# Dynamics of Bloch lines in a ferromagnet

A. K. Zvezdin and A. F. Popkov

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The motion of an isolated Bloch line (BL) induced in a domain wall of a ferromagnetic material by external magnetic fields of varying orientation is investigated. A reduced equation which is nonlinear in the BL velocity, obtained by perturbation theory for solitons, describes the dynamics of the BL as a mechanical particle (filament) whose state is determined by the position of the center and by the velocity. The nonlinear dependences of the BL mass and of the friction force acting on the BL on the velocity are determined and the action of the gyroscopic force acting on the BL is considered. The maximum BL velocity in a domain wall is investigated. A new mechanism for dynamically transforming a BL into a cluster as the limiting velocity is reached is discussed.

Bloch lines (BL) constitute an important element of the domain structure of magnetically ordered crystals.[1] They demarcate domain-wall sections (subdomains) with different spin directions. The study of BL is of fundamental importance in understanding the dynamic properties of domain walls (DW) and of magnetization reversal in magnets.[2–5] BL have lately attracted much attention as potential means of developing magnetic memories with extremely high information density.

One of the main problems in the study of BL is their dynamics. The linear dynamics (linear response to external fields) has by now been well investigated,[2,7–9] but nonlinear processes have hardly been touched upon. The latter include, e.g., the dependence of the steady-state velocity on the external fields, predicting the maximum velocity, nonlinear time-dependent processes, and others. These are problems of practical importance, since nonlinear motion of BL sets in many cases at relatively low velocities and in weak control fields.

We investigate here BL dynamics in materials with large uniaxial anisotropy, for which the condition $Q = K_u/2\pi M_s^2 \gg 1$ is satisfied, where $K_u$ is the uniaxial-anisotropy constant and $M_s$ is the magnetization. These are just the materials (usually single-crystal films) used in technological applications and in most physical investigations of the problem.[2,5,6]

From the viewpoint of mathematical physics, a BL moving along a domain wall is a solitary wave—a soliton. Strictly speaking, such BL motion is usually assumed to occur under ideal conditions, viz., when dissipation and external magnetic fields are absent and the parameters of the equations are independent of space and time. We are interested in BL motion under real conditions, however, when account must be taken of the field, of dissipation, and of the nonuniformity of the parameters. For this purpose we use soliton perturbation theory. In this approach, the perturbed soliton (the BL) is regarded as a particle whose state is determined by two parameters (the coordinate $x_0$ of the BL center and the velocity $u$) and by the "radiation." The "radiation" is produced in this case by the field of the near-wall magnons, which are flexural domain-wall oscillations excited by the moving soliton under the influence of perturbations. We pay principal attention in the present paper to the derivation of the reduced equations for the coordinates $x_0$ of the center and for the velocity $u$ of the BL, and also to the determination of the conditions for the existence of a moving BL as a particle. We consider also the maximum BL velocity $u_c$ and the peculiarities of its motion near $u = u_c$.

Figure 1 shows two typical experimental geometries in which BL are investigated. The easy-magnetization axis is perpendicular to the plane of the slab (film) in the first and is located in this plane in the second. Many BL experiments with iron-garnet slabs are performed in the geometry 1a. The second geometry, 1b, is the usual one for films with uniaxial anisotropy in the plane of the film, and was used in the well-known experiments of Nikitenko *et al.*[4]

## §1. BASIC EQUATIONS

Consider the geometry 1a. Let the DW be parallel to the $xz$ plane of a Cartesian coordinate frame. In accord with the usual approximation of DW theory, we regard the DW as a surface, i.e., we neglect its thickness.[2,3] The DW is then treated as an elastic membrane with internal degree of freedom, and its state is specified by two parameters, $q(\mathbf{r},t)$ and $\psi(\mathbf{r},t)$, where $q$ is the DW displacement from the equilibrium position and $\psi$ is the azimuthal angle that specifies the orientation of the magnetization at the DW center relative to the $x$ axis in the $xy$ plane. It is expedient to describe the DW dynamics by a Lagrangian formalism wherein the Lagrangian and the dissipative Rayleigh function can be represent-

