# Critical fluctuations in thin superconducting vanadium films in quantized magnetic fields

N. Ya. Fogel' and V. G. Cherkasova

*Low-Temperature Physicotechnical Institute, Academy of Sciences of the Ukrainian SSR*

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Crossover (change of effective fluctuation dimensionality) induced by a magnetic field $H$ in thin vanadium films during the superconducting transition is observed and investigated. In the critical region of reduced temperatures $\varepsilon_H = (T - T_{cH})/T_{cH}$ the normalized excess conductivity is described by the expression $\sigma'/\sigma_\eta \sim \exp\{\gamma_\eta \varepsilon_H^\eta\}$. The fluctuations are two-dimensional ($n = 1$) in weak magnetic fields, regardless of the orientation of $H$ with respect to the film. For strong quantized fields normal or parallel to the film, one has $n = 2$ and $n = 3/2$, respectively. The critical index $n$ varies with increasing $H$ from 1 to the values 2 and 3/2, respectively, the variation being quite sharp in a narrow range of field strength. The transitions also become broader with increasing $H$. Crossover is due to quantization of the fluctuation spectrum in the magnetic field. In a strong field $H_\perp$ the fluctuations are zero-dimensional, while in a field $H_\parallel$ they are quasi-one-dimensional.

Recent experiments[1–5] have shown that the superconducting transition in a thin film of a transition metal or intermetallide has several features that can be attributed to critical fluctuations in the order parameter. These include: 1) An exponential dependence of the normalized excess conductivity $\sigma'/\sigma_\eta$ on the reduced parameters $\varepsilon = (T - T_{c0})$ and $h = (H - H_c(T))/H_c(T)$ for temperatures close to the superconducting transition point $T_{c0}$ (here $\sigma' = \sigma - \sigma_\eta$, and $\sigma_\eta$ is the conductivity in the normal state); this dependence was predicted theoretically[6,7] for critical phenomena in two-dimensional superconductors. 2) The resistive transitions obey a similarity law. 3) The transition widths $\Delta H_\perp/H_{c\perp}$ and $\Delta T/T_{c0}$ are inversely proportional to the electron mean free path $l$. 4) The resistive behavior for $T \approx T_{c0}$ is insensitive to the orientation of the magnetic field relative to the film.

These experimental findings are in agreement with the theoretical predictions in Refs. 6, 7 for critical phenomena in superconducting films, whereas it was shown previously in Refs. 2 and 3 that they cannot be explained in terms of alternative mechanisms for quenching in superconductors (static variations in the parameters from one point to another in the superconductor, dissipative motion of ordinary Abrikosov vortices, topological transition in a system of thermally excited vortices).

The experimental observation of critical phenomena in transition-metal films would be impossible were it not for the low dimensionality of the system,[8] the extremely small electron mean free path ($\sim 10^{-7}$ cm), and the rather short correlation length $\xi_0$ characteristic of the transition metals. All of these factors ensure that the critical region is broad.

The behavior of the excess conductivity for $T \approx T_c$ and $H \approx H_c$, where the fluctuations are large,[1)] must depend on the dimensionality of the space in which the fluctuations develop. One thus expects the dependence $\sigma'(\varepsilon,h)$ for bulky three-dimensional ($3D$) superconductors to differ from $\sigma'(\varepsilon,h)$ for lower-dimensional systems (thin $2D$ films, fine $1D$ wires, systems of small $0D$ particles), for which one or more of the geometric dimensions is smaller than the superconducting correlation length $\xi(T)$. The critical indices for the films studied previously in Refs. 1–4 correspond to the $2D$ case. Since the correlation length is strongly temperature-dependent near the critical point $\varepsilon = 0$, the effective dimensionality may change as the critical point is approached. Such changes (or crossovers, as they are now called) have recently been investigated[9] for a wide variety of systems with second-order phase transitions.

