# Self-organization of spiral-vortex structures in shallow water with rapid differential rotation

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In shallow water with a tangential "discontinuity" of the rotation velocity "coherent" steady drifting structures have been found experimentally. They consist of extended spiral arms (crests) and banana-shaped vortices (craters) localized between them and elongated along the discontinuity of the velocity. The length of the spirals exceeds the Rossby-Obukhov scale by more than an order of magnitude.

In Refs. 1–3 it was found experimentally that there is a centrifugal instability of shallow water in which there is a jump ("discontinuity") of the rotation velocity between a rapidly rotating core and a periphery that rotates relatively slowly in the same direction. This instability generates a surface density wave in the form of spiral arms, which rotate steadily in the direction of rotation of the liquid at a velocity intermediate between the rotation velocities of the core and the periphery. The ends of the spirals are turned in the direction opposite to their intrinsic rotation (so-called trailing spirals). It was also noted (see also Ref. 4) that between the spiral arms there are localized vortices; in the nonlinear stage in the development of the instability, these apparently assist the generation of the spirals as "ship waves." The experiments of Refs. 1–3 were made with a view to simulating in the laboratory a hydrodynamic mechanism for generating the spiral structure of galaxies having a jump in the velocity of the rotation profile.

The work reported here is a fundamental development of the experiments of Refs. 1–3 and contains the following main results: 1) The first investigation of self-organized coherent spiral-vortex structures, which are of fundamental interest for nonlinear physics and its geophysical and astrophysical applications; 2) a demonstration that the structures remain stable even for such rapid rotation of the system that the maximal Rossby-Obukhov radius (the characteristic scale of dispersion) is at least an order of magnitude smaller than the length of the spiral arms; 3) a regime that is exotic from the point of view of astrophysics has been obtained; in it, the periphery rotates in the opposite direction to the core, and so-called leading spirals, which move with their ends in front, are formed.

The experiments were made using two modifications of the Spiral' apparatus (Fig. 1). The first of these differed from the apparatus of Refs. 1–3 by having a parabolic profile in the peripheral and much larger ratio $\Omega_2/\Omega_1$ of the angular frequencies of rotation of the periphery and core: $\Omega_2 \approx 0.2\Omega_1 \approx 3.6\ \text{sec}^{-1}$; the thickness of the layer of shallow water, measured in the vertical direction, was constant along the surface of the periphery: $H_0 \approx 1.5$ mm. The working liquid was a green solution of $NiSO_4$ in water, permitting high-contrast photographs of the spirals in red light illuminating the solution and reflected by the white bottom of the vessel. The second modification of the apparatus increased the dimensions by a factor 2 and consisted of two paraboloids whose shape corresponded to a layer of shallow water of constant thickness $H_0 = 1.5$–$3.5$ mm both in the core (with $\Omega_1 = 13\ \text{sec}^{-1}$) and in the periphery (with $\Omega_2 = 2.6\ \text{sec}^{-1}$). Glycerin was added to the $NiSO_4$ solution, and this increased its viscosity by two or three times (in all the experiments in which no special investigation was made of the influence of the viscosity, the total viscosity of the solution exceeded the viscosity of water under normal conditions by no more than a factor of 5–10); this addition of the glycerin greatly facilitated obtaining a spiral-vortex structure in the initial stage of its formation. For both modifications, the rotation regime was chosen to make the radius $D/2$ of the periphery much greater than the Rossby-Obukhov scale $r_R$ (which is analogous to the characteristic Lamor radius of plasma ions):