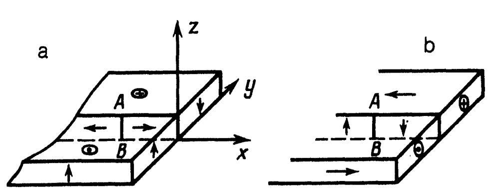


FIG. 1. Two geometries of formation of vertical Bloch lines ($AB$) in a domain wall of a uniaxial ferromagnet (the arrows indicate the directions of the magnetic moments): a—easy axis perpendicular to the plane of the film (plate); b—easy axis parallel to plane of film (plate).

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ed in the form

$$\mathscr{L}=-q\dot\psi-\sigma(\psi),\quad R={}^1/_2\alpha(\dot q^2+\dot\psi^2),\tag{1}$$

where $\alpha$ is the dimensionless damping constant of the Landau-Lifshitz equations and $\sigma(\psi)$ is the DW energy density given by

$$\sigma={}^1/_2(\nabla q)^2+{}^1/_2(\nabla\psi)^2+{}^1/_2\sin^2\psi+{}^1/_2b^2q^2-h_x\cos\psi-h_y\sin\psi-h_zq.\tag{2}$$

In the last equation we use the dimensionless quantities:

$$x_i\to x_i/\Delta_L,\quad t\to4\pi\gamma Mt,\quad q\to q/\Delta,\quad h_x=H_x/8M,$$
$$h_y=H_y/8M,\quad h_z=H_z/4\pi M,\quad b^2=H'\Delta/4\pi M,\quad \mathscr{L}\to\mathscr{L}/8\pi M^2\Delta,$$

where $H=(H_x, H_y, H_z)$ is the external magnetic field, $\Delta=(A/K_u)^{1/2}$ is the DW thickness, $\Delta_L=(A/2\pi M^2)^{1/2}$ is the BL thickness, $H'$ is the gradient of the magnetic fields that keeps the DW in the equilibrium position, $\gamma$ is the gyromagnetic ratio, and $A$ is the exchange-interaction constant.

To transform to geometry 1b, we must make the substitutions $H_x^{(a)}\to H_z^{(b)}$ and $H_z^{(a)}\to H_x^{(b)}$ everywhere, and set the $y$ component of the demagnetizing field equal to zero.

The Euler-Lagrange equations corresponding to the system (1) are of the form

$$\begin{aligned}&\dot\psi+\alpha\dot q=\nabla^2q-b^2q+h_z,\\&\alpha\dot\psi-\dot q=\nabla^2\psi-\sin\psi\cos\psi-h_x\sin\psi+h_y\cos\psi.\end{aligned}\tag{3}$$

The system (3) is known as the Slonczewski equations.[2] They constitute Landau-Lifshitz equations suitably averaged over the spin distribution within the DW. It is convenient to write them in vector form, putting $\mathbf{w}=(q,\psi)$

$$\hat T\partial_t\mathbf{w}-\mathscr{H}_0(\mathbf{w})=\mathscr{H}_1(\mathbf{w})+\alpha\partial_t\mathbf{w},\tag{4}$$

where

$$\hat T=\begin{pmatrix}0&-1\\1&0\end{pmatrix},\quad \mathscr{H}_0(\mathbf{w})=\begin{pmatrix}-\nabla^2q+b^2q\\-\nabla^2\psi+\sin\psi\cos\psi\end{pmatrix},$$
$$\mathscr{H}_1(\mathbf{w})=\begin{pmatrix}-h_z\\h_x\sin\psi-h_y\cos\psi\end{pmatrix}.$$

We shall investigate the BL dynamics in a one-dimensional approximation, i.e., assume the parameters describing the BL to be independent of the $z$ coordinate along the normal to the plane of the plate (film). This approximation is appropriate for the geometry of Fig. 1b, but is valid for the geometry 1a only for sufficiently thin films of thickness $d\lesssim\Delta_L$. If $d\gg\Delta_L$ the DW become "twisted" and the results of the one-dimensional theory apply to them only approximately.

