Characteristics of small-size tunnel junctions in the limit of zero external current

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(Submitted 27 December 1985)
Zh. Eksp. Teor. Fiz. **90**, 2226–2232 (June 1986)

The spectral density of equilibrium fluctuations of a quasiparticle conduction current in a low-capacitance tunnel junction is obtained, as well as the low-current conductivity of such a junction.

1. INTRODUCTION

Processes in tunnel junctions (between normal metals or superconductors) of low capacitance $C \lesssim e^2/T$ have been attracting considerable interest lately. This interest is caused to a considerable degree by the possible manifestation of new Coulomb and microscopic effects in such junctions.[1–4]

The present paper is devoted to the calculation of the fluctuations of the quasiparticle current and to the conductivity of such junctions in the absence of the Josephson component of the tunnel current. A similar problem (for the particular case of normal metals) was considered earlier in Refs. 1, but the results there pertained to the physically unrealistic case of a discrete spectrum of the electric charge Q of the junction. It will be shown in Sec. 2 that in all situations realizable in practice the spectrum of the charge is, on the contrary, continuous.

The paper consists of two main parts. In Sec. 3 is calculated the spectral density of the fluctuations of a quasiparticle current and the junction conductivity at zero average current through the junction, i.e., for thermal equilibrium between the junction and the heat bath. In Sec. 4 is calculated the conductivity of the junction likewise in the limit as $I \to 0$, but in the regime of single-electron oscillations,[4] i.e., in a patently nonequilibrium situation.

2. FUNDAMENTAL EQUATIONS

The Hamiltonian of the system in question and the standard Hamiltonian used to describe tunneling differ only in the allowance made for the Coulomb energy of the junction:

$$H = H_0 + H_T, \quad H_0 = H_1 + H_2 + Q^2/2C, \tag{1}$$

where H_1 and H_2 are the Hamiltonian of the metals forming the junctions, Q is the electric charge of the junction as a capacitor:

$$Q = -e(N_1 - N_2)/2 + Q_0, \quad N_i = \sum_{k_i} c_{k_i}^+ c_{k_i}, \tag{2}$$

and H_T is the usual tunneling Hamiltonian[5]

$$H_T = H_+ + H_-, \quad H_+ = \sum_{k_1 k_2} T_{k_1 k_2} c_{k_1}^+ c_{k_2}, \quad H_- = (H_+)^+. \tag{3}$$

The summation in (2) and (3) is over all the electronic states of the metals 1 and 2, while c_k^+ and c_k are the electron creation and annihilation operators. In Eq. (2), Q_0 is a constant that corresponds to the charge produced, say, by the difference between the Fermi levels of the metals.[6–8]

We shall need the commutation rules for the operator Q with the operators $H_\pm$ and $H_{1,2}$. Assuming, as usual, that the operators c_k^+ and c_k pertaining to different metals commute with each other, it is easy to verify by direct substitution that the following relations hold:

$$c_{k_1}^+ c_{k_2} Q = (Q + e) c_{k_1}^+ c_{k_2}, \tag{4a}$$

i.e.,

$$H_+ Q = (Q + e) H_+. \tag{4b}$$

Since Eq. (4b) leads to a similar relation for the operator Q^n

$$H_+ Q^n = H_+ \underbrace{Q \cdot \ldots \cdot Q}_{n} = \underbrace{(Q+e) \cdot \ldots \cdot (Q+e)}_{n} H_+ = (Q+e)^n H_+, \tag{5}$$

the following rule is valid for any analytic operator function $f(Q)$:

$$H_+ f(Q) = f(Q+e) H_+. \tag{6a}$$

The same procedure can be used to prove a similar relation for H_-:

$$H_- f(Q) = f(Q-e) H_-. \tag{6b}$$

Let the number N of the electronic states in metals be large, so that not too large a charge $|Q| \ll eN$ does not change their internal properties. It can then be assumed that

$$[H_{1,2}, Q] = 0. \tag{7}$$

We have already used the commutation relations (6) and (7) in Ref. 4, where we analyzed the dynamic properties of low-capacitance tunnel junctions.

