The role of incoherent scattering in radiation processes at small angles of incidence of particles on crystallographic axes or planes

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The intensity of multiple scattering of $e^\pm$ moving inside planes and tubes formed by atoms of a crystal is several orders of magnitude higher than in an amorphous material. It is shown that in spite of this, the Landau-Pomeranchuk effect (LPE) does not occur on incidence on crystallographic axes and planes at sufficiently small angles of high energy $e^\pm$ and γ. An expression is obtained for the probability of radiation of a γ ray by $e^\pm$ in a uniform field in the presence of uncorrelated multiple scattering of the $e^\pm$. It generalizes the expression obtained by Migdal for description of the Landau-Pomeranchuk effect for the probability of emission of a γ ray by $e^\pm$ moving in an amorphous beam, and the Klepikov-Nikishov-Ritus expression for the probability of emission of a γ ray by $e^\pm$ moving in a uniform electromagnetic field.

1. INTRODUCTION

It was established rather long ago[1,2] that uncorrelated scattering of $e^\pm$ in the coherence length can substantially influence the processes of radiation and pair production in an amorphous medium. The most important result of this effect is the decrease of the probability of these radiation processes, the so-called Landau-Pomeranchuk effect (LPE). In Ref. 3 Nasonov and Shul'ga showed that uncorrelated scattering of $e^\pm$ increases appreciably when they are incident onto crystal axes, as a result of which the LPE will appear at lower energies than in an amorphous medium. Recently it has been shown theoretically[4–10] and subsequently also experimentally[11,12] that on incidence of particles onto crystals at small angles with respect to planes (or axes) there is a new mechanism of radiation and pair production, which is accompanied by numerous polarization phenomena.[4,7,8] In this case an appreciable fraction of the processes as a whole occurs inside atomic planes and tubes where the density of scatterers is 10–10^3 times higher than their average density in the crystal. As a consequence it is natural to expect a still more distinct manifestation of the LPE and a substantial influence of coherent scattering of $e^\pm$ by inhomogeneities of the crystal potential on the effects discussed in Refs. 4–12. In the present work we have shown that the existence of a nonzero mean field of crystallographic axis and planes leads to absence of the LPE on incidence of $e^\pm$ and γ at small angles to crystallographic axes and planes. The reason for this can be made clear if one considers the evolution of the relation between the angle of multiple scattering of e^- in a coherence length l_{coh} and the characteristic radiation angle θ_{char}. We recall that in an amorphous medium $l_{coh} = \varepsilon\varepsilon'/m^2\omega$ and $\theta_{char} = \theta_\gamma = m/\varepsilon$ where ω and ε are the energies of the γ and e^-, $\varepsilon' = \varepsilon - \omega$, and m is the mass of the e^-. We shall use the system of units in which $\hbar = c = 1$. The mean square multiple scattering angle acquired by an e^- in a length l is (see for example Section 5 of Ref. 13)

$$\overline{\vartheta_s^2(l)} = 8\pi n \frac{Z^2\alpha^2}{\varepsilon^2} \ln\left(\frac{\vartheta_{max}}{\vartheta_{min}}\right) l, \qquad (1)$$

where Z is the atomic number, n is the density of atoms, $\alpha = 1/137$, and $\vartheta_{min} = 1/(R\varepsilon)$, where R is the screening radius of the atom. Multiple scattering will substantially influence the formation of the radiated γ ray if

$$\vartheta_{max}^2 = \overline{\vartheta_s^2(l_{coh})} \gtrsim \theta_\gamma^2. \qquad (2)$$

Assuming that $\omega \sim \varepsilon' \sim \varepsilon$, we have $\overline{\vartheta_s^2(l_{coh})} \propto \varepsilon^{-1}$, and, since $\theta_\gamma^2 \propto \varepsilon^{-2}$, at sufficiently large ε (in Pb at $\varepsilon = 10^3$ GeV; see Ref. 2) the inequality (2) will be satisfied. A similar situation is realized in incidence of e^- on crystallographic axes at angles $\psi > \tilde{\psi} = V_{max}/m$, where V_{max} is the amplitude of variation of the averaged potential of the axes (or planes) of the crystal. For $\psi < \tilde{\psi}$ the situation changes radically. Assume for simplicity that $\psi \ll \tilde{\psi}$. Then the formation of the γ ray will occur at a definite distance from the axis or plane, i.e., with practically constant multiple scattering and electric field. Radiation in the field E perpendicular to the momentum of the e^- is characterized by a parameter $\chi = eE\varepsilon/m^3$ (see Refs. 13–17). For example, at $\chi \gtrsim 1$ γ rays with energies $\omega \sim \chi\varepsilon/(1+\chi) \sim \varepsilon$ are radiated and the radiation process becomes substantially quantum. For $\chi \gg 1$ there is a modification of the characteristic radiation angle $\theta_{char} = \theta_E \propto \theta_\gamma \chi^{1/3}$ and the coherence length $l_{coh} = l_E \propto \omega/m\chi^{2/3}$ (we shall assume as before that $\omega \sim \varepsilon' \sim \varepsilon$). It is natural to expect that multiple scattering will greatly change the pattern of formation of the γ ray for

$$\vartheta_{max}^2 = \overline{\vartheta_s^2(l_E)} \gtrsim \theta_E^2, \qquad (3)$$

where $\vartheta_{min} = 1/(u\varepsilon)$, and u is the mean square amplitude of thermal vibrations of the nuclei of the crystal (the possibility of use of Eq. (1) in evaluation of the intensity of multiple scattering in crystals is discussed in the Appendix). Since $\chi \propto \varepsilon$, we have $\overline{\vartheta_s^2(l_E)} \propto \varepsilon^{-5/3}$, and $\theta_E^2 \propto \varepsilon^{-4/3}$, i.e., in contrast to the case of an amorphous medium the left-hand side of the inequality (3) falls off with increase of ε faster than the right side. Therefore with increase of ε and ω we should expect only a decrease of the influence of multiple scattering on the processes of radiation and pair production.