The field component $h_y$ will henceforth assumed to be zero, since the demagnetizing field $H_y(z)$ can in our approximation be disregarded, and an external field having this orientation does not act directly on the BL.

## §2. FREE MOTION OF BL IN A NONDISSIPATIVE MEDIUM ($\alpha=0$, $\mathbf{H}=0$)

Under the equilibrium conditions $\dot q=\dot\psi=0$ and at $\mathbf{H}=0$ Eq. (4) has a solution that describes immobile BL:

$$q=0,\quad \psi_0=\pi n+2\operatorname{arctg}\exp\eta(x-x_0),\tag{5}$$

where $n=\pm1, \pm2,..., \eta=\pm1$. This solution satisfies the boundary conditions

$$\partial_xq|_{x=\pm\infty}=\partial_x\psi|_{x=\pm\infty}=0.$$

Bloch lines having different values of $n$ and $\eta$ have equal energies, i.e., the solutions can be said to be degenerate in $n$ and $\eta$. It suffices to consider solutions with $n=0$ and 1; the only difference is in the sign of the magnetic charge produced when the subdomains are joined at the BL, and in the direction of the untwisting of the magnetic moment. It is important to take the magnetic charge into account in studies of interactions between BL. We, however, are considering an isolated BL and confine ourselves henceforth, for the sake of argument, to the case of $n=0$. The parameter $\eta$ is also called the topological charge of the BL. The concept of topological charge can be extended to include a solitary wave in the following manner:

$$\eta=\pi^{-1}\int_{-\infty}^{+\infty}d\psi(x).$$

The free motion of a BL is described by two-parameter functions of the form

$$\mathbf{w}=\mathbf{W}_0(x-ut-x_0,u),\tag{6}$$

where $u$ and $x_0$ are the velocity and coordinate of the BL center and satisfy the equations

$$-u\Psi_\xi=q_{\xi\xi}-b^2q,\tag{7a}$$
$$-uq_\xi=-\Psi_{\xi\xi}+\sin\Psi\cos\Psi,\qquad \xi=x-ut-x_0\tag{7b}$$

with boundary conditions

$$\Psi_\xi|_{\xi\to\pm\infty}=q_\xi|_{\xi\to\pm\infty}=0.$$

The phase space of the system (7) is defined by the four-dimensional vectors $\mathbf{a}=(\Psi,\Psi_\xi,q,q_\xi)$. We are particularly interested in the equilibrium points $\mathbf{a}_n=(\pi n,0,0,0)$, where $n=0, \pm1, \pm2,...$. They correspond physically to DW subdomains. For $u<u_-=1-b$, the equilibrium points are of the saddle-saddle type. The BL is described by an integral curve that joins neighboring points which it enters and leaves monotonically. In the region $u_-<u<u_+=1+b$ the points $\mathbf{a}_n$ are of the saddle-focus type. We know of no analytic solution of the nonlinear system (7), but can derive an approximate solution in terms of the small parameter $b\ll1$. It is just this situation, i.e., $b\ll1$, which is realized under typical experimental conditions.[1)]

From (7a) we have

$$q=u\int_{-\infty}^{+\infty}G(\xi-x')\Psi_{x'}(x')dx',\tag{8}$$

where

$$G(\xi)=(\pi/2b)\exp(-b|\xi|).\tag{9}$$

Substituting (8) in (7b) we get

$$\Psi_{\xi\xi}-\sin\Psi\cos\Psi=u^2\int_{-\infty}^{+\infty}G_\xi(\xi-x')\Psi_{x'}(x')\,dx'. \quad (10)$$

Assuming $u\ll 1$, we can solve Eqs. (8)–(10) by successive approximations:

$$\Psi=\psi_0(\xi)-\tfrac{1}{2}\pi[u^2+u^4(1-b|\xi|/2)+O(u^6)]\theta(\xi)\exp(-b|\xi|), \quad (11)$$

$$q=(\pi/2b)[u+u^3(1-b|\xi|)/2+O(u^5)]\exp(-b|\xi|), \quad (12)$$

where $\theta(\xi)=\xi/|\xi|$ and $\psi_0(\xi)$ is described by Eq. (5). The solutions obtained are valid in the region $b|\xi|\gg 1$. Recognizing that $b\ll 1$, we can regard (11) and (12) as a good approximation of the solution in the region $|\xi|\gtrsim 1$ of interest to us. Exact solutions of (7), obtained by numerical methods, are discussed below (see §5).