The possibility of crossover at sufficiently high magnetic field strengths has been considered theoretically.[10–14] The quantization of the fluctuation energy spectrum[2)] constraints the order parameter to fluctuate in a plane normal to the magnetic field, so that the effective dimensionality of the fluctuations is decreased. This strong-field effect acts in addition to the ordinary spatial quantization associated with the geometry of the specimen. Thus, for a bulk superconductor in a strong magnetic field the fluctuations should become one-dimensional,[10,11] while they should become "zero-dimensional" for a film in a perpendicular magnetic field[10,12] (in the latter case spatial quantization and quantization due to the magnetic field both occur together). On the other hand, if a strong magnetic field is applied parallel to the film, a transition from $2D$ to $1D$ fluctuations should occur.[14]

In this paper we report results from an experimental study of the excess conductivity for thin vanadium films which indicate that the dimensionality of the fluctuations is in fact decreased in strong quantized fields. We consider crossover in detail for both normal and in-plane magnetic fields and find that the fluctuations differ significantly for these two orientations. A dependence of the form $\ln \sigma_n/\sigma' \sim [(T - T_c)/T_c]^\eta$ is found, where the critical index $n$ changes in a narrow range of magnetic field strengths near the crossover point. The index $n$ is orientation-dependent for strong fields.

## EXPERIMENTAL RESULTS

The thin vanadium films used in the experiments were of thickness $d = 200$–$500$ Å. The experimental technique and the method used to grow the films are fully described in Ref. 2. Most of the measurements were carried out at a low transport current $100\,\mu$A, and the resistive transitions were reversible, i.e., there was no thermal hysteresis.

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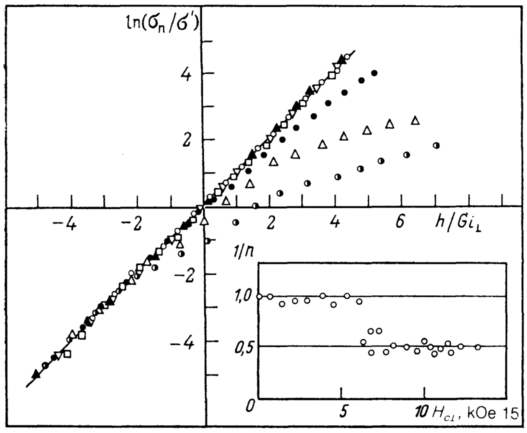


FIG. 1. $\ln(\sigma_n/\sigma')$ versus $h/\mathrm{Gi}_H$ for film specimen V-5 at seven different temperatures (in kelvins): ○, 3.650; □, 3.421; ▽, 3.270; ▲, 3.044; ●, 2.517; △, 2.244; ◑, 1.857. The insert shows $1/n(H_\perp)$ for film V-7.

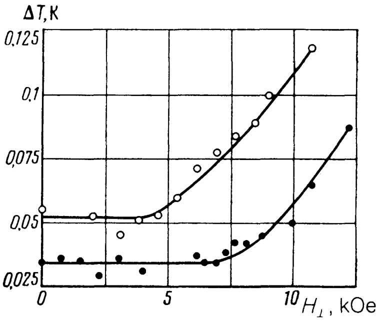


FIG. 2. Broadening $\Delta T$ as a function of $H_\perp$ for two films: ○, V-2; ●, V-7. The width $\Delta T$ is defined as the difference between the temperatures corresponding to $0.75R_\eta$ and $0.5R_\eta$ during the resistive transition.

We have already noted in the Introduction that the excess conductivity $\sigma'$ depends exponentially on the reduced magnetic field for temperatures close to the transition point $T_{c0}$. Figure 1 plots $\ln\sigma_n/\sigma'$ as a function for $h/\mathrm{Gi}_H$ for some typical resistive transitions when the magnetic field was normal to the film. The data show that for $T\approx T_{c0}$ the dependence $\sigma'(T,H)$ can be described by the expression[1,2]

$$\sigma_n/\sigma'=\exp\{h/\mathrm{Gi}_H\}, \qquad (1)$$

i.e., by a similarity law. Here

$$\mathrm{Gi}_H=\mathrm{Gi}_T/\varepsilon,$$

where $\mathrm{Gi}_T\approx\hbar^2/m_{\mathrm{eff}}^2 v_F{}^2 ld$ is the ordinary Ginzburg number for a two-dimensional superconductor,[15] $m_{\mathrm{eff}}$ and $v_F$ are the effective mass and the Fermi velocity of the electrons, and $d$ is the film thickness. The "magnetic" Ginzburg number $\mathrm{Gi}_H$ determines the width of the critical region for the phase transition in an external magnetic field. Formula (1) holds for $\sigma'(T\approx T_{c0})$ for both normal and in-plane fields, although $\mathrm{Gi}_H$ differs somewhat for these two orientations. However, (1) is valid only for a narrow range of temperatures, below which[3)] the transitions become much broader. This type of deviation from the general dependence (1) is particularly characteristic for the late stage of the resistive transition ($R\gtrsim 0.5R_n$).

