (9) and its maximum is located between ω_{ms} and ω_{ml}. The results of a numerical calculation of the SR spectrum in accordance with Eq. (9) for the cases when $\xi^2/qt = 0$, 1, 1, or 10 are plotted in Fig. 2.

4. ANGULAR DISTRIBUTION OF SYNCHROTON RADIATION INTENSITY

The frequency and angular distributions of the SR radiation intensity in the case when multiple scattering of a particle during the photon emission time is ignored can be described by the zeroth term of the expansion (3)–(5) in terms of the parameter $|a|\tau \ll 1$ and it is given by

$$\frac{dI_{n,\omega}}{d\omega\, d\theta^2} = \frac{2e^2\omega_H}{\pi qt} \operatorname{Re} \int_0^\infty dx \frac{\exp(-{}^1/_2 i\nu\xi^2 x)}{(\beta+ix)^2} \exp\left[-\frac{\nu\theta^2}{2} \times \frac{i\beta x - x^2 + 2(1-\cos x)}{(\beta+ix)}\right] \left\{\cos x + \frac{\nu\theta^2}{(\beta+ix)} \times [{}^1/_2\beta^2 + i\beta x - {}^1/_2 x^2 - i\sin x(\beta+ix) - \cos x(1-\cos x)]\right\}, \quad \nu = \frac{\omega}{\omega_H}. \quad (12)$$

If the radiation is formed in a small part of the trajectory of a particle ($\beta \ll 1$, i.e., $\Delta\varphi \sim \omega_H \tau_{Hs} \ll 1$), the main contribution of Eq. (12) comes from small values of x. Expanding in the integrand the preexponential factor and the argument of the exponential function up to the terms of order of x^3 inclusive, we obtain

$$\frac{dI_{n,\omega}}{d\omega\, d\theta^2} = \frac{2e^2\omega_H}{qt\pi^{1/2}} \exp\left(-\frac{\theta^2}{qt}\right) \left\{-\frac{\nu\xi^2}{2} \times \int_\Delta^\infty \Phi(z)\,dz + (\nu\theta^2)^{-1/3}\Phi(\Delta) - (\nu\theta^2)^{1/3}\Phi'(\Delta)\right\}, \quad (13)$$

where $\Delta = \nu\xi^2(\nu\theta^2)^{-1/3}$ and $\Phi(z)$ is the Airy function.[8] Integrating Eq. (13) over all the emission angles θ, we obtain an expression for the frequency spectrum of the radiation intenstity given by Eq. (10). The maximum of this expression is located at a frequency $\omega_m \approx 0.78\omega_t$. In the neighborhood of θ we have $\Delta \propto (qt/\theta^2)^{1/3}$. At high angles θ the radiation intensity is then exponentially low, whereas inside a cone with a vertex angle $\sim (qt)^{1/2}$ its distribution is given by

$$\frac{dI_{n,\omega}}{d\omega\, d\theta^2} = \frac{e^2\omega_H}{qt\pi^{1/2}} \exp\left[-\frac{\theta^2}{qt} - \frac{2}{3}\xi^3\left(\frac{\nu}{\theta}\right)\right] \{(\nu\xi\theta)^{-1/2} + (\nu\xi\theta)^{1/2}\}. \quad (14)$$

The maximum of Eq. (14) is located at angles $\theta_m(\omega) = (\xi^3 \nu qt)^{1/3}$. Hence, at frequencies $\omega \approx \omega_m$ we have $\theta_m \propto (qt)^{1/3}$, i.e., the radiation is concentrated mainly near the generators of a cone of characteristic multiple scattering angles of the investigated particle up to the moment t.

