Space-time interference of spontaneously emitted photons

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The space-time interference pattern observed in the detection of one or two photons from a system of two simultaneously excited and spontaneously radiating atoms is considered. It is shown that the well-known Brown-Twiss effect occurs in the absence of interaction between the emitters, and that this effect is accompanied by "classical" interference when the emitters interact with each other. The effect on the photon interference of a time lag between the initial moments of excitation of the atoms is considered.

The great interest in the results of photon interference experiments in which two light sources are used, and in which the quantum properties of electromagnetic radiation are observed has not abated since the development of quantum theory. A number of papers have reported the theoretical and experimental investigations of the space-time structure of the interference pattern from two light sources as a function of the conditions under which the photons are detected and of the various characteristics of the radiation sources (see Refs. 1–9 and the references cited therein).

In the present paper we derive within the framework of the standard quantum-mechanical formalism the space-time interference pattern for photons emitted by two identical atoms. It is assumed that the two atoms are simultaneously in excited states when their excitation occurs over a time period Δt satisfying the relation $2\Gamma_0\Delta t \leqslant 1$ ($\hbar = c = 1$, Γ_0 is the natural width of the level of the isolated atom). In 1954 Dicke[2] theoretically investigated phase coherence of spontaneously and simultaneously radiating atoms. The temporal evolution of an atomic ensemble consisting of many atoms is considered in Ref. 6. The results obtained in that paper will be used below.

Let us consider as a simple model two identical atoms that are in excited states at $t = 0$, and are separated by a distance $\mathbf{R}$. Let the difference between the energies of the excited and ground states of an atom be equal to Δ. The Hamiltonian of the system of two identical atoms + the electromagnetic field of the radiation can be written in the form[2]

$$H = \sum_{i=1}^{2} H_{0i} + \sum_{i=1}^{2} \int d\mathbf{r}\, \mathbf{A}(\mathbf{r}, t) \mathbf{J}_i(\mathbf{r}, t),$$

where H_{0i} is the Hamiltonian of the ith atom describing the c.m. motion and the interaction of the electrons with the nucleus; the second term describes the interaction of the ith atom with the quantized radiation field, whose vector-potential amplitude is denoted by $\mathbf{A}(\mathbf{r},t)$; and $\mathbf{J}_i(\mathbf{r},t)$ is the transition quantum current for the ith atom.[10] In this case the atoms interact via the common radiation field, i.e., through the emission and absorption of photons. As shown in Ref. 6, the interaction is characterized by a quantity Γ_{12} that depends primarily on the distance between the atoms and the wavelength of the emitted phonons.

Let us represent the wave function of the system of two atoms + the spontaneous radiation field in the form

$$\Psi(t) = A(t)\varphi_1^*\varphi_2^* + \sum_{\mathbf{k}_1} \alpha_{\mathbf{k}_1}^{(1)}(t)\varphi_1\varphi_2^*|\mathbf{k}_1\rangle + \sum_{\mathbf{k}_2} \alpha_{\mathbf{k}_2}^{(2)}(t)\varphi_1^*\varphi_2|\mathbf{k}_2\rangle + \sum_{\mathbf{k}_1,\mathbf{k}_2} \alpha_{\mathbf{k}_1\mathbf{k}_2}(t)\varphi_1\varphi_2|\mathbf{k}_1\mathbf{k}_2\rangle, \quad (1)$$

where $\varphi_{1,2}^*$ and $\varphi_{1,2}$ are the wave functions of the atoms located at the points with coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$ (the subscripts 1 and 2) in the excited and ground states respectively; $|\mathbf{k}\rangle$ and $|\mathbf{k}_1,\mathbf{k}_2\rangle$ are respectively the wave functions of the states with one and two photons; and k is the wave number.

To determine the coefficients $A(t)$, $\alpha_{\mathbf{k}_1}^{(1)}(t)$, $\alpha_{\mathbf{k}_2}^{(2)}(t)$, and $\alpha_{\mathbf{k}_1\mathbf{k}_2}(t)$, we used the Weisskopf-Wigner method,[11] which, in particular, allows us to directly compute the natural width of a transition line. The coefficients found by this method for the initial conditions

$$A(0) = 1, \quad \alpha_{\mathbf{k}_1}^{(1)}(0) = \alpha_{\mathbf{k}_2}^{(2)}(0) = \alpha_{\mathbf{k}_1\mathbf{k}_2}(0) = 0$$

can be written in the form[6]

