Current states and electron-phonon relaxation in point contacts in a magnetic field

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(Submitted 6 August 1986)
Zh. Eksp. Teor. Fiz. **92**, 730–744 (February 1987)

Geometrical trajectory effects in the electric conductivity of metallic points contacts (microcontacts) in a strong field are investigated. When either the elastic electron mean free path l_i or the electron cyclotron radius r_H is smaller than contact diameter d, the kinetics of the electron gas near the aperture can be described by a diffusion equation. In a strong field such that $r_H a \ll d$ and $l_i \gg d$, the electron motion is effectively one-dimensional, and transverse diffusion is the only means of restoring the three-dimensional nature of the current distribution near the contact. The contact resistance in a strong field is $R(H) = (\rho/d)(1+(\Omega\tau_i)^2)^{1/2}$ and increases linearly with the field at $\Omega\tau_i \gg 1$ (Ω is the cyclotron frequency and $\tau_i = l_i/v_f$). The inelastic current component in the microcontact is due to electron-phonon collisions and can increase in a strong field by a factor l_i/d compared with its value in a zero field. At $l_i \gg r_H \gg d$ the resistance ceases to depend on the field, but the potential distribution in the contact is substantially altered and is characterized by oscillations whose period is proportional to the cyclotron radius (analog of the Sondheimer effect).

§1. INTRODUCTION

Metallic point contacts are unique objects for the production and investigation of spatially localized strong current perturbations in an electronic system. The substantial spatial inhomogeneity of the transport perturbations is due to the concentration, near the microconstrictions, of an electric current whose density in real experiments reaches values $j \approx 10^{10}$ A/cm^2. The substantial decrease of the current density in regions far from the point contact is due to three-dimensional spreading of the carriers, which move along classical straight-line trajectories (Fig. 1). The nonequilibrium distribution of the electrons at some point **r** is shown in Fig. 1. It is easily seen that in the ballistic regime considered by way of example ($l > r$, where l is the carrier mean free path) the density of the nonequilibrium electrons decreases like $\delta n \approx n(d/r)^2$ with increasing distance from the contact. The disequilibrium of the electronic system is concentrated mainly near the contact in a region whose size is determined by the length of the spatial spreading of the current, a length that coincides in this case with the size d of the contact. As a result, the resistance of the structure is determined by the electron scattering processes that take place in the indicated region. In particular, the phonon increment to the resistance turns out to be proportional to the probability of electron-phonon scattering over a length r of the order of d. In the most interesting case, when the dimension of the contact is small compared with the electron-phonon mean free path l_ε, the main contribution to the phonon part of the resistance is made by single-quantum electron-phonon scattering processes, whose probability is proportional to d/l_ε. This contribution determines the nonlinear increment to the current-voltage characteristics (IVC) of the contact, analysis of which permits reconstruction of the electron-phonon interaction function (microcontact spectroscopy).

Application of a constant magnetic field H alters substantially the character of the spatial spreading of the current in the point contact. The finite character of the cyclotron motion of the electrons in a plane perpendicular to the magnetic-field direction prevents three-dimensional spreading of the current, as a result of which the density of the nonequilibrium electrons ceases to decrease with increasing distance from the contact, and the transport problem is more readily reminiscent of the one-dimensional one. The natural reason for the restoration of the three-dimensional character of the spreading is interaction of the electrons with the scatterers, as a result of which the characteristic scale of the localization region of the electron perturbation in the point contact turns out to depend on the mean free path and on the field strength. When the spreading region exceeds the size of the point contact, the current spreading turns out to be slower than in the case $H = 0$, and the electric conductivity of the point contact becomes substantially dependent on the magnetic field. In particular, the phonon increment to the resistance turns out to depend on the electron elastic-scattering length l_i even when this length is large compared with the contact size.

Recent experiments by Yanson, Gribov, and Shklyarevskiĭ[3] have revealed a substantial dependence of the point contact spectra on the magnitude and orientation of the

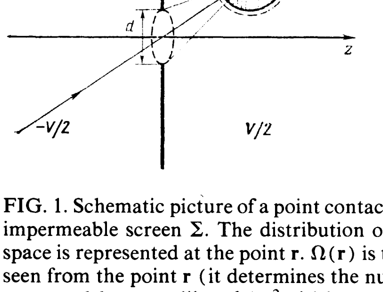

FIG. 1. Schematic picture of a point contact in the form of an orifice in an impermeable screen Σ. The distribution of the electrons in momentum space is represented at the point **r**. Ω(**r**) is the solid angle of the orifice as seen from the point **r** (it determines the number of nonequilibrium electrons, and decreases like $(d/r)^2$ with increasing distance from the orifice).

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magnetic field. This dependence was interpreted as a manifestation of trajectory effects of the cyclotron motions in the kinetics of electron-phonon relaxation.

The present paper is devoted to a theoretical analysis of the nonlinear electric conductivity of point contacts in a magnetic field. The presence of several parameters with the dimension of length, such as the cyclotron radius r_H, the point contact diameter d, the length l_i and l_ε of the elastic and inelastic scattering, lead to the feasibility of various regimes of nonlinear electric conductivity. The spreading of an electric current behaves quite differently in two cases: strong magnetic field, $r_H \ll d$, and weak ones, $r_H \gg d$. In the former the carrier motion is in the main uniform, and the current spreading is a slow diffusion process, carrier motion transverse to the magnetic field, with elementary steps $r_H \ll d$; it results from elastic scattering of the electrons (at $l_i \gg r_H$). The characteristic length of the current spreading in the direction of the magnetic field turns out to be equal to dl_i/r_H, i.e., substantially longer than d. In the case of extremely dirty metals, $l_i \ll r_H \ll d$, the magnetic field has little effect on the carrier motion and the spreading region coincides with the contact dimension d.