When the radiation is formed during many turns of a helical path of a particle in a magnetic field ($\beta \ll 1$, i.e., $\Delta\varphi \sim \omega_H \tau_{Hl} \gg 1$), it follows that going over in Eq. (14) to integration along the straight line $x = -i\omega_H\tau_{Hl} + z$, where $0 \leqslant z < \infty$, and then expanding both the preexponential factor and the argument of the exponential function in the integrand in terms of small quantities $z \ll 1$ up to the terms of the order of z^2 inclusive, we obtain

$$\frac{dI_{n,\omega}}{d\omega\, d\theta^2} = \frac{e^2\omega_H\nu\xi^2 \ln(\beta\ln\beta)}{4\pi^{1/2}\beta qt} \left(\frac{\nu\theta^2}{4}\ln\beta\right)^{-1/2} (\beta\ln\beta)^{-\nu\xi^2/2} \times \exp\left\{-\frac{{}^1/_2\nu(\theta^2\ln\beta - \xi^2)^2}{4\theta^2\ln\beta}\right\}, \quad \beta = \frac{2\omega_H}{\omega qt}. \quad (15)$$

The angles θ corresponding to the maximum of Eq. (15) are given by

$$\theta_m^2 = (\nu\ln\beta)^{-1}[(1+(\nu\xi^2)^2)^{1/2} - 1]; \quad (16)$$

if $\nu\xi^2 \ll 1$, the

$$\theta_m^2 = \frac{1}{2}(\nu\xi^2)\frac{\xi^2}{\ln\beta},$$

whereas if $\nu\xi^2 \gg 1$, then $\theta_m^2 = \xi^2/\ln\beta$. If follows from the last relationships that in the range of sufficiently low frequencies or on application of sufficiently strong magnetic fields ($\ln\beta \gg 1$), when the radiation is formed in many turns of a helical path in the applied magnetic field, the vertex angle of the cone of characteristic radiation emission angles is much smaller than $\theta_\xi = \xi = m/E$, and the spectrum is governed by the relationship between the angle of multiple scattering $\sim (qt)^{1/2}$ and the angle ξ within which an ultrarelativistic particle emits during a single collisional event.

5. MUTUAL INFLUENCE OF THE SCATTERING AND MAGNETIC FIELD ON THE SPECTRAL DISTRIBUTION OF RADIATION INTENSITY

The influence of multiple scattering on the SR spectrum or of the applied magnetic field on the BS spectrum is governed by the relationship between three parameters: $\tau_q = \omega^{-1}\xi^{-2}$ is the emission time of a BS photon in a single collision with a scatterer[4,9]; $\tau_q = (q\omega)^{-1/2}$ is the photon emission time in a dense medium[4,9]; τ_H is the emission time of a particle in a magnetic field [$\tau_H = \tau_{Hs} = 2\cdot 3^{1/3}(\omega\omega_H^2 qt)^{-1/3}$ if $\omega_H\tau_H \ll 1$, whereas $\tau_{Hl} = \omega_H^{-1}\ln(\beta\ln\beta)$, if $\omega_H\tau_H \gg 1$.]

We shall first consider the range of high frequencies $\omega \gg \omega_0 = q\xi^{-4}$ when the influence of the medium on the BS spectrum can be ignored.[4] Then, in the case of low values $\beta \ll 1$, if we expand Eq. (7), governing the frequency distribution of the radiation intensity, the functions $K(\tau)$ and $G(\tau)$ in terms of $|a|\tau \ll 1$ and $\omega_H\tau \ll 1$, and if we retain terms of not higher than the fourth order of smallness in respect of these parameters, we obtain

$$\frac{dI_\omega}{d\omega} = \frac{e^2\omega\xi^2}{\pi\gamma} \operatorname{Re} \int_0^\infty \frac{dx \exp(-i\gamma x)}{(x-i\varepsilon)} F_1(\Delta_a; \Delta_H; x), \quad \varepsilon \to +0, \quad (17)$$