## §3. PERTURBATION THEORY FOR SOLITONS

We consider now the influence of external fields and of dissipation on the BL motion. This can be done by using perturbation theory for solitons. We investigate first the influence of the field $h_x$, i.e., we put $h_y=h_z=0$ in Eqs. (4). The equilibrium points of the system (7) are then preserved for $h_x<1$. We represent the solution of Eqs. (3) in the form

$$\mathbf{w}(x,t)=\mathbf{w}_0\left(x-\int_0^t u\,dt-x_0,u\right)+\mathbf{w}_1, \quad (13)$$

where $\mathbf{w}_0$ is defined in (11) and (12), but assume here that the parameters $u$ and $x_0$ are slowly varying functions of $\mathbf{r}$ and $t$, where $\mathbf{r}$ is the coordinate in the DW plane. Assuming the perturbation to be small, i.e., $|\mathbf{w}_1|\ll|\mathbf{w}_0|$, and linearizing (4), we get

$$\hat{L}\mathbf{w}_1=\mathbf{f}, \quad (14a)$$

$$\mathbf{f}=\mathcal{H}_1(\mathbf{w}_0)+\alpha\partial_t\mathbf{w}_0-\dot{x}_0\hat{T}\partial_x\mathbf{w}_0-\dot{u}\hat{T}\partial_u\mathbf{w}_0, \quad (14b)$$

where the linear operator $L$ is of the form

$$\hat{L}=\hat{T}\partial_t-\begin{pmatrix}-\partial_x^2+b^2 & 0\\ 0 & -\partial_x^2+\cos 2\Psi\end{pmatrix}=\hat{T}\partial_t-\hat{L}_0. \quad (15)$$

The functions

$$\mathbf{V}_1=\partial_x\mathbf{W}_0,\qquad \mathbf{V}_2=\partial_u\mathbf{W}_0 \quad (16)$$

are[2)] solutions of the homogeneous equation $\hat{L}\mathbf{w}_1=0$. They are linearly independent, since the 2-form $\langle\partial_x\mathbf{W}_0\hat{T}\partial_u\mathbf{W}_0\rangle$ differs from zero. Here and henceforth $\langle\ldots\rangle=\int_{-\infty}^{+\infty}\ldots dx$.

We multiply (14a) from the left by $\mathbf{V}_j$ and integrate with respect to $x$; then

$$\langle\mathbf{V}_j\hat{T}\partial_t\mathbf{w}_1\rangle-\langle\mathbf{V}_j\hat{L}_0\mathbf{w}_1\rangle=\langle\mathbf{V}_j\mathbf{f}\rangle, \quad (17)$$

where $\mathbf{f}$ is defined in (14b) and $j=1$ and 2. Since the operator $\hat{L}_0$ is self-adjoint, and

$$\langle\mathbf{V}_j\hat{T}\partial_t\mathbf{w}_1\rangle=\partial_t\langle\mathbf{V}_j\hat{T}\mathbf{w}_1\rangle-\langle\partial_t\mathbf{V}_j\hat{T}\mathbf{w}_1\rangle=\partial_t\langle\mathbf{V}_j\hat{T}\mathbf{w}_1\rangle+\langle\mathbf{w}_1\hat{T}\partial_t\mathbf{V}_j\rangle$$

while $\hat{L}\mathbf{V}_j=0$, we get

$$\partial_t\langle\mathbf{V}_j\hat{T}\mathbf{w}_1\rangle=\langle\mathbf{V}_j\mathbf{f}\rangle. \quad (18a)$$