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2. PROBABILITY OF EMISSION OF γ RAYS BY AN ELECTRON MOVING IN A UNIFORM FIELD AND UNDERGOING UNCORRELATED MULTIPLE SCATTERING

The theory describing radiation processes in amorphous medium with inclusion of multiple scattering (the theory of the Landau-Pomeranchuk effect) was developed by Migdal.[2] An expression which describes radiation and pair production in a uniform external field was obtained by Klepikov[14] and in final form by Nikishov and Ritus.[15,16] The results of Refs. 14 and 16 can be reproduced most simply by means of the quasiclassical operator method developed by Baĭer and Katkov.[13,15] Therefore in considering the radiation of an e^- which undergoes simultaneously multiple scattering and the action of a uniform field, we also shall use the method of Ref. 13 and shall proceed from the expression for the probability of radiation of a γ ray by an electron passing through an arbitrary point $\mathbf{r}_0$:

$$\frac{dW}{d\omega\, d^2\theta}=\frac{e^2\omega}{8\pi^2}\int_{-\infty}^{\infty}\left[\frac{m^2\omega^2}{\varepsilon^2\varepsilon'^2}+\frac{(\varepsilon^2+\varepsilon'^2)}{\varepsilon'^2}(\mathbf{v}_0\mathbf{v}(\tau)-1)\right]\times\exp\left\{i\frac{\varepsilon}{\varepsilon'}[\mathbf{k}(\mathbf{r}(\tau)-\mathbf{r}_0)-\omega\tau]\right\}d\tau, \quad (4)$$

where $\varepsilon'=\varepsilon-\omega$, $\boldsymbol{\theta}$ is the solid angle indicating the direction of emission of the γ ray, $\mathbf{v}(\tau)$ and $\mathbf{r}(\tau)$ are the velocity and radius vector of the e^- at the moment of time τ, and $\mathbf{v}(0)=\mathbf{v}_0$ and $\mathbf{r}(0)=\mathbf{r}_0$. Introducing by means of the relation

$$\mathbf{v}(\tau)=\mathbf{v}_0\left(1-\frac{\boldsymbol{\vartheta}^2(\tau)}{2}\right)+\boldsymbol{\vartheta}(\tau),\quad \boldsymbol{\vartheta}(\tau)\mathbf{v}_0=0,\quad |\boldsymbol{\vartheta}|\ll 1 \quad (5)$$

the small-angle approximation and integrating in (4) over the directions of emission of the γ ray, we obtain

$$\frac{dW}{d\omega}=\frac{ie^2}{2\pi}\int_{-\infty}^{\infty}\left[\frac{m^2}{\varepsilon^2}+\frac{(\varepsilon^2+\varepsilon'^2)}{4\varepsilon\varepsilon'}\boldsymbol{\vartheta}^2(\tau)\right]\exp(-ia\tau)\times\prod_{i=x,y}\exp\left\{\frac{ib}{2}\left[\frac{1}{\tau}\left(\int_0^\tau dt\,\vartheta_i(t)\right)^2-\int_0^\tau dt\,\vartheta_i^2(t)\right]\right\}\frac{d\tau}{\tau}, \quad (6)$$

$$a=m^2\omega/2\varepsilon\varepsilon',\quad b=\omega\varepsilon/\varepsilon'. \quad (7)$$

Note that, generally speaking, change of the order of integration in (4) over τ and $\boldsymbol{\theta}$ is not allowed, as a result of which it is necessary to carry out a subtraction procedure in Eq. (6) (see below).

To find the probability of radiation in the presence of multiple scattering it is necessary to average (6) over all possible trajectories of the e^-. The transverse displacement of the e^- in the process of formation of a γ ray is of the order of the electron Compton wavelength $\lambda_e=\hbar/mc=3.862\cdot 10^{-11}$ cm. It is small in comparison with the characteristic scale $\Delta\rho\sim u$ of variation of the field of the axis (or plane) and the density of nuclei in the direction of the normal to it. Therefore on incidence on the axis (or plane) at angles $\psi\ll\tilde{\psi}$, an e^- undergoes in the course of formation of the γ ray (in the coherence length) the action of a practically constant electric field and uncorrelated multiple scattering which is constant in intensity at the deviations of the potential from the averaged potential, i.e., at the nuclei and electrons of the atoms forming the axis or plane. The effective (averaged) field of the axis or plane arises as the result of correlated peripheral (with impact parameters $\rho\gtrsim u$) collisions of the e^- with nuclei and electrons which are located asymmetrically with respect to the e^- trajectory. This asymmetry obviously leads also to an asymmetry of the multiple scattering of the e^- in the xy plane perpendicular to the e^- velocity. Since it follows from symmetry considerations that the intensity of multiple scattering is extremal in the directions of the axes x and y perpendicular and parallel to the intensity of the mean field, we shall characterize it by the mean squares of the angles of deviation, from the direction of motion of the e^- in the average potential, acquired by the e^- per unit length $\sigma_i=d\,\overline{\vartheta_{si}^2(z)}/dz$, $i=x,y$. We note that, generally speaking, the asymmetry of the multiple scattering is small: $|\sigma_x-\sigma_y|/(\sigma_x+\sigma_y)\lesssim 10$–$20\%$ (compare with Ref. 18). In a length Δz the uniform field deflects the e^- by an angle $\Delta\vartheta_x=w\Delta z$, where $w=eE/\varepsilon$ is the transverse acceleration of the e^-. Therefore in the small-angle approximation the distribution of e^- moving at the point $z=0$ in the direction of a vector $\boldsymbol{\vartheta}_1=(\vartheta_{1x},\vartheta_{1y})$, will be determined at a point $z>0$ by the relation

$$P(\boldsymbol{\vartheta})=\frac{1}{2\pi z(\sigma_x\sigma_y)^{1/2}}\exp\left\{-\frac{(\vartheta_x-wz-\vartheta_{1x})^2}{2\sigma_x z}-\frac{(\vartheta_y-\vartheta_{1y})^2}{2\sigma_y z}\right\}. \quad (8)$$

Since the distance between successive scatterings of the $e^\pm$ are equal to d—the interatomic distance in an axis or plane— and since at the energies of interest to us $\varepsilon>10$ (100) GeV the relation $d\ll l_{\rm coh}\approx\varepsilon/m^2$ is satisfied, in averaging of the expression (6) over various trajectories of the e^- it is possible to use the path integral approximation (see Ref. 19). For this purpose we shall break up an arbitrary interval z into N equal segments of length $\Delta=z/N$ and shall assume that the e^- undergo scattering at points $z_n=n\Delta$, $n=1,\ldots,N$, being deflected here by angles $\boldsymbol{\vartheta}_n=\boldsymbol{\vartheta}(z_n)$. From Eq. (8) it is easy to conclude that the probability density that these angles fall in intervals $(\boldsymbol{\vartheta}_n,\boldsymbol{\vartheta}_n+d\boldsymbol{\vartheta}_n)$, is given by the expression

$$d^2\mathscr{P}_N=\frac{d\boldsymbol{\vartheta}_1\ldots d\boldsymbol{\vartheta}_N}{(2\pi\Delta(\sigma_x\sigma_y)^{1/2})^N}\exp\left\{-\left[\frac{(\vartheta_{1x}-w\Delta)^2}{2\sigma_x\Delta}+\frac{\vartheta_{1y}^2}{2\sigma_y\Delta}\right]-\ldots-\left[\frac{(\vartheta_{Nx}-w\Delta-\vartheta_{N-1\,x})^2}{2\sigma_x\Delta}+\frac{(\vartheta_{Ny}-\vartheta_{N-1\,y})^2}{2\sigma_y\Delta}\right]\right\}. \quad (9)$$