where

$$F_1=\Big(1-2ix^3-i\Delta_a^2x^2-\frac{19}{10}\Delta_a^2x^5-\frac{2}{3}\Delta_a^4x^4+\frac{i}{6}\Delta_a^2\Delta_H^2x^4$$
$$+\frac{37i}{30}\Delta_a^4x^7+\frac{6}{5}\Delta_a^2\Delta_H^2x^7+\frac{6i}{5}\Delta_H^2x^5\Big)\Big[i-x^3+\frac{\Delta_a^2x^2}{3}$$
$$+\frac{13i}{60}\Delta_a^2x^5+\frac{2}{15}\Delta_H^2x^5-\frac{2i}{15}\Delta_a^4x^4-\frac{1}{15}\Delta_a^2\Delta_H^2x^4\Big]^{-2},$$
$$i\Delta_a^2=\frac{4a^2}{\omega^2}\gamma^2\xi^{-4},\qquad \Delta_H^2=\frac{\omega_H^2}{\omega^2}\gamma^2\xi^{-4},\qquad \Delta_{a,H}\ll 1$$

are the parameters of the expansion.

The function $F_1(\Delta_a;\Delta_H;x)$ in Eq. (17) has a pole at

$$x_0=-i-\frac{1}{30}\Delta_a^2+\frac{2i}{45}\Delta_H^2-\left(\frac{4}{45}\right)^2\Delta_H^4$$
$$+\frac{3}{50}\Delta_a^4-\frac{11i}{225}\Delta_a^2\Delta_H^2,$$

which because of the scattering is shifted to the left half-plane from the imaginary axis. The SR photon formation time is $\tau_H=\tau_{Hs}|\mathrm{Im}\,x_0|$.

The spectral density of the radiation intensity given by Eq. (17) in the case of arbitrary values of γ can be represented by a sum of two terms:

$$dI_\omega/d\omega=(dI_\omega/d\omega)_{\rm SR}+(dI_\omega/d\omega)_{\rm BS}, \tag{18}$$

the first of which described the spectrum of SR "modified" by scattering, whereas the second represents BS under conditions of bending of the trajectory of a multiply scattered particle by a magnetic field. In the case of arbitrary values of $\gamma\gtrsim 1$, we have

$$\frac{dI_\omega}{d\omega}=\left(\frac{dI_\omega}{d\omega}\right)_{\rm SR}=\frac{e^2\omega\xi^2}{3\gamma}(2+\gamma)\exp[-\gamma-\gamma\delta],$$
$$\delta=-\frac{2}{45}\Delta_H^2+\frac{3}{50}\Delta_a^4-\left(\frac{4}{45}\right)^2\Delta_H^4, \tag{19}$$

and $(dI_\omega/d\omega)_{\rm BS}$ makes only a small correction to Eq. (19). The factor

$$\exp(-\gamma\delta)\propto\exp\left(-\frac{6}{25}q^2\omega^{-2}\xi^{-4}\gamma^5\right)$$

in Eq. (19) determines the degree of suppression of SR because of multiple scattering of a particle in a medium.

However, if $\gamma\gg 1$, when Eq. (19) is exponentially small, the spectral density of the BS intensity is given by the expression

$$\frac{dI_\omega}{d\omega}=\left(\frac{dI_\omega}{d\omega}\right)_{\rm BS}$$
$$=\frac{2e^2q}{3\pi\xi^2}\left\{1+\frac{129}{5\gamma^3}+\frac{15408}{\gamma^6}+\frac{\Delta_H^2}{5\gamma^2}+\frac{327\cdot 9!}{10\gamma^{10}}\right\}, \tag{20}$$

where the first term is the BS intensity [Bethe-Heitler formula (Ref. 4)] and the remaining terms are the corrections to this intensity due to the presence of a magnetic field. Finally, in the limit $\omega\to 0$, BS is suppressed and the SR intensity is described by Eq. (11).