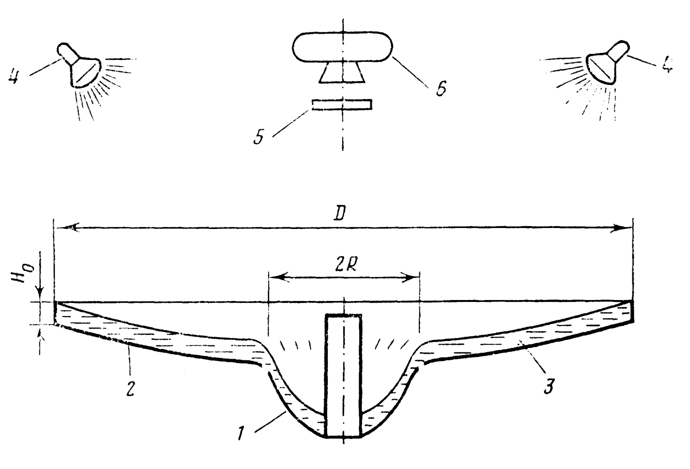


FIG. 1. The Spiral' apparatus: 1) core, 2) periphery, 3) layer of shallow water, 4) incandescent lamp, 5) red light filter, 6) camera. First modification: $D = 28$ cm, "discontinuity" radius $R = 4$ cm; a frustum of a cone with generator having angle of inclination 65° to the horizontal is used as core. The second modification: $D = 60$ cm, $R = 8$ cm. The core rotates clockwise when viewed from above.

  

$$r_R = (gH_0)^{1/2}/2\Omega_2, \quad (1)$$

where $g$ is the acceleration due to gravity. This choice of the regime made it possible to test whether the generated spirals were stable with respect to breakup into vortices of radius $\sim r_R$. We mention that the chosen ratios $\Omega_2/\Omega_1$ and $r_R/(D/2)$ are close to those actually observed in spiral galaxies.

A typical result, which unambiguously indicates stability of the spirals, is shown in Fig. 2. It corresponds to the following experimental conditions: second modification of the apparatus, core and periphery rotating in the same direction, and spiral arms rotating in the same direction with angular frequency $\Omega_p \approx 6$ sec$^{-1} \approx 0.8(\Omega_1 + \Omega_2)/2$. The ratio of the observed length of the arms to the characteristic scale of dispersion (1) is greater than an order of magnitude. The arms are very sharply delineated, their length is more than twice their length in Refs. 1–3, they are steady, and they do not exhibit signs of instability (in particular, there is no sign of breakup into smaller structures). The spiral arms cross the line of the jump in the rotation velocity; in order words, they exist not only in the periphery but also within the core. Between the spirals, along the jump in the rotation velocity, banana-shaped regions are localized, their number being equal to the number of spiral arms. These regions appear brighter, i.e., the thickness of the layer of liquid in them is significantly less than in the surrounding regions. They are anticyclonic vortices and form together with the spirals a single steady spiral-vortex structure that rotates with frequency $\Omega_p$.

A photograph of the motion of the liquid in the vortices obtained using the first modification of the apparatus by means of white test particles floating on the surface of the liquid is shown in Fig. 3 (in accordance with the requirements of the photography, the bright tracks of the test particles reduce the contrast of the spiral pattern; this "smearing" is especially pronounced at the tails of the spirals—cf. the photograph of the same structure without the test particles shown in Fig. 4). It can be seen that the vortices consist of trapped particles of liquid moving in closed trajectories of a banana shape drawn out along the jump in the rotation velocity. The direction of motion of the particles is anticyclonic, i.e., the liquid in the vortices rotates in the direction

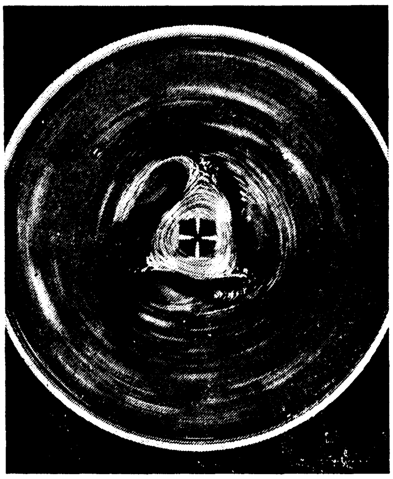

FIG. 3. Spiral-vortex structure for $\Omega_1 = 18$ sec$^{-1}$, $\Omega_2 = 3.6$ sec$^{-1}$, $H_0 = 1.5$ mm. First modification of the apparatus. The periphery rotates anticlockwise, in the opposite direction to the core. The camera is at rest relative to the structure; exposure time 1/8 sec.

opposite the rotation of the core. At the "ends" of the vortices, where the radius of curvature of the trajectories is approximately equal to the half-width $a$ of the bananas, there is a regime of equilibrium in which the centrifugal force due to the intrinsic rotation $F_c$ exceeds the Coriolis force $F_C$ (the difference between these forces is balanced by the pressure gradient force): $F_c > F_C$, or

$$\mathrm{Ro} \equiv \omega/2\Omega_p = F_c/F_C > 1, \quad (2)$$

where Ro is the Rossby number, $\omega \approx v_r/a$ is the characteristic rotation frequency of the vortex, and $v_r$ is the local radial velocity.