In weak magnetic fields, $r_H \gg d$, there are two characteristic dimensions in the region of the spatial spreading of the current. Near the microcontact, $r \sim d$, the electrons practically in straight lines and a three-dimensional trajectory spreading is realized, with a characteristic spatial scale $r \sim d$. At distances $r \sim r_H$ an important role is assumed by the twisting of the electron trajectories in the magnetic field, as a result of which the decrease of the electron disequilibrium with increase of r gives way to an oscillatory $\delta n(r)$ dependence. These oscillations, whose period is determined by the cyclotron radius r_H, cause an oscillatory distribution of the electric potential (analog of the Sondheimer effect[4]). Finally, at distances $r \gtrsim l_i \gg r_H$ the three-dimensional spreading is restored because of the interaction of the carriers with the scatterers. In this case the density of the nonequilibrium electrons decreases with the scattering in accordance with a law that is typical of the diffusion regime:

$$\delta n \sim n \frac{d^2}{r_H^2} \frac{l_i}{r}.$$

The feasibility of point-contact spectroscopy of electron-phonon interactions is connected with realization of a conservative carrier motion in the current-spreading region. To this end it is necessary that the time of motion in the spreading region be short compared with the inelastic-relaxation time τ_ε. In a zero magnetic field this condition is met in junctions with dimensions small compared with the electron-phonon scattering length; in the presence of a magnetic field this condition is substantially modified. In a strong magnetic field, $r_H \ll d$, the carrier diffusion-motion time in the spreading region (see Fig. 2) is equal to

$$\tau = (l_i/v_F)(d/r_H)^2,$$

therefore the condition for the absence of thermal heating can be written in the form

$$r_H \gg d(l_i/l_\varepsilon)^{1/2}, \quad r_H \ll d. \tag{1}$$

The distribution of the potential in the contact is shown for this case in Fig. 3.

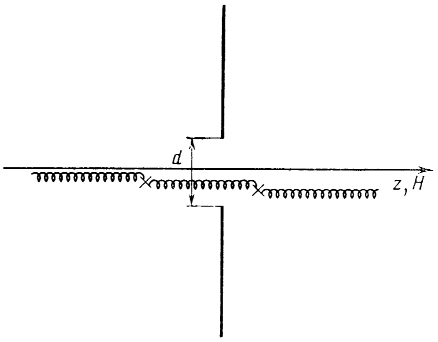

FIG. 2. Point contact in a strong magnetic field ($r_H \ll d$). The current spreads because of multiple elastic interaction of the electron with the scatterers (the scattering acts are represented by the crosses).

In weak magnetic fields, single scattering of carriers by impurities takes the electron out of the spatial spreading region (see Fig. 4), so that $\tau = l_i/v_F$. The analogous condition in the present case is therefore

$$l_\varepsilon \gg l_i, \quad r_H \gg d. \tag{2}$$

If inequalities (1) and (2) are satisfied, the inelastic increment to the contact resistance is determined by the single-quantum phonon-emission processes whose analysis is the subject of the present paper. In §2 we present a general formulation of the problem of nonlinear electric conductivity of a point contact in the form of an orifice in an impermeable partition $z = 0$, with the magnetic axis directed along the z axis. Conditions (1) and (2) allow us to formulate a perturbation theory in terms of the electron-phonon collision integral and to derive general relations for the elastic (zeroth order of perturbation theory) and inelastic (first order) components of the point-contact current. The actual analysis of these relations for strong ($r_H \ll d$) and weak ($r_H \gg d$) magnetic fields is carried out in §§3 and 4. We have confined ourselves in the investigation of the electric conductivity to classical phenomena in the magnetic field, assuming $\hbar\Omega < T$ (Ω is the cyclotron frequency and T is the temperature). Quantum effects in the contact resistance in a magnetic field are considered theoretically in Ref. 5 and were observed experimentally in Refs. 3 and 6.

§2. FORMULATION OF PROBLEM. GENERAL EXPRESSION FOR THE NONLINEAR COMPONENT OF THE POINT CONTACT CURRENT

Calculation of the current flowing through the point contact (see Fig. 1) entails as usual[7,8] solution of the Boltz-

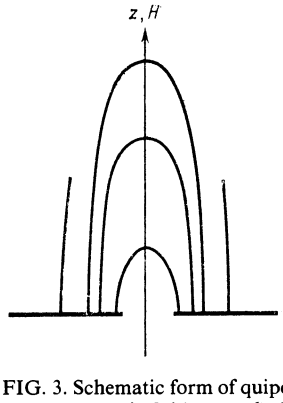

FIG. 3. Schematic form of quipotential surfaces near a point contact in a strong magnetic field ($r_H \ll d$). The parameter $\Omega\tau_i$ is assumed large.

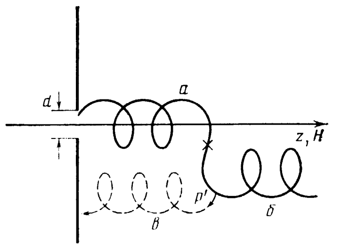

FIG. 4. Point contact in a weak magnetic field ($r_H \gg d$). The spreading of the current is due to single interaction with the scatterers (marked by a cross): a—ballistic trajectory, b—trajectory after scattering ($\times$), c—trajectory of possible return with initial momentum $\mathbf{p}'$.

mann equation for the electron distribution function

$$\mathbf{v}\frac{\partial f_\mathbf{p}}{\partial \mathbf{r}}+\left(e\mathbf{E}+\frac{e}{c}[\mathbf{vH}]\right)\frac{\partial f_\mathbf{p}}{\partial \mathbf{p}}-I_i\{f_\mathbf{p}\}=I_{e-ph}\{f_\mathbf{p}\} \quad (3)$$

with boundary conditions ($\mathbf{e} = -\nabla\Phi$),

$$f_\mathbf{p}(\mathbf{r}\to\infty)=n_F(\varepsilon_\mathbf{p}-\mu), \quad (4)$$

$$\Phi(\mathbf{r}\to\infty)=\frac{V}{2}\operatorname{sgn} z. \quad (5)$$

The electron-impurity and electron-phonon integrals, $I_i\{f_\mathbf{p}\}$ and $I_{e-ph}\{f_\mathbf{p}\}$ respectively, which enter in (3), have the standard forms

$$I_i\{f_\mathbf{p}\}=\frac{1}{(2\pi)^3}\int\frac{dS_{\mathbf{p}'}}{v_\perp'}W_{\mathbf{pp}'}^{(i)}(f_{\mathbf{p}'}-f_\mathbf{p}), \quad (6)$$

$$I_{e-ph}\{f_\mathbf{p}\}=\sum_\alpha\frac{1}{(2\pi)^3}\int d\mathbf{q}\,W_\mathbf{q}^\alpha$$
$$\times\{[f_{\mathbf{p}+\mathbf{q}}(1-f_\mathbf{p})(N_\mathbf{q}+1)-f_\mathbf{p}(1-f_{\mathbf{p}+\mathbf{q}})N_\mathbf{q}]$$
$$\times\delta(\varepsilon_{\mathbf{p}+\mathbf{q}}-\varepsilon_\mathbf{p}-\omega_\mathbf{q}^\alpha)$$
$$+[f_{\mathbf{p}-\mathbf{q}}(1-f_\mathbf{p})N_\mathbf{q}-f_\mathbf{p}(1-f_{\mathbf{p}-\mathbf{q}})(N_\mathbf{q}+1)]$$
$$\times\delta(\varepsilon_{\mathbf{p}-\mathbf{q}}-\varepsilon_\mathbf{p}+\omega_\mathbf{q}^\alpha)\}, \quad (7)$$

$W_{\mathbf{pp}'}^{(i)}$ is the square of the matrix element of electron-impurity scattering, W_q^α is the square of the matrix element of the interaction, $n_F(x) = (e^x+1)^{-1}$, and μ is the chemical potential.