It is not possible to obtain simple analytic expressions from Eqs. (6) and (7) for other limiting cases. However, an analysis based on a comparison of the coherence times,

$$\tau_e=\omega^{-1}\xi^{-2},\ \tau_q=(q\omega)^{-1/2},\ \tau_{Hs}$$
$$=2\cdot 3^{1/3}(\omega\omega_H^2qt)^{-1/3},\ \text{and}\ \tau_{Hl}=\ln(\beta\ln\beta)\omega_H^{-1},$$

makes it possible to understand the radiation pattern.

At very high frequencies $\omega\gg\max(\omega_{ms},\omega_{ml})$ (corresponding to the minimum coherence time τ_e) it is BS that predominates and its intensity is described by the Bethe-Heitler formula.

In the far long-wavelength region where $\omega\to 0$ (corresponding to the minimum coherence time τ_{Hl}), the emission spectrum becomes of the SR type. The frequency distribution of the radiation intensity is described by Eq. (11).

At arbitrary values of ω the frequency distribution of the radiation intensity is described by the general relationships (6) and (7).

6. SPECTRAL AND ANGULAR DISTRIBUTIONS OF THE RADIATION ENERGY EMITTED BY A PARTICLE

The spectral density of the radiation energy of a particle $E_\omega(T)$ in the case of an arbitrary thickness T of the scattering medium (naturally subject to the inequality $qT\ll 1$) can be found by direct integration of Eq. (6) with respect to the variable t between the limits $(0, T)$.

In the case when radiation is formed in a large part of the trajectory of a particle, i.e., if $\beta_T=2\omega_H/\omega qT\gg 1$, we then have

$$E_\omega(T)=\frac{e^2\omega\xi^2}{2}\frac{T}{(1+\xi^2\omega/2\omega_H)}\exp\left\{-\xi^2\frac{\omega}{2\omega_H}\ln\beta_T\right\}$$
$$\times\left\{1+O\left(\frac{1}{\ln\beta_T}\right)\right\}. \tag{21}$$

In the limit $\omega\to 0$ ($\beta_T\to\infty$) the spectral density of the radiation energy given by Eq. (21) vanishes linearly

$$E_\omega(T)=\frac{e^2\omega\xi^2}{2}T\exp\left\{-\xi^2\frac{\omega}{2\omega_H}\ln\beta_T\right\}\propto\omega T. \tag{22}$$

However, if the radiation is formed in a small section of a trajectory of a particle, i.e., if $\beta_T\ll 1$, then $E_\omega(T)$ can be written in the form

$$E_\omega(T)=(e^2\omega\xi^2T/\gamma_T)\{2E_5(\gamma_T)+\gamma_TE_4(\gamma_T)$$
$$+O(\beta_T\exp(-\gamma_T\beta_T^{1/2}))\}. \tag{23}$$

Here, $\gamma_T=(\omega/\omega_T)^{2/3}$; $\omega_T=3^{-1/2}\omega_H\xi^{-3}(qT)^{1/2}$; $E_p(\gamma_T)$ is the exponential integral.[13] The maximum of Eq. (23) lies at a frequency $\omega\sim\omega_T$. At low frequencies $\omega\ll\omega_T$ (i.e., if $\gamma_T\ll 1$), the asymptote of Eq. (23) gives

$$E_\omega(T)=\frac{e^2\omega\xi^2}{2\gamma_T}T\left\{1-\frac{2}{3}\gamma_T\right\},\qquad \gamma_T\ll 1. \tag{24}$$

In the opposite limiting case of high frequencies $\omega\gg\omega_T$ (i.e., if $\gamma_T\gg 1$), we find from Eq. (23) that

$$E_\omega(T)=\frac{e^2\omega\xi^2T}{\gamma_T}\exp(-\gamma_T)\propto\omega^{1/3}T^{4/3}\exp\left[-\left(\frac{3\omega^2\xi^6}{\omega_H^2qT}\right)^{1/3}\right],$$
$$\gamma_T\gg 1. \tag{25}$$