In order to average the probability (6) over all possible e^- trajectories it is necessary to multiply it by the probability density (9) and to integrate it over the scattering angles $\boldsymbol{\vartheta}_n$, $n=1,\ldots,N$. It would follow from physical considerations that we should set $\Delta=d$. However, in the limit $d=\Delta\ll z\sim l_{\rm coh}$ the result obtained will not depend on the value of Δ, which permits us to go over to the path-integral approximation $N\to\infty$, $\Delta\to 0$. Then (compare with Ref. 19) the expression for the average probability is written in the form of a functional integral over a Wiener measure $d_W^2\vartheta$:

$$\left\langle\frac{dW}{d\omega}\right\rangle=\int d_W^2\boldsymbol{\vartheta}\,\frac{dW\{\boldsymbol{\vartheta}(z)\}}{d\omega}=\lim_{N\to\infty}\int_{-\infty}^{\infty}\ldots\int d^2\mathscr{P}_N\frac{dW\{\boldsymbol{\vartheta}(z)\}}{d\omega}, \quad (10)$$

where $dW\{\vartheta(z)\}/d\omega$ is a functional of the scattering angles $\vartheta(z)$ and is determined by Eq. (6). Taking into account that the integration over the components ϑ_{nx} and ϑ_{ny} is carried out independently, we rewrite (10) in the form

$$\left\langle\frac{dW}{d\omega}\right\rangle=-\frac{e^2}{\pi}\operatorname{Im}\int_0^\infty\left[\frac{m^2}{\varepsilon^2}+\frac{\varepsilon^2+\varepsilon'^2}{4\varepsilon\varepsilon'}\frac{d}{d\mu}\right]$$
$$\times\exp(-iaz)\,Q_x(b,\sigma_x,E)\,Q_y(b,\sigma_y,0)\,\frac{dz}{z}\bigg|_{\mu=0},\qquad(11)$$

where

$$Q_i(b,\sigma_i,E)=\int d_{w'}\vartheta_i\exp\left\{\mu\vartheta_i^2+\frac{ib}{2z}\left[\int_0^z dz'\,\vartheta_i(z')\right]^2-\frac{ib}{2}\int_0^z dz'\,\vartheta_i^2(z')\right\}.\qquad(12)$$

The integrals Q_i, $i=x,y$, are Gaussian, which makes it possible to calculate them exactly. In the case $E=0$ this was done in Ref. 19. In the case $E\neq0$ the calculations are somewhat more cumbersome. They lead to

$$Q_x(b,\sigma_x,E,\mu)=\left\{D_x(0)\left[1-\frac{ib\sigma_x}{z}\int_0^z dz'\,D_x^{-2}(z')\times\left(\int_{z'}^z dz''\,D_x(z'')\right)^2\right]\right\}^{-1/2}\exp\left\{\frac{w^2z}{2\sigma_x}-\frac{w^2}{2\sigma_x}\int_0^z dz'\,D_x^{-2}(z')\right.$$
$$\left.-\frac{ibw^2}{2z}\left[\int_0^z dz'\,D_x(z')\int_0^{z'}dz''\,D_x^{-2}(z'')\right]^2\Big/\left[1-\frac{ib\sigma_x}{z}\int_0^z dz'\,D_x^{-2}(z')\left(\int_{z'}^z dz''\,D_x(z'')\right)^2\right]\right\},\qquad(13)$$

where

$$D_i(z')=\operatorname{ch}r_i(z'-z)+\frac{2\sigma_i\mu}{r_i}\operatorname{sh}r_i(z'-z),\qquad(14)$$

$$r_i=(ib\sigma_i)^{1/2},\quad w=eE/\varepsilon$$

is the acceleration of the e^-. From Eqs. (13) and (14) we easily obtain

$$Q_x(b,\sigma_x,E)\,|_{\mu=0}=\left(\frac{r_xz}{\operatorname{sh}r_xz}\right)^{1/2}\exp\left\{\frac{w^2}{\sigma_xr_x}\left[\frac{r_xz}{2}-\operatorname{th}\left(\frac{r_xz}{2}\right)\right]\right\},$$

$$\frac{d}{d\mu}Q_x(b,\sigma_x,E)\,|_{\mu=0}$$
$$=\left[\frac{2\sigma_x}{r_x}\operatorname{th}\left(\frac{r_xz}{2}\right)+\frac{4w^2}{r_x^2}\operatorname{th}^2\left(\frac{r_xz}{2}\right)\right]Q_x(b,\sigma_x,E)\,|_{\mu=0}.\qquad(15)$$

After substitution of (15) into (11), we have

$$\left\langle\frac{dW}{d\omega}\right\rangle=-\frac{e^2}{\pi}\operatorname{Im}\int_0^\infty\left\{\frac{m^2}{\varepsilon^2}+\frac{(\varepsilon^2+\varepsilon'^2)}{2\varepsilon\varepsilon'}\left[\frac{2w^2}{r_x^2}\operatorname{th}^2\left(\frac{r_xz}{2}\right)+\sum_{i=x,y}\frac{\sigma_i}{r_i}\operatorname{th}\left(\frac{r_iz}{2}\right)\right]\right\}$$
$$\times\left(\frac{r_xr_y}{\operatorname{sh}r_xz\,\operatorname{sh}r_yz}\right)^{1/2}\exp\left\{-iaz-\frac{w^2z}{2\sigma_x}+\frac{w^2}{r_x\sigma_x}\operatorname{th}\left(\frac{r_xz}{2}\right)\right\}dz.\,(16)$$

Going over in (16) to the limit of absence of uncorrelated multiple scattering ($\sigma_i\to0$), it appears that we should obtain an expression for the probability of radiation of a γ ray in a uniform field. However, as can easily be seen, in taking the limit in it of absence of an external field ($E\to0$), we will not obtain a zero result for the probability of radiation in vacuum. The reason for this, as we have already mentioned, is the unjustified change of the order of integration in (4). Frequently (see for example Section 20 of Ref. 13) this deficiency is corrected by introduction of a small shift of the integration contour in the variable τ. We, however, shall use a more lucid equivalent procedure of subtraction from (16) of this same expression taken in the limit of free motion ($E\to0$, $\sigma_i\to0$) (see for example Section 12 of Ref. 13). This procedure is chosen because it leads to expressions which are more convenient from the point of view of calculations by computer.