As noted in the Introduction, the final results depend substantialy on the properties of the eigenvalue spectra of the operator Q. Since, according to (3), tunneling leads only to discrete charge transfer ($\Delta Q = \pm e$), one might assume that the operator $Q = Q - Q_0$ takes on only discrete values ne. This assumption is actually correct when Q is the charge on an isolated conductor connected to the "outside world" only via the tunnel current. This is the case, for example, for a metal granule in the oxide layer of a tunnel junction,[6–8] which leads in particular to oscillatory (e-periodic) dependence of the properties of such structures on the values of Q_0 observed in experiment.[7]

A more realistic situation for an ordinary tunnel junction, however, is one in which it is shunted by an albeit small but finite metallic-type conductance G_s (this shunting is necessary at the very least for the measurement of the electrodynamic characteristics of the junction). The electric charge is transported through such a "shunt" as a result of

 0038-5646/86/061306-04$04.00

small displacements of a large number of carriers, so that this charge is not discrete in the scale of e.

Therefore, if the quantities characterizing the junctions are averaged over a time interval $\Delta\tau \ll C/G_0$ during which the action of the current through the shunt can be neglected, it can be assumed that the charge takes on only discrete value $Q_0 + ne$, where Q_0 is the initial value of the charge. On the other hand, if the averaging is over a time interval $\Delta\tau \gg C/G_s$ (as is in fact assumed when the equilibrium properties of the junctions are considered), it must be assumed that the charge takes on a continuous set of values[4] (see also Ref. 9). Therefore Refs. 1 should accordingly be reviewed, as will be done indeed later for the case $G_s \ll G_T$. Here

$$G_T = dI(V)/dV|_{V=0}, \tag{8}$$

and $I(V)$ is the average current through the junction in the classical situation, when the junction voltage V can be regarded as fixed (this situation is realized, in particular, for junctions of usual size with capacitance $C \gg e^2/T$).

3. EQUILIBRIUM CHARACTERISTICS OF JUNCTION

We calculate first the spectral density of the fluctuations of the quasiparticle conduction current at zero average current through the junction. In the first nonvanishing approximation in H_T we have at $\langle I \rangle = 0$ (Ref. 10)

$$(I^2)_\omega = (2\pi)^{-1} \int_{-\infty}^{+\infty} d\tau\, e^{-i\omega\tau} \langle I(\tau)I(0) + I(0)I(\tau)\rangle_{H_0}$$

$$= \frac{2e^2}{\pi\hbar^2} \operatorname{Re} \int_0^\infty d\tau \cos\omega\tau \langle H_+(\tau)H_-(0) + H_-(\tau)H_+(0)\rangle_{H_0}, \tag{9}$$

where the time dependences of the operators and the equilibrium density matrix over which the averaging is carried out are determined by the Hamiltonian H_0. Since the operators $H_{1,2}$ and Q commute, the averaging over the internal degrees of freedom of the metals and over Q can be carried out independently. In addition, it follows from (6) and (7) that

$$H_\pm(\tau) = \exp\left\{\mp \frac{ie\tau}{\hbar C}\left(Q \pm \frac{e}{2}\right)\right\} \exp\left\{\frac{i\tau}{\hbar}(H_1+H_2)\right\}$$

$$\times H_\pm(0) \exp\left\{-\frac{i\tau}{\hbar}(H_1+H_2)\right\}. \tag{10}$$

Thus, expression (9) takes the form

$$(I^2)_\omega = (2\pi CT)^{-1/2} \int dq \exp\left\{-\frac{q^2}{2CT}\right\}\left[\frac{2e^2}{\pi\hbar^2} \operatorname{Re} \int_0^\infty d\tau\right.$$

$$\times \cos\omega\tau \exp\left\{-\frac{ie^2\tau}{2\hbar C}\right\}\left(\exp\left\{-\frac{ieq}{\hbar C}\tau\right\}\langle H_+(\tau)H_-(0)\rangle_{H_1+H_2}\right.$$

$$\left.\left. + \exp\left\{\frac{ieq}{\hbar C}\tau\right\}\langle H_-(\tau)H_+(0)\rangle_{H_1+H_2}\right)\right]. \tag{11}$$