We note here that at low frequencies $\omega\ll\omega_T$ the formula for the spectral energy density (4.16) obtained by the method of estimates[4] in Ref. 12 differs from the exact formula (24) only by an unimportant numerical factor. However, at high frequencies $\omega\gg\omega_T$ the difference between the results of the present work and Ref. 12 becomes significant. For example, according to the estimate formula (4.13) of Ref. 12 we find that if $\gamma_T \gg 1$, then

$$E_\omega(T) \propto \omega^{-1} T^2 \exp[-2\omega\xi^3/3\omega_H (qT)^{1/2}],$$

whereas Eq. (25) gives

$$E_\omega(T) \propto \omega^{1/3} T^{4/3} \exp[-(3\omega^2\xi^6/\omega_H^2 qT)^{1/3}].$$

The discrepancy between the results obtained in Ref. 12 and those deduced above is due to the fact that the averaging method proposed by Landau and Pomeranchuk[4] and used in Ref. 12 can be used to estimate—as pointed out by the authors themselves—only the order of magnitude of the radiation intensity (energy). In an investigation of the radiation emitted by an ultrarelativistic particle traveling in a material medium subjected to a longitudinal magnetic field this method gives the qualitatively correct dependences only in a narrow range of frequencies:

$$\omega_H \xi (qT)^{1/2} \ll \omega \ll \omega_H \xi^{-3} (qT)^{1/2} \tag{26}$$

(and fairly large thicknesses of the scattering substance: $\beta_T \ll 1$). These inequalities determine the limits of validity of the results of Ref. 12. The averaging method of Ref. 4 gives in principle incorrect results when the emission line profile is calculated for arbitrary values and ω and T and for an arbitrary angular distribution of the radiation. The approach of Ref. 4 is essentially based on an "optional" replacement of the average (over the distribution of atoms) product of the functions by the product of the averages and a replacement of the average of the sine with the sine of the average. We then obtain inaccurate expressions for the correlation functions containing products of four or more multiple scattering angles, because in our averages the phases are similar and the results may vary from zero to unity depending on their difference.

Integrating Eq. (18) over the whole particle transit time in a scattering medium, we obtain a formula which relates the spectral density of the energy of SR and BS in the case when $\beta_T \ll 1$, that allows for the mutual influence of the scattering and magnetic field on this spectral density:

$$E_\omega(T) = \frac{e^2\omega\xi^2 T}{\gamma_T} [2E_5(\gamma_T) + \gamma_T E_4(\gamma_T)] + \frac{2e^2 q}{3\pi\xi^2} T \times \left\{1 + \frac{2322}{5\gamma_T^3} + \frac{416016}{\gamma_T^6} + \frac{3}{5}\frac{\xi^2}{qT\gamma_T^3} + \frac{12753\cdot 9!}{10\gamma_T^{10}}\right\}. \tag{27}$$

The spectral and angular distributions of the radiation energy of a particle can easily be obtained by integrating Eq. (3) with respect to the variable t within the limits $(0, T)$. Thus, if $\beta_T \gg 1$, we have

$$E_{\bar{n}\omega}(T) = \frac{2e^2\omega}{q\pi^{1/2}} \left\{ -\frac{\nu\xi^2}{2} \int_\Delta^\infty \Phi(z)\,dz - (\nu\theta^2)^{-1/2}\Phi(\Delta) - (\nu\theta^2)^{1/2}\Phi'(\Delta) \right\} E_1\left(\frac{\theta^2}{qT}\right). \tag{28}$$

The behavior of $E_{\mathbf{n},\omega}(T)$ described by Eq. (28) as a function of the frequency and the emission angle is similar to the behavior of the spectral density of the radiation intensity given by Eq. (13).