To simplify the expression obtained from (16) by the subtraction mentioned above, we shall introduce in accordance with the expressions

$$t=r_xz/2,\quad p=r_y/r_x=(\sigma_y/\sigma_x)^{1/2},\quad x=(m^3\omega/eE\varepsilon\varepsilon')^{2/3},$$
$$\chi=eE\varepsilon/m^3,\quad \nu=m^2\omega/\varepsilon\varepsilon'(2b\sigma_x)^{1/2}\qquad(17)$$

a new integration variable t, a parameter p which characterizes the asymmetry of multiple scattering of the e^-, parameters x and χ which are usually used in the theory of radiation in a uniform electromagnetic field (see Refs. 13–17), and a parameter ν which characterizes the influence of multiple scattering of the e^- on the process of formation of the γ ray. Note that in the case of symmetric multiple scattering ($p=1$) we have $\nu=4s$, where s is a parameter introduced for the same purpose by Migdal (Ref. 2 and Section 20 of Ref. 13). At zero angle of incidence onto crystallographic axes and planes $|p-1|\lesssim0.2$ (compare with Ref. 18). After carrying out the changes which are customary in such cases of the integration path in the complex plane of the variable t (see Refs. 2, 3, and 13) we obtain a final expression for the probability of radiation of a γ ray by an electron or positron moving in a uniform field and undergoing multiple scattering

$$\frac{dW_{E,s}}{d\omega}\equiv\left\langle\frac{dW}{d\omega}\right\rangle$$
$$=-\frac{e^2m^2}{\pi^{1/2}\varepsilon^2}\left[F_1(p,x,\nu)+\left(\frac{2}{x}+\frac{\omega}{\varepsilon}\chi x^{1/2}\right)F_3(p,x,\nu)+\frac{(\varepsilon^2+\varepsilon'^2)}{3\omega\varepsilon^2L(\sigma_x)}F_2(p,x,\nu)\right],\qquad(18)$$

where $L(\sigma_x)=\pi m^2/e^2\varepsilon^2\sigma_x$ is a quantity equal to the radiation length in the limits of an amorphous medium. The functions F_k, $k=1,2,3$ are defined by the expressions

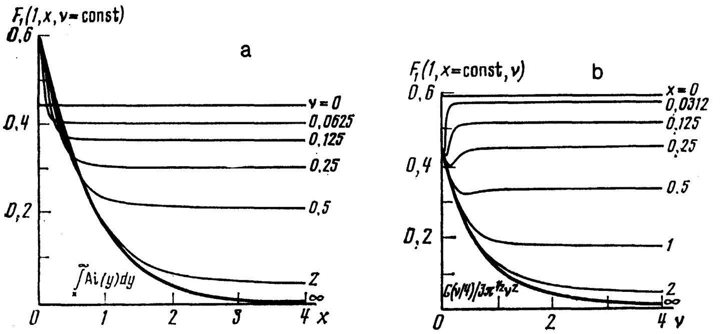

FIG. 1. (a) Dependence of the function $F_1(1, x, \nu)$ on the parameter x for fixed values of the parameter ν. For $1 \gtrsim x < \nu$ we have $F_1(1, x, \nu) \to \int_x^\infty \mathrm{Ai}(y)dy$. (b) Dependence of the function $F_1(1, x, \nu)$ on the parameter ν for fixed values of the parameter x. For $1 \gtrsim \nu < x$ we have $F_1(1, x, \nu) \to G(\nu/4)/3\pi^{1/2}\nu^2$.

$$F_1(p,x,\nu)=\frac{2}{\pi^{1/2}}\int_0^\infty e^{-\nu t}\left\{\frac{\sin \nu t}{2t}-f(p,t)\exp[\varphi(x,\nu,t)]\times\sin[\nu t+\varphi(x,\nu,t)]\right\}dt,$$

$$F_2(p,x,\nu)=3\sqrt{2}\,\nu\int_0^\infty [\mathrm{th}(t)+p\,\mathrm{th}(pt)]f(p,t)\exp[-\nu t+\varphi(x,\nu,t)]\times\cos\left[\nu t+\varphi(x,\nu,t)-\frac{\pi}{4}\right]dt, \qquad (19)$$

$$F_3(p,x,\nu)=-\frac{4\nu^2}{\pi^{1/2}x^2}\int_0^\infty \mathrm{th}^2(t)f(p,t)\exp[-\nu t+\varphi(x,\nu,t)]\times\cos[\nu t+\varphi(x,\nu,t)]dt,$$

$$f(p,t)=[p/\mathrm{sh}(2t)\,\mathrm{sh}(2pt)]^{1/2}, \qquad \varphi(x,\nu,t)=\frac{2\nu^3}{x^3}[\mathrm{th}(t)-t]. \qquad (20)$$

The exponential falloff of the integrands at large t and their not very rapid oscillations permit easy calculation of the function F_k by computer (see Figs. 1a and b). Note that in the case of the fields of crystallographic axes or planes the functions F_k, or more precisely all three of their parameters x, ν, and p, are functions only of the distance to the axis or plane.

The expressions (18)–(20) generalize the expression for the probability of radiation of a γ ray by an electron in an amorphous medium obtained by Migdal (Ref. 2 and Section 20 of Ref. 13) and the expression for the probability of radiation in a uniform field obtained by Klepikov[14] and in final form by Nikishov and Ritus.[15,16] Indeed, in the limit of dominance of a uniform field and symmetric multiple scattering $1 > x \ll \nu$ or $1 < x \ll \nu^{2/3}$, $p = 1$:

$$F_1(1,x,\nu)\to\int_x^\infty \mathrm{Ai}(y)dy, \qquad F_3(1,x,\nu)\to \mathrm{Ai}'(x),$$
$$F_2(1,x,\nu)/L(\sigma_x)\sim x/\nu^2\to 0, \qquad (21)$$

where

$$\mathrm{Ai}(x)=\int_0^\infty \cos(xt+t^3/3)\,dt/\pi^{1/2}$$

is the Airy function. Here. Eq. (18) goes over to the expression for the probability of radiation of a γ ray by an e^- moving in a uniform field (see Eq. (90.23) in Ref. 15):

$$\frac{dW_E}{d\omega}=-\frac{e^2m^2}{\pi^{1/2}\varepsilon^2}\left\{\int_x^\infty \mathrm{Ai}(y)\,dy+\left(\frac{2}{x}+\frac{\omega}{\varepsilon}\chi x^{1/2}\right)\mathrm{Ai}'(x)\right\}. \qquad (22)$$