Using variables in terms of which the dependences become linear, one sees readily that at the low temperatures for which (1) breaks down, the dependence $\sigma'(T)$ for a perpendicular magnetic field can be described empirically by

$$\sigma_n/\sigma'=\exp\{\gamma_2\varepsilon_H{}^2\}, \qquad (2)$$

where

$$\varepsilon_H=(T-T_{cH})/T_{cH}, \qquad T_{cH}=T_{c0}(1-H/H_{c\perp}'(0)) \qquad (b)$$

is the transition temperature in the magnetic field,

$$H_{c\perp}'(0)=T_{c0}dH_{c\perp}/dT|_{T_{c0}}, \qquad (c)$$

and the parameter $\gamma_2$ is independent of $\varepsilon_H$ and characterizes the width of the transition.

To compare the dependences $\sigma'(T)$ in weak and strong magnetic fields, we rewrite (1) in a form

$$\sigma_n/\sigma'=\exp\{\gamma_1\varepsilon_H\}, \qquad (3)$$

analogous to Eq. (2); here $\gamma_1=\mathrm{Gi}_T^{-1}(1-H/H_{c\perp}'(0))$. Formula (3) can be derived from (1) by using the above relations for $R_{cH}$ and $\mathrm{Gi}_H$.

Expressions (2) and (3) are both clearly of the same general form

$$\sigma_n/\sigma'=\exp\{\gamma_n\varepsilon_H{}^n\}, \qquad (4)$$

where the critical index $n$ determining the temperature dependence $\sigma'(T)$ depends on the magnetic field. For weak fields we have $n=1$, while for strong fields we have $n=2$. More detailed measurements on a series of films revealed that $n$ changes very abruptly within a field interval of width 1–2 kOe. The plot of $1/n(H_\perp)$ shown in the insert in Fig. 1 was found from a least-squares computer analysis of the experimental data [an arbitrary critical index $n$ was chosen for each curve $\sigma'(T)$, with $n$ varying from 0 to 3 in steps of 0.05, and the best value was selected]. A similar abrupt change in $1/n$ was observed for the other films. The field dependence of the transition width $\Delta T(H_\perp)$ differs greatly for $n=1$ and $n=2$ (Fig. 2).[4)] For weak fields $\Delta T$ is independent of $H_\perp$, while for strong fields $\Delta T$ increases appreciably with $H_\perp$. The increase in $\Delta T(H_\perp)$ begins for $H_\perp$ in the same interval in which $n$ changes. As for the inverse dependence $\Delta H_\perp(T)$, the above results show that $\Delta H_\perp=\mathrm{const}$ for $T\approx T_{c0}$, while $\Delta H_\perp$ increases with $1/T$ at low temperatures.

We now consider the case when the field is parallel to the film. Once again, the behavior of $\sigma'$ changes as $H_\parallel$ increases; Fig. 3 plots $\ln(\sigma_n/\sigma')$ as a function of temperature

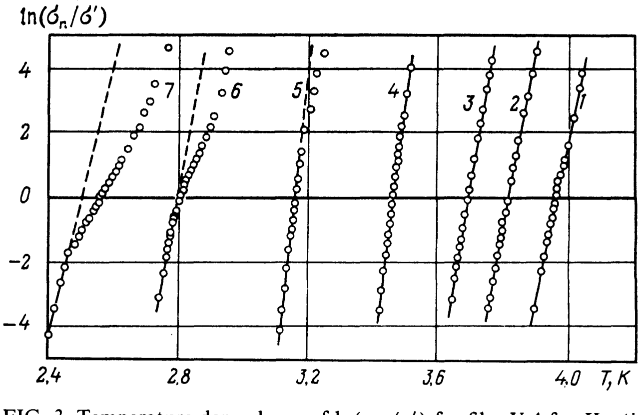


FIG. 3. Temperature dependence of $\ln(\sigma_\eta/\sigma')$ for film V-4 for $H_\parallel$ (in kOe) = 0(1), 4.62 (2), 6.16 (3), 10.8 (4), 13.9 (5), 16.17 (6), and 16.94 (7).