7. CONCLUSIONS

We developed a theory of the emission of radiation by a classical ultrarelativistic charged particle subjected to a longitudinal magnetic field and traveling in matter. A consistent transport analysis of the motion of the particle in a medium subjected to a magnetic field yielded the exact relationships for the spectral and angular distributions of the intensity and radiation energy in the case of low-angle elastic scattering of carriers by atoms in the medium being traversed.

It was found that as a particle penetrates deeper into matter it emits increasingly harder photons and the maximum of the SR spectrum shifts toward shorter wavelengths. The angular and frequency distributions of the intensity of this radiation and energy are very different at high and low frequencies, which is due to the difference between the mechanisms of formation of SR photons in the two parts of the frequency spectrum. It was also found that in the case of sufficiently low frequencies ω the cone of the characteristic emission angles becomes narrower, compared with the emission angles $\xi \sim m/E$ of an ultrarelativistic particle which experiences single scattering.[8]

In the short-wavelength part of the spectrum $(\omega \gg \omega_0 = q\xi^{-4})$ the radiation intensity represents a sum of the intensities of scattering-modified SR and BS in the case when the trajectory of a multiply scattered particle is bent by a magnetic field.

The assumption $E \gg \omega$ leads to a limit on the energy of a particle E and on the magnetic field $\mathbf{H}$: $E \ll m^3/eH(qt)^{1/2}$. In the case of astrophysical objects when we can have $H \sim 10^{11}$–10^{12} G and an emitting particle has a mass ~ 1 MeV, the above relationship reduces to the inequality

$$E \ll (qt)^{-1/2} \cdot 10 \text{ eV} \quad ((qt)^{1/2} \ll 1).$$

The mechanism of emission of radiation by a particle discussed by us presupposes the absence of quantization of its transverse motion. In this sense it is an alternative to the recently discussed[14] magneto-Coulomb emission mechanism. There is no quantization, i.e., as a result of the scattering a particle is transferred to highly excited Landau levels with $n \gg 1$ and during the process of photoemission the time is insufficient for "dropping" to the ground state if the inequality $\sigma_{01} n_i \gg \tau^{-1}$ is obeyed, where σ_{01} is the cross section for the transfer of a carrier to a Landau level after collision with an atom in a medium, τ is the lifetime of a carrier in this state in the laboratory system, and n_i is the concentration of scattering centers in matter.

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APPENDIX

In the case of small-angle multiple elastic scattering the function $W(\mathbf{r}, \mathbf{v}, t_0 + \tau, \mathbf{r}_0, \mathbf{v}_0, \mathbf{t}_0)$ satisfies the Fokker-Planck equation

$$\frac{\partial W}{\partial \tau} + \mathbf{v}\frac{\partial W}{\partial \mathbf{r}} + e[\mathbf{vH}]\frac{\partial W}{\partial \mathbf{v}} = \frac{qv_0}{4}\Delta_{\theta,\varphi} W, \tag{29}$$

where $\Delta_{\theta,\varphi}$ is the angular part of the Laplace operator in the $\mathbf{v}$ space.

Applying the Fourier transformation to W in

$$\Phi(\mathbf{v}, \tau, \mathbf{v}_0, \mathbf{k}) = \int d(\mathbf{r}-\mathbf{r}_0) \exp[i\mathbf{k}(\mathbf{r}-\mathbf{r})] W(\mathbf{r}, \mathbf{v}, t_0+\tau; \mathbf{r}_0, \mathbf{v}_0, t_0)$$

and going over to the space of velocities to the comoving reference system[15] in which the particle velocity in the absence of scattering is at all times directed along the Z axis, we find that the function $\Phi(\mathbf{v}, \tau, \mathbf{v}_0, \mathbf{k})$ obeys the equation