In the limit of dominance of symmetric multiple scattering $1 > \nu \ll x$ or $1 < \nu \ll x^{3/2}$, $p = 1$, we have

$$F_1(1,x,\nu)\to G(s)/48\pi^{1/2}s^2,$$
$$F_2(1,x,\nu)\to G(s)+2\Phi(s), \qquad (23)$$
$$F_3(1,x,\nu)\sim \nu^2/x^2\to 0,$$

where $s = \nu/4$, $G(s)$ and $\Phi(s)$ are the parameter and functions introduced by Migdal.[2] As a result Eq. (18) goes over into the expression obtained by him for the probability of radiation of a γ ray by an e^- or e^+ moving in an amorphous medium (compare with Eq. (54) in Ref. 2):

$$\frac{dW_s}{d\omega}=\frac{e^2m^2}{96\pi\varepsilon^3\varepsilon's^2}[G(s)\omega^2+2\Phi(s)(\varepsilon^2+\varepsilon'^2)]. \qquad (24)$$

Analysis of Eqs. (18)–(20) shows that the nature of radiation and pair production is determined completely by the relation of the parameters ν and x. Its evolution also determines the fate of the Landau-Pomeranchuk effect. Analysis of these equations shows that for $1 > x < \nu$, or $1 < x < \nu^{2/3}$ together with a decrease of the parameter x the processes of radiation and pair production rapidly acquire the features of the analogous processes in a uniform field. For example, the coherence length of the process, which is determined by the region of formation of the principal contribution to the integrals in (19), becomes

$$l_{E,s}=l_E(x)=\begin{cases} l_\gamma x, & 1>x<\nu \\ l_\gamma x^{1/2}, & 1<x<\nu^{2/3} \end{cases} \qquad (25)$$

where $l_\gamma = \varepsilon\varepsilon'/m^2\omega$. The characteristic angle of the radiation, which is equal to the angle of deflection of the e^- in l_E, is given here by the expression

$$\theta_{E,s}=\theta_E(x)=\frac{|e|E}{\varepsilon}l_E=\begin{cases} \theta_\gamma x^{-1/2}, & 1>x<\nu \\ \theta_\gamma, & 1<x<\nu^{2/3} \end{cases} \qquad (26)$$

where $\theta_\gamma = m/\varepsilon$. For $1 > \nu < x$ or $1 < \nu < x^{3/2}$ in addition to a decrease of the parameter ν the features of radiation and pair production in an amorphous medium begin to dominate and, if $\nu \lesssim 1$, the Landau-Pomeranchuk effect appears. In this case

$$l_{E,s}=l_s(\nu)=\begin{cases} l_\gamma\nu, & 1>\nu<x \\ l_\gamma, & 1<\nu<x^{3/2} \end{cases}; \tag{27}$$

$$\theta_{E,s}=\theta_s(\nu)=\begin{cases} (\sigma l_s)^{1/2}=\theta_\gamma\nu^{-1/2}, & 1>\nu<x \\ \theta_\gamma, & 1<\nu<x^{3/2} \end{cases} \tag{28}$$

(see Section 20 of Ref. 13). In determining the characteristic angle (28) we neglected the insignificant asymmetry of the uncorrelated multiple scattering and assumed that $\sigma=\sigma_x+\sigma_y$. In regard to the change of the nature of radiation processes at $x\approx\nu\lesssim 1$, this is illustrated by the plots of the function $F_1(1, x, \nu)$ shown in Figs. 1a and b. Just this change leads also to the absence of the Landau-Pomeranchuk effect, since both for $\omega\sim\varepsilon'\sim\varepsilon$ and for a fixed ratio of ω to ε we have $\nu\sim\varepsilon^{-1/2}$ and $x\sim\varepsilon^{-2/3}$, from which $x/\nu\sim\varepsilon^{-1/6}$ and $x/\nu^{2/3}\sim\varepsilon^{-1/3}$. The falloff of these ratios with increase of $\varepsilon(\omega)$, as was mentioned above, leads to dominance in radiation and pair-production processes of the features of the analogous processes in a uniform field over the features of these processes which appear in an amorphous medium. In particular, the probability of the radiation of pair-production process for $1>x<\nu$ will fall off as $\varepsilon^{-1/3}(\omega^{-1/3})$, and not as $\varepsilon^{-1/2}(\omega^{-1/2})$ with a Landau-Pomeranchuk effect which appears at $1>\nu<x$.

We recall that the expression for the probability, differential in the energy of the e^+ or e^-, of pair production by a γ ray can be obtained from Eq. (18) by the substitution $\omega\to-\omega, \varepsilon\to-\varepsilon_+, \varepsilon'\to\varepsilon_-$ and by multiplication by the ratio of the statistical weights $\varepsilon_\pm^2\, d\varepsilon_\pm/\omega^2 d\omega$ ($\varepsilon_\pm$ are the energies of the $e^\pm$). The total probability of pair production $W_{E,s}$ is then obtained by integration over the energy of the $e^+(e^-)$ between limits $\varepsilon_\pm=0$ and $\varepsilon_\pm=\omega$. In order to illustrate the statements made above in analysis of Eqs. (18)–(20), we have shown in Fig. 2 energy dependences of the probabilities $E_{E,s}$ calculated for the cases of pair production at distances $\rho=0$ and $\rho_1=0.02$ Å from the ⟨111⟩ axis of a tungsten crystal, taken at $T=293$ K ($u=0.0495$ Å) and the probability W_E for pair production in a uniform field $E(\rho_1)=2.3\cdot 10^{11}$ V/cm. Here $n(\rho_1)=2.17\cdot 10^{25}$

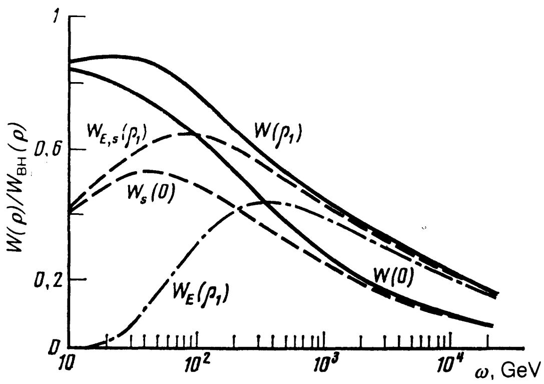

FIG. 2. Pair-production probability obtained after transition to the cross channel and integration over the energy of the $e^+(e^-)$ in the expressions (18), (22), and (33). The approach of the probability $W(\rho_1)$ to the probability $W_E(\rho_1)$ (and not to the probability $W_s(n(\rho_1))\approx W_s(n(0))$ (see Eq. (24)) illustrates the absence of the Landua-Pomeranchuk effect.