To exclude perturbations that increase linearly with the time $t$ (the secular terms), it suffices to put

$$\langle\mathbf{V}_j\mathbf{f}\rangle=0,\quad j=1,2. \quad (18)$$

These equations provide the actual reduced description of the soliton (BL) in terms of the coordinate $x_0$ of its center and of the velocity $u$.[3)]

Substituting expressions (14b), (16), (11), and (12) in (18) and integrating, we get

$$\partial_tP+2\alpha u(1+\pi^2u^2/8b)=2h_x, \quad (19)$$

$$\partial_uP\partial_tx_0=\langle h_x\sin\Psi\partial_u\Psi\rangle, \quad (20)$$

where $P$ is the adiabatic action invariant, defined as

$$P=\int_{-\infty}^{+\infty}q\partial_x\Psi\,dx=m_0u[1+u^2+O(u^4)],\quad m_0=\pi^2/2b. \quad (21)$$

The BL mass is defined by

$$m=\partial_uP=m_0[1+3u^2+O(u^4)]. \quad (22)$$

Equation (19) is the conservation law for the system's adiabatic invariant, which in the present case is the BL momentum density. A striking feature of this equation is that the viscous friction acting on the BL depends strongly on the velocity $u$, since $b\ll 1$. This dependence is due to the additional dissipation caused by the bending of the DW in the course of the BL motion. The kinetic nonlinearity (the velocity dependence of the BL mass) is weaker here.

Equation (20) determines the increase of the BL-center coordinate when the BL passes through an inhomogeneous field region $h_x$. If the field $h_x$ is independent of the coordinate $x$, we have $\partial_tx_0=0$.

## §4. BL MOTION DUE TO GYROSCOPIC FORCE

Turning on the field $h_z$ causes a DW displacement that leads in turn to gyroscopic motion of the BL. In the presence of $h_z$ the points $q=0$ and $\psi=0$ and $\pi$ are no longer equilibrium points of the system. Far from the BL center, when the derivatives with respect to $x$ can be neglected in Eqs. (3), these equations take the form

$$\dot{\bar{q}}=\bar{\psi}+\alpha\dot{\bar{\psi}}, \quad (23)$$

$$\dot{\bar{\psi}}=h_z-b^2\bar{q}-\alpha\dot{\bar{q}}, \quad (24)$$

where $\bar{q}$ and $\bar{\psi}$ are the values of the variables $q$ and $\psi$ as $|x|\to\infty$. Putting $w=w_0+\bar{w}+w_1$, where $\bar{w}=(\bar{q},\bar{\psi})$, assuming that $\bar{\psi}\ll\pi$, and then expanding Eq. (4) up to terms linear in $\bar{q}$ and $\bar{\psi}$ and eliminating these terms with the aid of Eqs. (23) and (24), we arrive at Eq. (14a) from which the field $h_z$ has been eliminated but whose right-hand side contains instead the additional external force

$$f_1=\begin{pmatrix}0\\ 2\bar{\psi}\sin^2\Psi+h_x\bar{\psi}\cos\Psi\end{pmatrix}. \quad (25)$$

Substituting this force in the solvability equation (18) and integrating with respect to $x$ in the weakly inhomogeneous case when $\partial_tx_0\approx 0$, we obtain the reduced BL equations:

$$\partial_tP+2\alpha u(1+\pi^2u^2/8b)=2h_x-\pi\eta\bar{\psi},\quad P=m_0u(1+u^2). \quad (26)$$

The term $-\pi\eta\dot{\bar{\psi}}$ on the right-hand side describes the gyroscopic pressure exerted on the BL by the DW. At $\alpha \ll 1$ Eqs. (23) and (24) can be transformed into

$$\dot{\bar{\psi}}=\dot{\bar{q}}, \qquad (27)$$

$$\ddot{\bar{q}}+\alpha\dot{\bar{q}}+b^2\bar{q}=h_z \qquad (28)$$

and the gyroscopic force takes in this case the usual form $-\pi\eta\dot{\bar{q}}$. Allowing for the smallness of the perturbing terms taken into account in Eqs. (3) and (14), the conditions for the validity of the reduced-description equations (19), (20), and (26)–(28) reduce to

$$h_x,\ h_z,\ u,\ \dot{u}/b,\ \dot{\bar{q}},\ \ddot{\bar{q}},\ \alpha,\ b,\ Q^{-1} \ll 1.$$