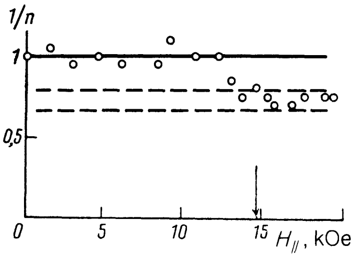


FIG. 4. Dependence $1/n(H_{\parallel})$ for film V-7. The arrow shows the critical field at $T = T_0$.

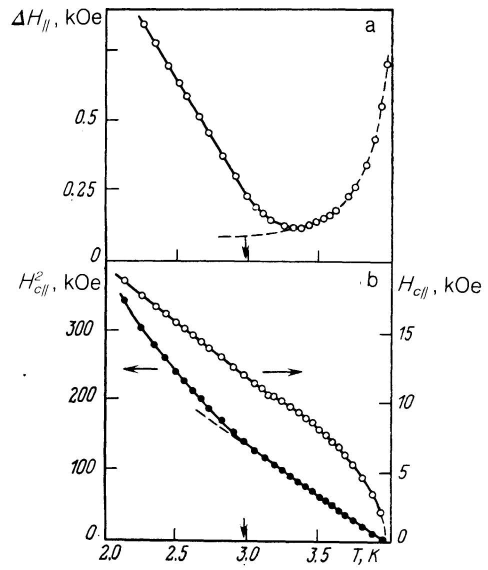


FIG. 5. a) Absolute broadening $\Delta H_{\parallel}$ of the transition as a function of temperature for film V-4 (the width $\Delta H_{\parallel}$ is defined in the same way as in Fig. 2). b) Critical field $H_{c\parallel}$ and its square $H^2_{c\parallel}$ as functions of $T$ for the same film. The arrow shows the temperature $T_0$.

for several values of $H_{\parallel}$. As in the case of strong fields $H_{\perp}$, Eq. (1) breaks down and $\ln(\sigma_n/\sigma')(T)$ becomes nonlinear.[5)] Computer analysis of the experimental data yielded the curves $1/n(H_{\parallel})$ shown in Fig. 4 for the reciprocal of the critical index. For large fields we find $n = 0.73 \pm 0.05$. This result differs both from the value $n = 1$ for $2D$ fluctuations near $T_{c0}$ and from the value of $n$ for strong perpendicular magnetic fields. The experimental results show that Eq. (4) with $n = 3/2$ accurately describes $\sigma'(T)$ for strong in-plane fields.

The temperature dependence $\Delta H_{\parallel}(T)$ of the absolute transition width is quite different from the case of a perpendicular field (Fig. 5a). At low temperatures $\Delta H_{\parallel}$ increases as $T$ decreases, just as for fields $H_{\perp}$; for $T \approx T_{c0}$, however, $\Delta H_{\parallel}$ decreases with decreasing $T$.[6)] Figure 5b shows how $H_{c\parallel}$ and $H^2_{c\parallel}$ depend on $T$, where $H_{c\parallel}$ is the critical field. We see that $\Delta H_{\parallel}(T)$ decreases to a minimum at some temperature $T_0$ and then increases, i.e., the fluctuation behavior changes markedly near $T_0$. For $T > T_0$ we have $H_{c\parallel} \sim \varepsilon^{1/2}$, while for $T < T_0, H_{c\parallel} \sim \varepsilon$. This behavior of $H_{c\parallel}(T)$ is due to the fact that a one-dimensional chain of vortices enters the film when the field is parallel.[16] The experimental results for $T_0$ in Table I are seen to agree closely with the values calculated in Ref. 16.

## DISCUSSION

We have already noted that for thin superconducting films, the quantization by the magnetic field accompanies the spatial quantization caused by the thinness of the film. The nature of the fluctuations will thus depend greatly on whether the field $H$ is normal or parallel to the film.

The experimental data presented in the previous section show that the strong- and weak-field resistive transitions differs greatly. The weak-field resistive behavior is the same for normal and parallel fields $H_{\perp}$ and $H_{\parallel}$ (the critical index $n$ is equal to 1), while for strong fields the values of $n$ for $H_{\perp}$ and $H_{\parallel}$ are not the same (Figs. 1 and 4).