cm^{-3} $=343n_0$ and $n(0)=2.35\cdot 10^{25}$ cm^{-3} $=372n_0$. In performing the numerical calculations illustrating the general properties of the probabilities W and $W_{E,s}$ we neglected the insignificant asymmetry of multiple scattering of the e^- ($p=1$) and assumed

$$\sigma_i=0.5\,\overline{d\vartheta_s^2(z)}/dz,\quad i=x, y,$$

where $\overline{\vartheta_s^2(z)}$ was calculated according to Eq. (1) with

$$\vartheta_{min}=1/(u\varepsilon) \text{ and } \vartheta_{max}^2=\overline{\vartheta_s^2(l_{E,s})}.$$

We emphasize that in a detailed description of the processes, the values of σ_i must be calculated by more accurate methods. Since at the center of an axis $E(\rho=0)=0$, the probability $W_{E,s}(\rho=0)$ coincides with the probability $W_s(n(0))$ obtained from Eq. (24). The substantial drop of this probability is due to the Landau-Pomeranchuk effect. Since $n(\rho_1)=n(0)$, the LPE could appear at $\rho=\rho_1$ at the same energies as at $\rho=0$ (these energies are two orders of magnitude lower than in the case of an amorphous material). With increase of ω the probability $W_{E,s}(\rho_1)$ would then approach the probability $W_s(n(0))$. However, (see Fig. 2), this does not occur. The probability $W_{E,s}(\rho_1)$ approaches the probability $W_E(\rho_1)$ for pair production in a uniform field of strength $E(\rho_1)$, illustrating thereby the conclusion that there is suppression of the Landau-Pomeranchuk effect in an external field and that the theory of radiation and pair production in a uniform field[13–17] is applicable for description of analogous processes in the fields of crystal axes and planes.

3. CONTRIBUTION OF LARGE ELECTRON-SCATTERING ANGLES TO THE PROBABILITY OF ELECTROMAGNETIC PROCESSES

In the previous section, as in Refs. 2 and 3, we averaged the probability of radiation of a γ ray along various trajectories of radiating electrons undergoing multiple scattering. It is important to understand that both the expression (18) obtained by us and the expression (24) obtained by Migdal do not always describe the total probability of radiation (or of pair production—after transition in them to the cross channel). Indeed, let us return to analysis of Eq. (24), which was obtained in Ref. 2 for the case of radiation in an amorphous medium. It is easy to show that for $\nu=4s>1$, i.e., in the limit of weak multiple scattering, Eq. (24) goes over to

$$\left[1-\frac{\ln\nu}{\ln(190Z^{-1/3})}\right]\frac{dW_{BH}}{d\omega}=\frac{\ln[\vartheta_s(l_\gamma)/\vartheta_{min}]}{\ln(\theta_\gamma/\vartheta_{min})}\frac{dW_{BH}}{d\omega}, \tag{29}$$

where $\vartheta_{min}=\theta_\gamma Z^{1/3}/190$, $\vartheta_s^2(l)\equiv\overline{\vartheta_s^2(l)}$ [see Eq. (1)], $l_\gamma=\varepsilon\varepsilon'/m^2\omega$, and $\theta_\gamma=m/\varepsilon$. Since in this limit $\vartheta_s(l_\gamma)<\theta_\gamma$, the probability (29) turns out to be less than the Bethe-Heitler probability $dW_{BH}/d\omega$ and therefore the probability (24) does not go over into $dW_{BH}/d\omega$ (see Fig. 2, where we have shown the energy dependence of the integrated probability for pair production $W_s(\rho=0)$ calculated for the density of nuclei $n(0)=372n_0$ achieved at the center of the ⟨111⟩ W axis at $T=293$ K). In Ref. 2 this deficiency was overcome by intoduction of a numerical factor in Eq. (24) [see Eq. (64) in Ref. 2]. We, however, cannot do without understanding the reason for this discrepancy. We recall that the averaging of the probability of radiation, both in the present work and in Ref. 2, was carried out with use of distribution functions of the e^- over the directions of motion obtained in solution of the Fokker-Planck equation. Distribution functions of this type, strictly speaking, are applicable only for description of the motion of e^- which experience at individual scatterers (nuclei) deviations by angles $\vartheta \ll \vartheta_s(l)$. It is true that in the logarithmic approximation it becomes sufficient to satisfy the condition $\vartheta < \vartheta_s(l)$. In the limit of weak multiple scattering $\nu = 4s > 1$ we have $\vartheta_s(l_\gamma) < \theta_\gamma$, while the important contribution to the probability of radiation is from angles of scattering of the e^- by nuclei $\vartheta < \theta_\gamma$. As a result, in averaging over a distribution function satisfying the Fokker-Planck equation, scattering of e^- by individual nuclei at angles $\vartheta_s(l_\gamma) < \vartheta < \theta_\gamma$ drop out of the discussion. Such scattering events occur rather rarely. It is easy to show that in the logarithmic approximation it is possible to neglect the probability of a second scattering of e^- within l_γ by an angle $\vartheta \gtrsim \vartheta_s(l_\gamma)$. In addition, in the logarithmic approximation one can neglect also the multiple scattering of e^- leading to a total deflection of the e^- by angles $\vartheta \lesssim \vartheta_s(l_\gamma)$. Therefore in calculation of the probability of radiation of a γ ray by an electron which has undergone scattering by a nucleus at an angle $\vartheta > \vartheta_s(l_\gamma)$, in the logarithmic aproximation one can assume that the scattering occurred at an isolated nucleus. The probability of radiation in scattering by angles $\vartheta_s(l_\gamma) < \vartheta < \theta_\gamma$ turns out then to be

$$\frac{\ln \nu}{\ln(190Z^{-1/3})}\frac{dW_{\rm BH}}{d\omega}=\frac{\ln[\theta_\gamma/\vartheta_s(l_\gamma)]}{\ln(\theta_\gamma/\vartheta_{min})}\frac{dW_{\rm BH}}{d\omega}, \quad \nu>1 \qquad (30)$$

and in sum with the probability (24) [see Eq. (29)] gives the probability $dW_{\rm BH}/d\omega$. In the case of strong multiple scattering ($\nu = 4s < 1$) we have $\vartheta_s(l_s) \equiv \theta_s = \theta_\gamma \nu^{-1/2}$ [see Eqs. (27) and (28)] and all scatterings by individual nuclei at angles $\vartheta < \vartheta_s(l_s)$ which give an important contribution to the probability of radiation of a γ ray are taken into account in averaging over a distribution function which satisfies the Fokker-Planck equation. Therefore for $\nu = 4s < 1$ the probability (24) completely describes the radiation process (and after transition to the cross channel also the pair-production process) in an amorphous medium.