## §5. VELOCITY LIMIT; DYNAMIC TRANSFORMATION OF A BL INTO A CLUSTER

We have considered so far the nonlinear properties of a BL at relatively low velocities, and have confined ourselves to corrections quadratic in the velocity. A question of fundamental importance is that of the velocity limit of an isolated BL and its structure at high velocity. This question was considered in Ref. 10, where it was indicated that two critical velocities $u_\pm = 1 \pm b$ exist, at which bifurcation takes place in the system (7). Bifurcation analysis, however, does not indicate existence of integral curves (separatrices) that join the equilibrium points of the system. We have investigated the separatrix solutions of Eqs. (7) by using a qualitative and numerical analysis method that is described in detail in Ref. 12 and provides the answer to our problem.[13]

We return to the dissipationless equations (7) that describe BL motion in the absence of a magnetic field, $\mathbf{H} = 0$. Figure 2a shows one of the solutions of the system (7), obtained for $b = 0.5$ and $u = 0.4$. The solution agrees in general form with the analytic equations (11) and (12). We call attention to the humps, which increase with velocity, on the tails of the magnetization distribution $\Psi(\xi)$. The topological charge is conserved in this case, but the amplitude of the untwisting of the angle $\Psi$ in the moving isolated BL exceeds $\pi$. On going through the bifurcation value $u = u_- = 1 - b$

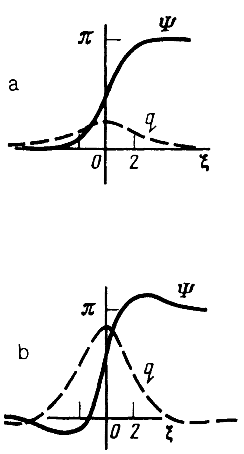


FIG. 2. Solitary wave describing an isolated Bloch line: a—for $u = 0.4$, $b = 0.5$; b—for $u = 1.0$, $b = 0.5$.

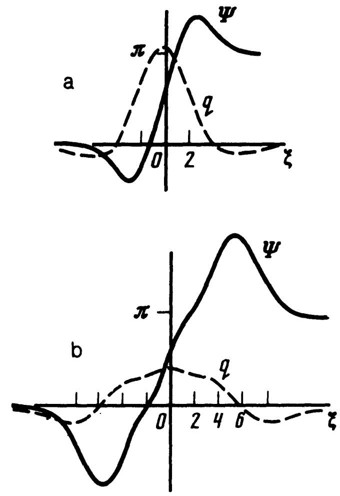


FIG. 3. Solitary wave describing a moving cluster of five Bloch lines: a—for $u = 1.0$, $b = 0.5$; b—for $u = 0.4$, $b = 0.5$.

the singular points $\mathbf{a}_0$ and $\mathbf{a}_1$ turn into saddle-focus singular points which the integral curves enter and leave in oscillating form. Numerical integration of the system (7) for $b = 0.5$ has shown that a very simple integral curve that joins the singular points $\mathbf{a}_0$ and $\mathbf{a}_1$ exists only in the velocity region $u < u_c < u_+$, where $u_c$ is the maximum BL velocity. At $b = 0.5$, in particular, we have $u_c = 0.5$. It can be seen that the humps on the $\Psi(\xi)$ tails are larger than in the solution for $u = 0.4$, but the topological structure remains on the whole unchanged.