$$\alpha_{\mathbf{k}_1}^{(1)}(t) = H_{k_1}^{(1)} \frac{\exp[-i(\Delta - k_1)t]}{(2k_1)^{1/2}} \times \{\beta_{k_1}^{(+)}(t) + \exp[i\mathbf{k}_1(\mathbf{r}_1 - \mathbf{r}_2)]\beta_{k_1}^{(-)}(t)\},$$

$$\alpha_{\mathbf{k}_1}^{(2)}(t) = H_{k_1}^{(2)} \frac{\exp[-i(\Delta - k_1)t]}{(2k_1)^{1/2}} \times \{\beta_{k_1}^{(+)}(t) + \exp[i\mathbf{k}_1(\mathbf{r}_2 - \mathbf{r}_1)]\beta_{k_1}^{(-)}(t)\}, \quad (2)$$

$$\alpha_{\mathbf{k}_1\mathbf{k}_2}(t) = H_{k_1}H_{k_2} \frac{\exp[-i(\Delta - k_1)t]\exp[-i(\Delta - k_2)t]}{(2k_1)^{1/2}(2k_2)^{1/2}} \times \exp(-i\mathbf{k}_1\mathbf{r}_1)\exp(-i\mathbf{k}_2\mathbf{r}_2)\{\beta_{k_1k_2}^{(+)}(1 + \exp[i(\mathbf{k}_2 - \mathbf{k}_1)(\mathbf{r}_2 - \mathbf{r}_1)]) + \beta_{k_1k_2}^{(-)}(\exp[-i\mathbf{k}_1(\mathbf{r}_2 - \mathbf{r}_1)] + \exp[i\mathbf{k}_2(\mathbf{r}_2 - \mathbf{r}_1)])\},$$

where

$$\beta_{k_1}^{(\pm)}(t) = \frac{1}{2}\left\{\exp[-\Gamma_0(1-\gamma)t] \times \frac{\exp[-\Gamma_0(1-\gamma)t] - \exp[i(\Delta - k_1)t]}{\Delta - k_1 - i\Gamma_0(1-\gamma)} \pm \exp[-\Gamma_0(1-\gamma)t] \frac{\exp[-\Gamma_0(1+\gamma)t] - \exp[i(\Delta - k_1)t]}{\Delta - k_1 - i\Gamma_0(1+\gamma)}\right\}$$

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$$
\begin{aligned}
\beta^{(\pm)}_{k_1k_2}(t)=&\frac{1}{2}\Bigg\{\frac{1}{\Delta-k_1-i\Gamma_0(1-\gamma)}\\
&\times\Bigg[\frac{\exp(-2\Gamma_0t)-\exp[i(\Delta-k_1+\Delta-k_2)t]}{2\Delta-k_1-k_2-2i\Gamma_0}\\
&-\exp[i(\Delta-k_1)t]\frac{\exp[-\Gamma_0(1+\gamma)t]-\exp[i(\Delta-k_2)t]}{\Delta-k_2-i\Gamma_0(1+\gamma)}\Bigg]\\
&+\frac{1}{\Delta-k_2-i\Gamma_0(1-\gamma)}\Bigg[\frac{\exp(-2\Gamma_0t)-\exp[i(\Delta-k_1+\Delta-k_2)t]}{2\Delta-k_1-k_2-2i\Gamma_0}\\
&-\exp[i(\Delta-k_2)t]\frac{\exp[-\Gamma_0(1+\gamma)t]-\exp[i(\Delta-k_1)t]}{\Delta-k_1-i\Gamma_0(1+\gamma)}\Bigg]\Bigg\}\\
&\pm\frac{1}{2}\Bigg\{\frac{1}{\Delta-k_1-i\Gamma_0(1+\gamma)}\\
&\times\Bigg[\frac{\exp(-2\Gamma_0t)-\exp[i(\Delta-k_1-\Delta-k_2)t]}{2\Delta-k_1-k_2-2i\Gamma_0}\\
&-\exp[i(\Delta-k_1)t]\frac{\exp[-\Gamma_0(1-\gamma)t]-\exp[i(\Delta-k_2)t]}{\Delta-k_2-i\Gamma_0(1-\gamma)}\Bigg]\\
&+\frac{1}{\Delta-k_2-i\Gamma_0(1+\gamma)}\Bigg[\frac{\exp(-2\Gamma_0t)-\exp[i(\Delta-k_1+\Delta-k_2)t]}{2\Delta-k_1-k_2-2i\Gamma_0}\\
&-\exp[i(\Delta-k_2)t]\frac{\exp[-\Gamma_0(1-\gamma)t]-\exp[i(\Delta-k_1)t]}{\Delta-k_1-i\Gamma_0(1-\gamma)}\Bigg]\Bigg\};
\end{aligned}
\qquad (3)
$$