The presence of an external field changes the situation somewhat. As before, the dominant role continues to be played by the relation between the characteristic radiation angle $\theta_{E,s}$ given by Eqs. (26) and (28) and the characteristic angle of uncorrelated scattering $\vartheta_s(l_{E,s}) = (\sigma l_{E,s})^{1/2}$ in the coherence length $l_{E,s}$ [see Eqs. (25) and (27)].

As in the case of an amorphous medium the averaged probability (18) describes the radiation of a γ ray by electrons which are scattered by individual nuclei only at angles $\vartheta < \vartheta_s(l_{E,s})$. In contrast to the case of an amorphous medium in the presence of an external field (and multiple scattering), the characteristic radiation angle $\theta_{E,s}$ can exceed the multiple scattering angle $\vartheta_s(l_{E,s})$ at arbitrarily high energies. In fact, for $x < \nu$, $x < 1$ we have

$$\vartheta_s(l_{E,s})=\vartheta_s(l_E(x)), \quad \theta_{E,s}=\theta_E(x),$$

while $\vartheta_s(l_E)/\theta_E = x/\nu < 1$ [see Eqs. (25)–(28)]. In the case in which this condition is satisfied, scatterings of e^- by individual nuclei at angles

$$\vartheta_s(l_{E,s})<\vartheta<\theta_{E,s}$$

make an important contribution to the probability of radiation of a γ ray but drop out of the discussion on averaging the radiation probability over the distribution function (9) which satisfies the Fokker-Planck equation. As in the case of an amorphous medium, in calculation in the logarithmic approximation of the probability of radiation of a γ ray in such scattering events it is possible to neglect all other uncorrelated scattering events of e^- in the process of formation of the radiated γ ray. In other words, one can assume that the radiation of a γ ray by an electron scattered at an angle $\vartheta > \vartheta_s(l_{E,s})$ occurs with the same probability as the radiation in scattering of an e^- by an individual nucleus located in a uniform external field. This probability was calculated by Zhukovskiĭ.[20] In carrying out the calculations by the simpler method given in the book by Baĭer *et al.* (Section 9 of Ref. 13), in the limit $l_E\vartheta_1\varepsilon \gg 1$ we obtained a somewhat different result for the cross section of the process:

$$\frac{d\sigma_{Z,E}}{d\omega}=\frac{4e^6Z^2}{15m^2\varepsilon^2\omega}\ln\left(\frac{\vartheta_2}{\vartheta_1}\right)\{\omega^2[x^3-7x^2\gamma'(x)-x^4\gamma(x)]$$

$$+(\varepsilon^2+\varepsilon'^2)[x^3+2x\gamma(x)-x^4\gamma(x)-10x^2\gamma'(x)]\}, \qquad (31)$$

where

$$\gamma(x)=\int_0^\infty \sin\left(xt+\frac{t^3}{3}\right)dt, \qquad \gamma'(x)=\frac{d\gamma(x)}{dx},$$

$$x=\left(\frac{m^3\omega}{eE\varepsilon\varepsilon'}\right)^{2/3};$$

here ϑ_2 and ϑ_1 are respectively the upper and lower limits of the region of angles of e^- scattering by the nucleus, and $\vartheta_2 \lesssim \theta_E(x)$, while the contribution of scattering angles $\vartheta > \theta_E(x)$ can be neglected in the logarithmic approximation. We note that the expression obtained in Ref. 20 can be reduced to the form of a linear combination of the same powers of the parameter x and their products with an epsilon function or its derivative, but with different coefficients. In spite of this, both of the expressions mentioned have the correct Bethe-Heitler limit in the absence of a fixed (for $x \gg 1$). We associate this discrepancy with the inclusion in Ref. 20 of the contribution of the so-called correction to the probability of single-photon pair production, arising as the result of use of the pseudophoton method for its calculation (in replacement of the field of the nucleus by an incoherent superposition of plane waves one loses information on its longitudinal localization, which is not important in discussion of the interaction of e^- with a nucleus only in the first order of perturbation theory). Returning to the problem of description of the process of radiation of a γ ray by an electron moving in an atomic tube or plane, we set $\vartheta_2 = \theta_{E,s}$, $\vartheta_1 = \vartheta_s(l_{E,s})$ (or with allowance for the scattering asymmetry $\vartheta_1 = [\vartheta_{sx}(l_{E,s}) + \vartheta_{sy}(l_{E,s})]/\sqrt{2}$), after which, neglecting the dependence of the density of nuclei on the transverse coordinate, we obtain for the differential probability of radiation

$$\frac{dW_{Z,E}}{d\omega}=\frac{n}{n_0}\frac{\ln[\theta_{E,s}/\vartheta_s(l_{E,s})]}{\ln(190Z^{-1/3})}\frac{1}{15\varepsilon^2\omega L}$$
$$\times\{\omega^2[x^3-7x^2\gamma'(x)-x^4\gamma(x)]$$
$$+(\varepsilon^2+\varepsilon'^2)[x^3+2x\gamma(x)-x^4\gamma(x)-10x^2\gamma'(x)]\}, \quad (32)$$

where n is the local concentation and n_0 the average concentration of nuclei and $L = L(n_0)$ is the radiation length. Note that in the limit $x \gg 1$ the expressions (31) and (32) go over respectively to the Bethe-Heitler expressions for the cross section and the probability of radiation of a γ ray by an electron in its scattering by a nucleus at the same angles.

Now we can write out the complete expression for the probability of radiation of a γ ray by an electron moving perpendicular to a uniform electric field and undergoing in addition uncorrelated scattering:

$$\frac{dW}{d\omega}=\frac{dW_{E,s}}{d\omega}+\theta(\theta_{E,s}-\vartheta_s(l_{E,s}))\frac{dW_{Z,E}}{d\omega}, \quad (33)$$

$$\theta(x)=\begin{cases}1, & x\geqslant 0\\ 0, & x<0\end{cases},$$

where $\vartheta_s(l_{E,s})$ is the mean square angle of uncorrelated multiple scattering of the e^- in a coherence length $l_{E,s}$ [see Eqs. (25) and (27)] and $\theta_{E,s}$ is the characteristic angle of radiation given by Eqs. (26) and (28); the probabilities $dW_{E,s}/d\omega$ and $dW_{Z,E}/d\omega$ are given by the expressions (18) and (32). Without going into detail, we recall that these probabilities, and with them also the total probability (33), go over in limiting cases to all of the known results with the exception of the result of Ref. 20. In Fig. 2 we have shown a plot of the energy dependence obtained from Eq. (33) by transition to the cross channel and integration over the energy of the $e^+(e^-)$ of the total probability for production of an e^+e^- pair by a γ ray. It is easy to see that the contribution of the second term in (33), although it does not disappear with increase of the γ-ray energy, nevertheless falls off for $x \ll 1$ rather rapidly; this can be seen also from Eq. (32) if one takes into account that for $x \ll 1$ $\gamma(x)$, $\gamma'(x) \sim 1$. Therefore taking into account the second term in (33) did not influence the conclusion that there is no Landau-Pomeranchuk effect at small angles of incidence of $e^\pm$ and γ on crystal axes.