Obviously, the velocity limit $u_c$ depends only on the parameter $b$. Since $1 - b < u_c < 1 + b$, recognizing that in a real situation $b \ll 1$, the assumption $u_c = 1$ is a good enough approximation. Of greater interest here is the following fundamental question: why cannot the solution that describes a single BL be continued into the region $u > u_c$? It is important to note that no anomalies occur at the singular points $\mathbf{a}_n$ when $u = u_c$. A numerical analysis for $b = 0.5$ has shown that there exists one more separatrix solution that joins the points $\mathbf{a}_0$ and $\mathbf{a}_1$ and merges at $u = u_c$ with the solution that describes a single BL. This solution, obtained for $u = 1.0$, is shown in Fig. 3a. This type of solution describes a moving cluster consisting originally of five BL. This is clearly seen in Fig. 3b, which shows the structure of this solution at $u = 0.4$. Since at $u = 0$ the separatrix solutions take the form (5), the solitary wave breaks up as $u \to 0$ into five isolated BL with arbitrarily large distances between them. This reason why it is impossible to continue the separatrix solution describing an isolated BL has its analog in the dynamics of Bloch and Néel domain walls in a ferromagnet. When the Walker velocity limit is approached in the latter, two initially (at $u = 0$) different solutions in the form of Bloch and Néel DW are merged. Thus, when a velocity limit is reached, solutions that have different structures at low velocities become dynamically degenerate.

Other separatrix solutions were also obtained; they describe a cluster of three or seven BL of maximum velocity $u_c' > u_c$.

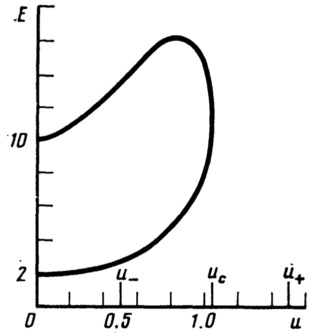


FIG. 4. Velocity dependence of the energy of moving Bloch lines.

What are the physical singularities acquired by a BL at $u = u_c$? We calculate the energy of a solitary wave, using the equation

$$E(u)=\int_{-\infty}^{+\infty} \sigma(\Psi, q)\,dx, \tag{29}$$

where $\sigma(\Psi,q)$ is given by Eq. (2) with $h_x = h_y = h_z = 0$. Figure 4 shows a plot of $E(u)$ obtained from Eq. (29) with $b = 0.5$. As $u \to 0$ the energy $E \to 2k$, where $k$ is the number of BL in the cluster. The mass $m_L = d^2E/du^2$ of an isolated BL increases with velocity and $m_L \to \infty$ as $u \to u_c$.[4)] The energy of a cluster of five BL has a maximum near $u \approx 0.8u_c$. In this velocity region the cluster has a negative mass, meaning that it is unstable at high velocities.

We estimate now the maximum magnetic field that blocks stationary motion of Bloch lines in a DW. This can be done by using the momentum-conservation (or energy-balance) equation for a system with the Lagrangian (1), which reduces in this case to

$$\frac{2h_x}{\alpha}=u\int_{-\infty}^{+\infty} (\Psi_\xi^2+q_\xi^2)\,d\xi. \tag{30}$$

Figure 5 shows the field dependence of the velocities of an isolated BL and of a cluster of five BL. This dependence was obtained from (30) by using numerical solutions of the system (70), and is valid for weakly dissipative systems, when $\alpha \ll 1$. The maximum value of the magnetic field that reverses the DW by the stationary motion of a BL, is in this case $h_x^{(c)} \approx 7.4\alpha$.

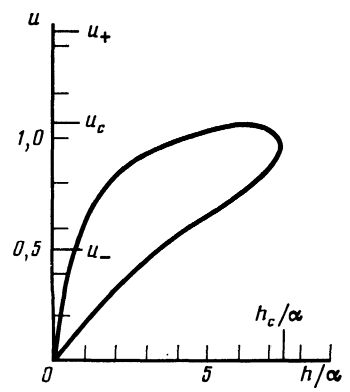


FIG. 5. Field dependence of the velocity of Bloch lines in a weakly dissipative medium.