If conditions (1) and (2) are met, the electron-phonon relaxation is weak and the electron-phonon interaction can be accounted for by perturbation theory. It is convenient to represent the distribution function in the elastic approximation ($I_{e-ph}\{...\}=0$) in the form[8]

$$f_\mathbf{p}^{(0)}=\alpha_\mathbf{p}(\mathbf{r})n_F(\varepsilon_\mathbf{p}+e\Phi-eV/2)+[1-\alpha_\mathbf{p}(\mathbf{r})]n_F(\varepsilon_\mathbf{p}+e\Phi+eV/2), \quad (8)$$

and then the probability $\alpha_\mathbf{p}(\mathbf{r})$ that the electron arrives at the point $\mathbf{r}$ with momentum $\mathbf{p}$ from the right-hand half-space also satisfies Eq. (3) with zero right-hand side. The boundary conditions for the function $(\mathbf{r})$ follow from relation (4) and take the form

$$\alpha_\mathbf{p}(\mathbf{r}\to\infty)=\theta(z), \quad (9)$$

where $\theta(z)$ is a function equal to unity at $z>0$ and to zero at $z<0$.

Just as at $H=0$, all the transport characteristics of the problem, and also the distribution of the potential, can be expressed in the form of functionals of $\alpha_\mathbf{p}(\mathbf{r})$. We consider below the case of low voltages typical of metallic contacts. In this approximation, the elastic component of the point-contact current satisfies Ohm's law

$$I=V/R, \quad (10)$$

$$\frac{1}{R}=\frac{2e^2S_F}{(2\pi\hbar)^3}\int d^2\rho\langle n_z\alpha_\mathbf{p}\rangle, \quad (11)$$

where S_F is the area of the Fermi surface $n_z = v_z/v_F$, and $\langle...\rangle$ is an average over the momentum direction at $\varepsilon_\mathbf{p}=\varepsilon_F$. The distribution of the potential $\Phi(\mathbf{r})$ takes the form

$$\Phi(\mathbf{r})={}^1/_2V[2\langle\alpha_\mathbf{p}(\mathbf{r})\rangle-1]. \quad (12)$$

The inelastic component of the point-contact current is determined by an addition $f_\mathbf{p}^{(1)}$ to the distribution function, calculated in first-order perturbation theory in $I_{e-ph}\{...\}$. It is convenient to express this increment, in analogy with Ref. 8, in terms of the Green's function $g_{\mathbf{pp}'}(\mathbf{r},\mathbf{r}';\mathbf{H})$

$$f_\mathbf{p}^{(1)}(\mathbf{r})=\int d\mathbf{r}'\int d\mathbf{p}'\,g_{\mathbf{pp}'}(\mathbf{r},\mathbf{r}';\mathbf{H})I_{e-ph}\{f_\mathbf{p}^{(0)}\}, \quad (13)$$

which is in turn the solution of the equation

$$\mathbf{v}'\frac{\partial g_{\mathbf{pp}'}}{\partial \mathbf{r}'}+\frac{e}{c}[\mathbf{v}'\mathbf{H}]\frac{\partial g_{\mathbf{pp}'}}{\partial \mathbf{p}'}+I_i\{g_{\mathbf{pp}'}\}=-\delta(\mathbf{p}-\mathbf{p}')\delta(\mathbf{r}-\mathbf{r}'), \quad (14)$$

$$I_i(g_{\mathbf{pp}'}\}=\frac{1}{(2\pi)^3}\int\frac{dS_{\mathbf{p}''}}{v_\perp''}W_{\mathbf{p}'-\mathbf{p}''}^{(i)}(g_{\mathbf{pp}''}-g_{\mathbf{pp}'}) \quad (15)$$

and satisfies the symmetry relation that reflects the reversible character of electron motion in a magnetic field:

$$g_{\mathbf{pp}'}(\mathbf{r},\mathbf{r}';\mathbf{H})=g_{-\mathbf{p}'-\mathbf{p}}(\mathbf{r}',\mathbf{r};-\mathbf{H}). \quad (16)$$

The necessary boundary condition for the Green's function follow from the corresponding conditions for the distribution function $f_\mathbf{p}(\mathbf{r})$

$$g_{\mathbf{pp}'}(\mathbf{r},\mathbf{r}'\to\infty)=0, \quad g_{\mathbf{pp}'}(\mathbf{r},\mathbf{r}'\in\Sigma)=g_{\mathbf{pp}_R'}(\mathbf{r},\mathbf{r}'\in\Sigma), \quad (17)$$

where $\mathbf{p}_R$ is the momentum of the electron reflected in a plane tangent to the contact surface at a point $\mathbf{r}\in\Sigma$. Introducing the function $G_p(\mathbf{r})$ defined as

$$G_\mathbf{p}(\mathbf{r})=\int dS'\int d\mathbf{p}'\,v_z'g_{\mathbf{p}'\mathbf{p}}(\mathbf{r}',\mathbf{r};\mathbf{H}), \quad (18)$$

we represent the expression for the inelastic addition to the point-contact current in the form

$$I'=\frac{2e}{(2\pi\hbar)^3}\int d\mathbf{r}\int d\mathbf{p}\,G_\mathbf{p}(\mathbf{r},\mathbf{H})I_{e-ph}\{f_\mathbf{p}^{(0)}(\mathbf{r})\}. \quad (19)$$