H_k is the matrix element of an atomic transition accompanied by the spontaneous emission of a photon; $\gamma\equiv\Gamma_{12}/\Gamma_0$;

$$
\begin{aligned}
\Gamma_{12}(\mathbf{r})=&3\Gamma_0\{\cos^2\theta_0[\sin(\Delta|\mathbf{r}|)-\Delta|\mathbf{r}|\cos(\Delta|\mathbf{r}|)](\Delta|\mathbf{r}|)^{-3}\\
&+{}^1/_2\sin^2\theta_0[\sin(\Delta|\mathbf{r}|)/\Delta|\mathbf{r}|\\
&-[\sin(\Delta|\mathbf{r}|)-\Delta|\mathbf{r}|\cos(\Delta|\mathbf{r}|)](\Delta|\mathbf{r}|)^{-3}]\};
\end{aligned}
\qquad (4)
$$

θ is the angle between the vector $\mathbf{k}$ and the axis of the dipole formed by the two emitter atoms (for the computation of Γ_{12}, see Ref. 6, Eq. (3.6)).

Now we can derive expressions for the packets describing the space-time picture of the propagation of one and two quanta[12]:

$$\Psi_{1,2}(r,t)=\sum_{\mathbf{k}}\exp[i(\mathbf{kr}-\omega t)]\alpha_{\mathbf{k}}^{(1),(2)}(t), \qquad (5)$$

$$\Psi(r,r',t)=\sum_{\mathbf{k}_1\mathbf{k}_2}\exp[i(\mathbf{k}_1\mathbf{r}-\omega_1t)]\exp[i(\mathbf{k}_2\mathbf{r}'-\omega_2t)]\alpha_{\mathbf{k}_1\mathbf{k}_2}. \qquad (6)$$

After simple, but tedious computations we find the following explicit expressions for Ψ:

$$
\begin{aligned}
\Psi_{1,2}(r,t)=&\frac{\Delta^{1/2}}{2r}H_\Delta\{\exp[-\Gamma_0(1-\gamma)t]\\
&\times\exp[-i(\Delta-i\Gamma_0(1+\gamma))(t-|\mathbf{r}-\mathbf{r}_1|)]\cdot\\
&\times(\exp[-\Gamma_0\gamma|\mathbf{r}-\mathbf{r}_1|]+1)\theta(t-|\mathbf{r}-\mathbf{r}_1|)+\exp[-\Gamma_0(1-\gamma)t]\\
&\times\exp[-i(\Delta-i\Gamma_0(1+\gamma))(t-|\mathbf{r}-\mathbf{r}_2|)]\\
&\times(\exp[-\Gamma_0\gamma|\mathbf{r}-\mathbf{r}_2|]-1)\theta(t-|\mathbf{r}-\mathbf{r}_2|)\},
\end{aligned}
\qquad (7)
$$

$$
\begin{aligned}
\Psi(r,r',t)=&\frac{\Delta^2}{8\pi rr'}H_\Delta^2\exp(-2i\Delta t)\{A\exp[i\Delta(r+r')]\\
&+B\exp[i\Delta(R_1+R_2)]+C\exp[i\Delta(r+R_2)]\\
&+D\exp[i\Delta(r'+R_1)]\},
\end{aligned}
\qquad (8)
$$

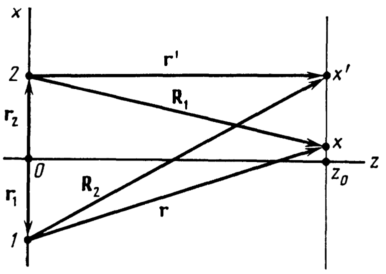

FIG. 1. Mutual disposition of the sources 1 and 2 and the points of detection of the photons (in the $z=z_0$ plane); $\mathbf{R}=\mathbf{r}_1+\mathbf{r}_2$, $|\mathbf{r}_1|=|\mathbf{r}_2|=x_0$.