$$\partial\Phi/\partial\tau - ikv_0[\sin\theta_0\sin\theta_k\cos\gamma_k(\tau)+\cos\theta_0\cos\theta_k]\Phi$$
$$-ikv_0\eta_x[\cos\theta_0\sin\theta_k\cos\gamma_k(\tau)-\sin\theta_0\cos\theta_k]\Phi - ikv_0\eta_y\Phi$$
$$\times\sin\theta_k\sin\gamma_k(\tau)+{}^1/_2ikv_0(\eta_x^2+\eta_y^2)\cos(\theta_0-\theta_k)\Phi$$
$$={}^1/_4qv_0(\partial^2\Phi/\partial\eta_x^2+\partial^2\Phi/\partial\eta_y^2), \tag{30}$$

where the terms $\sim\eta_x^2$ and η_y^2 describe bending of the particle trajectory. It is necessary to allow for these terms in discussing the problems of emission of radiation. The initial condition for Eq. (3) is

$$\Phi(0,\mathbf{k},\boldsymbol{\eta}) = \delta\left(\eta_z - 1 + \frac{\eta_x^2+\eta_y^2}{2}\right)\delta(\eta_x)\delta(\eta_y), \qquad \theta_0<1. \tag{31}$$

We have introduced here $\gamma_k(\tau) = -\gamma_0(\tau)+\varphi_k = \varphi_k - \varphi_0 + \omega_H\tau$; φ_k and θ_k are the polar and azimuthal angles in the $\mathbf{k}$ space; φ_0 and θ_0 are similar angles in the $\mathbf{v}$ space at a moment $\tau=0$; $\boldsymbol{\eta}(\tau)$ and $\mathbf{v}(\tau)$ are related by the transformation $\boldsymbol{\eta} = v_0^{-1}\hat{T}(\tau)\mathbf{v}$, where $\hat{T}(\tau)$ is the matrix of rotation by an angle (γ_0, θ_0) in the velocity space:

$$\hat{T}(\tau) = \begin{bmatrix} \cos\gamma_0\cos\theta_0 & \sin\gamma_0\cos\theta_0 & -\sin\theta_0 \\ -\sin\gamma_0 & \cos\gamma_0 & 0 \\ \cos\gamma_0\sin\theta_0 & \sin\gamma_0\sin\theta_0 & \cos\theta_0 \end{bmatrix}.$$

Solving Eq. (30) with the initial condition (31), we find (as demonstrated by direct substitution) that

$$\Phi(\tau,\mathbf{k},\boldsymbol{\eta}) = \frac{c\delta(\eta_z-1+(\eta_x^2+\eta_y^2)/2)}{\pi v_0^4 q\,\mathrm{sh}\,a\tau}$$
$$\times\exp\{-C(\eta_x^2+\eta_y^2)-A\eta_x-B\eta_y+Q\},$$

$$c=v_0[{}^1/_2iqk\cos(\theta_0-\theta_k)]^{1/2},\quad C=(a\,\mathrm{cth}\,a\tau)q^{-1},$$

$$B=\frac{iv_0k\sin\theta_k}{(a^2+\omega_H^2)}\Big\{\omega_H\cos(\omega_H\tau+\varphi_k-\varphi_0)-a\,\mathrm{cth}\,a\tau\sin(\omega_H\tau$$
$$+\varphi_k-\varphi_0)+\frac{a\sin(\varphi_k-\varphi_0)}{\mathrm{sh}\,a\tau}\Big\},$$

$$A=\cos\theta_0\frac{dB}{d(\varphi_k-\varphi_0)}+\frac{iv_0k\cos\theta_k\sin\theta_0(\mathrm{ch}\,a\tau-1)}{a\,\mathrm{sh}\,a\tau},$$

and Q is related to A and B by the simple expression

$$dQ/d\tau={}^1/_4qv_0(A^2+B^2)$$
$$+ikv_0[\sin\theta_0\sin\theta_k\cos\gamma_k(\tau)+\cos\theta_0\cos\theta_k].$$

[1)]The photon formation time or the coherence time is the interval during which a photon can travel away from the charge a distance of the order of the emission wavelength.[3]

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