The equation satisfied by the function $G_p(r)$ and also the corresponding boundary conditions follow from expressions (14)–(17):

$$\mathbf{v}\frac{\partial G_\mathbf{p}}{\partial \mathbf{r}}+\frac{e}{c}[\mathbf{vH}]\frac{\partial G_\mathbf{p}}{\partial \mathbf{p}}+I_i\{G_\mathbf{p}(\mathbf{r})\}=-v_z\delta(z), \quad (20)$$

$$G_\mathbf{p}(\mathbf{r}\to\infty)=0, \quad G_\mathbf{p}(\mathbf{r}\in\Sigma)=G_{\mathbf{p}R}(\mathbf{r}\in\Sigma). \quad (21)$$

Comparison of these equations with the analogous problem for elastic electric conductivity (3)–(5) (in the case $I_{e-ph}\{...\}a=0$) yields the connection between the function $G_\mathbf{p}(r)$ and the previously introduced function $\alpha_\mathbf{p}(\mathbf{r})$:

$$G_{\mathbf{p}}(\mathbf{r},\mathbf{H})=\alpha_{-\mathbf{p}}(\mathbf{r},-\mathbf{H})-\theta(z) \quad (22)$$

Further transformations of the expression for the inelastic addition to the current lead to the following relation for the normalized second derivative of the IVC:

$$\frac{1}{R}\frac{\partial R}{\partial V}=\frac{32}{3}\frac{ed}{v_F}\int_0^\infty \frac{d\omega}{T}G(\omega)S\left(\frac{\hbar\omega-eV}{T}\right),$$

$$S(x)=\frac{d^2}{dx^2}\frac{x}{e^x-1}, \quad (23)$$

where $R=dV/dI$ and

$$G(\omega)=\sum_\alpha\int\frac{dS_{\mathbf{p}}}{v_\perp}\int\frac{dS_{\mathbf{p}'}}{v_\perp'}\frac{1}{(2\pi)^3}$$
$$\times W^\alpha_{\mathbf{p}-\mathbf{p}'}K(\mathbf{p},\mathbf{p}')\delta(\omega-\omega^\alpha_{\mathbf{p}-\mathbf{p}'})\Big/\int\frac{dS_{\mathbf{p}}}{v_\perp}. \quad (24)$$

The intensity of the point-contact spectrum is determined by the transport factor $K(\mathbf{p},\mathbf{p}')$ contained in (14) and connected with the functions $\alpha_{\mathbf{p}}(\mathbf{r})$ by the relation

$$K(\mathbf{p},\mathbf{p}')=\frac{3\pi}{32}\frac{v_F}{d}\frac{\int d\mathbf{r}[\alpha_{\mathbf{p}}(H)-\alpha_{\mathbf{p}'}(H)][\alpha_{-\mathbf{p}}(-H)-\alpha_{-\mathbf{p}'}(-H)]}{\int d^2\rho\langle v_z\alpha_{\mathbf{p}}\rangle}. \quad (25)$$

The substantial difference between the expression obtained for the K-factor and its value in a zero magnetic field (see Eq. (2.29) of Ref. 8) is due to the fact that reversal of electron motion in a magnetic field calls for simultaneous replacement of $\mathbf{v}$ by $-\mathbf{v}$ and of $\mathbf{H}$ by $-\mathbf{H}$. This effect of the magnetic field on electron motion manifests itself similarly in the Onsager symmetry principle for kinetic coefficients.

Expressions (10)–(12) and (23)–(25) contain the complete solution of the nonlinear point-contact conductivity problem, reducing it to finding the probabilities $\alpha_{\mathbf{p}}(\mathbf{r})$ of elastic motion of the carriers. The actual form of this quantity depends on the relation between the parameters d, r_H, and l, which determine the different electric-conductivity regimes. The values of $\alpha_{\mathbf{p}}(\mathbf{r})$ in the case of weak and strong magnetic fields will be calculated in the sections that follow.

§3. DIFFUSIVE SPREADING OF ELECTRIC CURRENT IN A STRONG MAGNETIC FIELD ($r_H \ll d$)

The quasi-one-dimensional trajectories of the electrons in a strong magnetic field are shown in Fig. 2. A specific feature of this problem is that account must be taken of scattering by impurities, which leads to drift of the carriers in the XY plane, with spacing r_H. Assuming that the trajectory displacement during a collision time τ_i is r_H, we obtain the coefficient of diffusion in a direction transverse to the field

$$D_\perp\sim r_H^2/\tau_i, \quad (26)$$

while the coefficient of electron diffusion along the field is

$$D_\parallel={}^1/_3 v_F l. \quad (27)$$

Their ratio is

$$D_\parallel/D_\perp\sim(\Omega\tau_i)^2.$$

At $r_H=0$ the carrier motion is strictly one-dimensional, and for any value of the elastic mean free path the structure resistance is equal to that of a cylinder of diameter d and of infinite length, i.e., is infinite. The fact that the cyclotron radius is not zero leads to a finite resistance and to its dependence on the magnetic field. The Boltzmann equation for the function $\alpha_{\mathbf{p}}(\mathbf{r})$ is[1)]

$$\mathbf{v}\frac{\partial\alpha_{\mathbf{p}}}{\partial\mathbf{r}}+\frac{e}{c}[\mathbf{vH}]\frac{\partial\alpha_{\mathbf{p}}}{\partial\mathbf{p}}+\frac{1}{\tau_i}(\alpha_{\mathbf{p}}-\langle\alpha_{\mathbf{p}}\rangle)=0, \quad (28)$$

where $\langle\alpha_{\mathbf{p}}(\mathbf{r})\rangle$ is the value of $\alpha_{\mathbf{p}}(\mathbf{r})$ averaged over the momentum directions. The differential equation (28) can be replaced by an equivalent integral equation in which the integration is over the time of particle motion along the cyclotron trajectory

$$\alpha_{\mathbf{p}}(\mathbf{r})=\frac{1}{\tau_i}\int_{-\infty}^0 d\tau\, e^{\tau/\tau_i}\langle\alpha_{\mathbf{p}}(\mathbf{r}_\tau)\rangle. \quad (29)$$

Those particle trajectories in the magnetic field which satisfy the initial condition $\mathbf{r}(\tau=0)=\mathbf{r}$ are of the form

$$x_\tau=x+(v_\perp/\Omega)[\sin(\Omega\tau+\varphi)-\sin\varphi],$$
$$y_\tau=y+(v_\perp/\Omega)[\cos(\Omega\tau+\varphi)-\cos\varphi], \quad (30)$$
$$z_\tau=v_z\tau.$$