We will show below that the observed change in the critical behavior of the excess conductivity in strong perpendicular fields occurs because the field restricts the possible configurations of the fluctuating order parameter, thereby decreasing the effective dimensionality of the fluctuations from $2D$ to $0D$. The behavior for fields parallel to the film can be explained similarly (in this case, however, a $2D \to 1D$ crossover occurs). This interpretation, which is based on quantization of the fluctuation spectrum by the magnetic field, can account for all of the experimental findings—the dependence of the critical index $n$ on the magnitude and orientation of the magnetic field, the deviation from the law of corresponding states, and the characteristic features of the dependence $\Delta H(T)$.

Crossover can be analyzed most easily for the case of small fluctuations of the order parameter in a thin film perpendicular to a magnetic field. The Ginzburg-Landau (GL) functional is given by

$$F\{\psi\} = \int d\mathbf{r} \left\{ \alpha |\psi|^2 + \frac{1}{2m} \left| \left( \mathbf{p} - \frac{2e}{c}\mathbf{A} \right) \psi \right|^2 \right\} \tag{5}$$

provided the fluctuations are small enough so that their interaction may be neglected. Expanding the order parameter $\psi$ in terms of eigenfunctions, we can reduce the functional (5) to

$$F = \sum_l E_l |\psi_l|^2, \tag{6}$$

TABLE I.

| Specimen | $T_{c0}$ | $d$, Å | $T_0$, exp | $T_0$, theor. | $b$ | $2b/\varepsilon_{max}$ |
|---|---|---|---|---|---|---|
| V-2 | 4.121 | 420 | 3.585 | 3.537 | 0.204 | 16 |
| V-3 | 4.475 | 230 | – | 1.21 | 0.115 | 2.6 |
| V-4 | 3.978 | 380 | 3.080 | 3.024 | 0.321 | 29 |
| V-5 | 3.987 | 420 | 3.095 | 3.016 | 0.256 | 37 |
| V-7 | 3.684 | 320 | 2.580 | 2.604 | 0.237 | 13 |
| V-8 | 3.861 | 420 | 2.935 | 2.949 | 0.320 | 32 |

where $E_l$ is the energy spectrum for the order parameter fluctuations (see Ref. 17). The eigenvalues $E_l$ can be found by solving the linearized GL equation obtained from (5). Dimensional quantization and Landau quantization ensure that the spectrum $E_l$ is discrete. Since the dimensional quantization levels for a thin film are well separated, by an energy $\pi^2\hbar^2/md^2 \gg kT$ (Ref. 18), the terms corresponding to nonzero dimensional quantization levels in the energy spectrum may be omitted, so that the spectrum is of the form

$$E_l=\alpha(T)+\hbar\omega_H(l+{}^1/_2)=E_0+\hbar\omega_H l. \qquad (7)$$

Here $\omega_H = 2eH/mc$ is the cyclotron frequency, $l = 0, 1, 2, \ldots$, and $\alpha(T) = \hbar^2/2m\xi^2(T)$. Since the state with $l = 0$ has the lowest energy, the condition

$$E_0=\alpha(T)+\hbar\omega_H/2=0 \qquad (8)$$

determines the field dependence of the critical temperature $T_{cH}$.[12] Formula (6) implies[17] that the mean square fluctuation amplitude

$$\langle|\psi_l|^2\rangle=T/2E_l \qquad (9)$$

decreases as $E_l$ increases.

The effective dimensionality of the fluctuations decreases when the energy $E_0$ of the lowest Landau level ($l = 0$) is much less than the distance $\hbar\omega_H$ between the levels, so that $E_0 \ll E_{l\neq 0}$. In this case $\langle|\psi_0|^2\rangle \gg \langle|\psi_l|^2\rangle$, so that by (9) we have

$$\frac{\hbar^2}{2m\xi^2(0)}\frac{\delta T}{T_{c0}} \ll \hbar\omega_H, \qquad \delta T=T-T_{cH},$$

or equivalently,

$$\delta T \ll \frac{2H}{H_{c\perp}(0)}T_{c0}. \qquad (10)$$

When (10) is satisfied, the fluctuations in the film are zero-dimensional,[10,12,13] while when the inequality in (10) is reversed the magnetic field does not change the dimension of the fluctuations, which remain $2D$.