Measurement of the track lengths of the test particles corresponding to the photograph exposure time showed that $v_r \approx (gH_0)^{1/2}$. Using this, we find from (2) that the characteristic radial scale of the vortices is several times less than the Rossby-Obukhov radius (1) in the frame of reference rotating with the vortices:

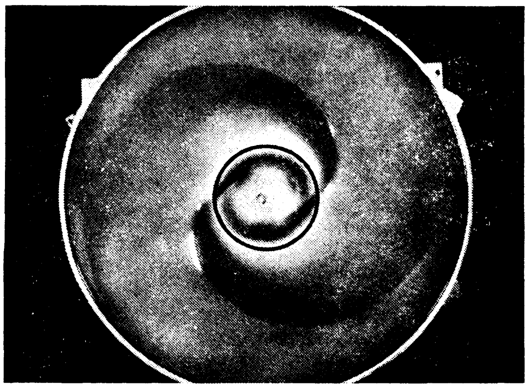

FIG. 2. Spiral surface density waves for $\Omega_1 = 13$ sec$^{-1}$, $\Omega_2 = 2.6$ sec$^{-1}$, $H_0 = 3.5$ mm. $r_R = 3.5$ cm. Apparatus of the second modification. The core, periphery, and spiral pattern rotate clockwise. The dark circle intersected by the spiral arms is the line of the "discontinuity" of the rotation velocity.

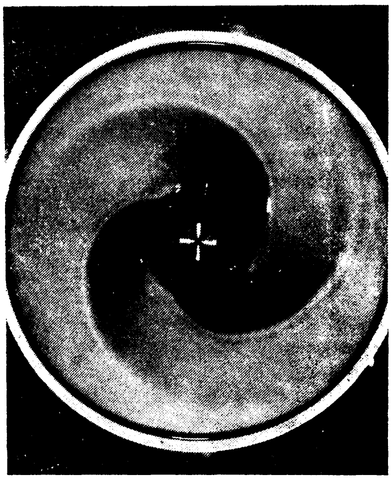

FIG. 4. Surface density waves of the same spiral-vortex structure and under the same conditions as in Fig. 3 but without visualization of the motion of the liquid by test particles.

$$a \lesssim r_R = (gH_0)^{1/2}/2\Omega_p. \quad (3)$$

The result (3) has a simple theoretical interpretation. One can show (for more details, see Ref. 4) that if in shallow water in a state of equilibrium differential rotation there is a local pertubation of the free surface in the form of a depression which rotates steadily with frequency $\Omega_p$ then fluid particles are trapped within this perturbation and describe about its center elliptic trajectories (similar to the observed bananas) only under the condition

$$\partial^2 z/\partial r^2 > \Omega_p^2/g, \quad (4)$$

where $z$ and $r$ are the vertical and radial coordinates of the points of the surface of the liquid. If the amplitude of the depression is $\Delta H \approx H_0$, the condition (4) can be approximately rewritten in the form $\Delta H/a^2 \approx H_0/a^2 > \Omega_p^2/g$, and this, apart from a coefficient 2, is equivalent to the condition (3).

There is much interest in comparing the vortices described in this paper with the Rossby vortices, also anticyclonic, observed in a different exerimental situation.[5-8] The experimental conditions differ in that all motions in the Rossby vortices are slow compared with the rotation of the system as a whole, i.e., in contrast to the relation (2) there is a Rossby regime in which $\mathrm{Ro} < 1$ and the centrifugal force due to the intrinsic rotation of the particles in the vortices is small compared with the Coriolis force; accordingly, the Rossby anticyclones are not wells but elevations. Another difference is that the long-lived Rossby vortices of Refs. 5–8 have scales several times as large as the Rossby-Obukhov radius (1), in contrast to the relation (3). Finally, these Rossby vortices are soliton attractor, into which an arbitrary extended disturbance of the liquid breaks up. The existence of such attractors enabled certain theoreticians to suggest that spiral structures which are long compared with the Rossby-Obukhov radius will break up into Rossby vortices (solitons). The results obtained in the present work do not confirm this suggestion. They show that in the present system very extended spiral arms, for which the length $L$ along the crests exceeds the Rossby-Obukhov radius by least an order of magnitude, are steady and do not exhibit any tendency to decay. It is also important to note that the face of experimental observation of long spiral arms ($L \gg r_R$) is in complete agreement with a computer result obtained in Ref. 9.