Here $v_x=v_\perp\cos\varphi$ and $v_y=-v_\perp\sin\varphi$ are the components of the velocity $\mathbf{v}$ at the instant $\tau=0$. already noted, if $r_H\ll d$ and $r_H\ll l_i$ the carriers spread away from the contact to distances larger than l_i. (note that if $l_i\ll r_H\ll d$ the influence of the magnetic field is negligible and the diffusive spreading length d is also large compared with l_i). The function $\langle\alpha_{\mathbf{p}}(\mathbf{r})\rangle$ changes therefore slowly over the free-path times τ. This allows us to replace the integral equation (29) for $\alpha_{\mathbf{p}}$ by a differential one. Expanding $\langle\alpha_{\mathbf{p}}\rangle$ up to terms of first order in l_i we obtain

$$\alpha_{\mathbf{p}}(\mathbf{r})=\langle\alpha_{\mathbf{p}}(\mathbf{r})\rangle-\tau_i v_z\frac{\partial\langle\alpha_{\mathbf{p}}(\mathbf{r})\rangle}{\partial z}$$
$$+\frac{v_\perp}{\Omega}[1+(\Omega\tau_i)^2]^{-1}\Big\{[-\Omega\tau_i\cos\varphi-(\Omega\tau_i)^2\sin\varphi]\frac{\partial\langle\alpha_{\mathbf{p}}(\mathbf{r})\rangle}{\partial x}$$
$$+[\Omega\tau_i\sin\varphi-(\Omega\tau_i)^2\cos\varphi]\frac{\partial\langle\alpha_{\mathbf{p}}(\mathbf{r})\rangle}{\partial y}\Big\}. \quad (31)$$

Retaining terms of second order in l_i and averaging (31) over the momentum directions, we obtain a diffusion equation for $\langle\alpha_{\mathbf{p}}(\mathbf{r})\rangle$

$$\left[\frac{\partial^2}{\partial z^2}+\frac{1}{1+(\Omega\tau_i)^2}\left(\frac{\partial}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]\langle\alpha_{\mathbf{p}}(\mathbf{r})\rangle=0. \quad (32)$$

The boundary condition (9) should be supplemented by the condition that the current cannot leak through the point-contact boundary

$$\frac{\partial\langle\alpha_{\mathbf{p}}(\mathbf{r}\in\Sigma)\rangle}{\partial\mathbf{n}}=0, \quad (33)$$

where $\mathbf{n}$ is the normal to the contact surface $z=0$. The solution of the boundary-value problem (9), (32), (33) is (Ref. 8)[2)]

$$\langle\alpha_{\mathbf{p}}(\mathbf{r})\rangle=\theta(z)-\varphi_0[x,y,(1+\Omega^2\tau_i^2)^{-1/2}z]\,\mathrm{sgn}\,z, \quad (34)$$

$$\varphi_0(r)=\mathrm{arctg}\left\{\left[\frac{2r^2}{d^2}-\frac{1}{2}+\left(\left(\frac{2r^2}{d^2}-\frac{1}{2}\right)^2+\frac{4z^2}{d^2}\right)^{1/2}\right]^{-1/2}\right\},$$

where $r = (x^2+y^2+z^2)^{1/2}$. Equations (32) and (33) are valid if at least one of the quantities, r_H or l_i, is small compared with the contact diameter d. If $l_i \ll d$ and $H=0$ we obtain from (34) a potential distribution corresponding to the diffusion limit.[8]

The solution (34) leads to an expression for the point-contact resistance in a strong magnetic field

$$R(H)=\frac{1}{\sigma d}(1+(\Omega\tau_i)^2)^{1/2}, \quad r_H \ll d, \tag{35}$$

where $\sigma = ne^2\tau_i/m$. Note that in strongly contaminated contacts ($\Omega\tau_i = l_i/r_H \ll 1$) Eq. (35) leads to a correction to the well known expression for the Maxwell resistance $R_M = 1/\sigma d$ (Ref. 9):

$$R(H)=R_M[1+{}^1/_2(\Omega\tau_i)^2]. \tag{36}$$

In pure contacts ($\Omega\tau_i \gg 1$), on the other hand, the resistance becomes independent of the electron elastic relaxation length; furthermore, this takes place at $l_i \gg d$ in a zero magnetic field and at $l_i \gg r_H$ in a strong magnetic field ($r_H \ll d$. The resistance is in this case

$$R(H)=\frac{3\pi}{16}R_k\frac{d}{r_H}, \tag{37}$$

where

$$R_k=\frac{2(2\pi\hbar)^3}{e^2SS_F}=\frac{16}{3\pi}\rho_{\text{ост}}\frac{l}{d^2}$$

is the point-contact resistance in the Knudsen limit in a zero magnetic field (the Sharvin resistance[10,11]). Since expression (35) for the resistance was obtained at $V=0$, when thermal heating in the contact is negligible, condition (1) imposes in fact no limit on the validity of Eq. (35). Figure 3 shows the form of the equipotential surfaces corresponding to the potential distribution [see (12)]:

$$\Phi(\mathbf{r})=\frac{V}{2}\{2\theta(z)-2\varphi_0[x,y,(1+\Omega^2\tau_i^2)^{-1/2}z]\,\mathrm{sgn}\,z-1\}. \tag{38}$$

Note the strongly anisotropic form of the equipotential surfaces, which corresponds to quasi-one-dimensional spreading of the carriers in a strong magnetic field.

The expressions (31) and (34) that define $\alpha_\mathbf{p}(\mathbf{r})$ can be used to calculate the transport K-factor (25). Carrying out the required integration, we get

$$K(\mathbf{p},\mathbf{p}')=\frac{9\pi}{128}\frac{l_i}{d}\left\{2(n_z-n_z')^2+\frac{1-(\Omega\tau_i)^2}{1+(\Omega\tau_i)^2}(n_\perp-n_\perp')^2\right\}, \tag{39}$$

where $\mathbf{n}=\mathbf{v}/v_F$. An interesting feature of the result is the dependence of the K factor on the transport length l_i. The result depends also on the magnetic field, and coincides as $H\to 0$ with the corresponding expression $l_i \ll d$ for the form factor in dirty point-contacts.[8]

The relation obtained here, however, is valid also at $l_i \gg d$ (if the condition (1) is met), and can correspond, since the K-factor averaged over the Fermi surface is zero, to either decrease or increase of the intensity of the point-contact spectra, and even to a reversal of their sign in pure contacts in the limit of strong field (these questions are treated in greater detail in the last section). Note also that the results obtained in the present section are valid not only for a spherical Fermi surface, but also for a Fermi surface in the form of an ellipsoid with one crystallographic axis aligned with the point-contact axis.