where

$$
\begin{aligned}
A=&\exp[-\Gamma_0(1-\gamma)(t-r)]\exp[-\Gamma_0(1-\gamma)(t-r')]\cdot\\
&\times\{\exp[-2\Gamma_0\gamma(t-r)]+\exp[-2\Gamma_0\gamma(t-r')]\}\theta(t-r)\theta(t-r'),\\
B=&\exp[-\Gamma_0(1-\gamma)(t-R_1)]\exp[-\Gamma_0(1-\gamma)(t-R_2)]\\
&\times\{\exp[-2\Gamma_0\gamma(t-R_1)]\\
&+\exp[-2\Gamma_0\gamma(t-R_2)]\}\theta(t-R_1)\theta(t-R_2),\\
C=&\exp[-\Gamma_0(1-\gamma)(t-r)]\exp[-\Gamma_0(1-\gamma)(t-R_2)]\\
&\times\{\exp[-2\Gamma_0\gamma(t-R_2)]-\exp[-2\Gamma_0\gamma(t-r)]\}\theta(t-r)\theta(t-R_2),\\
D=&\exp[-\Gamma_0(1-\gamma)(t-R_1)]\\
&\times\exp[-\Gamma_0(1-\gamma)(t-r')]\{\exp[-2\Gamma_0\gamma(t-r')]\\
&-\exp[-2\Gamma_0\gamma(t-R_1)]\}\theta(t-R_1)\theta(t-r'),
\end{aligned}
\qquad (9)
$$

and $\theta(t)$ is the Heavyside theta function. The definitions of the spatial coordinates in Eqs. (8) and (9) are clear from Fig. 1.

From the expressions (7) and (8) we obtain by the standard method the corresponding probabilities for propagation of the quanta:

$$
\begin{aligned}
W_1(r,t)=&|\Psi_1(r,t)|^2=\frac{\Delta}{4r^2}|H_\Delta|^2\exp[-2\Gamma_0(1-\gamma)t]\\
&\times\{\exp[-2\Gamma_0(1+\gamma)(t-|\mathbf{r}-\mathbf{r}_1|)](1+\exp[-\Gamma_0\gamma|\mathbf{r}-\mathbf{r}_1|])^2\\
&\times\theta(t-|\mathbf{r}-\mathbf{r}_1|)+\exp[-2\Gamma_0(1+\gamma)(t-|\mathbf{r}-\mathbf{r}_2|)]\\
&\times(1-\exp[-\Gamma_0\gamma|\mathbf{r}-\mathbf{r}_2|])^2\theta(t-|\mathbf{r}-\mathbf{r}_2|)\\
&-2\exp[-\Gamma_0(1+\gamma)(t-|\mathbf{r}-\mathbf{r}_1|)]\\
&\times\exp[-\Gamma_0(1+\gamma)(t-|\mathbf{r}-\mathbf{r}_2|)][1+\exp(-\Gamma_0\gamma|\mathbf{r}-\mathbf{r}_1|)]\\
&\times[1-\exp(-\Gamma_0\gamma|\mathbf{r}-\mathbf{r}_2|)]\cos[\Delta(|\mathbf{r}-\mathbf{r}_1|-|\mathbf{r}-\mathbf{r}_2|)]\\
&\times\theta(t-|\mathbf{r}-\mathbf{r}_1|)\theta(t-|\mathbf{r}-\mathbf{r}_2|)\}.
\end{aligned}
\qquad (10)
$$

From this expression for W_1 it follows that an interference pattern can be observed even for the single-photon state when the atoms interact with each other (i.e., when $\gamma\neq0$) and the relation $2\Gamma_0(1-\gamma)t\lesssim1$ holds at those points in space up to which the signal has already propagated ($\theta_1=\theta_2=1$).

The nature of this phenomenon is fairly simple: if the atoms can exchange quanta, i.e., if they interact via the radiation field ($\gamma\neq0$), then both atom 1 and atom 2 can each emit a photon, and as a result of this exchange interference occurs, which is described by the third term in Eq. (10). Naturally, the interference for the single-photon state disappears in the absence of interaction between the atoms (i.e., when $\gamma=0$).

A similar expression for the probability in the case of the two-photon state can be obtained from (8):

$$W_2(r,r',t)=\left(\frac{\Delta^2}{8\pi}\right)^2\frac{|H_\Delta|^4}{(rr')^2}\{A^2+B^2+C^2+D^2$$
$$+2AB\cos[\Delta(r+r'-R_1-R_2)]+2AC\cos[\Delta(r'-R_2)]$$
$$+2AD\cos[\Delta(r-R_1)]+2BC\cos[\Delta(R_1-r)]+$$
$$+2BD\cos[\Delta(R_2-r')]+2CD\cos[\Delta(r+R_2-R_1-r')]\}. \quad (11)$$