The data in Table I give an idea of the extent to which (10) is satisfied experimentally during crossover when the quantization of the fluctuation spectrum becomes important. Values of $b = H/H'_{c\perp}(0)$ at which $n$ jumps from 1 to 2 are listed for several films, together with the ratio $2b/\varepsilon_{max}$, where $\varepsilon_{max} = [(T - T_{cH})/T_{c0}]$ is the reduced width of the temperature interval near $T_{cH}$, within which the fluctuations are zero-dimensional ($\varepsilon_{max}$ increases with $H$). Table I shows that $2b \gg \varepsilon_{max}$ for all of the films (except $V-3$, for which $2b/\varepsilon_{max} > 1$), i.e., the crossover condition (10) is satisfied.

Since no theory is yet available for resistive phenomena in quantized magnetic fields in the critical region, we will base our discussion of the experimental findings on the theoretical results for crossover in Refs. 10, 12–14, 20, where the change in the behavior of $\sigma'$ was analyzed for small fluctuations in the order parameter, and on the results in Refs. 6, 7, 21, 22, where both large and small fluctuations were considered for superconductors of various dimensions in the absence of a magnetic field.

If we compare the results of Aslamazov and Larkin[21] with the ones in Refs. 6, 7, 22, where critical fluctuations were studied, we see that $\sigma'$ may vary either exponentially or as a power of the reduced temperature, depending on the relative magnitude of the fluctuations. On the other hand, the critical index $n$ for the conductivity is the same for both large and small fluctuations and depends only on the spatial dimension $D$. One finds $n = 1/2$, 1, and 2 for $D = 3$, 2, and 0, respectively.

In our experiments using fields of various strengths perpendicular and parallel to the films, we obtained the values $n = 1$, 3/2, and 2, which correspond to $D = 2$, 1, and 0. According to the theory in Ref. 12, the transition from $n = 1$ to $n = 2$ in a field $H_\perp$ corresponds to a crossover in which the effective dimensionality of the fluctuations decreases from $2D$ to $0D$. Similarly, the change in the critical index from $n = 1$ to $n = 3/2$ in a parallel field should correspond to a $2D \rightarrow 1D$ crossover.[14]

The weak-fluctuation theory thus predicts that the magnetic field should decrease the effective dimensionality, while our experimental results demonstrate that this actually occurs quite generally—the critical indices for $\sigma'$ depend on the quantized field in the same way for both large and small fluctuations. Moreover, the dependence $\sigma'(T)$ for a parallel field, for which the fluctuations should become one-dimensional, has the same form as $\sigma'(T)$ for critical fluctuations in one-dimensional wires when $H = 0$ (Ref. 22; for obvious reasons, no such comparison is possible for $0D$ fluctuations). In view of the foregoing, we conclude that crossover due to quantization of the fluctuation spectrum in the magnetic field is responsible for the radical change in the fluctuation behavior in strong fields. This conclusion is further supported by the abrupt broadening $\Delta T(H)$ that accompanies the change in $n$. According to the theory in Refs. 10 and 11, a sharp dependence $\Delta T(H)$ should result from a decrease in the dimensionality.

We observe that the empirical formula (2), which describes the behavior of $\sigma'$ for $0D$ fluctuations, becomes meaningless for negative $\varepsilon_H$ (i.e., when $T < T_{cH}$), because $\sigma'$ changes monotonically with temperature. In Refs. 10 and 12–14, the theoretical results on crossover were derived for small fluctuations ($T > T_{cH}$) far from the transition points. When the transition temperature is approached from above, the fluctuations in the modulus of the order parameter increase and their interaction becomes important; however, the spatial constraints on the fluctuating order parameter are the same as for noninteracting fluctuations. The effects of the quantized field on the fluctuation behavior of $\sigma'$ are qualitatively the same as for the case of small fluctuations. In particular, the dimensionality decreases in the same way. However, the situation differs qualitatively for large $\sigma' \gg \sigma_n$, i.e., at the very beginning of the resistive transition. In this case the fluctuations in the phase of the order parameter are important for $2D$ systems.[23] It is not clear how a strong magnetic field will affect the behavior under these conditions, when nucleation of the normal phase occurs in the superconductor. For the range of magnetic fields investigated in our experiments, the early stage of the resistive transition was unchanged when the field was applied.