§4. TRAJECTORY EFFECTS IN THE ELECTRIC CONDUCTIVITY OF POINT CONTACTS IN A WEAK MAGNETIC FIELD ($r_H \gg d$)

The electric conductivity of point contacts in a weak magnetic field depends on the ratio of elastic mean free path and the cyclotron radius. In $l_i \ll r_H$, the trajectory effect of cyclotron motion is weakly pronounced and the magnetic field leads to only a small correction to the electric conductivity. We therefore restrict ourselvs in the present section to the case

$$l_i \gg r_H, \tag{40}$$

which we shall assume to hold together with relation (2). The motion along the trajectories in the magnetic field determines then for the most part the spreading of the electrons over distances that are large compared with the contact diameter. The carrier motion in the region $r<l_i$ is ballistic, and the function $\alpha_\mathbf{p}(\mathbf{r})$ in (8) can be easily represented by classifying the electron trajectories r_τ in accordance with their ability to move through the point contact. The function $\alpha_\mathbf{p}(\mathbf{r})$ is equal to unity of the trajectory goes off, under time reversal, from the point into the interior of the right-hand bank of the contact, and is equal to zero in the opposite case. Using the explicit trajectories of the electron in a magnetic field (30), we obtain for $\alpha_\mathbf{p}(\mathbf{r})$ the expression

$$\alpha_\mathbf{p}(\mathbf{r})=\theta(z)-\theta[(d/2)^2-x^2(\tau)-y^2(\tau)]\theta(zv_z)\,\mathrm{sgn}\,z, \quad \tau=-|z/v_z|. \tag{41}$$

With the aid of this function we can calculate the elastic resistance of the contact (11). We obtain (S is the contact area)

$$R(H)=R(0)=2(2\pi\hbar)^3/e^2SS_F. \tag{42}$$

As seen from (42), in a weak magnetic field (in the principal approximation in r_H/l_i) the resistance is independent of the field. The reason is that as $l_i\to\infty$ the electrons in the point contact plane are classified as arriving from the right and left banks in accordance with the sign of the z component of the velocity, just as at $H=0$. Although the resistance of the point contact does not change in a weak magnetic field, the potential distribution is radically altered. The differences occur in regions relatively far from the contact, $r\gg d$, when the twisting of the electron trajectories in the magnetic field becomes substantial. The potential distribution in the region $r<l_i$ is given by

$$\Phi(\mathbf{r})=\frac{V}{2}[1-J(\mathbf{r})],$$
$$J(\mathbf{r})=\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^1 dt\,\theta\Big\{\Big(\frac{d}{2}\Big)^2-\Big[r_H(1-t^2)^{1/2}\Big(\sin\Big(-\frac{z}{r_Ht}+\varphi\Big)-\sin\varphi\Big)+x\Big]^2+\Big[r_H(1-t^2)^{1/2}\Big(\cos\Big(-\frac{z}{r_Ht}+\varphi\Big)-\cos\varphi\Big)+y\Big]^2\Big\}. \tag{43}$$

At $z\ll r_H$ the trigonometric functions in (43) can be expanded in series and we obtain for J a relation that coincides with that for $H=0$ (Ref. 7). In particular, a substantial voltage decrease amounting to $V[1-(d/r_H)^2]$ occurs in the region of the three-dimensional spreading of the current at distances r on the order of d from the contact. Note the following features of the behavior of the function $\Phi(\mathbf{r})$ at distances comparable with the cyclotron radius. First, at

$$\rho=(x^2+y^2)^{1/2}>2r_H+d/2$$

the potential $\Phi(\mathbf{r})$ is equal to its limiting values $V/2\ \mathrm{sgn}\, z$. Such a rigid localization of the inhomogeneity of the potential distribution in a direction transverse to the magnetic field is due to the existence of a "geometric shadow" $\rho>2r_H+d/2$ which the electrons passing through the contact cannot penetrate because of the twisting of their trajectories in the magnetic field. Second, as $z\to\pm\infty$ the function $\Phi(\mathbf{r})$ defined by Eq. (43) is not equal to the limiting value $V/2\ \mathrm{sgn}\, z$, but differs from it by an amount of the order of $(d/r_H)^2$ (at $\rho\sim r_H$). More accurate asymptotic values of the potential at $z\gg r_H$ are

$$J(\rho,z)=g(\rho)\left[1-\left(\frac{2r_H}{\pi z}\right)^{1/2}\sum_{k=1}^{\infty}\frac{1}{k^{1/2}}\cos\left(\frac{kz}{r_H}+\frac{\pi}{4}\right)\right]. \quad (44)$$

Here

$$g(\rho)=\frac{d}{2\pi r_H}E\left(\frac{2\rho}{d},f(\rho)\right),\quad E(x,y)=\int_0^y d\varphi(1-x^2\sin^2\varphi)^{1/2},$$

$E(x,y)$ is an elliptic integral of the second kind, $f(\rho)=\arcsin d/2\rho$ at $d/2\rho<1$, and $f(\rho)=\pi/2$ at $d/2\rho>1$. Using asymptotic expressions for the function $E(x,y)$, we get

$$g(\rho)=\begin{cases}(d/4r_H)(1-\rho^2/d^2), & \rho\ll d\\ d/2\pi r_H, & \rho=d/2.\\ d^2/16r_H\rho, & r_H\gg\rho\gg d\end{cases} \quad (45)$$

An expression for J in the form (45) becomes invalid near $z=2\pi r_H n$ (n are integers) in a region having a width of order d. As seen from (44), the distribution of the electric potential $\Phi(\mathbf{r})$ is an oscillating function of z with a period $\Delta z=2\pi r_H$. The reason for these oscillations is that the nonequilibrium electrons, moving along a helix in the magnetic field, periodically "hover" above one and the same point in the $z=0$ plane at distances along z that are multiples of the pitch of the helix. (An effect similar to longitudinal focusing of electron beams in a magnetic field[10] and leading, in particular, to Sondheimer oscillations in the resistance of thin plates.[4])