Using now the known relation[11]

$$\operatorname{Re} \int_0^\infty d\tau\, e^{-i\omega\tau} \langle H_+(\tau)H_-(0)\rangle_{H_1+H_2} = \exp\{-\hbar\omega/T\}$$

$$\times \operatorname{Re} \int_0^\infty d\tau\, e^{-i\omega\tau} \langle H_-(0)H_+(\tau)\rangle_{H_1+H_2}, \tag{12}$$

we can express the right-hand side of (11) in terms of the aformentioned $I(V)$ dependence of the average current through the junction at a fixed junction voltage[12]

$$I(V) = \frac{2e}{\hbar^2} \operatorname{Re} \int_0^\infty d\tau \exp\left\{-\frac{ieV\tau}{\hbar}\right\} \langle [H_-(0), H_+(\tau)]\rangle_{H_1+H_2}. \tag{13}$$

From (11)–(13) we obtain ultimately

$$(I^2)_\omega = (2\pi CT)^{-1/2} \int dq \exp\left\{-\frac{q^2}{2CT}\right\}$$

$$\times \frac{e}{\pi} \sum_\pm \left[I\left(\frac{q+e/2}{C} \pm \frac{\hbar\omega}{e}\right)\right.$$

$$\times \left(\exp\left\{\frac{1}{T}\left[\frac{e(q+e/2)}{C} \pm \hbar\omega\right]\right\} - 1\right)^{-1} + I\left(\frac{q-e/2}{C} \pm \frac{\hbar\omega}{e}\right)$$

$$\left.\times \left(1 - \exp\left\{-\frac{1}{T}\left[\frac{e(q-e/2)}{C} \pm \hbar\omega\right]\right\}\right)^{-1}\right]. \tag{14}$$

Using the fluctuation-dissipation theorem and the Kramers-Kronig relation, we can obtain from the spectral density of the conduction-current fluctuations the real and imaginary parts of the junction conductance $Y(\omega)$, and consequently calculate also the spectral densities of the small voltage fluctuations across the junction

$$(V^2)_\omega = (I^2)_\omega \{[\omega C + \operatorname{Im} Y(\omega)]^2 + [\operatorname{Re} Y(\omega)]^2\}^{-1}. \tag{15}$$

In the low-temperature limit

$$T \ll e^2/C, \tag{16}$$

we obtain from (14) the following asymptotic expression for the junction conductance at zero frequency:

$$Y(0) = G_T(\pi^3 CT/2e^2)^{1/2} \exp\{-e^2/8CT\}, \tag{17}$$

which differs substantially from the corresponding expression of Ref. 1 both in the argument of the exponential and in the pre-exponential factor. At high temperatures, $T \gg e^2/C$, the effects connected with the Coulomb energy of the transition become insignificant and the junction conductance ceases to depend on temperature: $Y(0) = G_T$.

Equation (17) is valid for a junction between superconductors if the characteristic scale of the junction voltage e/C is much smaller than the nonlinearity scale Δ/e of the function $I(V)$, i.e., if

$$e^2/C \ll \Delta. \tag{18}$$

Obviously, inequalities (16) and (18) can be simultaneously satisfied only if $T \ll T_c$.

A more stringent restriction on the region of applicability of Eqs. (14) and (17) follows from the necessary requirement that the voltage-fluctuation spectrum (15) obtained be compatible with the Gibbs charge probability distribution used in its derivation. This is possible only in the case of classical fluctuations $(V^2)_\omega$ at all the essential frequencies $\omega \lesssim Y(0)/C$, i.e., when

$$\hbar Y(0)/C \ll T. \tag{19}$$

In the low-temperature limit (16), the condition (19) takes the form

$$G_T R_Q \ll (CT/e^2)^{1/2} \exp\{e^2/8CT\}, \tag{20a}$$

and for high temperatures

$$G_T R_Q \ll CT/e^2, \quad (20b)$$

where $R_Q = \pi\hbar/2e^2$ is the quantum unit of resistance.