Here we should mention the work of Usadel,[24] whose results differ from the ones obtained in the other theoretical papers cited above. He found that the critical index for $\sigma'$ in a strong quantized field is the same as in the limit $H \rightarrow 0$, and that only the coefficient of $\varepsilon_H$ changes (it depends on the magnetic field). This discrepancy can be traced to the fact that Usadel allowed for flows of fluctuations in his treatment.[20]

The fact that our experimental results agree with Bergmann's theory[12] and not with Usadel's results would appear to indicate that there was no flow of fluctuations in our films. To rule out the possibility that pinning effects might have prevented us from detecting such a flow, we carried out measurements for resistive transitions at much higher currents. We found that the fluctuation behavior in strong perpendicular fields remained qualitatively the same even when the transport current was increased by three orders of magnitude. The maximum current (10 mA) was certainly greater than the critical pinning current for fields near the critical value. Usadel's theoretical results are thus in conflict with the experimental findings for large currents also. It is possible that the lifetime of the fluctuations was too short to permit a steady flow to become established.

We will now consider in more detail the case when the magnetic field is parallel to the film. Crossover in this geometry was analyzed theoretically by Imry in Ref. 14. It is not obvious that a "pure" crossover can be observed in a parallel field, because the spatial and the Landau quantizations are intertwined in a complicated way. The energy levels of the fluctuations for a plate in a parallel field ($H \| z$) are given by[14]

$$E=\left(l+\frac{1}{2}\right)\hbar\omega_H+\frac{\hbar^2k_z^2}{2m}+\frac{\hbar^2(k_y-k_0)^2}{2m_y} \qquad (11)$$

through second order in the components $k_i$ of the wave vector ($i=x,y,z$). Here $m$ is the mass of a free electron, $m_y=(2eH/c)^2\partial^2E/\partial x_0^2|_{\min}$ is the analog of the effective mass in the equation from which the spectrum (11) was derived, $k_0=2x_0eH/\hbar c$ is the value of $k_y$ for which the energy of the fluctuations is a minimum, and $x_0$ is the distance from the surface, on which superconducting regions are nucleated in the parallel field. The fluctuations will be two-dimensional if their energy depends on two components ($k_z$ and $k_y$) of the wave vector. The ratio $A=m/m_y$ depends on the magnetic field strength and on the thickness of the film[25]; it vanishes [i.e., the third term in (11) disappears] for a certain value of the dimensionless parameter $\alpha=H^2_{c\|d}/\Phi_0$. Thompson[25] has shown that this singular point corresponds to the field at which a single chain of vortices enters the film in a parallel field. In our approximation, the vanishing of the coefficient $1/m_y$ is equivalent to a one-dimensional fluctuation spectrum. Away from the singular point, $1/m_y$ increases and the fluctuations can be regarded as nearly one-dimensional only as long as $1/m_y$ remains small. A one-dimensional spectrum is thus possible only for $T$ close to $T_0$, the temperature at which a single chain of vortices enters the film. The dependence of $m_y$ on the surface conditions[14] is also important, i.e., $m_y$ depends on the extrapolation length $L$ appearing in the boundary condition $\psi^{-1}\partial\psi/\partial x|_{x=0}=L^{-1}$ for the modulus of the order parameter $\psi$. For an ideal surface (i.e., a superconductor/dielectric interface) we have $L=\infty$, and in the thick-film limit $A=0.58$ holds and crossover effects are of little significance. On the other hand, $L$ may be quite small if the superconductor is bounded by a normal metal, or if paramagnetic ions are present at the surface. If $L\sim\xi(T)$ then the ratio $H_{c3}/H_{c2}$ is considerably less than its value 1.7 for an ideal surface (Ref. 16), and $m_y$ is larger. If surface superconductivity is completely suppressed then $H_{c3}=H_{c2}$ and $m_y=\infty$, and the fluctuations are strictly one-dimensional (their energy is independent of $k_y$). The fluctuation behavior in this case should be similar to what is observed in bulk superconductors when $H\approx H_{c2}$. Thus, $1D$ (or quasi-$1D$) fluctuations can be observed only when $T\approx T_0$ and the surface conditions are favorable. In addition, the film thickness must be essentially constant everywhere, since otherwise the singular point at which $m_y$ vanishes will be spread out.