The absence of the LPE considerably simplifies the quantitative description of radiation processes in the case of zero angle of incidence of high-energy particles onto crystal axes or planes. As a result of this, over almost the entire crystal (but see below) the probabilities (18) and (22) will differ only in the corrections, the derivation of which we will postpone to subsequent papers. Here incoherent scattering processes will appear most clearly as a result of the contribution (32) from individual nuclei. It is important that, in contrast to the region of applicability of the theory of Ref. 23, incoherent radiation processes are substantially modified by the presence of coherent scattering of $e^\pm$. In fact, neglecting this factor and following the theory of Ref. 23 outside the region of its applicability, the authors of Refs. 21 and 22 approximated the total probability for production of a pair by a γ ray incident at a small angle onto crystal axes by the sum of the probability for pair production in a uniform field, averaged over the crystal cross section, and the amorphous part of the pair-production probability calculated on the basis of the theory of coherent pair production.[23] In the case of a tungsten crystal at $T = 293$ K we have $W_{am} = 0.88 W_{BH}$. Without performing an average of the expression (33) over the cross section of the crystal, let us compare the "local" probability of pair production at a distance $\rho_1 = 0.02$ Å from the ⟨111⟩ axis with the sum of the probability for pair production in a uniform field of strength $E = E(\rho_1)$ and the Bethe-Heitler probability, multiplied by 0.88, calculated for a concentration of nuclei $n = n(\rho_1)$. Since all of the probabilities given in Fig. 2 have been normalized to the corresponding "local" Bethe-Heitler probability, it is easy to see that at $\omega = 100$ GeV we have

$$W(\rho_1)=W_E(\rho_1)+0.44W_{BH}(\rho_1),$$

at $\omega = 300$ GeV—

$$W(\rho_1)=W_E(\rho_1)+0.12W_{BH}(\rho_1),$$

and at $\omega = 10^3$ GeV—

$$W(\rho_1)=W_E(\rho_1)+0.057W_{BH}(\rho_1),$$

which indicates the limited applicability of the approximation of the probability W by the sum $W_E + W_{am}$ and confirms the necessity of a systematic approach to description of radiation and pair production in the fields of crystal axes and planes, which led us to the expressions (18) and (33).

The efficiency of suppression of the Landau-Pomeranchuk effect by the action of a coherent (averaged) potential is weakened with decrease of the intensity of the averaged field on approach to a line (or plane) on which the averaged potential of the axis or plane reaches a maximum. For example, in tubes (planes) of width $\lesssim 0.01$ Å surrounding the maxima of the potentials of the axes or planes of W, with increase of the energies of the $e^\pm$ or γ in the region of hundreds or thousands of GeV there will be a gradual suppression of the LPE which is described by Eqs. (18) and (19). Observation of this new phenomenon is considerably facilitated in the case of radiation, since near a maximum of the averaged potential the deviation of the direction of motion (and consequently of the emission of γ rays) reaches a maximum (or minimum) for e^- (or e^+), as a consequence of which the γ rays radiated near a maximum of the potential can be distinguished from the entire mass of radiated γ rays.

APPENDIX

Let us consider in more detail the motion of $e^\pm$ at a small angle to an axis or plane. It is well known that the potential of interaction of $e^\pm$ with a crystal can be represented in the form of the sum of a coherent part (averaged over the ground state of the crystal) and an incoherent part. Scattering in a coherent potential (see for example Ref. 24) is well described by the model of an averaged potential of the axes (planes) [i.e., by a coherent potential which is further averaged along the axes (planes)], since the contribution of the oscillating components of the coherent potential can be neglected.[24,25] In addition to the averaged potential, the motion of $e^{\pm}$ inside nuclear tubes and planes is greatly affected also by incoherent scattering of the $e^{\pm}$ by nuclei (see for example Refs. 18, 24, and 26). In fact, in scattering of $e^{\pm}$ at angles $\theta \gtrsim 1/u\varepsilon$ a momentum $p_{\perp} \approx \varepsilon\theta > 1/u$ is transferred to the nucleus, which leads, for example, to the excitation of phonons. In other words, scattering processes of this type are incoherent and can be described only by the incoherent component of the crystal potential. At the same time, for radiation and pair production, deflection of $e^{\pm}$ at angles right up to $\theta \sim m/\varepsilon \gg 1/u\varepsilon$ is important. Therefore in description of these radiation processes it is necessary to take into account the contribution of incoherent scattering of $e^{\pm}$ by nuclei. At transverse distances which are important for scattering at angles $\theta \sim m/\varepsilon \gg 1/u\varepsilon$, $\Delta\rho \ll u$, the density of the transverse distributions of nuclei of the axes and planes is practically constant. Therefore correlations in the locations of nuclei having such close transverse displacements disappear, and scattering of $e^{\pm}$ at angles $\theta \sim m/\varepsilon \gg 1/u\varepsilon$ will be uncorrelated (random) and essentially will not differ at all from scattering at the same angles in an amorphous medium of the same density, which is described by Eq. (1) with $\vartheta_{\min} = 1/u\varepsilon$. Since the density of nuclei in crystal planes and tubes is tens of hundreds of times greater than their density in an amorphous material, the question arises of whether this scattering, like in the case of an amorphous medium,[1] influences the radiation and pair production in that region of energies and angles of incidence of $e^{\pm}$ and γ in which they are described by the electrodynamics of phenomena in an intense uniform field.[13-17] We note that already in construction of a theory of radiation and pair production in crystals on the basis of the Born approximation[23] it was discovered that a single coherent potential for interaction of $e^{\pm}$ with crystals cannot satisfactorily describe these processes. Transfers to individual nuclei of transverse momenta $1/u \lesssim p_{\perp} \lesssim m$, which are extremely important for the occurrence of radiation processes, are described in the theory of Ref. 23 by the incoherent component of the crystal potential and lead to appearance of a significant "amorphous" component in the cross sections for radiation processes.

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