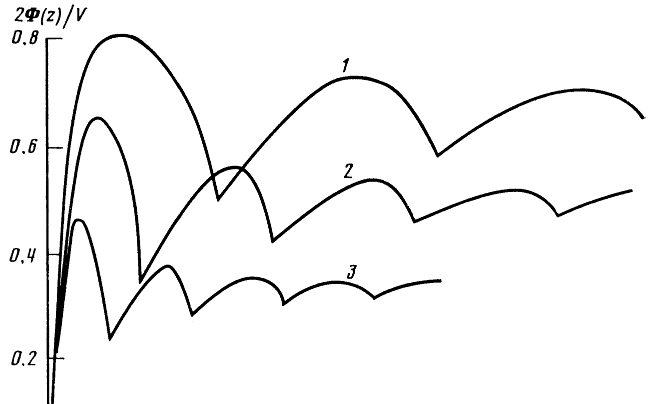

FIG. 5. Distribution of electric potential in a weak magnetic field along the z axis ($\rho=0$). Curves 1, 2, and 3 correspond to the parameter values $r_H/d=0.75$, 0.5, and 0.375.

Figure 5 shows the distribution of the electric potential for finite values of z/r_H at $\rho=0$, obtained by numerically calculating (43). From the form of the plots it follows that the oscillatory dependence of the potential at $z\sim r_H$ is more pronounced that in the asymptotic expression (44), and is not due to the strong inequality $r_H\gg d$. The difference between the limiting values of the potential $\Phi(\mathbf{r})$ as $z\to\pm\infty$ and $V/2\ \mathrm{sgn}\, z$ is due to the absence of transverse spreading over distances r of order l_i, where the collisions between the carriers and the scatterers become substantial. To track the asymptotic approach of the function $\Phi(\mathbf{r})$ to the limiting values $V/2\ \mathrm{sgn}\, z$, we consider the spreading of the current in the most remote regions of space, $r>l_i$. At these distances the change of the function $\Phi(\mathbf{r})$ in the scale of the electron mean free path and therefore the connection of the current density with the electric field strength $\mathbf{E}(\mathbf{r})=-\nabla\Phi(\mathbf{r})$ can be regarded as local:

$$j_i(\mathbf{r})=\sigma_{ik}E_k(\mathbf{r}). \quad (46)$$

The distribution of the potential in the system is obtained by solving the continuity equation $\mathrm{div}\,\mathbf{j}=0$ under the condition that the total current flowing through a closed surface is equal to the current I calculated on the section of the contact and equal to V/R. Solution of such a boundary-value problem is equivalent to solving the equation

$$\left\{\sigma_\perp(H)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+\sigma_\parallel\frac{\partial^2}{\partial z^2}\right\}\Phi(\mathbf{r})=2I\delta(\mathbf{r})\,\mathrm{sgn}\, z, \quad (47)$$

where the diagonal components of the conductivity tensor are

$$\sigma_\perp(H)=\sigma(\Omega\tau_i)^{-2},\quad \sigma_\parallel=\sigma. \quad (48)$$

Analysis of (47) leads to the following expression for the potential distribution at large distances:

$$\Phi(\mathbf{r})=\frac{V}{2}\mathrm{sgn}\, z\left[1-\frac{d^2}{4r_H(x^2+y^2+z^2(\Omega\tau_i)^2)^{1/2}}\right]. \quad (49)$$

It follows from the form of (49) that at $z\sim l_i$ and $\rho\sim r_H$ we have

$$\Phi(\mathbf{r})-\frac{V}{2}\mathrm{sgn}\, z\sim\frac{d^2}{r_H^2}\frac{V}{2}, \quad (50)$$

which agrees in order of magnitude with the asymptotic potential (44),(45) obtained in the ballistic region.

As mentioned in §2, the inelastic addition to the current is determined by the form factor $K(\mathbf{p},\mathbf{p}')$. To calculate it we must substitute the function $\alpha_\mathbf{p}(\mathbf{r})$ in (25). If we confine ourselves to the value of $\alpha_\mathbf{p}(\mathbf{r})$ in the ballistic motion region (40), the integral in the definition of $K(\mathbf{p},\mathbf{p}')$ can be reduced to the form

$$K(\mathbf{p},\mathbf{p}')=\frac{3\pi}{4d}|v_z|\frac{\theta(-v_zv_z')}{S}\times\int d^2\rho\int d\tau\,\theta\left[\left(\frac{d}{2}\right)^2-x^2(\tau;\mathbf{p},\mathbf{p}')-y^2(\tau;\mathbf{p},\mathbf{p}')\right], \quad (51)$$

$$
\begin{aligned}
x(\tau;\mathbf{p},\mathbf{p}') &= x + r_H{}^{\perp}[\sin(\Omega\tau+\varphi)-\sin\varphi] \\
&\quad + r_H{}'^{\perp}\left[\sin\left(-\Omega\tau\left|\frac{v_z}{v_z{}'}\right|+\varphi'\right)-\sin\varphi'\right], \\
y(\tau;\mathbf{p},\mathbf{p}') &= y + r_H{}^{\perp}[\cos(\Omega\tau+\varphi)-\cos\varphi] \\
&\quad + r_H{}'^{\perp}\left[\cos\left(-\Omega\tau\left|\frac{v_z}{v_z{}'}\right|+\varphi'\right)-\cos\varphi'\right],
\end{aligned}
\tag{52}
$$

where $r_H^{\perp} = cp_{\perp}/eH$, $r_H'^{\perp} = cp_{\perp}'/eH$, $p_{\perp}\cos\varphi = p_x$, $p_{\perp}\sin\varphi = -p_y$, $p_{\perp}'\cos\varphi' = p_x'$, $p_{\perp}'\sin\varphi' = -p_y'$. The meaning of the integral with respect to τ in the definition of the K factor in (51) is that it describes the electron-phonon relaxation as the charge moves along a trajectory that terminates in a certain point $\boldsymbol{\rho}$ in the contact plane, corresponding to the value of the momentum $\mathbf{p}$. Account is taken here only of those trajectory segments within which the electron, scattered into a state $\mathbf{p}'$, can land as it moves farther in the orifice of the contact and by the same token decrease the total current calculated without allowance for scattering (the so-called return current). It must be noted that the integration with respect to τ in (51) must be carried out over all times of electron motion on the trajectory, i.e., from zero to infinity. As $H\to 0$ we obtain from (51) a K-factor value

$$K_0=\theta(-v_z v_z')\,|v_z v_z'|/|v_z'\mathbf{v}-v_z\mathbf{v}'|, \tag{53}$$

which coincides with the ballistic K factor.[7]