Finally, these results are valid for a junction in thermodynamic equilibrium, i.e., when the external current that causes the deviation from equilibrium is small. In the case $G_s \ll G_T$ considered, the smallness of the current is determined not by the conductance G_T of the junction itself, but by the conductance of the shunt

$$\bar{I} \ll eG_s/C, \quad (21)$$

since failure to satisfy the last condition leads to the onset of one-electron oscillations,[4] i.e., to a substantial deviation from equilibrium.

4. JUNCTION CONDUCTANCE IN THE ONE-ELECTRON OSCILLATION REGIME

The spectral density of the voltage fluctuations on a junction for one-electron oscillations (for the case $T=0$) was calculated in Ref. 4. We confine ourselves in this section to calculation of the conductance of the junction to the current that causes these oscillations, $\bar{I} > eG_s/2c$, but is small in the following sense:

$$\bar{I} \ll eG_T/C. \quad (22)$$

To this end we must obtain a stationary solution for the kinetic equation that describes the dynamics of the charge on the junction.[4] Under the additional condition

$$\bar{I} \gg eG_s/C, \quad (23)$$

this equation can be expressed in a particularly simple form

$$\bar{I}\frac{\partial\sigma}{\partial q} = S(q+e) - S(q), \quad (24a)$$

$$S(q) = \frac{G_T}{eC}\left(q-\frac{e}{2}\right)\left(1-\exp\left\{-\frac{e(q-e/2)}{CT}\right\}\right)^{-1}$$
$$\times\left(\sigma(q)-\sigma(q-e)\exp\left\{-\frac{e(q-e/2)}{CT}\right\}\right). \quad (24b)$$

To calculate the conductance, it suffices to solve the equation accurate to first order in $\bar{I}$.

It is obvious from (24a) that the following relation holds:

$$\bar{I}\frac{\partial}{\partial q}\left[\sum_{n=-\infty}^{+\infty}\sigma(q+en)\right]=0. \quad (25)$$

Whereas at $\bar{I}=0$ the condition (25) is identically satisfied with respect to $\sigma(q)$, at $\bar{I}\neq 0$ Eq. (35) leads to a relation that must be satisfied by the solution of Eq. (24) and can be written, with allowance for the normalization of the function $\pi(q)$, in the form

$$\sum_{n=-\infty}^{+\infty}\sigma(q+en) = e^{-1}. \quad (26)$$

It is easily verified by direct substitution that a solution satisfying (26) can be written for (24) in the zeroth approximation in $\bar{I}$ in the form

$$\sigma^0(q) = \left[e\sum_{n=-\infty}^{+\infty}\exp\{-(q+en)^2/2CT\}\right]^{-1}\exp\{-q^2/2CT\}. \quad (27)$$

In the limit of low temperature (16) and a weak current (22), the probability density $\sigma(q)$ differs from zero only on the interval $[-e/2, +e/2]$ and in its nearest vicinities. In this case it is easy to find the correction in first order in $\bar{I}$ to the solution (27), and the following expression is obtained for the junction conductance in the one-electron-oscillation regime

$$Y(0) = \left[\frac{\partial}{\partial\bar{I}}\int dq\left(\frac{q}{C}\right)\sigma(q)\right]^{-1} = \lambda G_T\frac{CT}{e^2}, \quad (28)$$

$$\lambda = \left[2\int_0^\infty dx\frac{(1-e^{-x})e^{-x}}{x(1+e^{-x})^3}\right]^{-1} \approx 2.346.$$

At high temperatures, the amplitude of the one-electron oscillations falls off rapidly and the junction conductance tends to its classical value $Y(0) = G_T$.

Thus, at low temperatures the junction conductance (28) for one-electron oscillations is linear in the temperature and, unlike the equilibrium conductance, contains no exponentially small factor. This result is a reflection of the fact that in one-electron oscillations the charge on the junction becomes "smeared out" by the external current over the entire interval $[-e/2, +e/2]$, in contast to the equilibrium case, when the charge is concentrated near the point $q=0$ and the probability of electron tunneling that increases the junction energy by $\Delta E \sim e^2/C$ is exponentially small.