It can be seen from the present work, as from Ref. 9, that the real length of the spiral arms is in no way limited by the effect of screening of perturbations on scales $\sim r_R$ discussed in Ref. 10. In this connection, we note the following: In the second modification of the apparatus (differing from the one used in Refs. 1–4 by a doubling of the dimensions and a more "regular" core) the spiral arms were in the case of sufficiently rapid rotation of the periphery ($\Omega_2 = 0.2\Omega_1$) fairly long not only along the crests ($L \gtrsim 10 r_R$, Fig. 2) but also along the radius of the system (in other words, they were essentially "open," i.e., they were not tightly wound around the core); at the same time, they had approximately the same form as in the case of a periphery at rest ($\Omega_2 = 0$).

Detailed experiments showed that in the second modification one observes the same qualitative features of alternation of azimuthal instability modes (with different numbers of spiral arms) as a function of the Mach number at the velocity jump as in the earlier version.[1,2] These features also remain virtually unchanged despite considerable variation of the viscosity of the working solution, for example, for an increase of the viscosity up to 50 times the viscosity of water. The regime in which a particular mode (number $m$ of spiral arms) predominates depends somewhat on the viscosity of the solution; when the viscosity, like any other parameter of the system, is varied, hysteresis is observed. From these experiments it can be concluded that although the viscosity is in principle necessary, since it is through the viscosity that the differential rotation is transmitted to the layer of shallow water from the botton of the vessel, the outcome of the experiments depends little on the value of the viscosity provided there is differential rotation of the shallow water.

This situation is similar to the one found by observing spiral waves of a quite different nature, namely, those associated with the instability of an Ekman layer in which the rotation velocity of a liquid decreases vertically (see, for example, Ref. 11). "Ekman" spirals are very tightly wound, are observed under quite different experimental conditions, and have a quite different excitation mechanism, on which we shall not dwell here.

**The results obtained in the present work essentially confirm the point of view that in the experiments begun in Refs. 1–3 and continued here and in Ref. 4 we have the physical simulation of a hydrodynamic mechanism of generation of the spiral structure of galaxies. In these experiments, as in a galactic gas disk, the length of the spiral arms is much greater than the Rossby-Obukhov radius.** The spiral structure is generated by a shear (centrifugal) instability of differentially rotating shallow water with anticyclonic shear of the rotation velocity. Under the different conditions of the experiment of Refs. 5–8, this instability generates a rotational Rossby autosoliton, which is a physical laboratory analog of natural vortices like Jupiter's Great Red Spot. It is therefore very probable that natural structures that at the first glance seem very different, for example, the spiral arms of the galaxies and the planetary vortices of the type of Jupiter's Great Red Spot, are generated by a common mechanism, namely, the shear instability of differentially rotating shallow water.

We note that Figs. 3 and 4 correspond to a regime of differential rotation of the shallow water in which the periphery and core rotate in opposite directions. Changing slightly the value of $\Omega_1$ in this regime, we can obtain spiral-vortex structures that rotate one way or the other relative to the laboratory frame or are fixed (as in Figs. 3 and 4). It is interesting that the spirals which rotate in the opposite direction from the core (their form hardly differs from the form of the spirals in Figs. 3 and 4) move with their ends forward, corresponding to the case, rarely observed in astrophysics, of "leading" spirals, which are sometimes observed in systems of interacting galaxies (isolated galaxies have trailing spirals).[12a]

The present results (see also Ref. 4) make it possible to predict a vortex form for the motion of the matter between the spiral arms of galaxies, and they are in good agreement with observational data.[12b] Following Ref. 4, we can also suggest that leading spirals are observed in binary systems in which the orbital angular momentum of the "satellite" has the opposite sign to the intrinsic angular momentum of the spiral galaxy.

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