The situation in a magnetic field is much more complicated. The integral in the definition of the K factor diverges. The reason is that in view of the periodic recurrence of the coordinate ρ on the helical trajectory the electron can repeatedly return from the scattered state to the contact area. It is obvious at the same time that only elastic scattering can limit the integration with respect to τ. Attention must be called in this connection to the fact that even a single collision with an impurity under the condition $r_H \gg d$ (see Fig. 4) places the electron on a trajectory from which ballistic return to the opening is impossible. The integration with respect to the time of motion τ must be carried out up to the instant of the nearest collision with the scatterers. As a result, to estimate the order of magnitude of the K factor it suffices to integrate with respect to τ in (51) from zero to the time τ_i of the carrier elastic scattering. Calculation of the K factor beyond this is possible, albeit very cumbersome. Compact estimates can be obtained only by averaging over the possible orientations of the vectors $\mathbf{p}$ and $\mathbf{p}'$. We then obtain

$$\langle K(\mathbf{p},\mathbf{p}')\rangle \approx \langle K_0\rangle\left(1+\eta\frac{dl_i}{r_H{}^2}\right),\qquad r_H \ll l_i, \tag{54}$$

where $\langle K_0\rangle$ is the averaged value of the ballistic K factor in a zero magnetic field and is equal to 1/4, while η is a coefficient of the order of unity.

Note that, just as for strong fields, the form factor depends on the elastic-relaxation length l_i. In weak fields, however, it increases as the square of the magnetic field. At $r_H \sim d$ the obtained K factor is matched to the results of the preceding section. In addition, the characteristic l_i values in (54) are bounded from above by the condition (2) of one-phonon relaxation of the electrons.

5. CONCLUSION

The analysis in the present paper has demonstrated the substantial influence of a magnetic field on the character of spreading of the current in point contacts. Helical electron motion oriented along the magnetic field does not ensure three-dimensional spreading of the current, so that the characteristic length over which the electronic system remains substantially inhomogeneous is determined only by the elastic-scattering length l_i. In pure contacts, $l_i \gg d$, the effective phonon-generation region increases steeply compared with the case $H=0$. The intensity of the point-contact spectra is determined by the value of the K factor. The expression (39) obtained for the K factor in a strong magnetic field is highly anisotropic, and its mean value in the limit as $H\to\infty$ is zero. The amplitude and sign of the spectrum depend, first, on whether the excited phonons are transverse or longitudinal relative to the field direction and, second, on the character of the electron scattering (intra- or intervalley). The last circumstance is vital for semimetals[3] and requires clarification.

Intravalley scattering can excite phonons with arbitrary wave vectors $\mathbf{q}(0<q<k_F)$. The electron-phonon interaction has strong dispersion, and the K factor is not averaged out in (24) (even when $\langle K\rangle \ll 1$). The intensity of the spectrum in a magnetic field can then increase by a factor l_i/d. In intervalley scattering, the momentum change is $\Delta q \sim k_D \gg k_F$ (k_D is the Debye momentum), and the relative scatter of the values of q is small, of the order of k_F/k_D, i.e., the dispersion is weak. The main contribution to the point-contact spectrum is therefore made by the averaged value of the K factor. The amplitude of the spectrum depends in this case on the parameter ratios l_i/d and k_F/k_D, and can both increase and decrease with increase of field. Such a behavior of the inter- and inter-valley peaks was observed in experiments with antimony.[3] The transition from an increase of the peak intensity in weak magnetic fields ($r_H \gg d$) to the strong-field regime described above takes place in fields satisfying the condition $r_H \sim d$. This makes it possible in principle to determine the dimensions of the contact by an independent method from the "size effect" in the intensity of the point-contact spectra.

Although we have considered the case of field orientation along the contact axis (along the current), it can be shown that the effects in a strong field, $r_H \ll d$, remain unchanged also for another orientation. The reason is that the picture of "one-dimensional" motion in the field (Fig. 2) is not sensitive to the orientation of the vector $\mathbf{H}$ (except when the field is strictly parallel to the surface).

We note in conclusion that the use of extremely strong magnetic fields with $r_H \ll d$ increases the length of the spreading of the current and makes it commensurate with the length of the electron–phonon interaction. This should change the conditions of charge transport in the point contacts—a transition from the ballistic regime of point-contact spectroscopy to the thermal regime.[12,13] In this transition region, the point-contact spectra should broaden with increase of field, owing to the additional heating in the point contact. These effects (as well as one that does not follow from the present paper, a shift of the intervalley spectra in a strong field into a region of higher energies[3]) were observed in experiment[3] for phonons due to intervalley scattering of electrons in antimony. The absence or smallness of such effects for intravalley transitions may be attributed to the fact that in the latter case the magnetic field is not strong enough to violate condition (1) (e.g., in view of the large inelastic length l_ε in the region of small displacements). The characteristic value of the magnetic field H_c at which the change of regimes occurs is determined by relation (1), viz.,

$$H_c=(cp_F/ed)(l_\varepsilon/l_i)^{1/2}. \quad (55)$$

Study of the nonlinear electric conductivity in such strong fields offers a unique possibility of observing a transformation, continuous in the field, of point-contact spectra on going from a conservative to a thermal regime of electric conductivity.

Oscillations of the potential in a contact in fields $r_H \gg$, which appear at sufficiently large mean free paths $l_i, l_\varepsilon \gg r_H$, will lead to interesting effects, particularly to a resonant behavior of the electric conductivity of a point contact in an alternating field.

[1)] We have neglected in (28) the influence of the electric field on the electron motion ($eV \ll \mu$) and chose for simplicity a collision integral in a model form corresponding to an elastic-scattering cross section that is independent of the momenta.

[2)] Substitution of (34) in (31) confirms the validity of this expansion. The expansion parameter is $r_H/d \ll 1$.

[3)] Since the electron loses energy by collision, the maximum energy acquired in the field at $l_\varepsilon \sim d$ should be somewhat less than eV; this causes, besides broadening, also a shift of the spectrum towards higher energies.

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Translated by J. G. Adashko