The effect described here, dynamic transformation of an isolated BL into a cluster of five BL when a velocity limit is reached, can serve as a new mechanism for multiplication (generation) of BL in a moving DW.[5)] Obviously, the action of such a mechanism can be facilitated by the presence of defects in the sample. It is possible that this effect is the cause of the BL multiplication observed by Nikitenko *et al.*[14] in $Y_3Fe_5O_{12}$ acted upon by high-frequency magnetic fields.

## §6. CONCLUSION

We present the reduced equations for a BL in dimensional form:

$$\dot{\mathscr{P}}+m_L\tau_L^{-1}v[1+(\pi^2/8b)(v^2/s^2)]=2M_sH_x\pi\Delta-2M_s\pi\gamma^{-1}\eta\dot{q}, \tag{31}$$

$$m_D\ddot{q}+\beta\dot{q}+kq=2M_sH_z, \tag{32}$$

where $v$ is the BL velocity and $q$ the DW displacement, in dimensional units, from the equilibrium position far from the BL,

$$\mathscr{P}=m_Lv(1+v^2/s^2),\ s^2=8\gamma^2\pi A,\ \tau_L^{-1}=(16/\pi)\alpha b\gamma M_s,\ m_L=\pi(4b\gamma^2Q^{1/2})^{-1},$$

(the expression obtained for $m_L$ agrees with that obtained for the mass earlier[9)], $\eta$ is the topological charge defined in Eq. (5a), $\beta = 2M_s/\mu$, $\mu = \gamma\Delta a^{-1}$ is the DW mobility, $m_D$ is the Döring mass of the DW, and $k = 2M_sH'$. The velocity and the magnetic field limits are $v_c \approx s$ and $h_x^{(c)} \approx 60\alpha M_s$. We list below the main BL parameters calculated for typical experimental conditions:

$$A=4\cdot10^{-7}\ \text{erg/cm},\ K=5\cdot10^{4}\ \text{erg/cm}^3,\ M_s=50\ \text{G},$$
$$d=10^{-4}\ \text{cm},\ Q=3,\ \alpha=0{,}1,\ b=0{,}08,$$
$$\Delta=3\cdot10^{-6}\ \text{cm},\ \Delta_L=5\cdot10^{-6}\ \text{cm},\ s=6\cdot10^{4}\ \text{cm/s},$$
$$m_L=10^{-14}\ \text{g/cm},\ \tau_L=2\cdot10^{-8}\ \text{s},\ H_x^{(c)}=300\ \text{Oe}.$$

The gradient of the displacement field is given by $H_z' = 2M_sd$, where $d$ is the film (plate) thickness, assumed equal to $10^{-4}$ cm.

Equations (31) and (32) are our main results. They contain two nonlinearities: kinetic—the dependence of the BL mass on the velocity, and dissipative—the dependence of the viscous deceleration force on the velocity. The second type of nonlinearity is stronger than the first. The physical cause of these nonlinear dependences is the local bending of the domain wall by the moving BL. One manifestation of such nonlinearity can be generation of multiple harmonic oscillations of the BL in an harmonically oscillating DW. We have also determined the limiting values of the dynamic characteristics of the BL, such as the velocity limit and the critical DW-magnetization-reversal field that limits the stationary steady-state motion of the BL. The singularities observed in the motion of a BL near the velocity limit demonstrate the possibility of dynamic transformation of an isolated BL into a cluster, i.e., the possibility of BL multiplication in the dynamic regime.

[1)]The other limiting case $b \gg 1$ was considered in Ref. 10.

[2)]The function $W_0$ is defined here by Eq. (6). We note also that $\partial_u W_0 = t\partial_x W_0 + \partial_u W_0$, where the second term depends on the time only via $x - ut$. The first term in the conditions (18) can be omitted, since the convolution $t\langle \partial_x W_0 f\rangle$ vanishes by virtue of the first condition (18), i.e., at $j = 1$ (see also Ref. 11).

[3)]The derivation of Eqs. (18) is equivalent to the approach described in Ref. 11.

[4)]The mass definition given here coincides with (22).

[5)]A well-known mechanism of BL generation in magnetic films with perpendicular anisotropy is initiation of horizontal BL (Ref. 2).

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