Expression (28) obtained for the junction conductance pertains to the case of a substantial disequilibrium. If, however, the average junction voltage is low, $eV \ll T$, a situation is possible when the current is subject to a stronger restriction than condition (22), viz.,

$$\bar{I} \ll (TC/e^2)^2(eG_T/C), \quad (29)$$

the spectrum of the voltage fluctuations can be assumed to differ little from the equilibrium spectrum. In this case the condition under which (28) is valid can again be obtained from Eq. (19), namely,

$$G_T R_Q \ll 1. \quad (30)$$

Comparison of conditions (20) and (30) shows that at low temperatures coherent one-electron oscillations are suppressed by strong dissipation more rapidly than the classical Gibbs distribution of the charge. It is interesting to note that condition (30) coincides with the condition for the suppression of coherent Bloch oscillations in Josephson junctions.[3]

5. CONCLUSION

It can be seen from our results that the properties of low-capacitance tunnel junctions in a weak external field differ substantially from those of high-capacitance junctions. The qualitative meaning of this difference is that the conductance of the junction depends on temperature and decreases with it. A similar effect was observed in experiments on tunneling through metallic granules in an oxide layer.[6] Notwithstanding the substantial difference between that case and the case considered here (the principal difference is due to the discrete spectrum of the electric charges on the granules), the behavior of $Y(0)$ in the two systems has features in common. If the electric charge is localized near a fixed point inside the interval $[-e/2, +e/2]$, the conductance decreases exponentially with decreasing temperature. If for some reason the charge is distributed over the entire interval $[-e/2, +e/2]$ and the characteristics of the junction must be averaged over this distribution, the conductance decreses linearly with temperature. In the case considered, this averaging is ensured by the one-electron oscillations,[4] while for an ensemble of granules in an oxide layer[6-8] it is determined by statistical averaging over the positions of the effective Fermi levels of the different granules.

The author is grateful to K. K. Likharev for help and interest in the work, and to A. I. Larkin and Yu. N. Ovchinnikov for a helpful discussion of the results.

[1]T.-L. Ho, Phys. Rev. Lett. **51**, 2060 (1983). E. Ben-Jacob, E. Motolla, and G. Schön, *ibid*. **51**, 2064 (1983).
[2]Yu. M. Ivanchenko and Yu. V. Medvedev, Fiz. Nizk. Temp. **11**, 537 (1985) [Sov. J. Low Temp. Phys. **11**, 294 (1985)].
[3]D. V. Averin, A. B. Zorin, and K. K. Likharev, Zh. Eksp. Teor. Fiz. **88**, 692 (1985) [Sov. Phys. JETP **61**, 407 (1985)]. K. K. Likharev and A. B. Zorin, J. Low Temp. Phys. **59**, 347 (1985).
[4]D. V. Averin and K. K. Likharev, Zh. Eksp. Teor. Fiz. **90**, 733 (1986) [Sov. Phys. JETP **63**, No. 2 (1986)].
[5]M. H. Cohen, L. M. Falicov, and J. C. Phillips, Phys. Rev. Lett. **8**, 316 (1962).
[6]I. Giaever and H. R. Zeller, *ibid*. **20**, 1504 (1968).
[7]J. Lambe and R. C. Jaklevic, *ibid*. **22**, 1371 (1969).
[8]I. O. Kulik and R. I. Shekhter, Zh. Eksp. Teor. Fiz. **68**, 623 (1975) [Sov. Phys. JETP **41**, 308 (1975)].
[9]G. Schön, Preprint, "Macroscopic quantum effects and quantum noise in Josephson elements," 1985.
[10]A. J. Dahm, A. Denenstein, D. N. Langerberg, *et al*., Phys. Rev. Lett. **22**, 1416 (1969).
[11]N. N. Bogolyubov and N. N. Bogolyubov, Jr., Introduction to Quantum Statistical Mechanics [in Russian], Nauka, 1984, Part 1, Ch. 4
[12]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 1969, **51**, 1535 (1969) [Sov. Phys. JETP **24**, 1035 (1970)].

Translated by J. G. Adashko