Let us simplify the expression for the arguments of the cosines, assuming the distance between the planes of the sources (atoms) and the radiation detectors to be fixed (see Fig. 1) and equal to z_0, and, retaining the first terms of the power series expansion in (x/z_0), derive for W_2 the following expression:

$$W_2(r,r',t)=\left(\frac{\Delta^2}{8\pi}\right)^2\frac{|H_\Delta|^4}{(rr')^2}\{A^2+B^2+C^2+D^2$$
$$+2AB\cos\left[\Delta\frac{2x_0}{z_0}(x-x')\right]+2(AC+BD)\cos\left(\Delta\frac{2x_0}{z_0}x'\right)$$
$$+2(AD+BC)\cos\left(\Delta\frac{2x_0}{z_0}x\right)+2CD\cos\left[\Delta\frac{2x_0}{z_0}(x+x')\right]\Bigg\}. \quad (12)$$

In the $\gamma=0$ limit, when the atoms do not interact via the radiation field, $C=D=0$. Then there remains in the expression for W_2 only one interference term $\sim AB\cos[\Delta 2x_0(x-x')/z_0]$, which depends only on the difference between the coordinates of the points of observation of each quantum. This natural result corresponds to the well-known Brown-Twiss effect[13] (two independent sources located at a distance $|R|\gg\pi/\Delta$ from each other and two radiation detectors).

If $\gamma\neq 0$, all the four interference terms in the expression for the probability remain: in this case there appear, besides the term proportional to AB, which determines the Brown-Twiss effect, terms that depend only on the path difference for the rays from the two sources. Thus, the expression obtained for the spatial probability for detection of two photons includes, besides the normal Brown-Twiss effect, "classical" interference between two photons emitted by different atoms interacting via the proper field of the spontaneously emitted quanta: the two sources become tuned in to one other.

It should be noted that, although our arguments are for a system of two emitter atoms, the entire procedure and all the conclusions can easily be generalized to the case of an arbitrary number of emitters (see, for example, Refs. 6 and 9).

Let us now consider how the interference pattern changes if one of the two atoms (for any pair from a large ensemble of atoms) is excited some time τ after the first. Let us derive the time-averaged probability for the two-photon state:

$$\tilde{W}_2(r,r',t)=n\int_0^T d\tau\, W_2(r,r';t,t+\tau). \quad (13)$$

Here the normalization factor n is the number of atoms excited in unit time (for a given density of the atoms of the requisite kind this quantity depends only on the power of the generator exciting the atoms) and T is the excited-state production time ($T\gg 1/\Gamma_0$). Let us, for definiteness, assume that atom 2, with coordinate $\mathbf{r}_2$ (see Fig. 1), is excited after some delay.

Then in the expression we must for W_2, make the substitution $t\to t+\tau$ everywhere in those exponents which contain the coordinates (r',R_1) characterizing this source.

The expression for $\tilde{W}_2(r,r',t)$ is quite unwieldy, but can be obtained from (12) through elementary integration. Therefore, we simply note that $\tilde{W}_2$ has the same form (12) as W_2, with the only difference that, instead of exponential functions of the type $\exp[-a(t-r)]$ appearing in (9), we will have $\{\exp[-a(t-r)]-\exp(-aT)\}/a$, where $r=r',R_1$. A general factor for W_2 will be n/Γ_0. From this it follows that only those pairs of emitter atoms for which the time shift τ between the instants of excitation satisfies the condition $\tau\lesssim 1/\Gamma_0$ function effectively (i.e., produce an interference pattern).

Under steady emission conditions (i.e., for $T\to\infty$), the second exponential function tends to zero, and the interference pattern is similar to the pattern obtained from two emitters with account taken of the rate of generation of these states.

Thus, a consistent quantum-mechanical calculation of the space-time dependence of the probability for detection of the two-photon state allows us to naturally obtain, without making additional assumptions about the radiation sources, the conditions for the appearance of an interference pattern from two atoms, and to describe its nature.

It should be emphasized that this approach, in contrast to those discussed in Refs. 14 and 15, allows us to obtain the space—time picture of the propagation of wave packets for the one-and two-photon states without making additional assumptions about the quantum states of the photons in the radiation field. It is shown that an interference pattern arises as a result of the exchange of photons between the atoms even in the single-photon state if the parameter characterizing the interaction of the atoms via the radiation field has a value $\gamma\sim 1$. For the two-photon state there occurs, besides the Brown-Twiss interference effect,[15] which manifests itself in the $\gamma=0$ case as well, interference between two photons emitted by different atoms interacting via the radiation field ($\gamma\sim 1$). In Refs. 14 and 15 the effect of the interaction of the atoms via the radiation field is ignored.

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