The values of $H_{c\|}/H_{c\perp}$ measured for $T<T_0$ were greater than 1.7 for all of our films. This indicates that $H_{c\|}/H_{c\perp}$ can approach the theoretical value 1.7 only for critical fields $H_{c\|}\gtrsim 2H_{FE}$ (Refs. 26, 27), where $H_{FE}$ is the entrance field for a single vortex chain. Since our films were relatively thin, we were unable to reach such critical fields even at the extreme low end of the accessible temperature range. It is thus difficult to assess the surface condition of the films from our experimental data on the critical fields. However, we found that $H_{c\|}/H_{c\perp}$ was between 1.1 and 1.3 for thicker $3D$ films ($d\gg\xi(T)$) grown under the same conditions. This indicates that the extrapolation length was quite small even for the thin films, i.e., the surface conditions were favorable for observing crossover.

Figures 4 and 5 show that as predicted by the theory in Ref. 14, crossover occurs in a neighborhood of the temperature $T_0$. The dashed lines in Fig. 4 show the theoretical values for $1/n$. Although the experimental error was too large to permit us to choose between the values $n=3/2$ and $n=5/4$ (Ref. 14), it is obvious that $n$ does change in a strong parallel field. It is also clear that the change in $n$ differs for perpendicular and parallel magnetic fields (cf. Figs. 1 and 4). For both parallel and normal fields, $n$ changes in a relatively narrow field interval. The change in $n$ in a parallel field is also accompanied by a pronounced field-dependent broadening of the transition (Fig. 5). In weak fields, for which there is little quantization of the fluctuation spectrum and the fluctuation are two-dimensional, the width of the critical region for the phase transition in a magnetic field is proportional to $\varepsilon^{-1}$, as predicted by the Shmidt theory[8] for $2D$ superconductors, while the absolute broadening $\Delta H_\|$ is $\varepsilon^{-1/2}$ (dashed curve in Fig. 5a). We see that deviations from the curve $\Delta H_\|\sim\varepsilon^{-1/2}$ begin at a temperature close to (but slightly higher than) $T_0$. This can also be seen from the plot of $1/n(H_\|)$ in Fig. 4. The value of the critical index and the size of the temperature interval within which the fluctuation behavior changes both suggest that the observed behavior is due to crossover (the fluctuations become $1D$).

Finally, we point out that since crossover in a parallel field is sensitive to the condition of the surface, it can be used to analyze film surfaces (in particular, e.g., thin-film Josephson junctions based on transition metals). Analysis of fluctuation effects in strong parallel fields may yield information on the boundary conditions for the order parameter at the surface of a superconductor.

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1)The interaction between the fluctuations is important for reduced parameters $\varepsilon$ and $h$ in this interval, which we call the critical region.

2)Since the linearized Ginzburg-Landau (GL) equation is formally analogous to the Schrödinger equation for an electron, the quantization problem for the fluctuating order parameter is similar to that for an electron moving in a magnetic field (see, e.g., Refs. 12, 14).

3)Since we are considering the excess conductivity near a line of critical points $H_c(T)$ [or equivalently, $T_c(H)$], the phase diagram for superconductors in a magnetic field implies that low-temperature transitions correspond to strong fields $H$.

4)The qualitative form of the dependence $\Delta T(H_\perp)$ (and also of $\Delta H_\parallel(T)$, discussed below) does not depend on the choice of the points on the resistive transitions in terms of which the widths $\Delta T$ or $\Delta H$ are defined.

5)One can show that Eq. (3) (with a different value of $\gamma_1$) also describes $\sigma'(\varepsilon_H)$ in a parallel magnetic field when the deviation from the phase transition line is small, i.e., $[T-T_{cH}]/T_{c0} \ll H^2/H^2_{c\parallel}(0)$. The latter condition was well satisfied experimentally for a wide range of fields.

6)The dependences $\Delta H_\perp(T)$ and $\Delta H_\parallel(T)$ differ for $T \approx T_{c0}$ even though $\sigma'(\varepsilon,h)$ is given by (1) in both cases. This is due to the completely different dependences of the critical fields $H_{c\perp}$ and $H_{c\parallel}$ near the critical temperature ($H_{c\perp} \sim \varepsilon$ throughout the range of temperatures investigated).


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